JETP Letters, Vol. 77, No. 7, 2003, pp. 366–370. Translated from Pis'ma v Zhurnal Éksperimental'noĭ i Teoreticheskoĭ Fiziki, Vol. 77, No. 7, 2003, pp. 434–438.

Dynamically Induced Kondo Effect in Double Quantum Dots

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Received February 18, 2003

A new mechanism of resonance Kondo tunneling via a composite quantum dot (QD) is proposed. It is shown that, owing to the hidden dynamic spin symmetry, the Kondo effect can be induced by a finite voltage eV applied to the contacts at an even number N of electrons in a QD with zero spin in the ground state. As an example, a double QD is considered in a parallel geometry with $N = 2$, which possesses the SO(4) type symmetry characteristic of a singlet–triplet pair. In this system, the Kondo peak of conductance appears at an eV value compensating for the exchange splitting. © *2003 MAIK "Nauka/Interperiodica".*

PACS numbers: 72.10.Fk; 72.15.Qm; 73.63.Kv

The Kondo effect, originally observed in the form of an anomalously strong resonance scattering of electrons by magnetic impurities in metals, has proved to be a universal mechanism of interaction between an electron gas and localized quantum objects possessing internal degrees of freedom [1]. In particular, a magnetic impurity within the barrier between two metal contacts, as well as a quantum dot (QD) with uncompensated spin occurring in a tunneling junction between metal electrodes, can account for anomalously high tunneling transparency of the barrier for electrons from the contacts [2, 3]. Shortly after the experimental discovery of such Kondo resonances in the tunneling via planar QDs [4], it was established that the spectrum of phenomena related to the effective magnetic exchange in QDs is by no means restricted to simple passage from the problem of Kondo scattering to that of Kondo transfer. In particular, it was found that the resonance Kondo tunneling via QDs with an even number of electrons and zero total spin is possible under the action of an external magnetic field [5] or the electric field of a gate [6].

The large variety of manifestations of the Kondo effect in QDs is related to the fact that these nanodimensional objects are essentially a kind of artificial atom possessing complicated spectra. The tunneling of electrons from outside via QDs breaks their spin symmetry and induces transitions to low-lying excited states. The transitions at energies comparable with the Kondo temperature (T_K) are involved into resonance interactions and modify the pattern of the Kondo scaling as compared to that typical of the canonical Kondo effect in metals. Thus, within the limits of the Kondo energy scale, it is necessary to take into account the dynamic symmetry of a given QD [7]. This symmetry is determined both by the spin and by other vectors involved in the algebra of the corresponding dynamic group. As a result, the effective Hamiltonian describing the Kondo tunneling acquires a more complicated form than that of the *sd*-exchange Hamiltonian describing the Kondo effect in metals: all the above vectors contribute to the cotunneling with spin reversal via the QD. A theory of the dynamic symmetry of composite (double and triple) QDs has been recently developed in [8], where it is also demonstrated how an external magnetic field or the gate electric field can influence the dynamic symmetry.

Below, we consider a new class of phenomena related to the dynamic symmetry of QDs. It will be demonstrated that violation of the thermodynamic equilibrium between contacts may induce resonance Kondo tunneling not observed in the equilibrium system. The nonequilibrium Kondo effect in QDs at a finite voltage applied between the source and sink has been extensively studied (see, e.g., [9]). In most cases, however, researchers were interested in the influence of nonequilibrium conditions on the Kondo effect existing in the equilibrium state. In such a situation, relaxation of the system related to the finite lifetime of excited states probably hinders attaining a strong coupling regime (see, e.g., the discussion in [10, 11]).

We are interested in a different situation, whereby no channel of nonequilibrium induced spin relaxation exists in the ground state (e.g., for $S = 0$). In this case, the spin degrees of freedom are excited only in QDs possessing a dynamic symmetry. Such a symmetry is inherent, for example, in a double quantum dot (DQD) structure experimentally realized and studied recently [12]. In the simplest nontrivial case, the DQD contains two electrons occupying energy levels according to the Heitler–London scheme (Fig. 1). The system occurs under the conditions of a strong Coulomb blockade Q

suppressing the tunneling v between potential wells. The spectrum of spin states represents a singlet–triplet (S–T) pair with zero spin in the ground state (because the effective exchange between the two valleys, $I \approx v^2/Q$, has an antiferromagnetic character). As demonstrated previously [6, 8], an isolated DQD represents a quantum spin rotator with the $SO(4)$ symmetry, in contrast to the $SU(2)$ symmetry of a QD with odd occupation usually considered in the theory of the Kondo type tunneling. The $SO(4)$ group is generated by the spin vector $\mathbf{S}$ and the vector $\mathbf{P}$ describing the S–T transition matrix.

The Hamiltonian of an isolated QD can be written as

$$H_d = E_S|S\rangle\langle S| + \sum_\eta E_T|T\eta\rangle\langle T\eta|, \qquad (1)$$

where $E_T = E_S + \delta$; $\eta = \pm, 0$ are the projections of the spin total $S = 1$; and $\delta = I$ is the exchange coupling. Since the direct tunneling W of electrons from contacts to the DQD is suppressed by the Coulomb blockade Q, the charge transfer is possible only by means of second-order processes (cotunneling). The effective Hamiltonian describing these processes is as follows [6, 8]:

$$H_{\text{int}} = J\sum_{\alpha\alpha'}(\mathbf{S}+\mathbf{P})\mathbf{s}_{\alpha\alpha'},$$
$$\mathbf{s}_{\alpha\alpha'} = \sum_{kk'} c^\dagger_{k\alpha\sigma}\hat{\tau}c_{k'\alpha'\sigma'}, \quad n_{\alpha\alpha'} = \sum_{kk'} c^\dagger_{k\alpha\sigma}\hat{1}c_{k'\alpha'\sigma}, \qquad (2)$$

where the subscript $\alpha = L, R$ denotes electrons in the left- and right-hand contacts, respectively; $\hat{\tau}$ are the Pauli matrices; $\hat{1}$ is the unit matrix; and $J \approx W^2/(\epsilon_F - E_S/2)$is the effective constant of exchange between the DQD and contacts (we neglect a difference between tunneling parameters in the S and T states of the QD). The $\mathbf{S}$ and $\mathbf{P}$ vectors defined above are written in matrix form as follows:

$$S^+ = \sqrt{2}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad S^z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$P^+ = \sqrt{2}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \quad P^z = -\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

where a singlet state corresponds to the last row. The corresponding algebra (o_4) is described by the commutations relations

$$[S_j, S_k] = ie_{jkl}S_l, \quad [P_j, P_k] = ie_{jkl}S_l,$$
$$[P_j, S_k] = ie_{jkl}P_l,$$

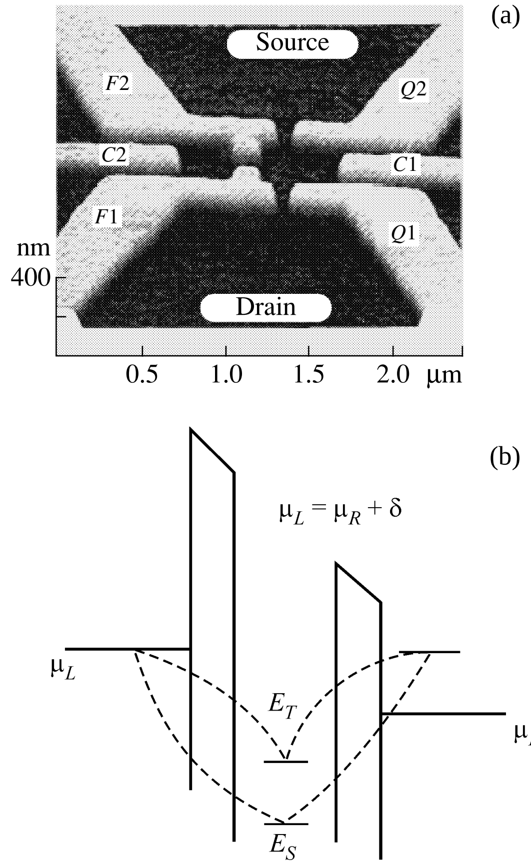

Fig. 1. (a) A parallel double quantum dot configuration (Hofmann *et al.* [12]) and (b) an energy band diagram illustrating tunneling and cotunneling processes contributing to the differential conductance in this system.

where j, k, l are the coordinate indices and e_{jkl} is the Levi–Civita tensor.

As can be seen, both $\mathbf{S}$ and $\mathbf{P}$ vectors are involved in the tunneling with spin reversal. However, since the threshold energy for excitation of the spin degrees of freedom is δ, the spin scattering under equilibrium conditions is effective only provided that $T_K > \delta$. It will be shown below that this threshold can also be surmounted in the opposite limit, $T_K \ll \delta$, at a finite source–sink voltage $eV \approx \delta$ compensating for the S–T splitting energy. In the weak coupling regime, $T > T_K$, we use the thermodynamic perturbation theory and assume that electrons in the contacts obey the Fermi statistics with the chemical potentials μ_R and $\mu_R + eV$ in the right- and left-hand contacts, respectively. We can also assume that weak tunneling currents do not violate thermodynamic quasi-equilibrium (the validity of this approach is discussed below).

In order to construct the perturbation theory, let us perform fermionization of the generators of $SO(4)$

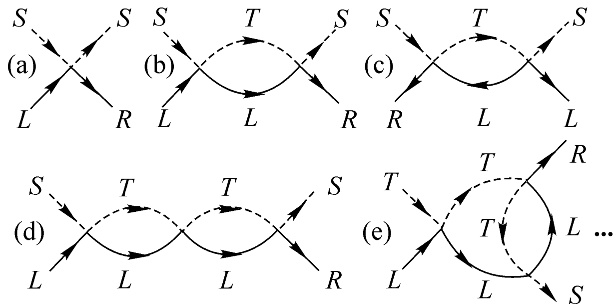

Fig. 2. Perturbation theory diagrams determining (a) J, (b, d) the main corrections to J, and (c, e) corrections containing a lower power of the logarithm. Solid curves refer to electron between contacts; dashed curves refer to the QD states.

group—a generalizing procedure originally suggested for $SU(2)$ group [13, 14]:

$$S^+ = \sqrt{2}(f_0^\dagger f_{-1} + f_1^\dagger f_0), \quad S^- = \sqrt{2}(f_{-1}^\dagger f_0 + f_0^\dagger f_1),$$
$$P^+ = \sqrt{2}(f_1^\dagger f_s - f_s^\dagger f_{-1}), \quad P^- = \sqrt{2}(f_s^\dagger f_1 - f_{-1}^\dagger f_s),$$
$$S_z = f_1^\dagger f_1 - f_{-1}^\dagger f_{-1}, \quad P^z = -(f_0^\dagger f_s + f_s^\dagger f_0). \tag{3}$$

Here, $f_{\pm 1}^\dagger$ are the operators of creation for fermions with the spin projections 1 and –1; f_0^+ and f_s are the operators of creation for zero-spin fermions in the triplet and singlet states, respectively. Representation (3) automatically takes into account the local kinematic constraint $\sum_\Lambda f_\Lambda^\dagger f_\Lambda = 1$. A diagram technique of the perturbation theory is provided by the temperature Green's functions for electrons between contacts, $G_{L,R}(\mathbf{k}, \tau) = -\langle T_\tau c_{L,R\sigma}(k, \tau) c_{L,R\sigma}^\dagger(k, 0)\rangle$, and for fermions in the DQD, $\mathcal{G}_\Lambda(\tau) = -\langle T_\tau f_\Lambda(\tau) f_\Lambda^\dagger(0)\rangle$. The Fourier transform in imaginary time yields

$$G_{k\alpha}^0(\epsilon_n) = (i\epsilon_n - \epsilon_{k\alpha} + \mu_{L,R})^{-1},$$
$$\mathcal{G}_\eta^0(\omega_m) = (i\omega_m - E_T)^{-1}, \quad \eta = -1, 0, 1, \tag{4}$$
$$\mathcal{G}_s^0(\epsilon_n) = (i\epsilon_n - E_S)^{-1},$$

where $\epsilon_n = 2\pi T(n + 1/2)$, $\omega_m = 2\pi T(m + 1/3)$ [13, 14].

Figure 2 shows the main diagrams of the perturbation theory. The first diagrams (Figs. 2b and 2c) determining the Kondo tiling in a standard theory give the following expressions for the renormalized exchange vertices:

$$\Gamma_{LR}^{(2b)}(\omega) \sim J^2 \sum_{\mathbf{k}} \frac{1 - f(\epsilon_{kL} - eV)}{\omega - \epsilon_{kL} + \mu_L - \delta},$$
$$\Gamma_{LR}^{(2c)}(\omega) \sim J^2 \sum_{\mathbf{k}} \frac{f(\epsilon_{kL} - eV)}{\omega - \epsilon_{kL} + \mu_L + \delta}. \tag{5}$$

Replacing ϵ_{kL} by $\epsilon_{kL} - eV$ at the vertex (Fig. 2b), we obtain

$$\Gamma_{LR}^{(2b)}(\omega) \sim J^2 \nu \ln(D/\max\{\omega, (eV - \delta), T\}).$$

where $D \sim \varepsilon_F$ is the truncation parameter determining the effective width of continuum between contacts, ν is the density of states on the Fermi level, and $f(\varepsilon)$ is the Fermi function. As can be seen, a bias compensating for the exchange splitting $|eV - \delta| \ll \max[eV, \delta]$ gives rise to a logarithmic singularity (typical of the Kondo effect) independent of eV. In the second vertex correction (Fig. 2c), the compensation is absent and the corresponding contribution for $eV \sim \delta \gg T$, ω can be estimated as

$$\Gamma_{LR}^{(2c)}(\omega) \sim J^2 \nu \ln(D/(eV + \delta)) \ll \Gamma_{LR}^{(2b)}(\omega).$$

Analogous estimates for the diagrams in Figs. 2d and 2e give

$$\Gamma_{LR}^{(2d)}(\omega) \sim J^3 \nu^2 \ln^2(D/\max\{\omega, (eV - \delta), T\}),$$
$$\Gamma_{LR}^{(2e)}(\omega) \sim J^3 \nu^2 \ln(D/\max\{\omega, (eV - \delta), T\}) \tag{6}$$
$$\times \ln(D/\max\{\omega, eV, T\}).$$

Only the first of these contributions survives in the main logarithmic approximation. Thus, a logarithmic singularity in the tunneling amplitude is actually restored by applying an electric field to a DQD with zero spin in the ground state, whereby a sequence of divergent tiling diagrams degenerates into the sequence of ladder diagrams.

The perturbation theory diagrams at $T > T_K$ can be summed using the renorm group method, which is applicable under both equilibrium and nonequilibrium conditions [15]. The set of renorm group equations for the tunneling vertices $J_{\alpha\alpha'}^{\Lambda\Lambda'}$ obtained upon reduction of the high-energy part of the spectrum is as follows:

$$\frac{dJ_{LL}^T}{d\ln D} = -\nu(J_{LL}^T)^2, \quad \frac{dJ_{LL}^{ST}}{d\ln D} = -\nu J_{LL}^{ST} J_{LL}^T,$$
$$\frac{dJ_{LR}^T}{d\ln D} = -\nu J_{LL}^T J_{LR}^T, \quad \frac{dJ_{LR}^{ST}}{d\ln D} = -\nu J_{LL}^{ST} J_{LR}^T, \tag{7}$$
$$\frac{dJ_{LR}^S}{d\ln D} = \frac{1}{2}\nu\left(J_{LL,+}^{ST} J_{LR,-}^{TS} + \frac{1}{2} J_{LL,z}^{ST} J_{LR,z}^{TS}\right).$$

Solving these equation with the boundary conditions $J_{\alpha\alpha'}^{\Lambda\Lambda'}(D) = J$, we obtain

$$J_{\alpha,\alpha'}^{TT} = \frac{J}{1 - \nu J \ln(D/T)}, \quad J_{\alpha,\alpha'}^{ST} = \frac{J}{1 - \nu J \ln(D/T)},$$
$$J_{LR}^{SS} = J - \frac{3}{4}\nu J^2 \frac{\ln(D/T)}{1 - \nu J \ln(D/T)}. \tag{8}$$

The structure of the renorm group equations (Fig. 3) shows that the Kondo singularity arising in the T-channel influences conductance in the S-channel. This influence is related to the presence of the operator **P** in the tunneling Hamiltonian, which breaks the spin symmetry of the DQD. Thus, the Kondo effect in this system exists only due to the dynamic symmetry inherent in the DQD.

The differential conductance $G(eV, T)/G_0 \sim \left|J_{LR}^{ST}\right|^2$ [15], where $G_0 = e^2/\pi\hbar$, is a function of the universal parameters T/T_K and eV/T_K,

$$G/G_0 \sim \ln^{-2}(\max[(eV-\delta), T]/T_K), \qquad (9)$$

with a maximum at $eV - \delta = 0$ (see Fig. 4). Thus, in contrast to the usual situation [9], whereby the Kondo peak (representing a zero bias anomaly) exhibits evolution or splitting at finite eV values, the Kondo peak in our case appears at a threshold bias of $eV_0 = \delta$. Owing to this threshold character, the peak is asymmetric (cf., e.g., [11]).

The asymmetry, as well as the broadening of the Kondo resonance are related to nonequilibrium character of the tunneling process. In contrast to the usual situation [11, 16], when relaxation takes place in the ground state of the QD, the spin triplet in our case appears only as a virtual state (Fig. 3) and, hence, the nonequilibrium effects are not as destructive. Both relaxation and asymmetry are determined by the imaginary part of the self-energy of the Green's function $\mathcal{G}_T(\omega)$. Figure 5 shows diagrams describing the self-energy in the same order as the renormalization of vertices. The diagrams of Figs 5a and 5b determine the main contribution to the imaginary part $\hbar/\tau_d$. For $\omega \sim eV$, this contribution amounts to $\sim(eV)(J/D)^2$ and contains no logarithmic corrections. Such corrections appear in the third order, but still outside the limits of the main logarithmic approximation, and are estimated as $eV(J/D)^3\ln(D/eV)$. As a result, we obtain as an estimate

$$\hbar/\tau_d \sim eV(\nu J_0^{ST})^2[1 + O(J_0^S\ln(D/(eV)))].$$

Comparing this damping to T_K and taking into account that (under the resonance conditions) $eV \sim \delta \sim J$, we arrive at the following condition for the existence of the anomalous Kondo peak at a finite bias:

$$\delta(\delta/D)^2 \ll T_K \ll \delta. \qquad (10)$$

Here, the right-hand inequality resembles the Doniach criterion for the stability of a Kondo singlet with respect to antiferromagnetic correlations (see, e.g., [1]). The conditions (10) are satisfied in a broad range of parameters since $\delta/D \ll 1$.

Another contribution, related to the reoccupation of levels as a result of the tunneling of nonequilibrium electrons, leads to asymmetry of the resonance line. These processes are described by the diagrams in Figs. 5e and 5f, in which at least one of the virtual sates is triplet. Such transitions, as well as the corresponding

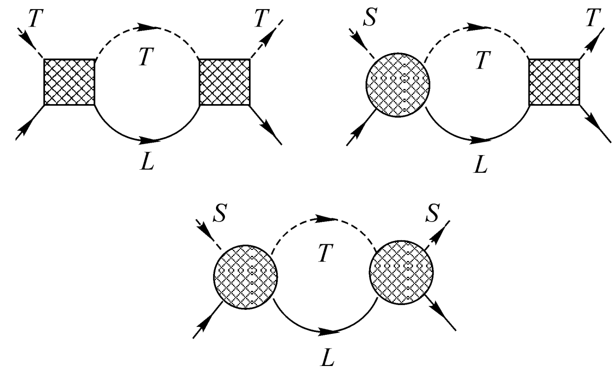

Fig. 3. Irreducible diagrams determining renorm group equations. Cross-hatched squares and circles represent vertices of the T–T and S–T transitions, respectively; other notations as in Fig. 2.

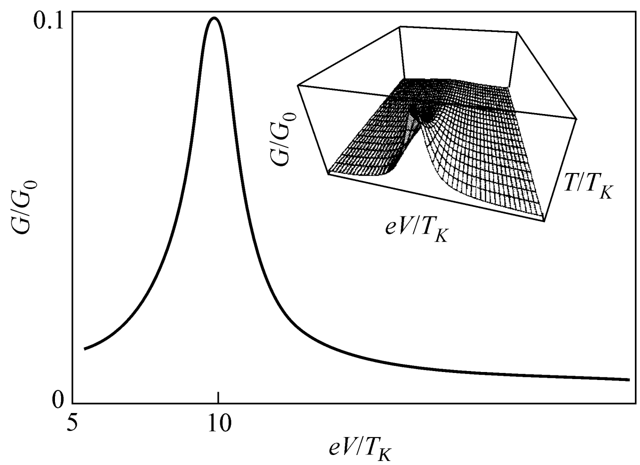

Fig. 4. The Kondo peak of the differential conductance as a function of two universal parameters eV/T_K and T/T_K (inset) and a curve for $\delta/T_K = 10$ and $\hbar/\tau T_K = 0.1$.

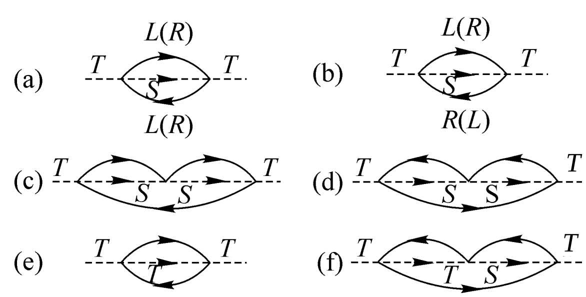

Fig. 5. The diagrams determining the main contribution to $\hbar/\tau T_K$ (a–d) (see text). The diagrams (e–f) describe threshold processes leading to the Kondo peak asymmetry.

second-order processes, possess a threshold character and give small contributions to $\hbar/\tau_d$. As can be seen from Fig. 4, the asymmetry is small even at a significant damping.

Thus, we have described a situation in which the Kondo effect exists only under nonequilibrium conditions and is induced by an external voltage applied to electrodes in tunneling contact with a composite QD. In this case, the Kondo-type tunneling is induced by dynamic processes of the excitation of low-lying spin states of the QD, the ground state of which is a spin singlet. The simplest example of such a system is offered by a double QD with even occupation under the conditions of strong Coulomb blockade. The spin symmetry of such a QD is essentially that of a quantum spin rotator. Since a singlet ground state in quantum mechanics is always accompanied by triplet excitations, this situation is not very unusual and can probably be manifested as a peak in the differential conductance, observed at a nonzero bias in a Coulomb window with an even number of electrons. Such a peak should be distinguished from a maximum corresponding to the cotunneling via excited electron states [17].

This study was supported in part (M.N.K.) by the European Commission (LF Project) via the Weizman Institute (Israel) (contract #HPRI-CT-1999-00069) and by the Deutsche Forschungsgemeinschaft (SFB-410). The authors are grateful to P. Wölfle for fruitful discussions.

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Translated by P. Pozdeev