Explicit and Hidden Symmetries in Quantum Dots and Quantum Ladders

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The concept of dynamical hidden symmetries in the physics of electron tunneling through composite quantum dots (CQD) and quantum ladders (QL) is developed and elucidated. Quite generally, dynamical symmetries are realizable in the space of low energy excited states in a given charge sector of nanoobjects, which involve spin variables and/or electron-hole pairs. While spin multiplets in an individual rung of a QL or in an isolated CQD form a representation space of the usual rotation group, this $SU(2)$ symmetry is broken due to spin transfer (in QL) electron cotunneling through CQD. Dynamical symmetries in the space of spin multiplets are then unravelled in these processes. The corresponding symmetry groups are described by $SO(n)$ or $SU(n)$ depending on the origin of rotation group symmetry breaking. The effective spin Hamiltonians of QL and CQD are derived and expressed in terms of the pertinent group generators. We employ fermionization procedure for analyzing the physical content of these dynamical symmetries, including Kondo tunneling through CQD and Haldane gap formation in QL.

PACS numbers: 72.10.-d, 72.15.-v, 73.63.-b

INTRODUCTION

Symmetry considerations play a central role in the physics of low-dimensional systems, and serves as a key for understanding their peculiar properties [1]. It predetermines their thermodynamics, response to external fields, transport properties, phase diagrams, etc. In many cases, formulating the physics of strongly interacting electrons in low dimensional systems (especially nano-objects) should be constructed by employing group theoretical concepts (among them non-commutative algebras), whose specific structure have direct consequences for observable physical properties. It appears that not only the symmetry of a given Hamiltonian but also the *dynamical symmetry* of low-energy excitations is relevant.

In order to clarify the above statements, let us consider a system with Hamiltonian $\mathcal{H}_0$ whose eigenstates $|\Lambda\rangle = |M\mu\rangle$ form a basis for an irreducible representation of some Lie group G (μ enumerates the lines of this representation). It is convenient to express the generators of Lie algebras via Hubbard operators $X^{\Lambda\Lambda'} = |\Lambda\rangle\langle\Lambda'|$. Then the Hamiltonian under consideration is expressed in terms of diagonal Hubbard operators

$$\mathcal{H}_0 = \sum_{\Lambda=M\mu} E_\Lambda |\Lambda\rangle\langle\Lambda| = \sum_\Lambda E_M X^{\Lambda\Lambda} \ , \qquad (1)$$

so that

$$[X^{\Lambda\Lambda'}, \mathcal{H}_0] = -(E_M - E_{M'})X^{\Lambda\Lambda'}. \qquad (2)$$

The symmetry group of the Hamiltonian is then generated by the operators $X^{M\mu,M\mu'}$, which commute with $\mathcal{H}_0$, whereas the dynamical symmetry of $\mathcal{H}_0$ is generated by the whole set of operators $\{X\}$. This dynamical symmetry may be revealed, when $\mathcal{H}_0$ describes a quantum object, which is part of larger system with Hamiltonian $\mathcal{H}$, and its symmetry is violated by interaction with this environment. If the interaction scale is characterized by some energy $\mathcal{E}$, then the dynamical symmetry is determined by transitions between those states from the manifold E_Λ, which fall into the interval $\mathcal{E}$. We divide the Hubbard operators acting within this low-energy interval into subsets $\{S\}$ and $\{R\}$. Here the S-operators generate the symmetry group G, whereas the S- and R-operators together generate the dynamical group D. In this paper we study spin properties of quantum dots and quantum ladders, so the group G is in fact $SU(2)$, namely, the group of spin angular momentum. It will be shown that the dynamical symmetry of this object is that of the $SO(n)$ group. We will construct the corresponding algebras by means of Hubbard operators, rewrite the corresponding Hamiltonians $\mathcal{H}$ in terms of the group generators, discuss the possible ways of fermionization of these Hamiltonians and consider some specific properties of quantum dots and quantum ladders possesing these symmetries.

FROM SPIN ROTATOR TO KONDO TUNNELING

The symmetry of spin rotator is an intrinsic property of many low-dimensional spin systems. As was shown in [2], this symmetry predetermines the low-energy dynamics of zero-dimensional quantum dots with even occupation in tunneling contact with metallic reservoirs. Let us consider a double quantum dot (DQD)occupied by two electrons in a neutral state in a T-shaped parallel geometry (Fig.1) as a representative example. In this geometry, two valleys of DQD are coupled by tunneling V. In a limit of strong Coulomb blockade Q, such that $V \ll Q$, the energy spectrum of an isolated DQD consists of a

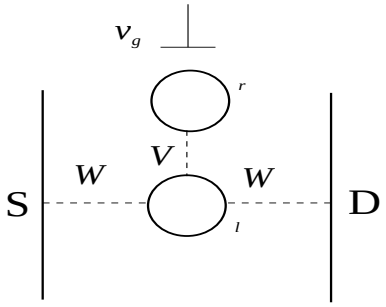

FIG. 1: Parallel Double Quantum Dots in contact with source (S) and drain (D) metallic leads. V and W are tunneling coupling constants, v_g is a gate voltage.

ground state singlet with energy E_S, a spin triplet with the energy E_T separated by an exchange gap $\delta = 2V^2/Q$ above E_S and two charge transfer excitons with large excitation energies $\sim Q$, the charging energy for a given well of DQD. Thus the indices Λ in the Hamiltonian $\mathcal{H}_0$ (1) stand for $\Lambda = S, T\mu$ with $\mu = 1, 0, \bar{1}$ denotes the three projections of the spin $S = 1$.

The dynamical symmetry of the $\{S, T\}$ manifold is that appropriate for the $SO(4)$ group. Two vectors generating this group are constructed by means of Hubbard operators (2) in the following way:

$$\begin{aligned} S^+ &= \sqrt{2}\left(X^{10} + X^{0-1}\right),\ S^- = \sqrt{2}\left(X^{01} + X^{-10}\right), \\ S_z &= X^{11} - X^{-1-1}. \end{aligned} \tag{3}$$

$$\begin{aligned} R^+ &= \sqrt{2}\left(X^{1S} - X^{S-1}\right), R^- = \sqrt{2}\left(X^{S1} - X^{-1S}\right), \\ R_z &= -\left(X^{0S} + X^{S0}\right). \end{aligned} \tag{4}$$

The first one, $\mathbf{S}$ is the conventional spin 1 operator, while the second vector is the R-operator describing S/T transitions. The spin algebra is o_4, which is characterized by the commutation relations

$$[S_\alpha, S_\beta] = ie_{\alpha\beta\gamma}S_\gamma,\ [R_\alpha, R_\beta] = ie_{\alpha\beta\gamma}S_\gamma,\ [R_\alpha, S_\beta] = ie_{\alpha\beta\gamma}R_\gamma \tag{5}$$

(α, β, γ are Cartesian coordinates, $e_{\alpha\beta\gamma}$ is a Levi-Civita tensor). These vectors are orthogonal, $\mathbf{S}\cdot\mathbf{R} = 0$,, the Casimir operator is $\mathbf{S}^2 + \mathbf{R}^2 = 3$.

A gate voltage v_g applied to DQD, makes the level positions essentially asymmetric, and the charging energy Q may be nearly compensated for at least one of the charge transfer singlet excitons (say, the right one, $\Lambda = E_r$). In this case we encounter a "Coulomb resonance" excitations, where the spin singlet and charge transfer exciton (also a singlet!) are strongly intermixed, but the spin triplet is untouched by this resonance tunneling. This means that the corresponding manifold is $\{S, T, E_r\}$. Besides, one more R-vector $\mathbf{R_1}$ and a scalar A operators should be included in the set of group generators. These generators are expressed in terms of Hubbard operators as follows:

$$\begin{aligned} &R_1^+ = \sqrt{2}\left(X^{1E_r} - X^{E_r1}\right), R_1^- = \sqrt{2}\left(X^{E_r1} - X^{-1E_r}\right), \\ &R_{1z} = -\left(X^{0E_r} + X^{E_r0}\right), \\ &A = i(X^{SE_r} - X^{E_rS}). \end{aligned} \tag{6}$$

To close the algebra the commutation relations (5) which are valid also for $R_{1\alpha}$ should be completed by

$$\begin{aligned} &[R_{l\alpha}, R_{1\beta}] = i\delta_{\alpha\beta}A, [R_{1\alpha}, A] = iR_{l\alpha}, \\ &[A, R_{l\alpha}] = iR_{1\alpha}, \quad [A, S_{l\alpha}] = 0. \end{aligned} \tag{7}$$

The system of commutation relations (5), (7) is that of the o_5 algebra, and the manifold $\{S, T, E_r\}$ obeys $SO(5)$ dynamical symmetry, provided all three levels are involved in the interaction within the Hamiltonian H. The Casimir operator for the $SO(5)$ group in this representation is $\mathbf{S}^2 + \mathbf{R}^2 + \mathbf{R_1}^2 + A^2 = 4$.

In terms of these operators $\mathcal{H}_0$ has the form

$$\mathcal{H}_0 = \frac{1}{2}\left(E_T\mathbf{S}^2 + E_S\mathbf{R}^2\right) + Q(\hat{N} - 2)^2. \tag{8}$$

and

$$\mathcal{H}_0 = \frac{1}{2}\left(E_T\mathbf{S}^2 + E_S\mathbf{R}^2 + E_{E_r}\mathbf{R}_1^2\right) + Q(\hat{N} - 2)^2. \tag{9}$$

for the $SO(4)$ and the $SO(5)$ groups, respectively. The last terms in (8) and (9) control the number of electrons given by the operator $\hat{N}$ in the DQD.

As is seen from this equation, spin is still conserved in the isolated DQD. However, a tunnel contact with metallic leads breaks the spin rotation invariance and reveals the dynamical symmetry of the DQD. The physical mechanism of this symmetry breaking is *electron cotunneling* with spin flips, when an electron with spin σ enters the DQD, whereas another electron with spin σ' leaves it. This process is known to be a source of Kondo effect in tunnel barriers and quantum dots [3]. Eliminating charge degrees of freedom by means of the Schrieffer-Wolff transformation, one usually arrives at an exchange-like cotunneling Hamiltonian of the type $J_{cot}\mathbf{S}\cdot\mathbf{s}$, where $J_{cot} \sim W^2$, and W is a lead-dot tunneling amplitude (which is antiferromagnetic, $J_{cot} > 0$) .

Since $E_T - E_S = \delta > 0$, the Kondo effect seems to be irrelevant in DQD with even occupation. However, one should remember that the tunneling W induces additional contribution of indirect exchange between the two wells of the DQD. As is shown in Refs. [2] this contribution may change the sign of δ provided the excitation E_r is soft enough, but the condition $V/(E_T - E_S) \ll 1$ is still valid. Then the exciton E_r is eliminated from the manifold, the symmetry of the DQD reduces from $SO(5)$ to $SO(4)$ and the Schrieffer-Wolff transformation yields the effective spin Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + J_{cot}^T\mathbf{S}\cdot\mathbf{s} + J_{cot}^{ST}\mathbf{R}\cdot\mathbf{s}, \tag{10}$$

where J_{cot}^{T} and J_{cot}^{ST} are two indirect exchange coupling parameters which are renormalized by Kondo screening. This screening affects both vectors $\mathbf{S}$ and $\mathbf{R}$.

The problem of Kondo tunneling within the framework of the Hamiltonian (10) has been solved in the weak coupling limit (see [2] and references therein), so we do not elaborate upon it here. For further progress it is important to note that this example demonstrates how the dynamical symmetry is realized in the S/T subspace due to interaction with the electrons in the leads, which breaks the rotational symmetry of the isolated spin system. This interaction introduces its own energy scale $\mathcal{E}$ into the problem (in example considered above it is just the Kondo temperature T_K), and the dynamical symmetry of the DQD as a spin rotator becomes relevant when the T/S energy splitting is comparable with T_K. In more complicated quantum dots the spin manifolds consist of several S/T pairs, and the dynamical symmetry of such dots is described by the $SO(n)$ groups (see Ref. [4] where the cases of $n = 3, 5, 7$ are discussed).

FROM SPIN ROTATOR TO SPIN LADDER

In this section we show that an $SO(4)$ symmetry is an intrinsic property of S=1/2 spin ladders and decorated spin chains (defined below) . A generic Hamiltonian for the spin systems under consideration is of the Heisenberg-type, consisting of spin 1/2 moments residing on the sites of a two leg ladder,

$$H^{(SL)} = J_t \sum_{\langle i1,i2\rangle} \mathbf{s}_{i1}\cdot\mathbf{s}_{i2} + J_l \sum_{\alpha}\sum_{\langle i\alpha,j\alpha\rangle} \mathbf{s}_{i\alpha}\cdot\mathbf{s}_{j\alpha}. \quad (11)$$

Here, the index $\alpha = 1, 2$ enumerates the legs of the ladder, and the sites $\langle i1, i2\rangle$ belong to the same rung (Fig.2a).

A chain of dimers of localized spins illustrated by Fig. 2b is described by the simplified version of this Hamiltonian

$$H^{SRC} = J_t \sum_{\langle i1,i2\rangle} \mathbf{s}_{i1}\cdot\mathbf{s}_{i2} + J_l \sum_{\langle ij\rangle} \mathbf{s}_{i1}\cdot\mathbf{s}_{j1} \quad (12)$$

The geometry of alternate rungs is chosen in a system (12) to avoid exchange interaction between spins $\mathbf{s}_{i2}$ and $\mathbf{s}_{j2}$. The transverse coupling may emerge either from direct exchange (in case of localized spins) or from indirect Anderson-type exchange induced by tunneling (similarly to the case encountered in QD). In the latter case the sign of J_t is antiferromagnetic (AFM), in the former case it may be ferromagnetic (FM) as well. The same is valid for J_l.

It is useful to start with diagonalization of the Hamiltonian of a perpendicularly aligned dimer (cf. Ref. [5]). The $SO(4)$ symmetry stems from the obvious fact that the spin spectrum of a dimer $\{i1, i2\}$ is formed by the same singlet-triplet (ST) pair as the spin spectrum of DQD studied in the previous section. This analogy prompts us a canonical transformation connecting two pairs of spin vectors, $\{\mathbf{s}_{i1}, \mathbf{s}_{i2}\}$ and $\{\mathbf{S}_i, \mathbf{R}_i\}$: The two sets of spin operators are connected by a simple rotation

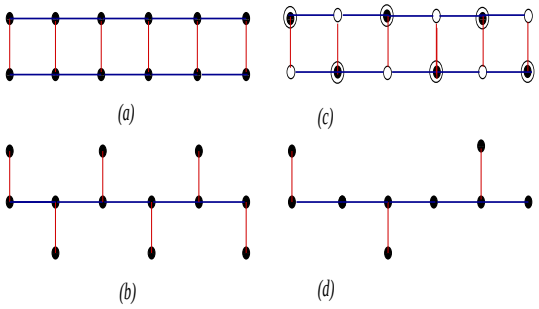

FIG. 2: Spin Ladder (a), Spin Rotator Chain (b), Spin ladder in the CDW phase (c) and Alternate Spin Rotator Chain (d)

$$\mathbf{s}_{i1} = \frac{\mathbf{S}_i + \mathbf{R}_i}{2}, \quad \mathbf{s}_{i2} = \frac{\mathbf{S}_i - \mathbf{R}_i}{2}, \quad (13)$$

Then the Hamiltonian $\mathcal{H}_i$ of a single dimer i is the same as the Hamiltonian (8) of DQD. The total spin of a dimer is not conserved in such a spin chain, so the dynamical symmetry of an individual rung is revealed by the modes propagating along the chain [5]. Applying the rotation operation (13) to the Hamiltonians (11) and (12), we transform them into an equivalent form

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{int}. \quad (14)$$

Here $\mathcal{H}_0 = \sum_i \mathcal{H}_i$ is common for both models. It is useful to include the Zeeman term in $\mathcal{H}_i$,

$$\mathcal{H}_i = \frac{1}{2}\left(E_S\mathbf{R}_i^2 + E_T\mathbf{S}_i^2\right) + hS_{iz}. \quad (15)$$

We confine ourselves by a charge sector $N_i = 2$ and omit the Coulomb blockade term for the sake of brevity. The interaction part of the SL Hamiltonian transforms under rotation (13) to the following expression

$$\mathcal{H}_{int}^{SL} = \frac{1}{4}J_l \sum_{\langle ij\rangle}(\mathbf{S}_i\mathbf{S}_j + \mathbf{R}_i\mathbf{R}_j) \quad (16)$$

The interaction part of the SRC Hamiltonian is

$$\mathcal{H}_{int}^{SRC} = \frac{1}{4}J_l \sum_{\langle ij\rangle}(\mathbf{S}_i\mathbf{S}_j + 2\mathbf{R}_i\mathbf{S}_j + \mathbf{R}_i\mathbf{R}_j) \quad (17)$$

One may also consider the *alternate SRC model* (ASRC, see Fig. 2(c)). After an appropriate rotation (13)its interaction Hamiltonian acquires the form

$$\mathcal{H}_{int}^{ASRC} = \frac{1}{4}J_l \sum_{\langle ij\rangle}(\mathbf{S}_i\mathbf{S}_j + \mathbf{S}_i\mathbf{R}_j). \quad (18)$$

Now we see that all three effective Hamiltonians belong to the same family. In all cases the initial ladder or "semi-ladder" Hamiltonian is transformed into a one-dimensional spin-chain Hamiltonian, which, however, takes into account the hidden symmetry of a dimer. The effective Hamiltonians (16), (17), (18) contain operators $\mathbf{R}$ describing the dynamical symmetry of the dimers. This dynamical symmetry turns the spectrum of this Hamiltonians to be richer than that of a standard Heisenberg chain. Like in many other cases, rotation transformation eliminates the antisymmetric combination of two generators.

Thus, the transformation (13) reveals the hidden symmetry of a spin 1/2 ladder (16). It maps the Hamiltonian onto a pair of coupled chain Hamiltonians: one is the conventional spin 1 chain, while the other is a pseudospin chain. A spin $\mathbf{S}_i$ and a pseudospin $\mathbf{R}_i$ are kinematically coupled by the commutation relations and by the local Casimir constraint

$$\mathbf{S}_i^2 + \mathbf{R}_i^2 = 3. \tag{19}$$

One may also compare the Hamiltonian (16) with the effective Hamiltonian of a spin 1 chain, which arises after decomposition of spin-one operators into a pair of spin 1/2 operators, $\mathbf{S}_i = \mathbf{s}_i + \mathbf{r}_i$ [6]. This decomposition operation transforms the initial Hamiltonian into a form similar to H^{SRC} but for spin-one-half operators $\mathbf{s}_i,\ \mathbf{r}_i$. The difference between the two cases is that these effective spins commute, (unlike operators $\mathbf{S}_i,\ \mathbf{R}_i$). In other words, the difference is that the local symmetry of spin-one chain is $SO(3)$ whereas the local symmetry of SRC is $SO(4)$. The spin rotator chains (17), (18) are in some sense intermediate between spin chains and spin ladders. In these cases the spin-pseudospin symmetry is obviously broken by the cross terms $2\mathbf{S}_i\mathbf{R}_j$.

The excitation spectrum of spin ladders may be calculated in terms of operators $\mathbf{S}_i$ and $\mathbf{R}_i$. For example, the well known expression for a gap ΔE in excitation spectrum in the limit of strong transversal exchange $J_t \gg J_l$ for AFM interaction [5] looks like

$$\Delta E = J_t + \frac{(J_l/4)^2 \sum_{ij,\alpha\beta} \left(\langle T_{ij}^{\alpha\beta} | \mathbf{R}_i \mathbf{R}_j | S_i S_j \right)^2}{(E_T - E_S)} = J_t + \frac{3J_l^2}{8J_t} \tag{20}$$

(here $T_{ij}^{\alpha\beta}$ and S_iS_j stand for possible triplet projections and spin states at the sites i, j, respectively). The singlet-triplet excitations above this gap are given by the dispersion law $\omega(k) = \Delta E + J_l \cos k$.

In all cases the simplified versions of Heisenberg Hamiltonians may be considered. The simplified SL models are well known [7]. The anisotropic versions of the Hamiltonian (17) are: *Ising-like SRC model:*

$$H = \frac{1}{4} J_l \sum_{\langle ij \rangle} (S_i^z S_j^z + 2 S_i^z R_j^z + R_i^z R_j^z). \tag{21}$$

Anisotropic SRC model:

$$H = \frac{1}{4} J_l \sum_{\langle ij \rangle} \left[(S_i^+ S_j^- + S_i^+ R_j^- + S_i^- R_j^+ + R_i^+ R_j^-) \right. \tag{22}$$

$$\left. + \Delta (S_i^z S_j^z + 2 S_i^z R_j^z + R_i^z R_j^z) \right].$$

SRC in strong magnetic field: SO(4) group reduces to SU(2) group in magnetic field, when the Zeeman splitting *exactly* compensates the exchange gap in a single dimer, $h_0 = |E_T - E_S|$. Then at low T, the states $|i0\rangle$ and $|i-1\rangle$ are quenched, and only two components, $R^{\pm}$ survive in the manifold (3), (4). As a result, the Hamiltonian (17) is mapped onto a XY-model for spin 1/2:

$$H_{XY}^{(R)} = \frac{1}{4} J_l \sum_{\langle ij \rangle} (R_i^+ R_j^- + H.c.). \tag{23}$$

This means that starting from a singlet ground state for $J_t \equiv E_T - E_S > 0$, one may induce development of spin liquid-like excitations by applying strong magnetic field. In a near vicinity of this point of degeneracy, H_{int} acquire the features of *XY model in transverse magnetic field.*

FERMIONIZATION

To describe the elementary excitations in SRC, one should generalize the $SU(2)$-like semi-fermionic representation for S operators [8]

$$S^+ = \sqrt{2}(f_0^\dagger f_{\bar{1}} + f_1^\dagger f_0), \quad S^- = \sqrt{2}(f_{\bar{1}}^\dagger f_0 + f_0^\dagger f_1),$$

$$S_z = f_1^\dagger f_1 - f_{\bar{1}}^\dagger f_{\bar{1}}, \tag{24}$$

where $f_1^\dagger$, $f_{\bar{1}}^\dagger$ denote creation operators for fermions with spin "up" and "down" respectively whereas f_0 stands for spinless fermion [8, 9]. Fermionization of $SO(4)$ group is completed by introducing one more spinless fermion f_s which represents the singlet state. As a result, P-operators are given by the following equations:

$$P^+ = \sqrt{2}(f_1^\dagger f_s - f_s^\dagger f_{\bar{1}}), \quad P^- = \sqrt{2}(f_s^\dagger f_1 - f_{\bar{1}}^\dagger f_s),$$

$$P^z = -(f_0^\dagger f_s + f_s^\dagger f_0). \tag{25}$$

Then the single-site Hamiltonians may be represented in a form

$$H_i = -\delta f_{is}^\dagger f_{is} + h(f_{i1}^\dagger f_{i1} - f_{i\bar{1}}^\dagger f_{i\bar{1}}) \tag{26}$$

The Casimir operator $\mathbf{S}^2 + \mathbf{P}^2 = 3$ transforms to the local constraint

$$\sum_{\Lambda = \pm, 0, s} f_\Lambda^\dagger f_\Lambda = 1.$$

We start the studies of elementary excitations in SRC with the anisotropic XXZ version. of general effective Hamiltonian. The simplest of all is the case (18). The problem is reduced to a standard XY-model for spin one half, and the spinon spectrum may be easily obtained either by bosonization or by spinon-type fermionisation. In former case one deals with hard-core bosons, and in latter one the problem is mapped onto the non-interacting incompressible fermions at half-filling.

We concentrate on a more complicated case of XXZ-SRC model (22) specifically on its simplified alternate version, which is obtained from the Hamiltonian (18). The Hamiltonian of this model is

$$H = \frac{1}{4} J_l \sum_{\langle ij \rangle} (S_i^+ S_j^- + S_i^+ P_j^- + S_i^- P_j^+ + \Delta (S_i^z S_j^z + 2 S_i^z P_j^z). \tag{27}$$

The S-S part of this Hamiltonian describes the S=1 chain, with the Haldane gap in the excitation spectrum (see, e.g.,[10, 11]). The question is, how do the S-P interaction modifies the gap. We consider the case of FM dimers, when the triplet is the ground state. In this case one has one more gap mode, where the gap equals J_t. This mode is coupled to Haldane branch only via S-P exchange terms in (27).

The spin liquid fermionization approach adopted here is a convenient tool for description of Haldane spectrum. Unlike the S=1/2 model, where the spin-liquid state is easily described by globally U(1) invariant modes $T_{ij} T_{ji} = \sum_\sigma f_{i\sigma}^\dagger f_{j\sigma}|^2$, in case of S=1, one deals with variables which effectively break this symmetry. One can rewrite the effective Hamiltonian of SRC model with $\Delta = 0$ in a form

$$H = \frac{1}{4} J_l \sum_{ij} \left(f_{i1}^\dagger f_{j1} + f_{i\bar{1}}^\dagger f_{j\bar{1}} \right) \bar{B}_j^{0S} B_i^{0S}$$
$$+ f_{i\bar{1}}^\dagger f_{j1}^\dagger C_j^{0S} B_i^{0S} + \bar{B}_i^{0S} \bar{C}_j^{0S} f_{j1} f_{i\bar{1}}, \tag{28}$$

where $B_j^{0S} = f_{0j} + f_{Sj}$, $C_j^{0S} = f_{0j} - f_{Sj}$. The terms in the first line of Eq. (28) describe coherent propagation of spin fermions accompanied by a backflow on neutral fermions, whereas the terms in the second line are "anomalous" (they do not conserve spin fermion number). For example the propagator $\langle S_i^+ S_j^- \rangle$ contains anomalous components $f_{i1}^\dagger f_{j\bar{1}}^\dagger f_{j0} f_{i0} \to F_{ij,1\bar{1}}^* F_{ji,00}$ along with normal ones $f_{i1}^\dagger f_{j1} f_{j0}^\dagger f_{i0}$. Here $F_{ij,\Lambda\Lambda'} = f_{j\Lambda} f_{i\Lambda'}$. The first term in (28) describes the kinetic energy spinon excitations, and two last anomalous term breaking U(1) symmetry are responsible for the Haldane gap. To reveal the contribution of dynamical symmetry on the Haldane gap, one have to note that the terms B^{0S} and B^{0S} appear both as a counterflow in the first term and as gauge symmetry breaking terms in the second line. In spin 1 ladder the counterflow term $\sim f_{i0}^\dagger f_{j0}$ predetermines the width of spinon band described by the first line of Eq. (28). Apparently, the one more channel (tripet/singlet transitions in B^{0S}) enhances this effect, because in this case the local constraint imposes more restrictions of phase fluctuations. The gap itself is due to anomalous correlations described by the second line of Eq. (28). Here the appearance of second channel of spinless excitations results in formation of even and odd operators B_j^{0S} and C_j^{0S}. The Haldane gap closes when the $|0\rangle$ and $|S\rangle$ states are degenerate (the odd operator C_j^{0S} nullifies the anomalous terms responsible for its formation). This means that appearance of $0S$ channel *favors* closing of the Haldane gap.

In a strong coupling case of $J_t \gg J_l$ both above trends may be considered at least in the lowest order of a perturbation theory. In case of spin ladders [5] the 1-st and 2-nd-order in $g = J_l/J_t$ anomalous diagrams are represented in Fig.3.

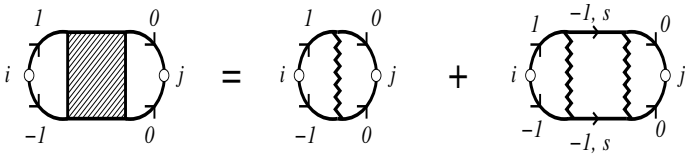

FIG. 3: Lowest order contributions to anomalous propagator.

SO(N) DYNAMICAL SYMMETRIES FOR A TWO-LEG QUANTUM LADDER

It was mentioned in Section II that the dynamical symmetry of DQD becomes $SO(5)$, if the charge transfer excitonic state is involved (see Eq. 9). In this section we discuss the origin of this symmetry in spin ladders. This problem arose in a context of $SO(5)$ symmetric $t-J$ model of 2D cuprate superconductors [12]. Later on the version of this theory was formulated for cuprate two-leg ladders [13]. Here we show that the *dynamical* $SO(5)$ group arises in description of Heisenberg ladder, but excitonic states are involved in this symmetry instead of Cooper states.

Let us consider a two-leg quantum ladder depicted in Fig.2a under condition of strong Coulomb blockade imposed on each rung i. We allow electron tunneling t_{ij}^α along both legs. This tunneling is described by the Hamiltonian

$$\mathcal{H}_{tun} = \sum_{ij} \sum_{\alpha\sigma} t^\alpha d_{i\alpha\sigma}^\dagger d_{j\alpha\sigma}. \tag{29}$$

(only a nearest-neighbor hopping along the leg is allowed). This hopping results in appearance of *charged* rungs because each hopping act creates a hole on a rung j and an electron on a rung i. To treat this charging properly the Coulomb blockade term in the Hamiltonian $\mathcal{H}_i$ (15) should be restored (see Eq. 9), and the terms with excess electron and excess hole should be added. It

is more convenient to represent the Hamiltonian $\mathcal{H}_i$ of individual rung i in terms of diagonal Hubbard operators [see (1)]

$$\mathcal{H}_i = \sum_i \left[\sum_\Lambda E_\Lambda X_i^{\Lambda\Lambda} + \sum_\gamma E_\gamma X_i^{\gamma\gamma} + \sum_\Gamma E_\Gamma X_i^{\Gamma\Gamma} \right] \quad (30)$$

Here index $\gamma = \alpha\sigma$ stands for the states with one electron with spin σ on a site $i\alpha$ of the rung i, index $\Gamma = \alpha\sigma$ stands for three-electron states of a rung, where two electrons occupy site $i\alpha$ and one electron with spin σ is located in a site $i\bar{\alpha}$ ($\bar{\alpha} = 2$ if $\alpha = 1$ and v.v). The energy levels E_γ and E_Γ are separated by a Coulomb blockade gap $\sim Q$ from the two-electron states E_Λ. The Hamiltonian (29) in these terms is

$$\mathcal{H}_{tun} = \sum_{ij,\alpha} \sum_{\gamma\Gamma\Lambda} t^\alpha X_i^{\Gamma\Lambda} X_j^{\gamma\Lambda} + H.c. \quad (31)$$

It is seen from (31) that the intersite hopping "charges" two neighboring rungs in a ladder, which was initially neutral, and one should pay the energy $\sim Q$ for each hopping act, like in the generic Hubbard model at half-filling. This energy loss is reduced if an electron-hole pare is created at a given rung i. In this case the electron-hole attraction $V < 0$ partially compensates charging energy Q. Let us assume the hierarchy $Q \gg Q - |V| \gg t$. Then the states $|\Gamma\rangle$ may be excluded from the manifold in favor of excitonic states $|iE_\alpha\rangle$ similar to the states $|E_r\rangle$ introduced in (6). Here $\alpha = 1(2)$ for the electron occupying site $i1(i2)$. If the ground state of a rung is singlet, $|iS\rangle$, then electron and hole have antiparallel spins and the excitation energy is $Q' = Q - |V|$. Even combination of two states $|iE_{(1,2)}\rangle$ form a singlet exciton $|iE\rangle$. Such exciton can propagate coherently along the ladder unlike single electron, whose tunneling leaves a trace of charged states according to (31). Indeed, translation of e-h pair from a rung i to a neighboring rung $i+1$ can be presented as coherent tunneling of electron from a site $i\alpha$ to a site $i+1, \alpha$ and another electron in the opposite direction (from $i+1, \bar{\alpha}$ to $i, \bar{\alpha}$. The exciton propagation is described by the following term in effective Hamiltonian:

$$H_{ex} = \sum_i K^S X_i^{SE} X_{i\pm1}^{ES} \quad (32)$$

with effective exchange coupling constant $K^S = |t_1 t_2|/Q'$, and the dispersion law describing coherent exciton propagation is $\epsilon_S(k) = 2K^S \cos k$. As was shown in Section 2, the manifold $\{iS, iT, iE\}$ possesses the local dynamical symmetry $SO(5)$ [see Eqs. (6), (7)], and this symmetry allows existence of coherent collective singlet exciton mode. The Hamiltonian (32) acquires a form $H_{ex} = (K^S/4) \sum_{ij} \widetilde{A}_i \widetilde{A}_j$ in terms of generators of SO(5) group (6), where $\widetilde{A}_i = (\mathbf{R}_i^2 - 1) A_i$. There is one more collective mode, namely triplet exciton $|E_\mu\rangle$ ($\mu = \pm1, 0$) separated by the gap $\sim J_t$ from the singlet exciton. In case of triplet ground state ($J_t < 0$), this mode becomes the lowest one, and the Hamiltonian similar to (32) may be derived for triplet exciton propagation with operators $X_i^{TE_\mu}$ replacing X_i^{SE}. In this case the manifold $\{iS, iT, iE_\mu\}$ consists of one singlet and two triplets, and the corresponding dynamical group is $SO(7)$ [4]. If exchange and excitonic gaps are comparable in magnitude, then the interplay between exciton and magnon modes is possible, and dynamical symmetry will result in observable physical effects. Like in cuprate ladder, [14], the excitonic instability can develop for certain values of model parameters, which results in phase separation and, in particular in formation of CDW phase illustrated by Fig. 1c (where double and empty circles stand for doubly occupied and empty sites respectively.

CONCLUDING REMARKS

We rederived a family of Hamiltonians for quantum dots and quantum ladders in terms of $SO(4)$ group, which describes the dynamical symmetry of spin rotator [2]. We exploited the fact that in case, when the Hamiltonian $\mathcal{H}$ contains blocks $\mathcal{H}_i$ formed by two sites occupied by spins 1/2, one may use its eigenstates (singlet-triplet manifolds) as a basis for representing the spin invariants entering $\mathcal{H}$. These invariants contain the Runge-Lenz-like vectors $\mathbf{R}_i$ along with the usual spin vectors $\mathbf{S}_i$. If the electron-hole pairs are also included in the set of eigenstates, then the local dynamical symmetry of $\mathcal{H}_i$ is characterized by the $SO(n)$ group with $n = 5$ or 7 for a singlet and triplet ground state of $\mathcal{H}_i$, respectively. The elementary excitations in quantum dots and quantum ladders are described by means of generators of $SO(n)$ groups and the interplay between different branches of excitation spectra is a direct manifestation of local dynamical symmetry violated by non-local interactions.

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