



**M.N.Kiselev**

# **Strongly Correlated Systems:**

**High Temperature Superconductors**

**Heavy Fermion Compounds**

**Organic materials**

**Strongly Correlated Systems:  
High Temperature Superconductors**

# Superconductivity: what and what for

- **Zero resistance at finite (but low) temperatures**
  - Discovered in 1911 by Kamerlingh-Onnes (Nobel Prize in 1913): Hg superconducting at 4.2 K
  - Later observed in other metals like Nb, Al, ... but the critical temperature,  $T_c < 23\text{K}$
- **Applications (examples):**



**MRI (without need for liquid He cooling?)**



**Loss free power transmission (without cooling?)**

# Superconducting magnetically levitated train

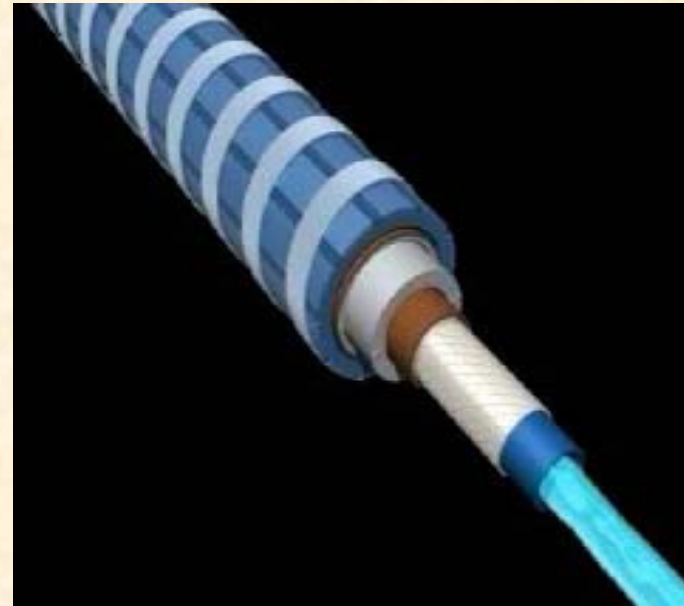
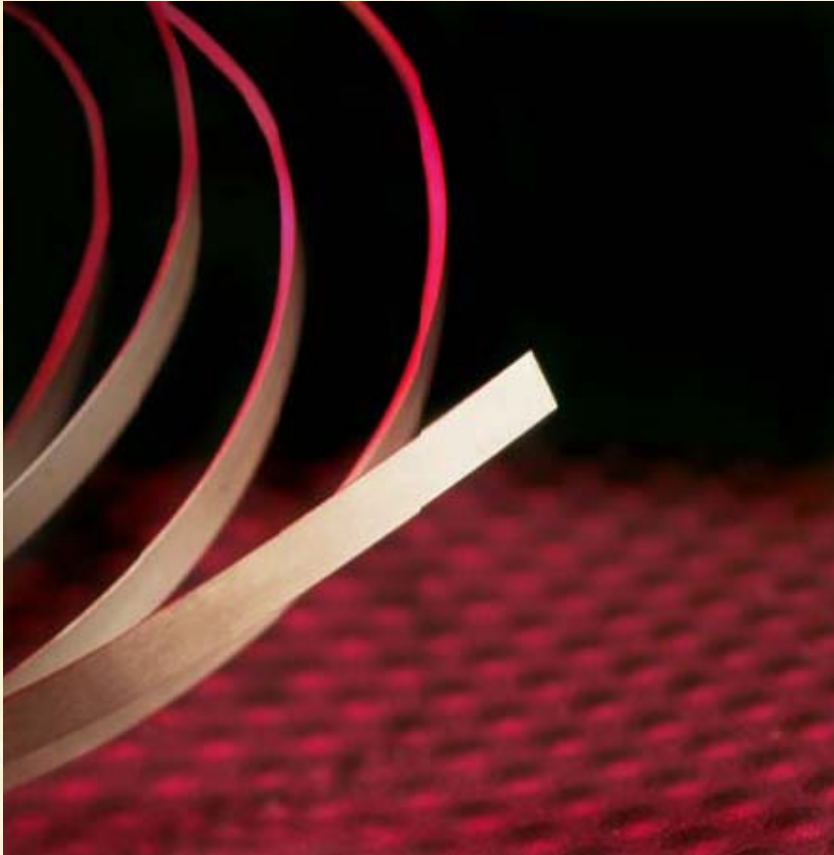


# Superconducting magnet used to detonate mines





# Superconducting Cables



# KNOWN SUPERCONDUCTIVE ELEMENTS

1	IA	1	H	IIA	2	He	0																														
2		3	Li	4	Be	5	B	6	C	7	N	8	O	9	F	10	Ne																				
3		11	Na	12	Mg	13	Al	14	Si	15	P	16	S	17	Cl	18	Ar																				
4		19	K	20	Ca	21	Sc	22	Ti	23	V	24	Cr	25	Mn	26	Fe	27	Co	28	Ni	29	Cu	30	Zn	31	Ga	32	Ge	33	As	34	Se	35	Br	36	Kr
5		37	Rb	38	Sr	39	Y	40	Zr	41	Nb	42	Mo	43	Tc	44	Ru	45	Rh	46	Pd	47	Ag	48	Cd	49	In	50	Sn	51	Sb	52	Te	53	I	54	Xe
6		55	Cs	56	Ba	57	*La	72	Hf	73	Ta	74	W	75	Re	76	Os	77	Ir	78	Pt	79	Au	80	Hg	81	Tl	82	Pb	83	Bi	84	Po	85	At	86	Rn
7		87	Fr	88	Ra	89	+Ac	104	Rf	105	Ha	106	106	107	107	108	108	109	109	110	110	111	111	112	112												

■ BLUE = AT AMBIENT PRESSURE  
■ GREEN = ONLY UNDER HIGH PRESSURE

SUPERCONDUCTORS.ORG

\* Lanthanide Series

58	59	60	61	62	63	64	65	66	67	68	69	70	71
Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu

+ Actinide Series

90	91	92	93	94	95	96	97	98	99	100	101	102	103
Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr

## Short history of superconductivity

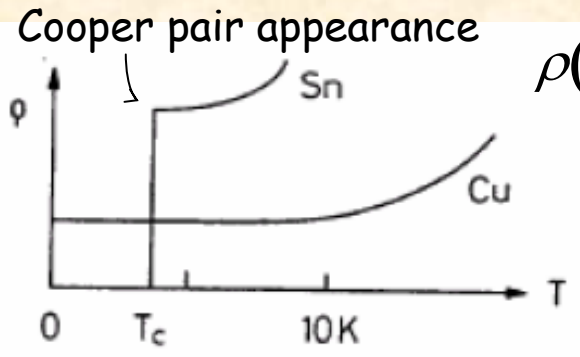
- ◆ 1908 Heike Kamerlingh Onnes achieves very low temperature producing liquid He ( $< 4.2$  K)
- ◆ 1911 Onnes and Holst observe sudden drop in resistivity to essentially zero **SC era starts**
- ◆ 1914 Persistent current experiments (Onnes)
- ◆ 1933 Meissner-Ochsenfeld effect observed
- ◆ 1935 Fritz and London theory
- ◆ 1950 Ginsburg - Landau theory
- ◆ 1957 BCS Theory (Bardeen, Cooper, Schrieffer)
- ◆ 1962 Josephson effect is observed
- ◆ 1967 Observation of Flux Tubes in Type II superconductors (Abrikosov, Ginsburg, Leggett)
- ◆ 1980 Tevatron: The first accelerator using superconducting magnets
- ◆ 1986 First observation of Ceramic Superconductor at 35 K (Bednorz, Muller)
- ◆ 1987 first ceramic superconductor at 92 K (above liquid Nitrogen at 77 K!) **HTS era starts**
- ◆ 2003 discovery of a metallic compound the  $B_2Mg$  superconducting at 39 K ( **$\times 2 T_c$  of  $Nb_3Sn$** )
  - It took  $\sim 70$  years to get first accelerator from conventional superconductors.
  - How long will it take for HTS or  $B_2Mg$  to get to accelerator magnets? Have patience!





# What is a superconductor

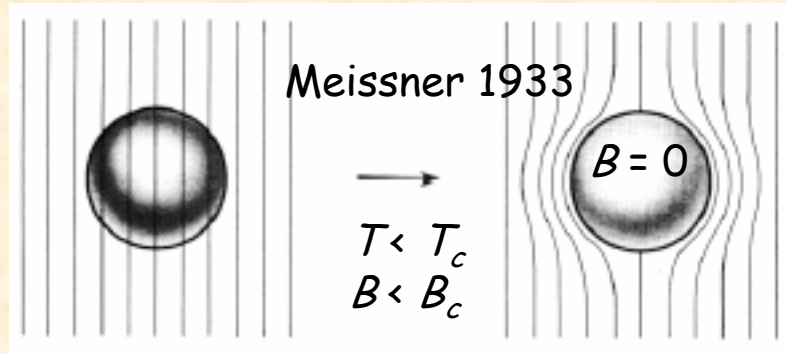
Below the *critical temperature*  $T_c$  the resistivity drops



$$\rho(T) = \rho_0 + cT^5$$

phonon- $e^-$  interaction

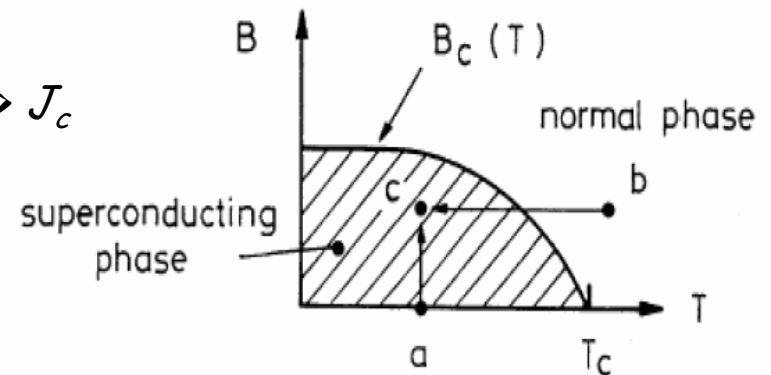
Below  $T_c$  the B-field lines are expelled out of a superconductor (perfect diamagnetic behaviour)



## Type I superconductors

the superconductivity disappears as  $T > T_c$  |  $B > B_c$  |  $J > J_c$

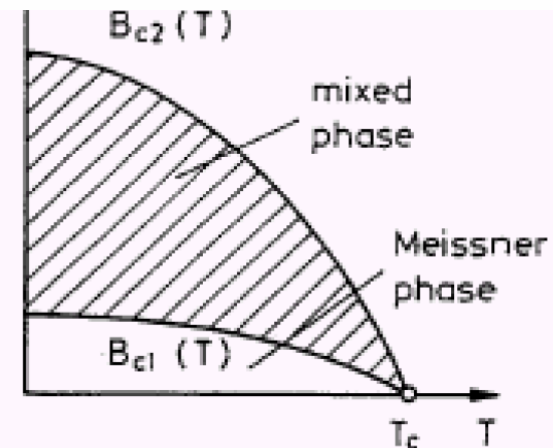
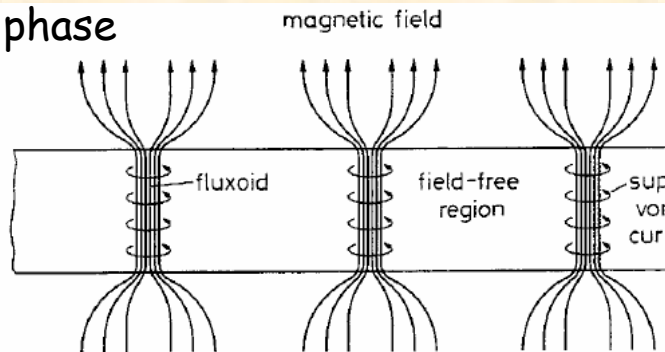
SC de type I	Ti	Al	Hg	Sn	Pb
$B_c$ [mT] à 0 K	10	10,5	41,2	30,9	80,3



## Type II superconductors

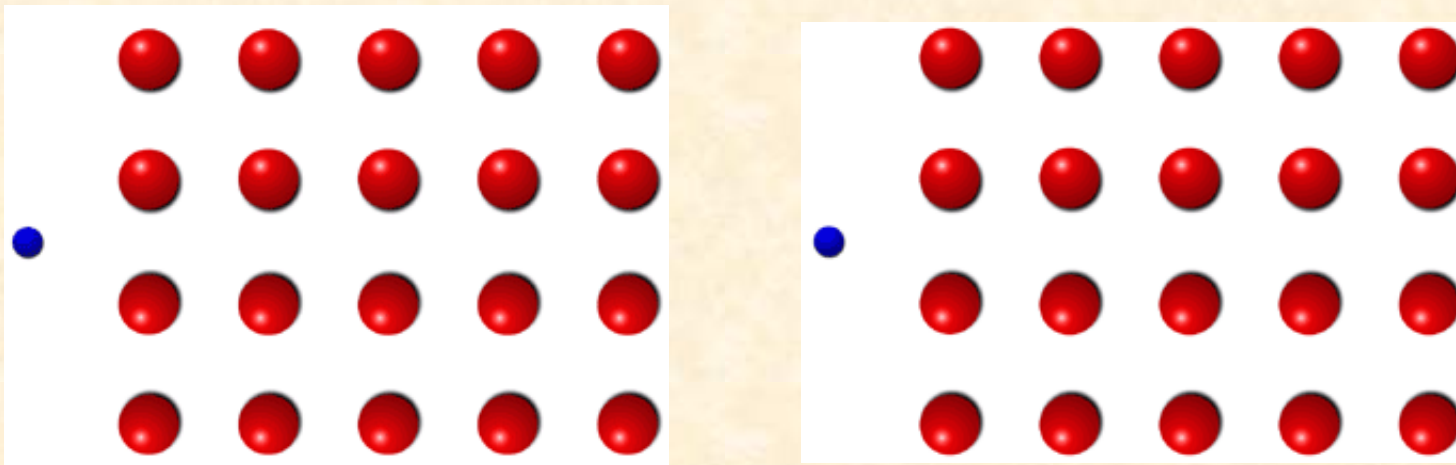
For  $B_{c1} < B < B_{c2}$  there is a partial flux penetration through fluxoid vortices and a mixed phase

Nb (type II)	$B_{c1}$	$B_{c2}$
$B_c$ [mT] à 0 K	~170	~240



# BCS Pairing mechanism

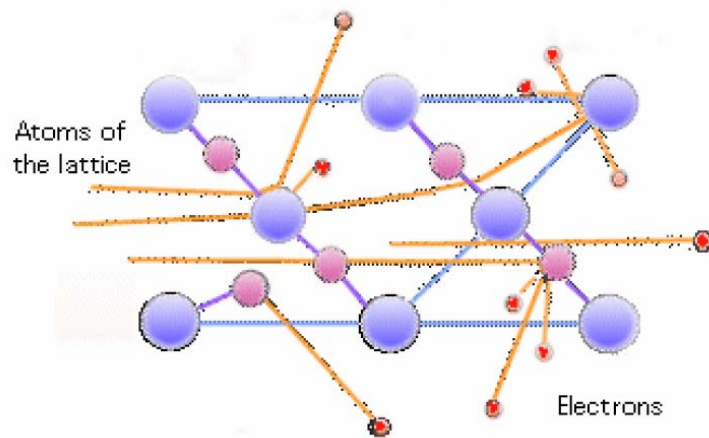
- When electrons form pairs, they behave like bosons and can condensed into a macroscopic quantum state
- Bardeen, Cooper, and Shriver (BCS) develop rigorous description of pairing mechanism
  - Theory developed in the 1950s, Nobel Prize in 1973



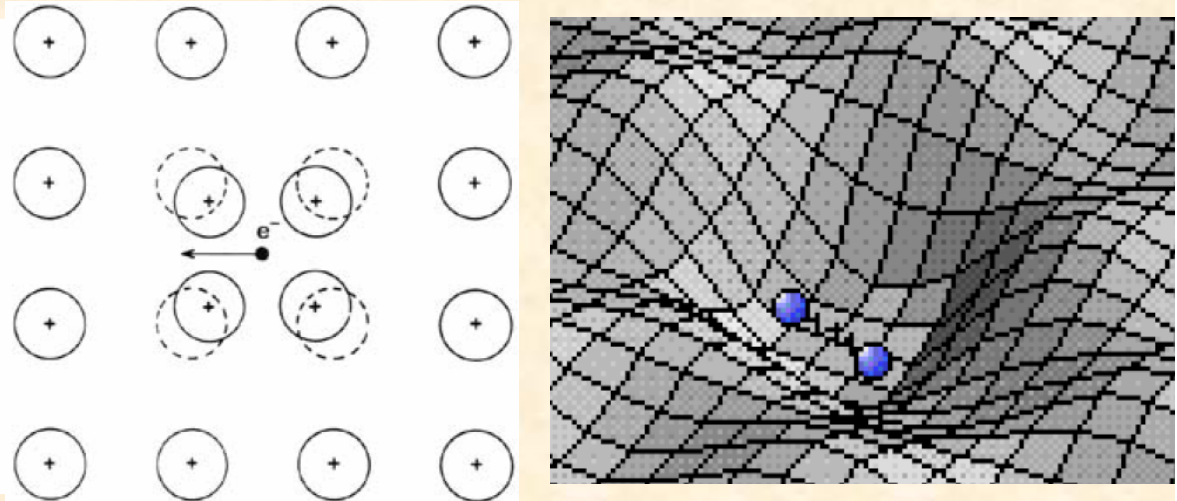
- At  $T > 20\text{K}$ , lattice vibrations are strong and destroy pairs, superconductor becomes a normal metal

# BCS theory

## Normal conducting state



## Superconducting state

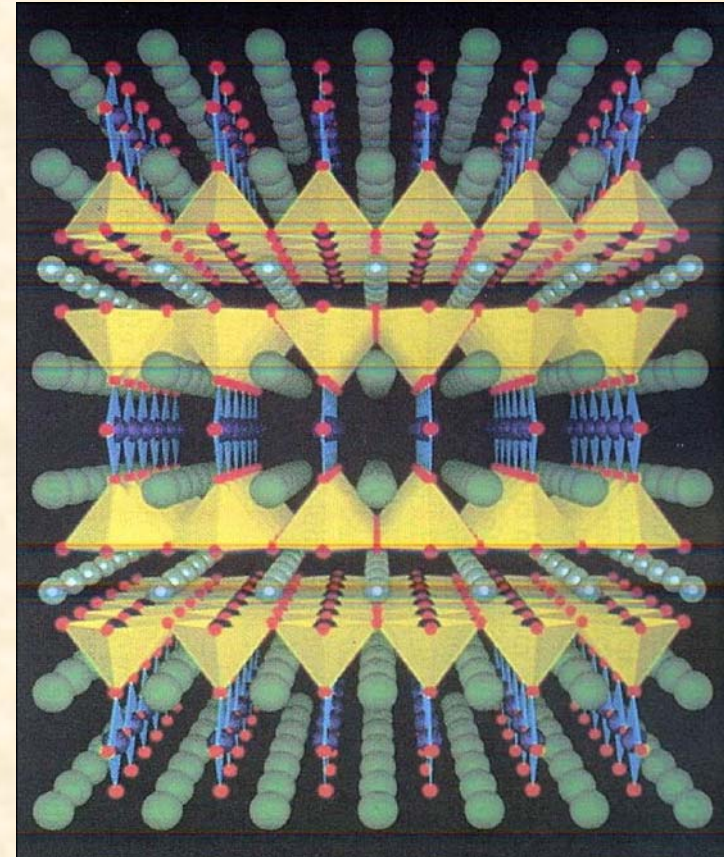


- ◆  $T_c \sim 1/\sqrt{M_{isotopic}}$  → **phonons** should play a role in superconductivity
- ◆ Creation of **Cooper pairs** (over-screening effect)
  - An  $e^-$  attracts the surrounding ion creating a region of increased positive charge
  - The lattice oscillations enhance the attraction of another passing by  $e^-$  (Cooper pair)
  - The interaction is strengthened by the surrounding sphere of conduction  $e^-$  (Pauli principle)
- ◆ In a superconductor the net effect of  $e^-e^-$  attraction through phonon interaction and the  $e^-e^-$  coulombian repulsion is attractive and the Cooper pair becomes a **singlet state** with zero momentum and zero spin
- ◆ To break a pair the excitation energy is  $\Delta E = 2\Delta$



# High Temperature Superconductivity

- **Doped  $\text{YBa}_2\text{Cu}_3\text{O}_7$** 
  - Normal states is insulating / poor metal
  - Discovered to be a SC with  $T_c=30\text{K}$  in 1986 (75 years after Kamerlingh-Onnes)
  - 1987 Nobel Prize for Bednorz and Muller
  - Within years, other transition metal oxides were discovered with  $T_c>100\text{K}$  (liquid Nitrogen cooled SC)
- There is general agreement that the pairing mechanism is not phonon mediated





# Some examples of HTSC Compounds

- **Mostly compounds**

- **Record holder:**

**$T_c = 138 \text{ K}$**

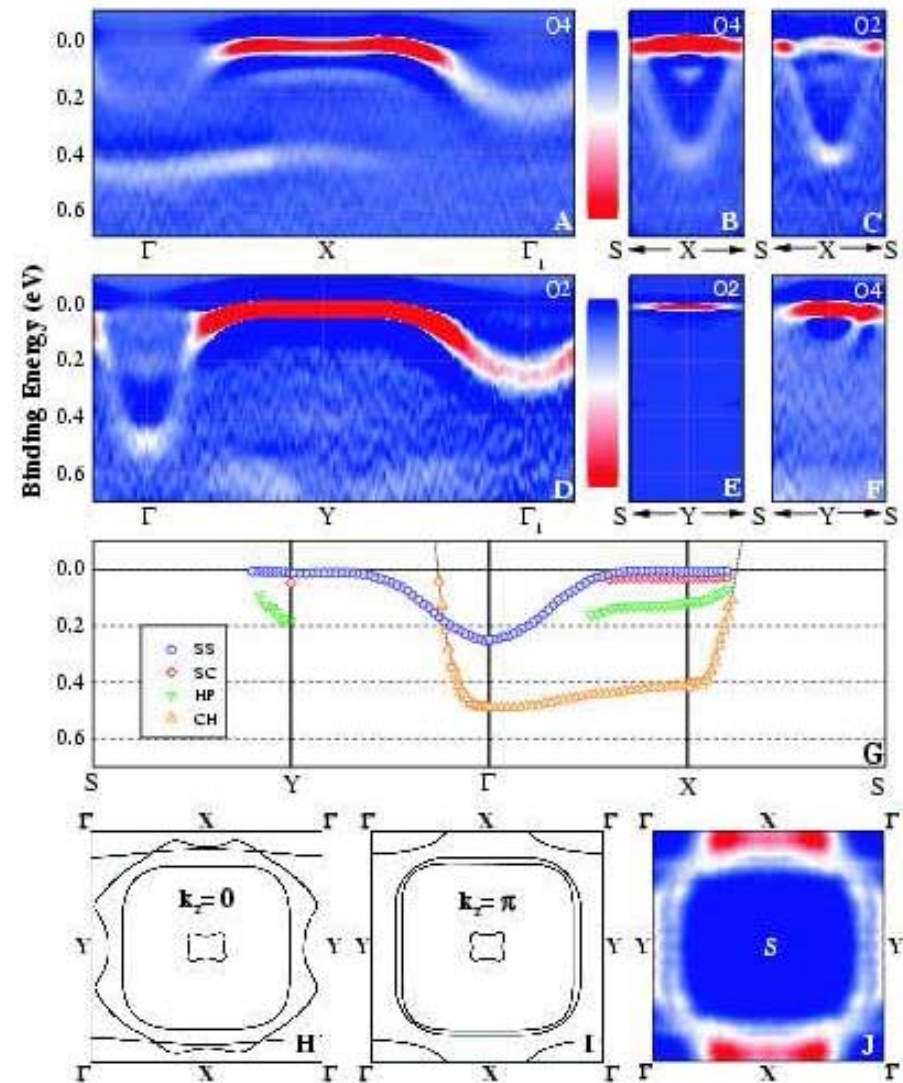
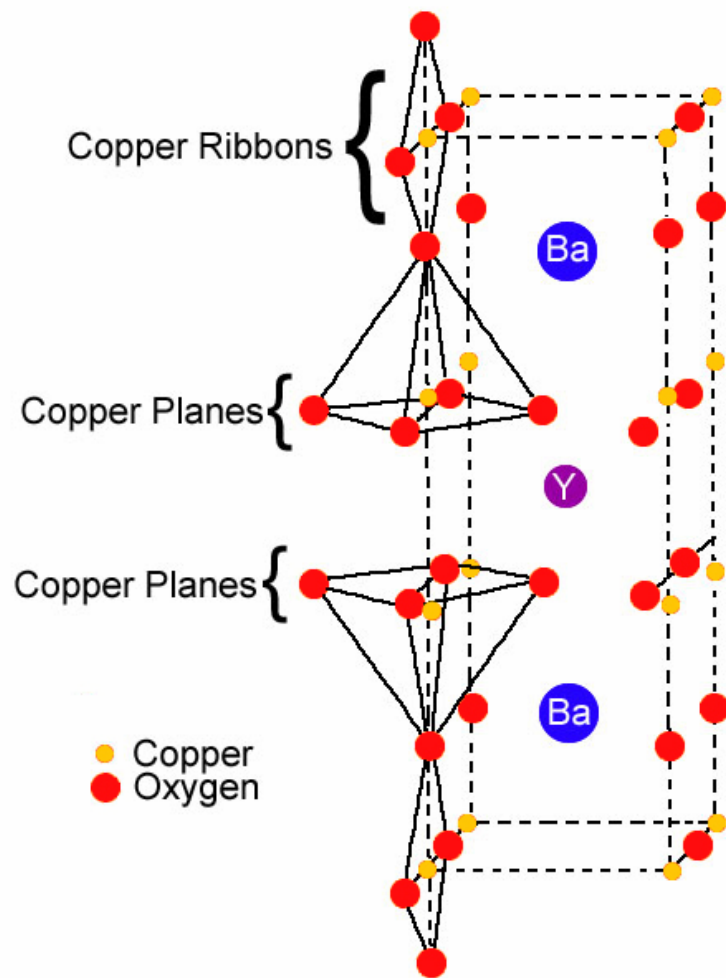
- **High  $H_{c2}$ :**

**$H_{c2} > 1000 \text{ 000 G}$**

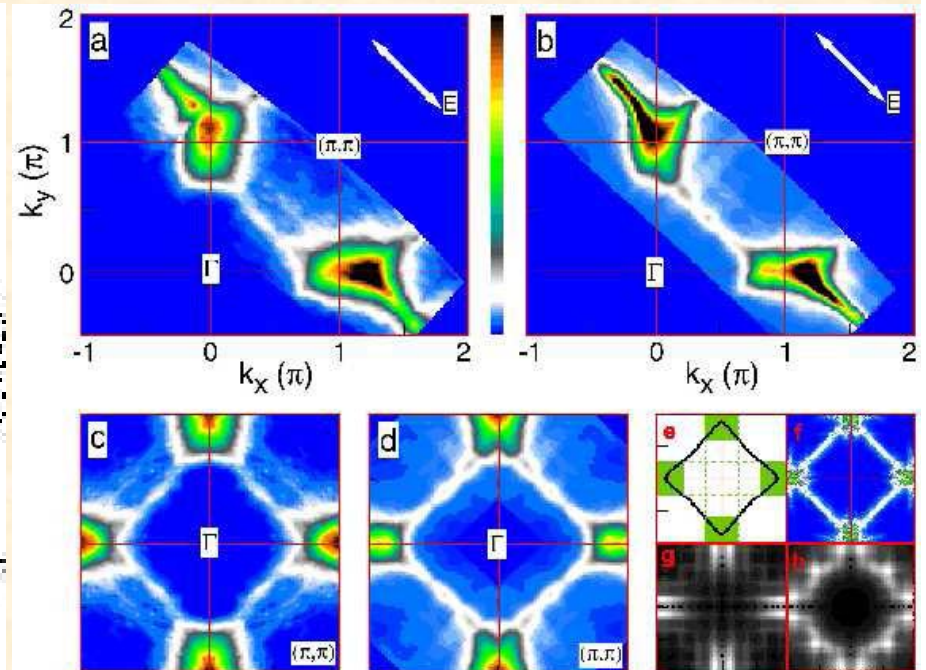
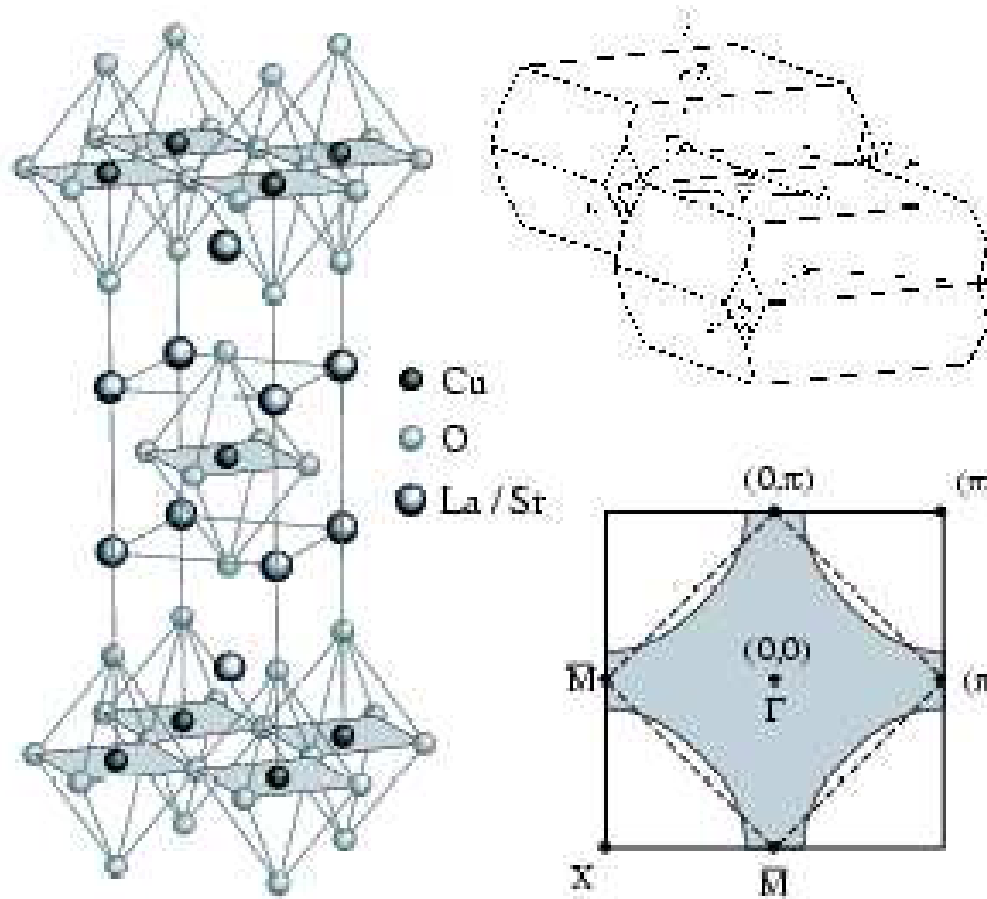
**(YBCO)**

Element	$T_c$ (K)
Tc	7.80
Nb	9.25
$\text{La}_{1.85}\text{Ba}_{.15}\text{CuO}_4$	30
$\text{YBa}_2\text{Cu}_3\text{O}_{7+}$	93
$\text{Ca}_{1-x}\text{Sr}_x\text{CuO}_2$	110
$\text{Tl}_2\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{10}$	128
$\text{Hg}_{0.8}\text{Tl}_{0.2}\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{8.33}$	138

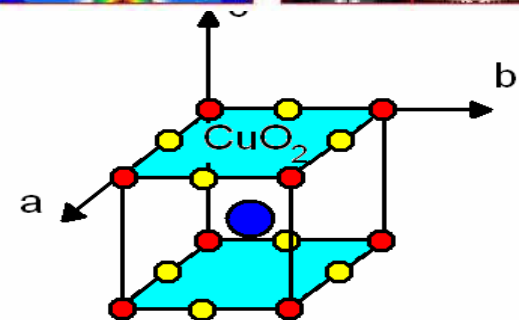
# Crystal Structure and Fermi Surface



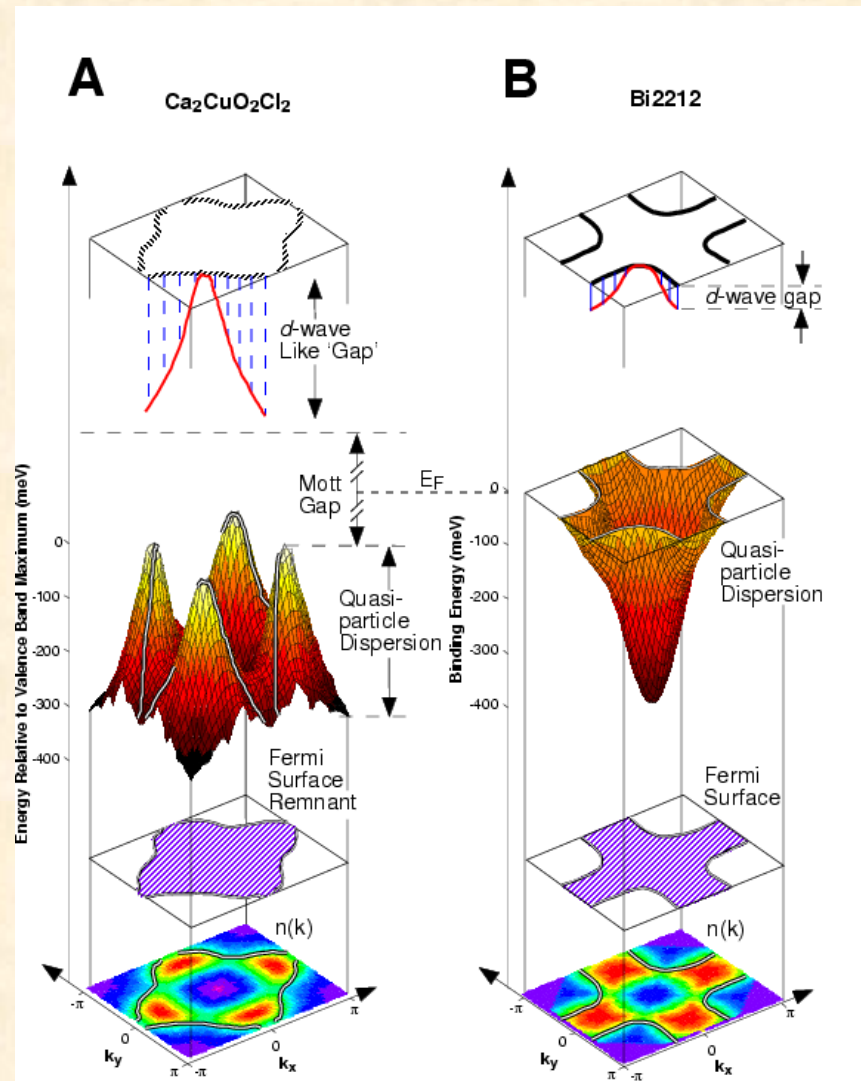
# Crystal Structure and Fermi Surface



- (Sr, La)
- Cu
- O



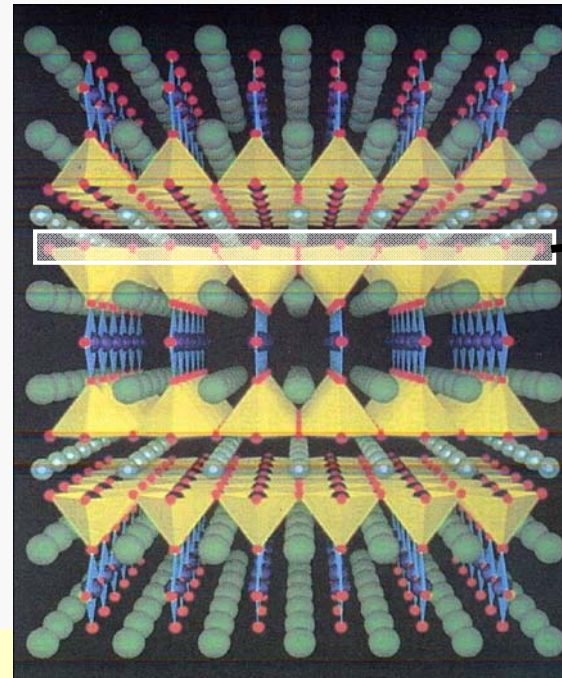
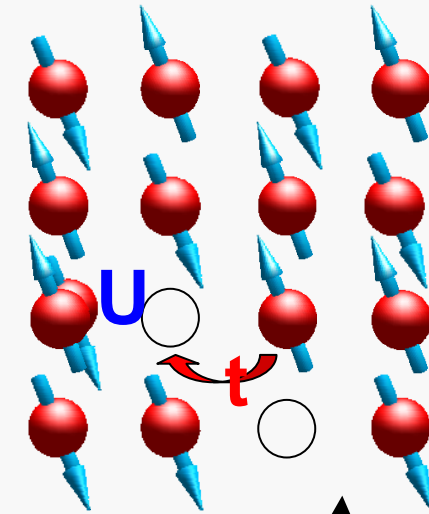
# Crystal Structure and Fermi Surface



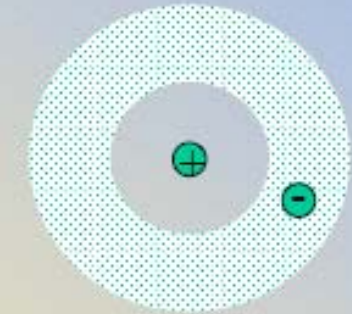


# High Tc Superconductivity (conventional wisdom)

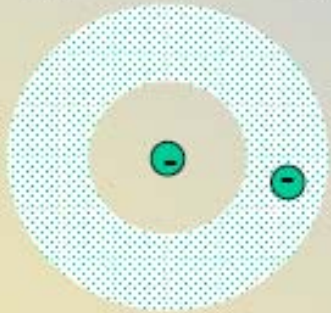
- **Model Copper oxide planes with single band 2D Hubbard models (Zhang & Rice, PRB 1989)**
- **Still not solvable, but thousands of papers published every year**
- **David Pines: “arguably the major problem in physics today”**



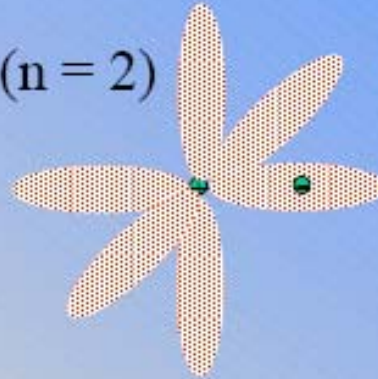
Understanding the mechanism --- Where are we ?



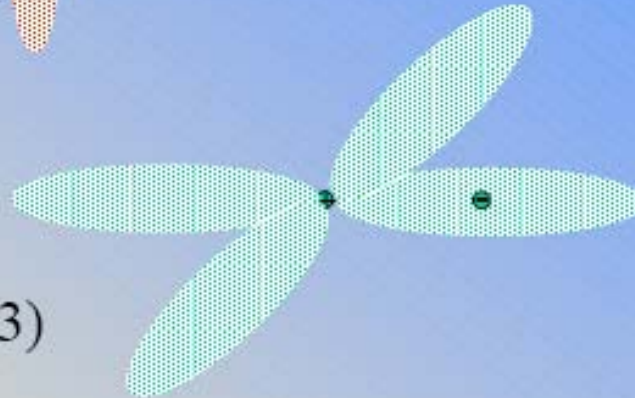
Type *s* ( $n = 1$ )




Type *p* ( $n = 2$ )




Type *d* ( $n = 3$ )



$n=3$  

$n=2$  

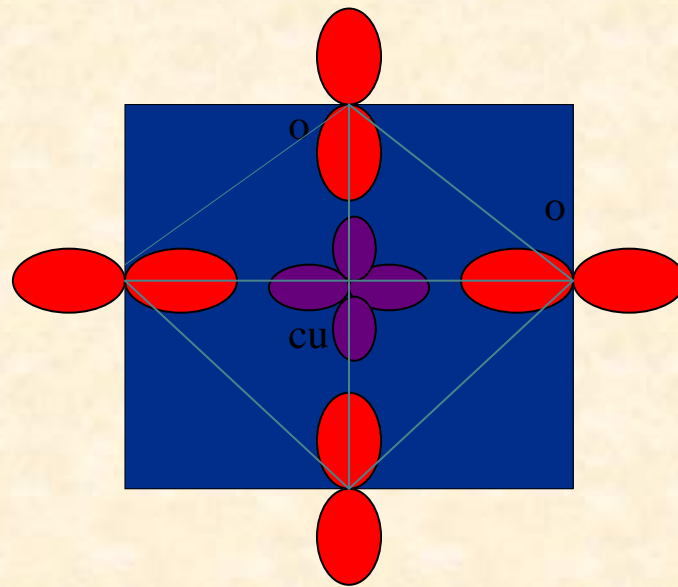
$n=1$  



## Microscopic Model:

### Why are Cuprates Unique? (1987)

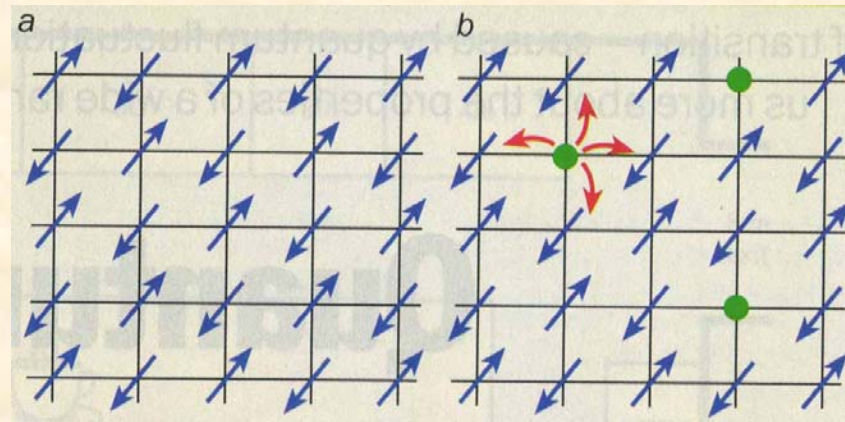
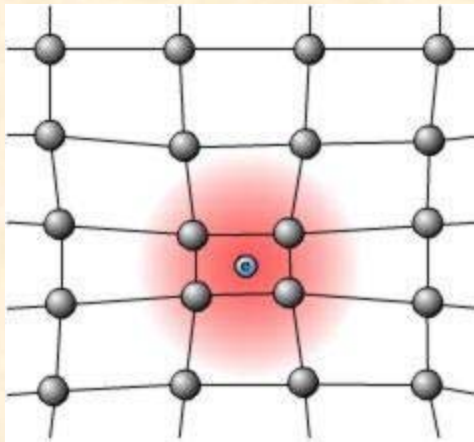
*Minimum Hamiltonian : Space of three orbitals ( $d, p_x, p_y$ ) per unit cell :  
 $\mathcal{H} = K.E.(t_{pd}, t_{pp}) + \text{Local Repulsions } (U_d, U_p) + \text{Ionic Interactions } (V).$*



Cannot be reduced to a Hubbard Model because the ionization energy of Cu is nearly the same as the ionization energy of oxygen.

# What interactions/orders exist in High-Tc Superconductor?

- **Electron-phonon interaction**

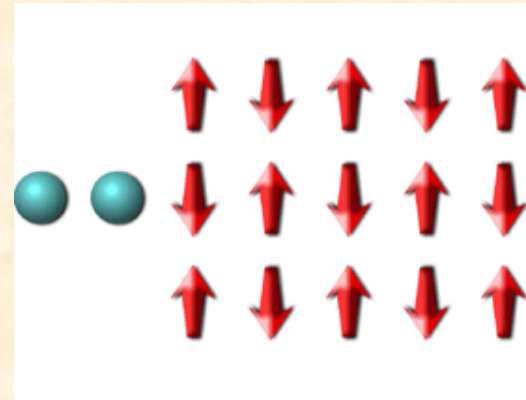
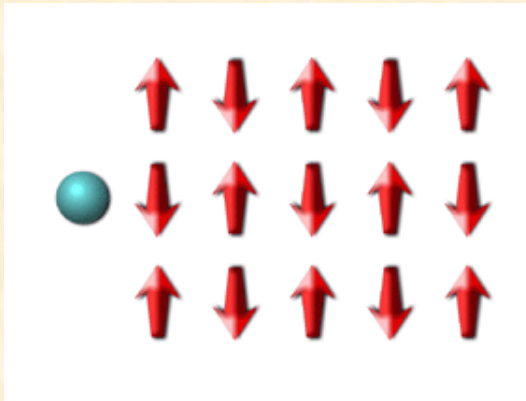


- **Spin exchange interaction-antiferromagnetic order**
- **Charge density waves, spin density waves and other competing orders.**

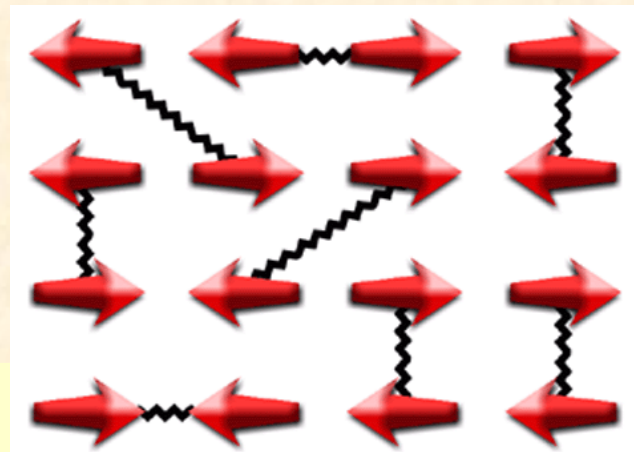


Of the many proposed pairing mechanisms, few remain likely:

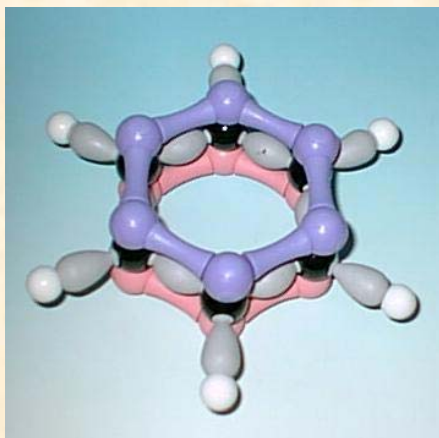
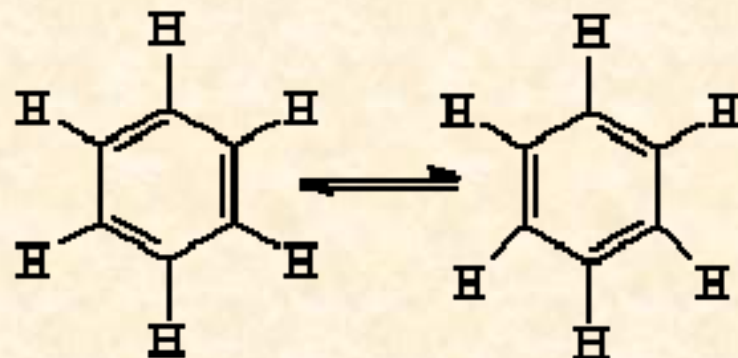
- **Quasi particles in AF background (Hirsch 02)**



- **Resonating valence bond (Anderson 87)**

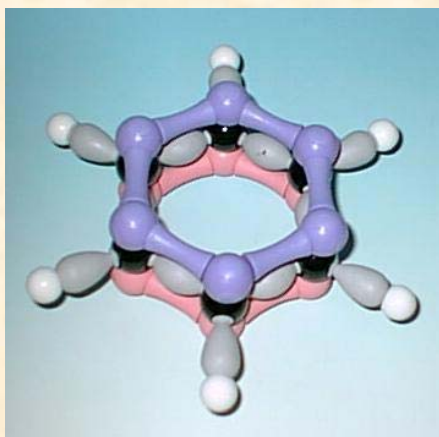
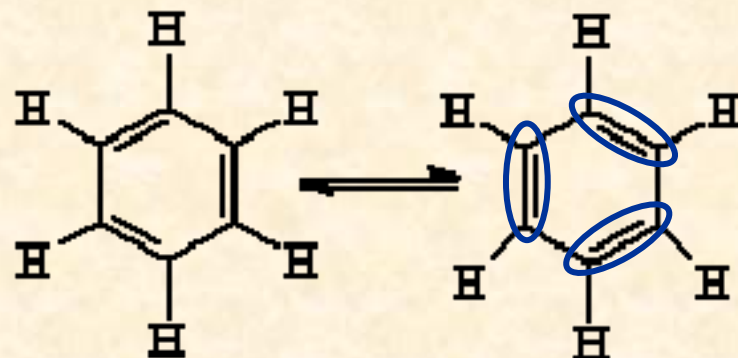


# Valence bonds in benzene



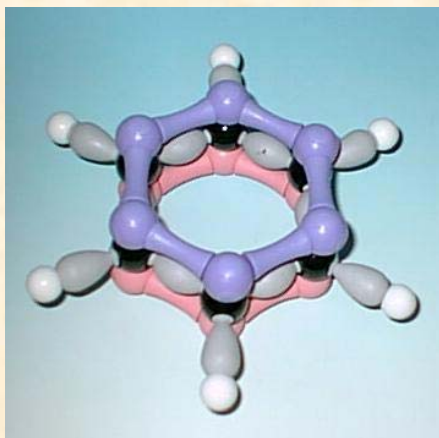
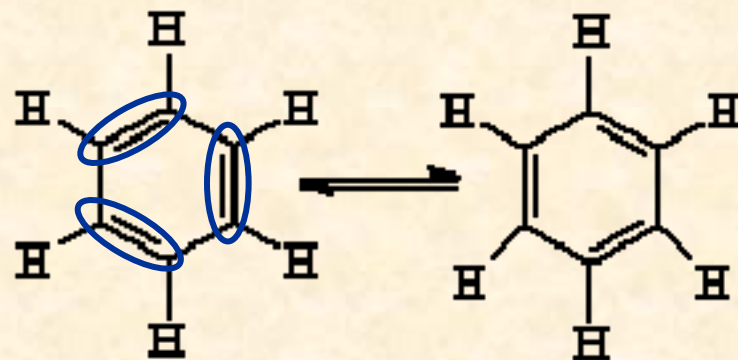
Resonance in benzene leads to a symmetric configuration of valence bonds  
(*F. Kekulé, L. Pauling*)

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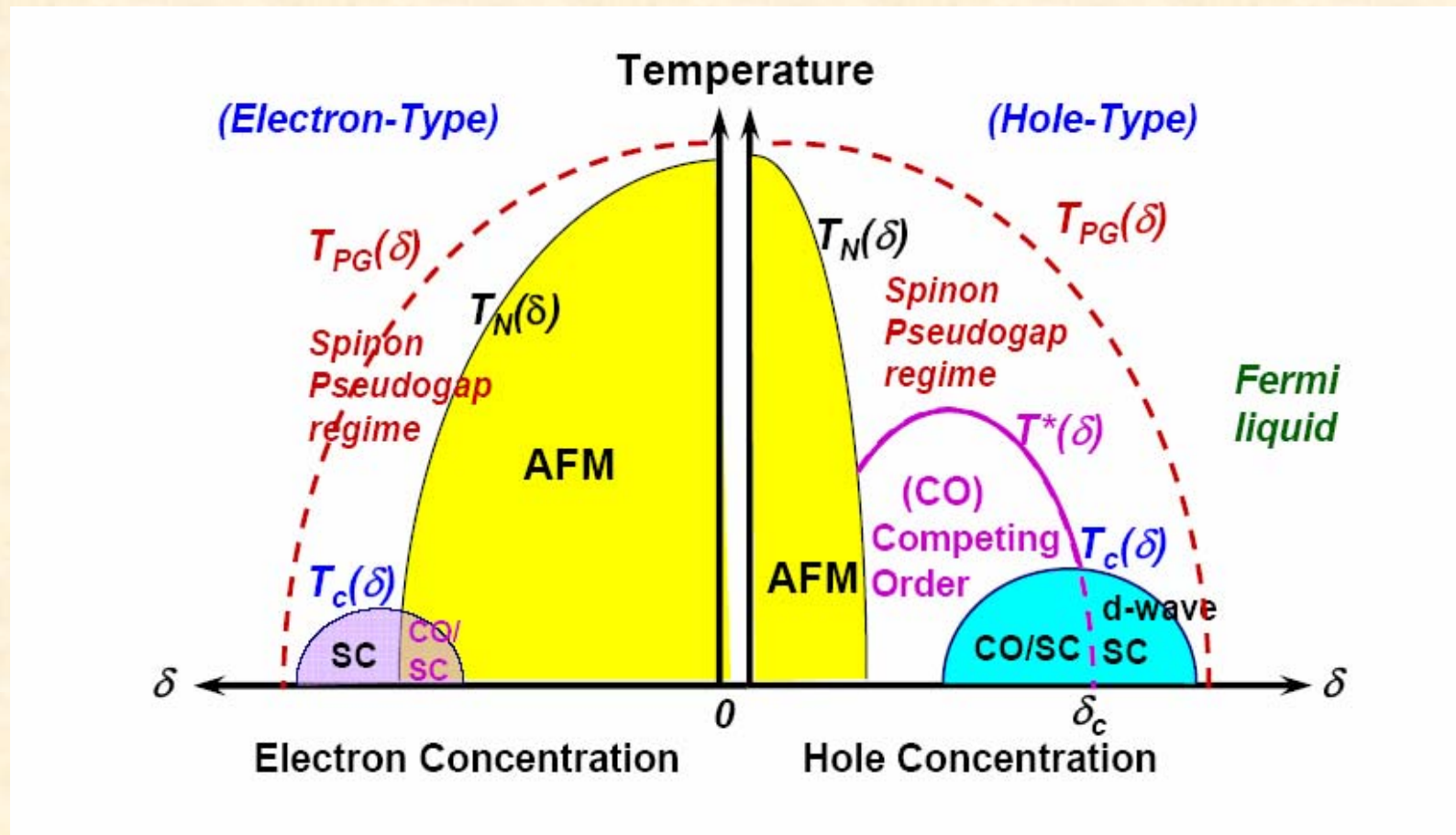
# Valence bonds in benzene



Resonance in benzene leads to a symmetric configuration of valence bonds  
(*F. Kekulé, L. Pauling*)



## Phase Diagram and Competing Orders



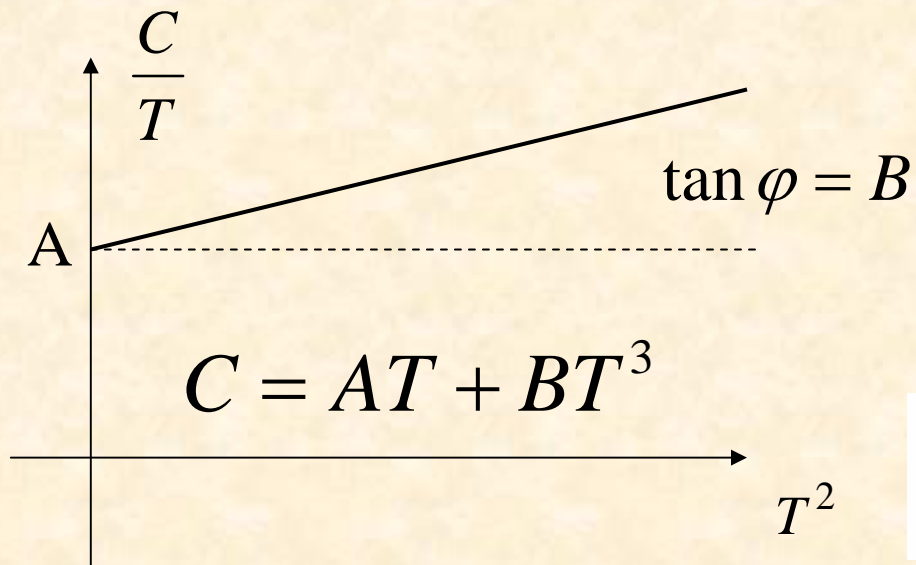
PG: Pseudogap, SC: Superconductivity, CO: Competing order, AFM: Antiferromagnetic

# **Strongly Correlated Systems: Heavy Fermions**

# Heavy Fermions

Type	Material	$T^*$	$T_c, x_c, B_c$	Properties	$\rho$	$\gamma_n$ $mJmol^{-1}K^{-2}$	Ref.
Metal	$CeCu_6$	10K	-	Simple HF Metal	$T^2$	1600	[1]
Super-conductors	$CeCu_2Si_2$	20K	$T_c=0.17K$	First HFSC	$T^2$	800-1250	[2]
	$UBe_{13}$	2.5K	$T_c=0.86K$	Incoherent metal $\rightarrow$ HFSC	$\rho_c \sim 150\mu\Omega cm$	800	[3]
	$CeCoIn_5$	38K	$T_c=2.3K$	Quasi 2D HFSC	$T$	750	[4]
Kondo Insulators	$Ce_3Pt_4Bi_3$	$T_\chi \sim 80K$	-	Fully Gapped KI	$\sim e^{\Delta/T}$	-	[5]
	$CeNiSn$	$T_\chi \sim 20K$	-	Nodal KI	Poor Metal	-	[6]
Quantum Critical	$CeCu_{6-x}Au_x$	$T_0 \sim 10K$	$x_c = 0.1$	Chemically tuned QCP	T	$\sim \frac{1}{T_0} \ln(\frac{T_0}{T})$	[7]
	$YbRh_2Si_2$	$T_0 \sim 24K$	$B_\perp=0.06T$ $B_\parallel=0.66T$	Field-tuned QCP	T	$\sim \frac{1}{T_0} \ln(\frac{T_0}{T})$	[8]
SC + other Order	$UPd_2Al_3$	110K	$T_{AF}=14K$ , $T_{sc}=2K$	AFM + HFSC	$T^2$	210	[9]
	$URu_2Si_2$	75K	$T_1=17.5K$ , $T_{sc}=1.3K$	Hidden Order & HFSC	$T^2$	120/65	[10]

# Specific heat



$$E = 2V \int \epsilon(p) n(\epsilon_p) \frac{d^3p}{(2\pi\hbar)^3}$$

$$C = V^{-1} \left( \frac{\partial E}{\partial T} \right)_V = 2 \int \epsilon(p) \left( \frac{\partial n(\epsilon_p)}{\partial T} \right) \frac{d^3p}{(2\pi\hbar)^3}$$

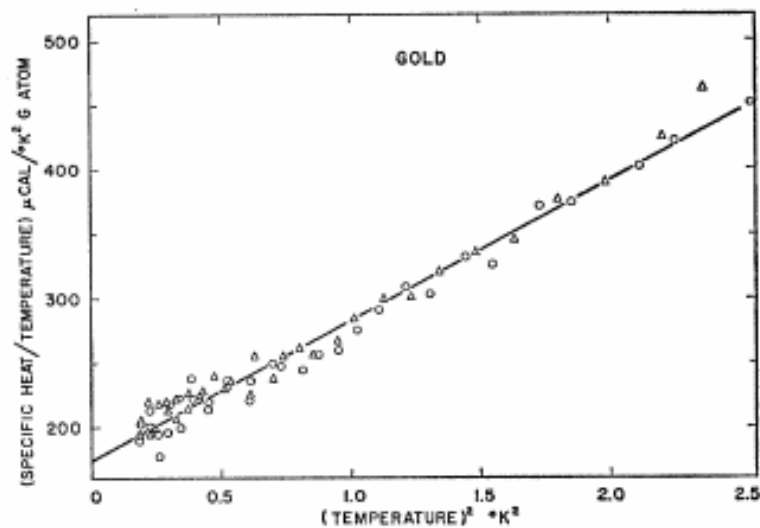


FIG. 1. Plot of (specific heat/temperature) versus (temperature<sup>2</sup>) for gold.

$$C = \frac{\pi^2}{3} \rho(0) T = \gamma T$$

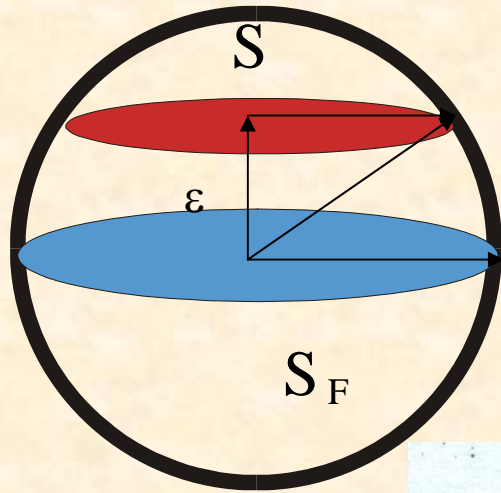
$$\gamma = \frac{p_F m^*}{3\hbar^3}$$

$$\rho(0) = \frac{p_F m^*}{\pi^2 \hbar^3}$$

	N/V [cm <sup>-3</sup> ]	$\epsilon_F$ [eV]	$T_F$ [K]	$m^*/m$
<i>Li</i>	$4.6 \times 10^{22}$	4.7	$5.5 \times 10^4$	2.3
<i>Na</i>	2.5	3.1	3.7	1.3
<i>K</i>	1.34	2.1	2.4	1.2
<i>Cu</i>	8.5	7.0	8.2	1.3
<i>Ag</i>	5.76	5.5	6.4	1.1
<i>Au</i>	5.9	5.5	6.4	1.1

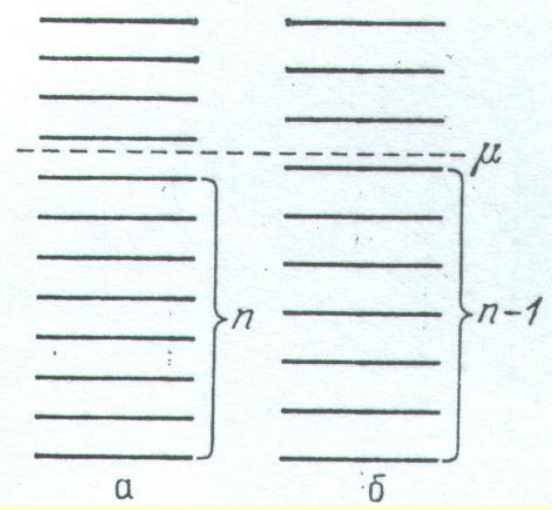
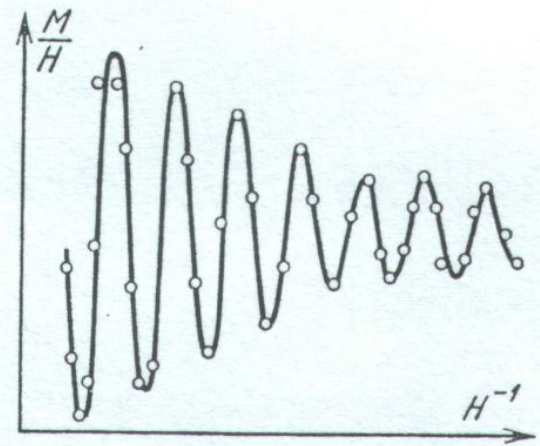


# de Haas-van Alphen Effect (dHvA)

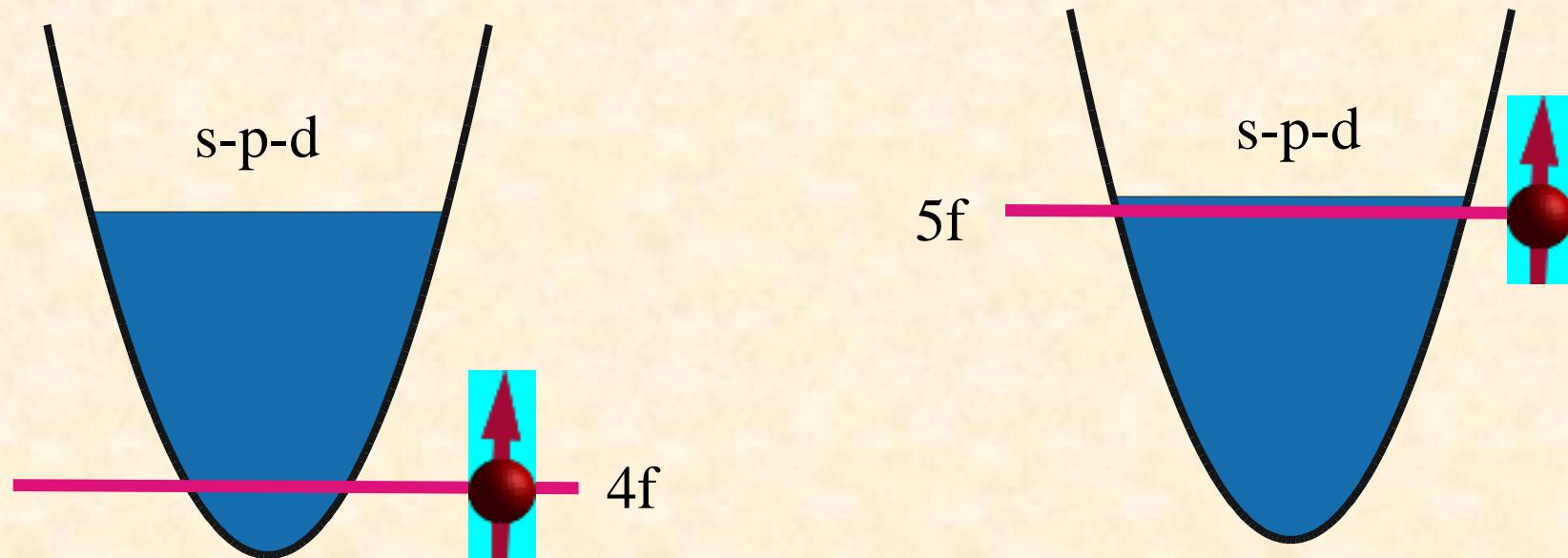


$$M \approx T \sqrt{\frac{2e\hbar}{\pi^3 c H}} \sum_m S_m \left| \frac{\partial^2 S_m}{\partial p_z^2} \right| \exp\left(-\frac{2\pi^2 T c m^*}{e\hbar H}\right) \sin\left(\frac{c S_m}{e\hbar H} \pm \frac{\pi}{4}\right) \cos\left(\frac{\pi m^*}{m}\right)$$

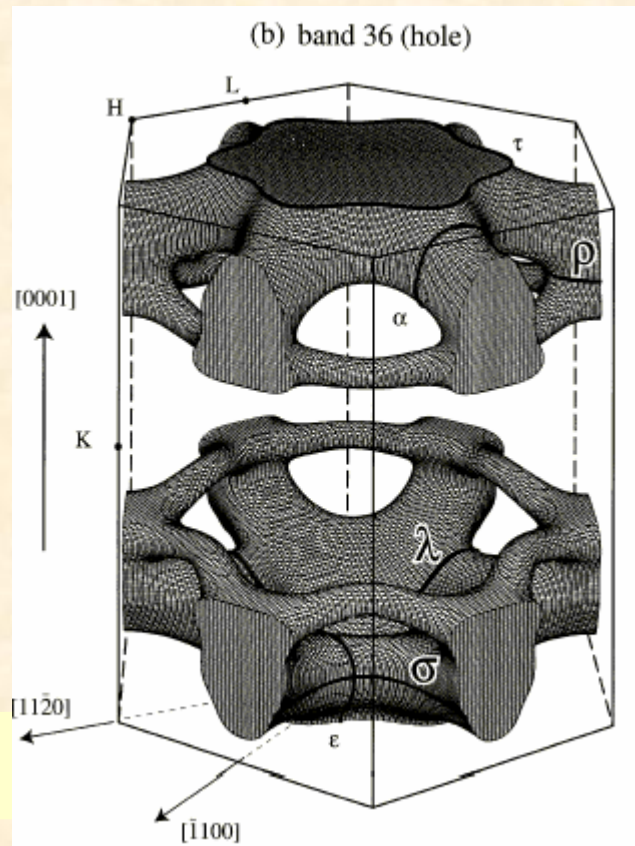
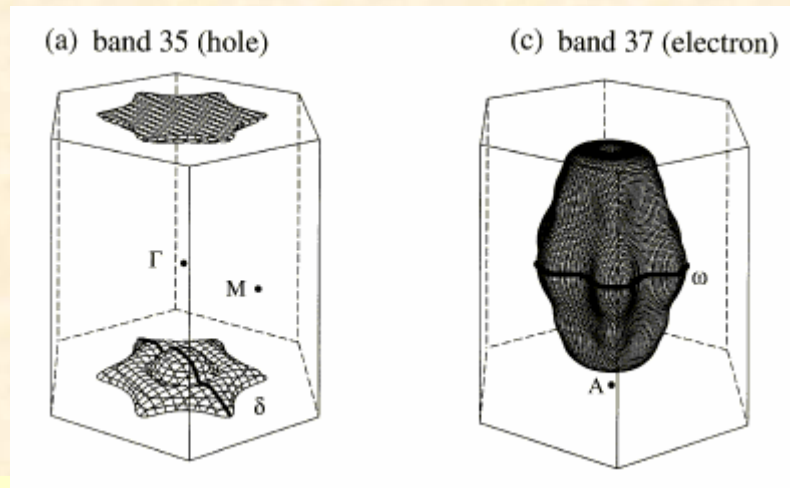
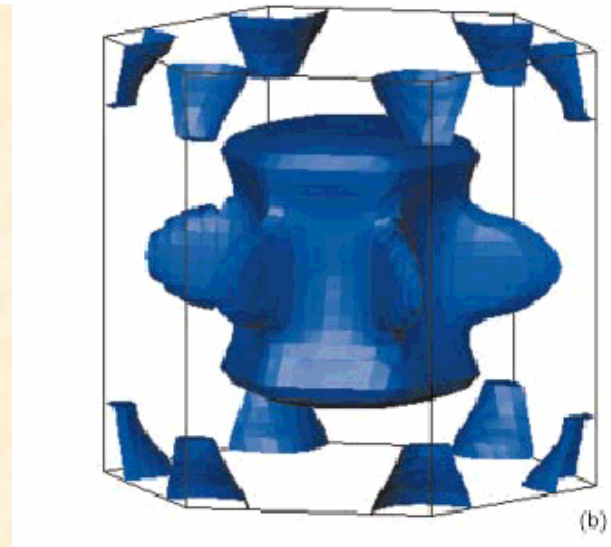
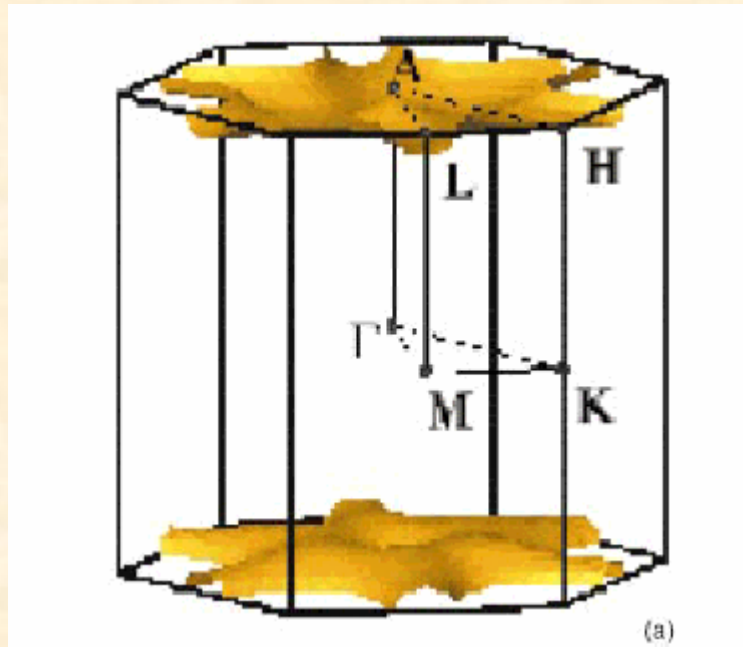
$$m^* = \frac{1}{2\pi} \frac{\partial S}{\partial \epsilon}$$



# Heavy Fermion Compounds

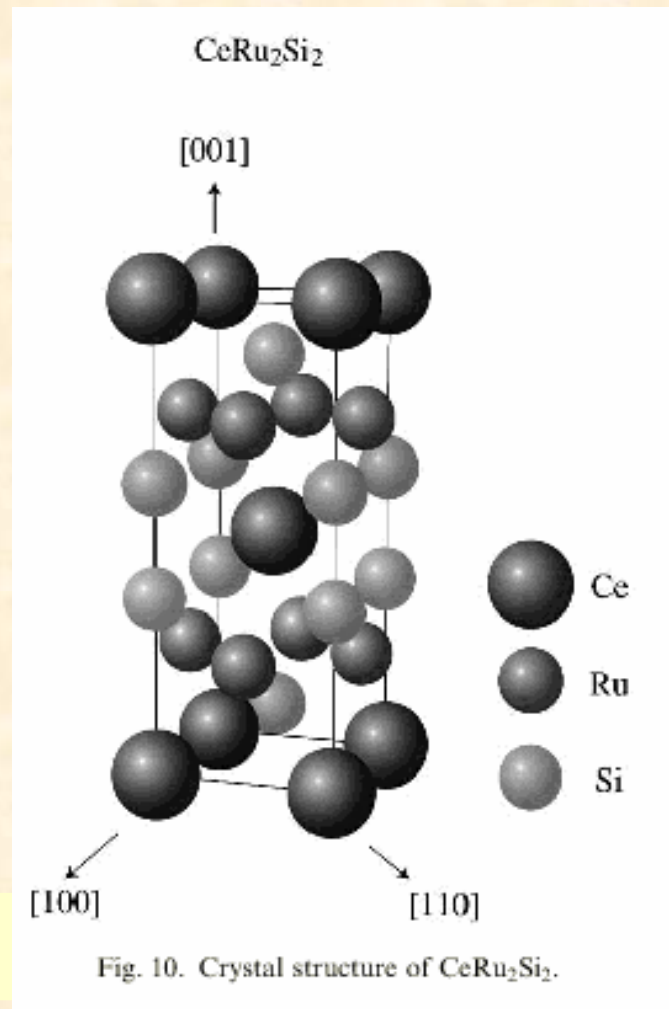


System	$\gamma_0$ ( $J/K^2mol$ )	$T_F$ K	$m^*/m$	$T_c$ K	$T_K$ K
<i>CeCu<sub>6</sub></i>	1.6	300	500	-	5
<i>CeAl<sub>2</sub></i>	0.14	4000	42	-	3.9
<i>CeCu<sub>2</sub>Si<sub>2</sub></i>	1.1	350	450	0.8	10
<i>CeRu<sub>2</sub>Si<sub>2</sub></i>	0.35	1200	100	-	20
<i>UBe<sub>13</sub></i>	0.72	700	260	-	8
<i>UPt<sub>3</sub></i>	0.42	1100	180	5	80
<i>UCd<sub>11</sub></i>	1.42	400	425	5	-
<i>U<sub>2</sub>Zn<sub>17</sub></i>	0.42	1100	180	10	-



# Models: Kondo Lattice

$$H = \sum_k \varepsilon(k) c_{k,\sigma}^+ c_{k,\sigma} + J \sum_i \vec{S}_i \vec{S}_i + \sum_{ij} I_{ij} \vec{S}_i \vec{S}_j$$

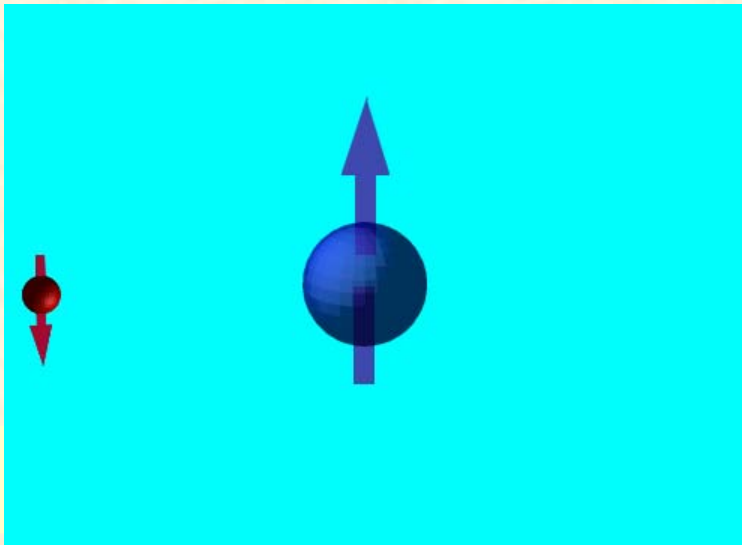
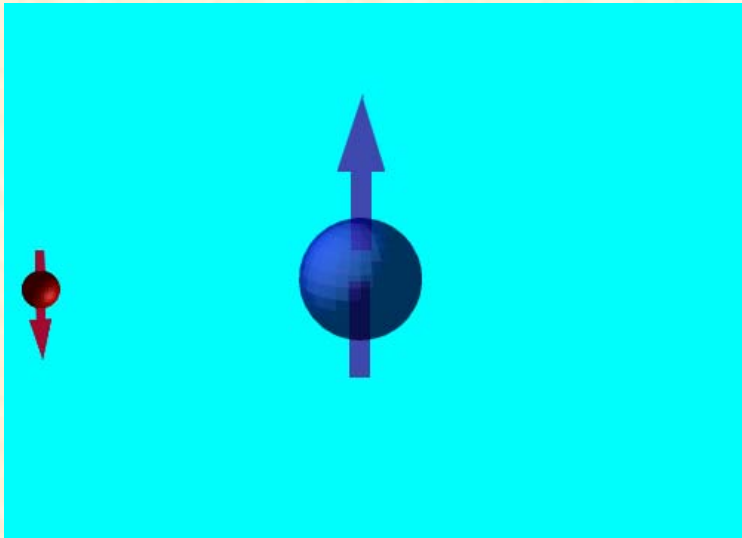
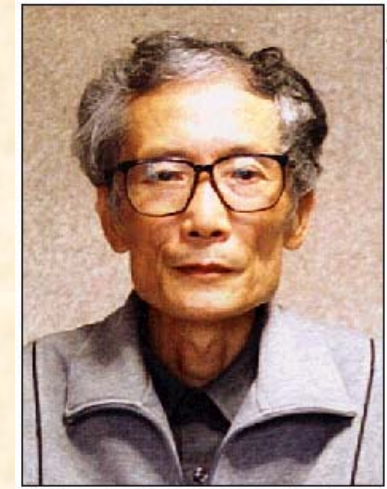


$$H_{Kondo} = J \sum_i \vec{S}_i \vec{S}_i$$

$$H_{RKKY} = \sum_{i,j} I_{ij} \vec{S}_i \vec{S}_j$$



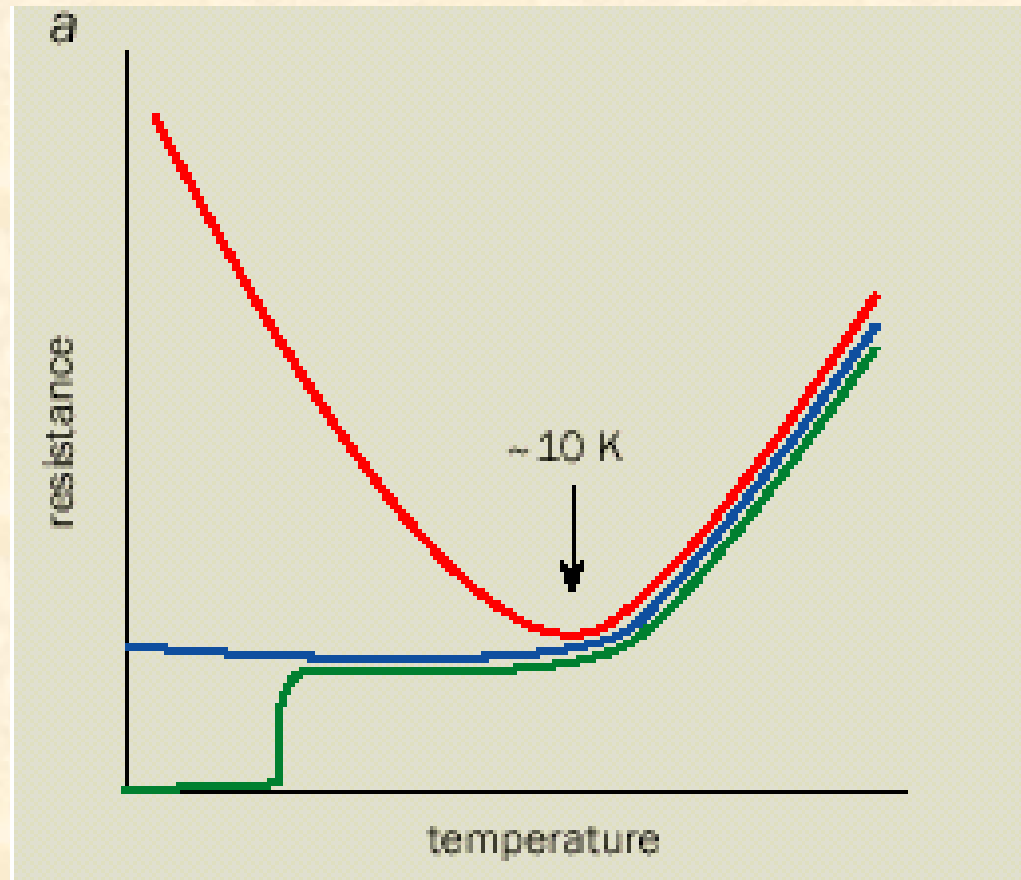
# Kondo-Effect



$$H_{Kondo} = J \sum_i \vec{S}_i \vec{s}_i$$

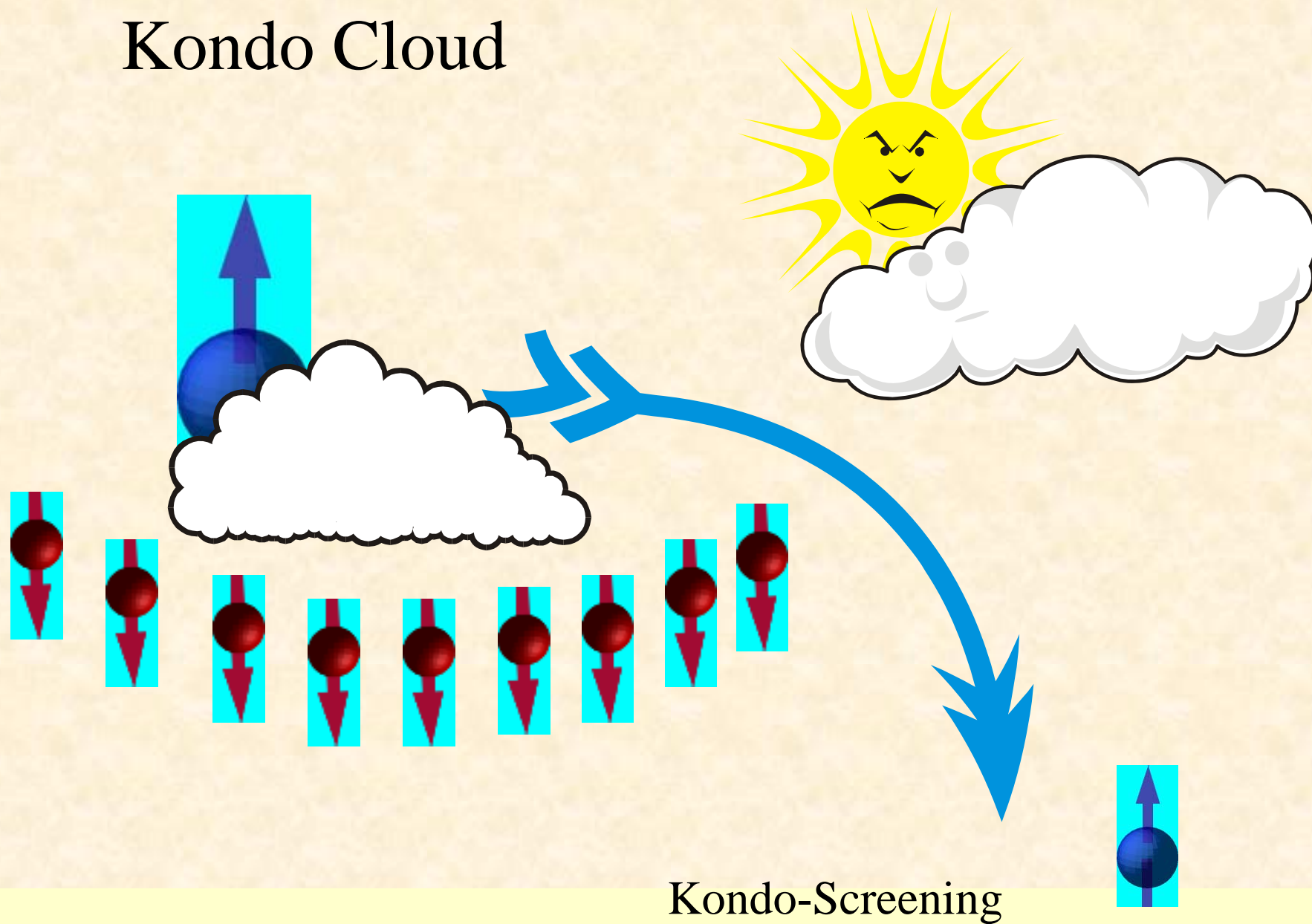
$$\rho = \rho_v + c_m a \ln(\mu/T) + bT^5$$

## Resistance at low temperatures

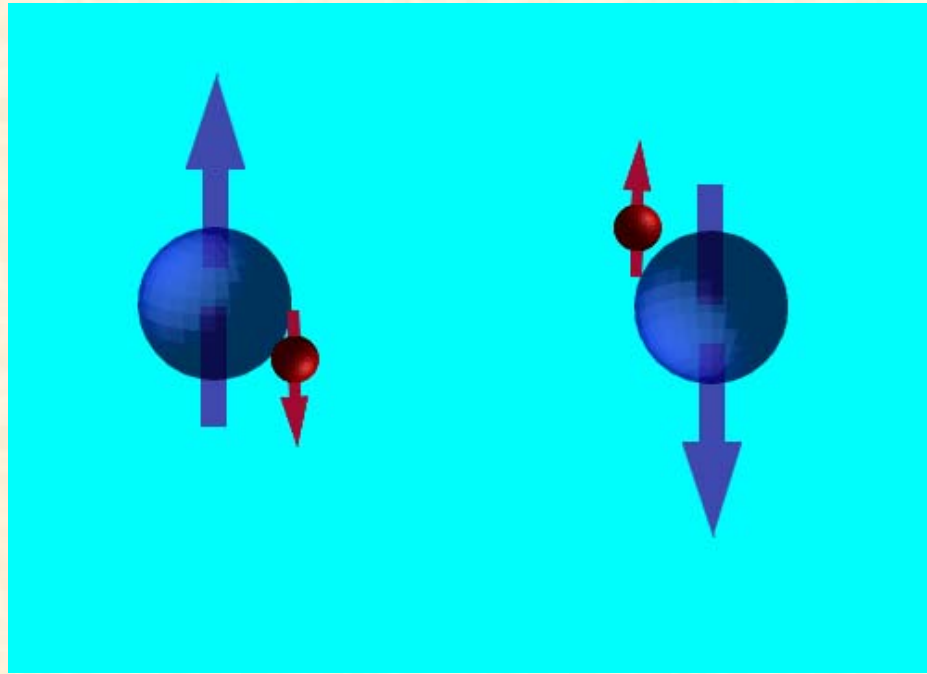


$$\rho = \rho_v + c_m a \ln(\mu/T) + bT^5$$

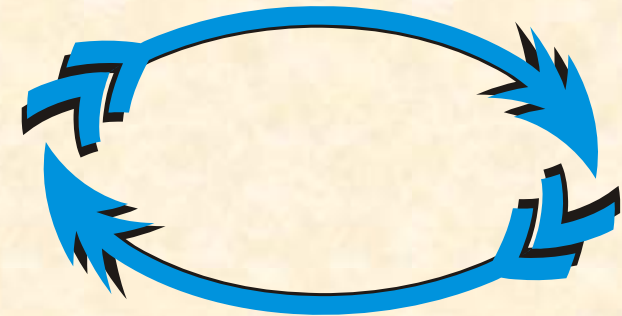
# Kondo Cloud



# Ruderman-Kittel-Kasuya-Yosida (RKKY) Wechselwirkung



$$H_{RKKY} = \sum_{i,j} I_{ij} \vec{S}_i \cdot \vec{S}_j$$

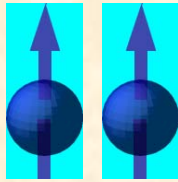


$$I_{RKKY} = 2 \left( \frac{J}{n} \right)^2 \sum_{p,p'} \frac{n_p(1 - n_{p'})}{\epsilon_p - \epsilon_{p'}}$$

$$I_{RKKY}(q) = -\frac{m^* p_F}{4\pi^2 \hbar^3} \left( \frac{J}{n} \right)^2 \left[ 1 + \frac{(2p_F)^2 - q^2}{4p_F q} \ln \left| \frac{2p_F - q}{2p_F + q} \right| \right]$$



$I_{RKKY}$

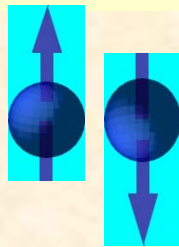


Ferromagnetic Ordering

$$I_{RKKY}(R) = -\frac{J^2}{\epsilon_F} \left[ \frac{\cos(2p_F R/\hbar)}{(2p_F R/\hbar)^3} - \frac{\sin(2p_F R/\hbar)}{(2p_F R/\hbar)^4} \right]$$

$2p_F R$

Antiferromagnetic Ordering



## Antiferromagnetic Ordering

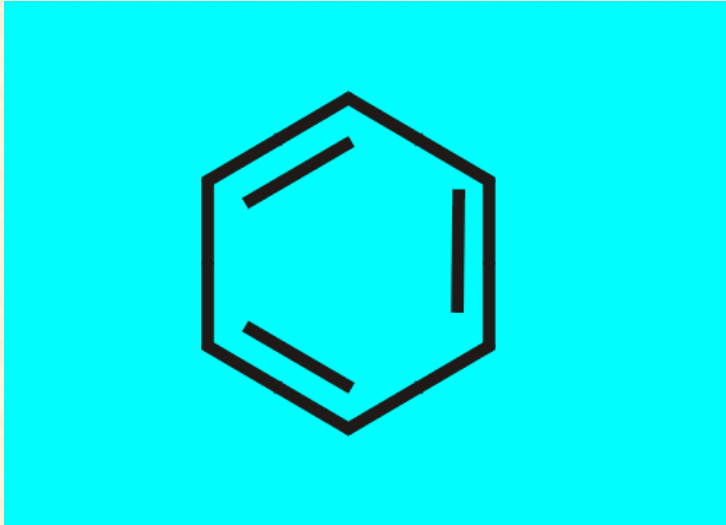
$$T > T_c$$

$$\mathcal{N} = \tanh\left(\frac{I_Q \mathcal{N}}{2T}\right)$$

$$T < T_c$$

$$\mathcal{N} = \tanh\left(\frac{I_Q \mathcal{N}}{2T}\right) \left[ 1 - \frac{a_N}{\ln(T/T_K)} \frac{\cosh^2(\beta I_Q \mathcal{N}/2)}{\cosh^2(\beta I_Q \mathcal{N})} \right]$$

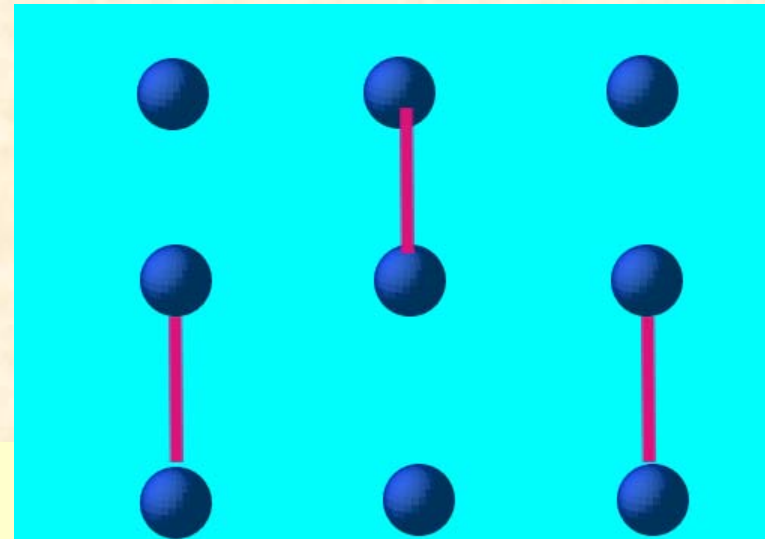
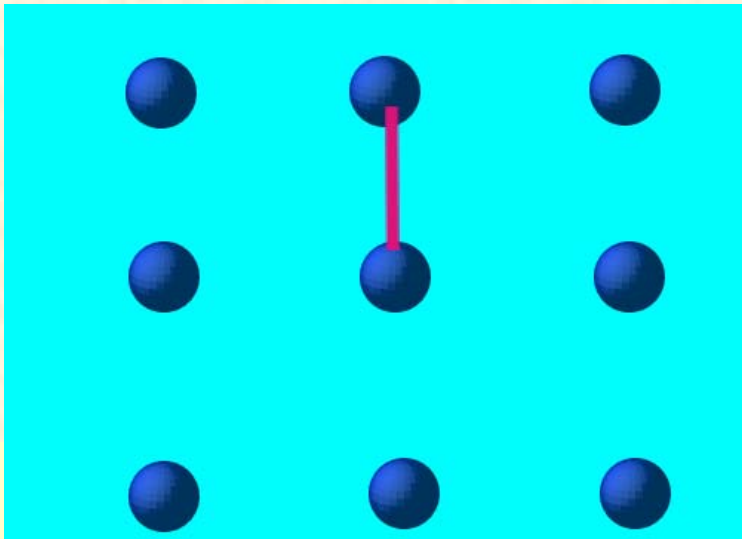
# Spin Liquid



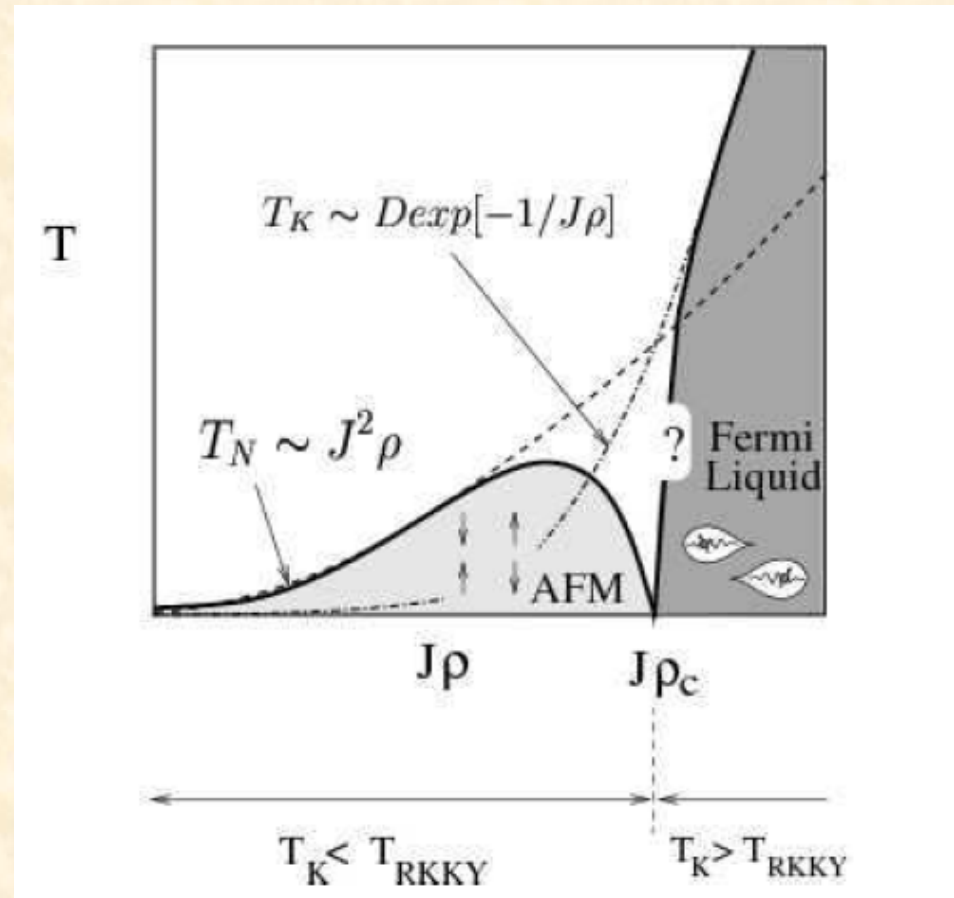
## Resonating Valence Bonds

$$\Delta = - \sum_{\mathbf{q}} \nu(\mathbf{q}) \tanh \left( \frac{I_{\mathbf{q}} \Delta}{T} \right)$$

$$\Delta = - \sum_{\mathbf{q}} \nu(\mathbf{q}) \left[ \tanh \left( \frac{I(\mathbf{q}) \Delta}{T} \right) + a_{sl} \frac{I_{\mathbf{q}} \Delta}{T \ln(T/T_K)} \right]$$



# Heavy Fermions: Phase Diagram

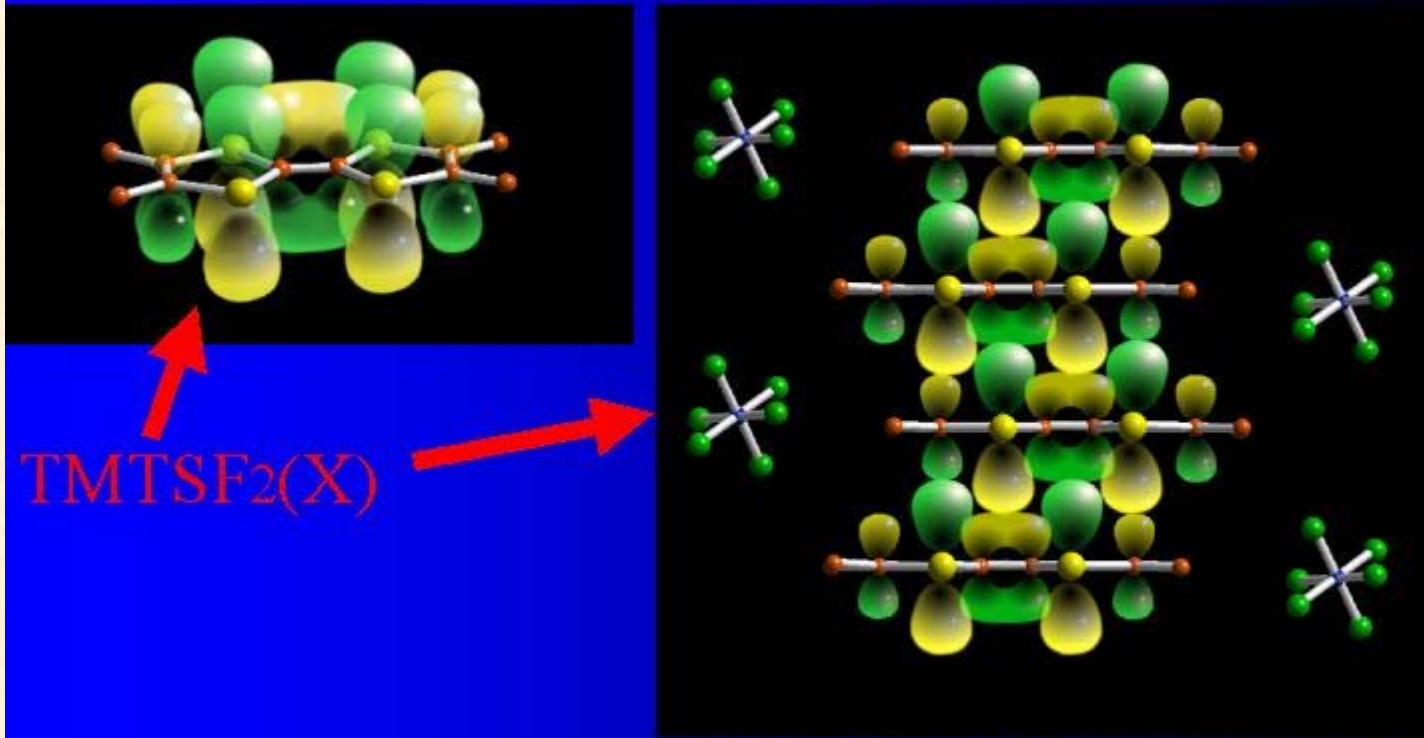


# **Strongly Correlated Systems: Organic Conductors**



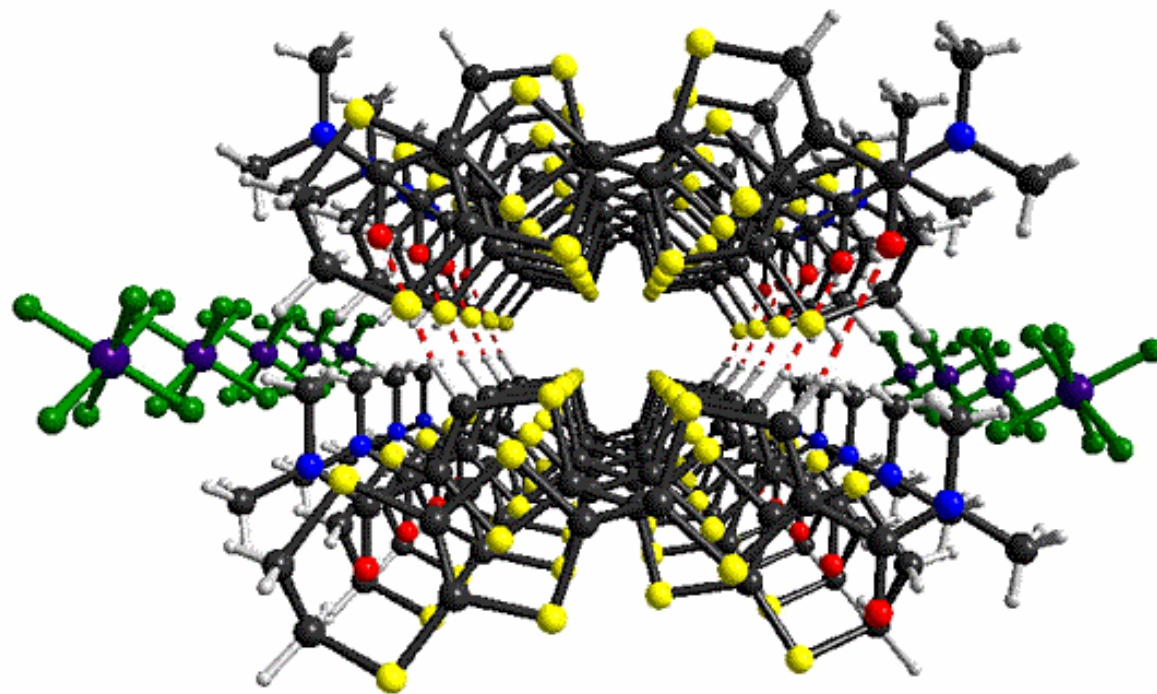
# Bechgaard salts

## Organic (super-) conductors



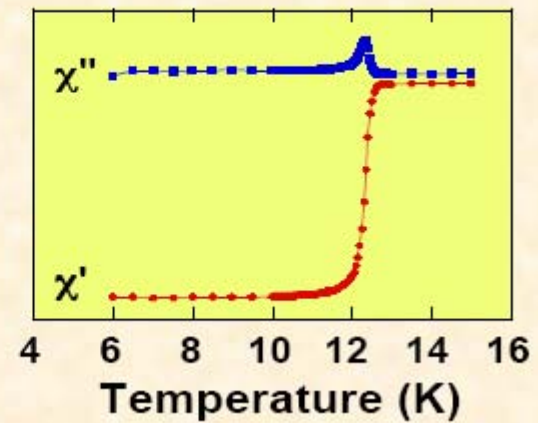
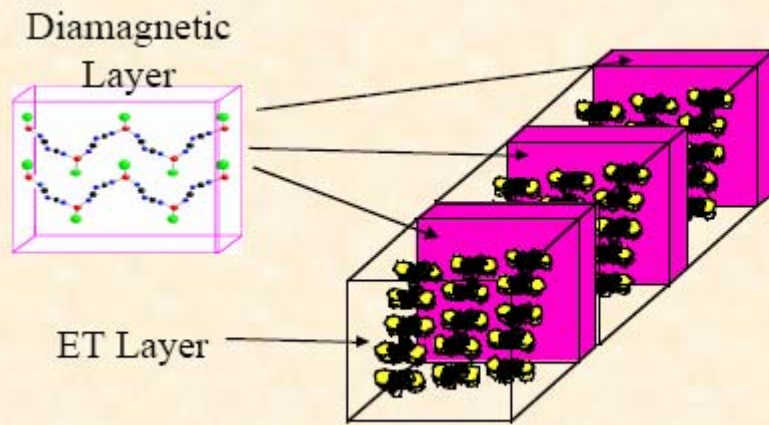
TMTSF<sub>2</sub>(X)

- Propagation of electrons along the chains



A perspective down the criss-cross stacks in [EDT-TTF-CONMe<sub>2</sub>]<sub>2</sub>AsF<sub>6</sub>

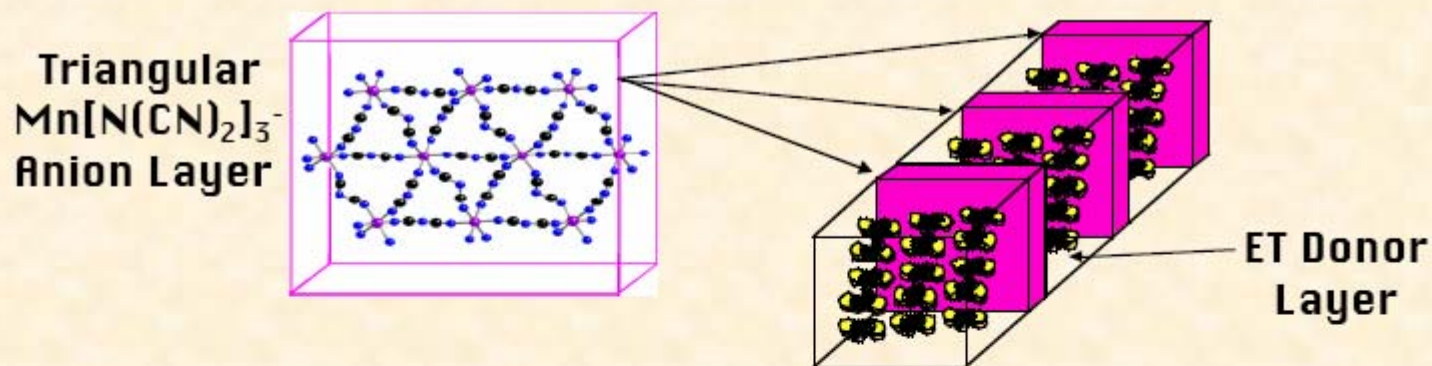
# Molecular Superconductors



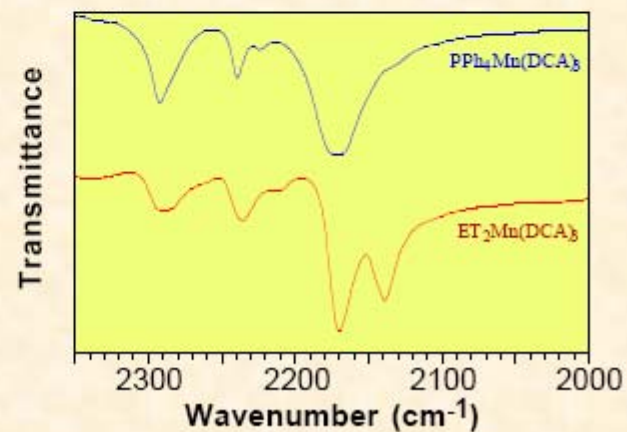
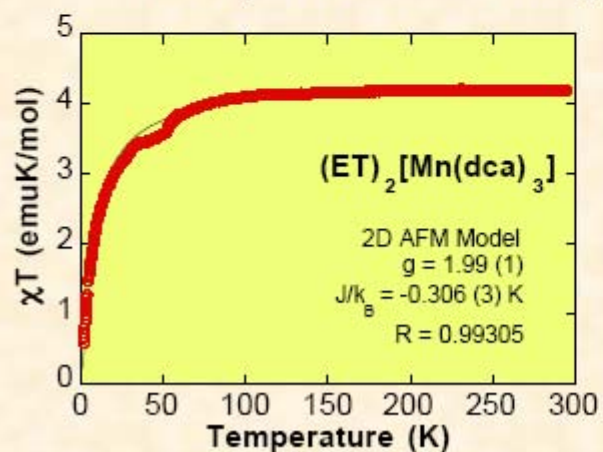
**ET=BEDT-TTF**

Bis(EthyleneDiThio) TetraThiaFulvalene

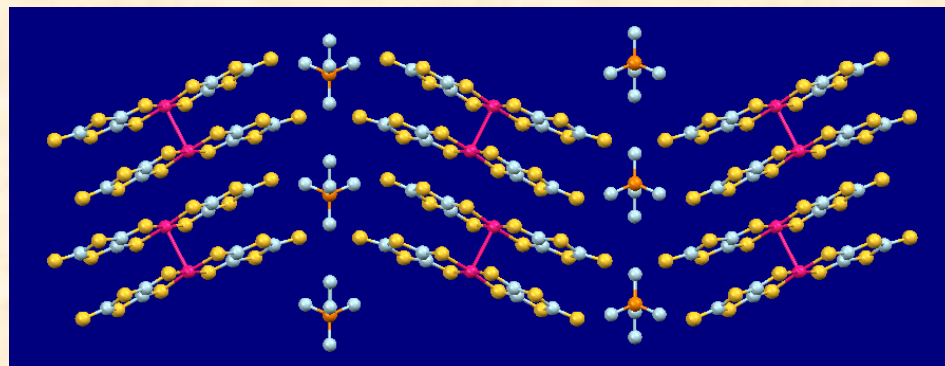
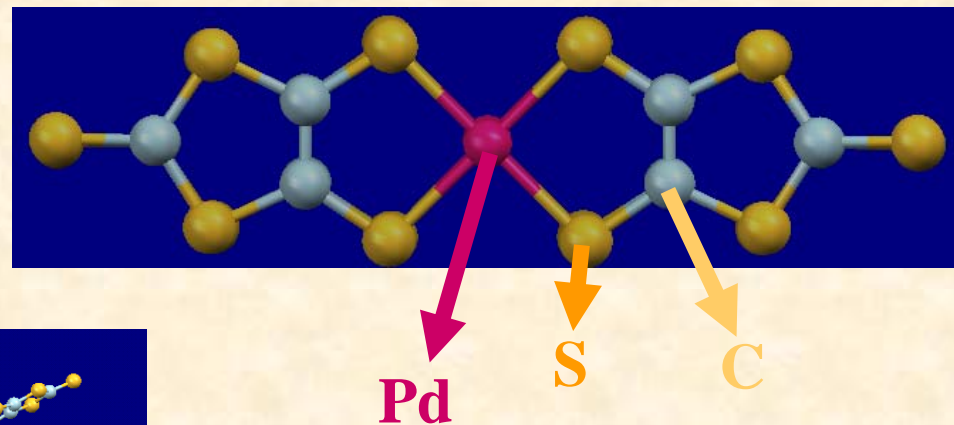
# Conducting/Magnetic Hybrid Molecular Solids



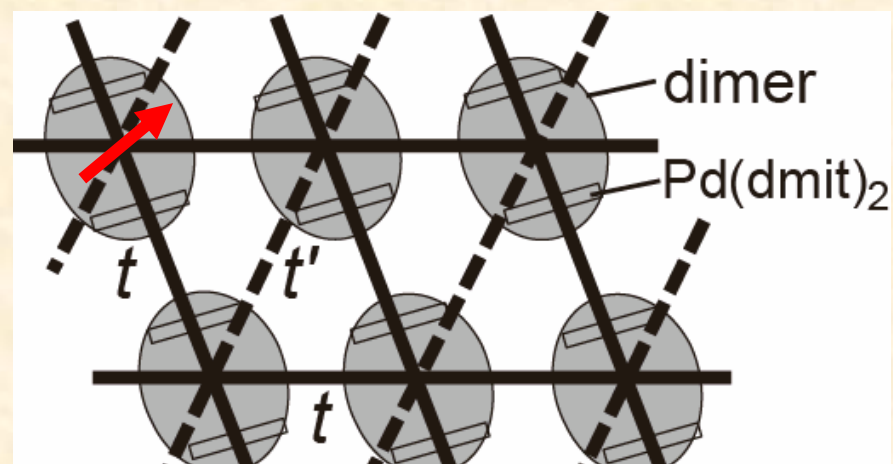
Incorporation of magnetic layer between ET donor layers





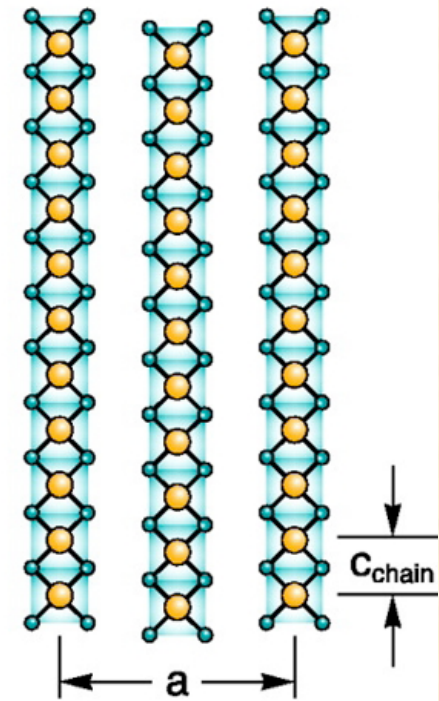
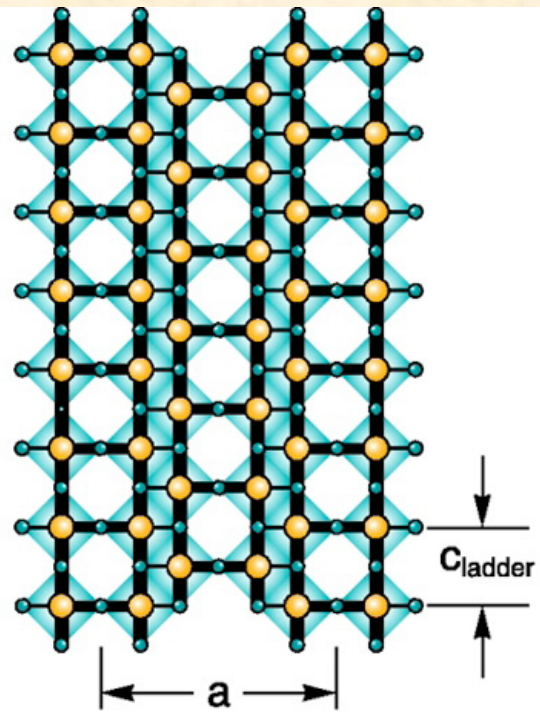
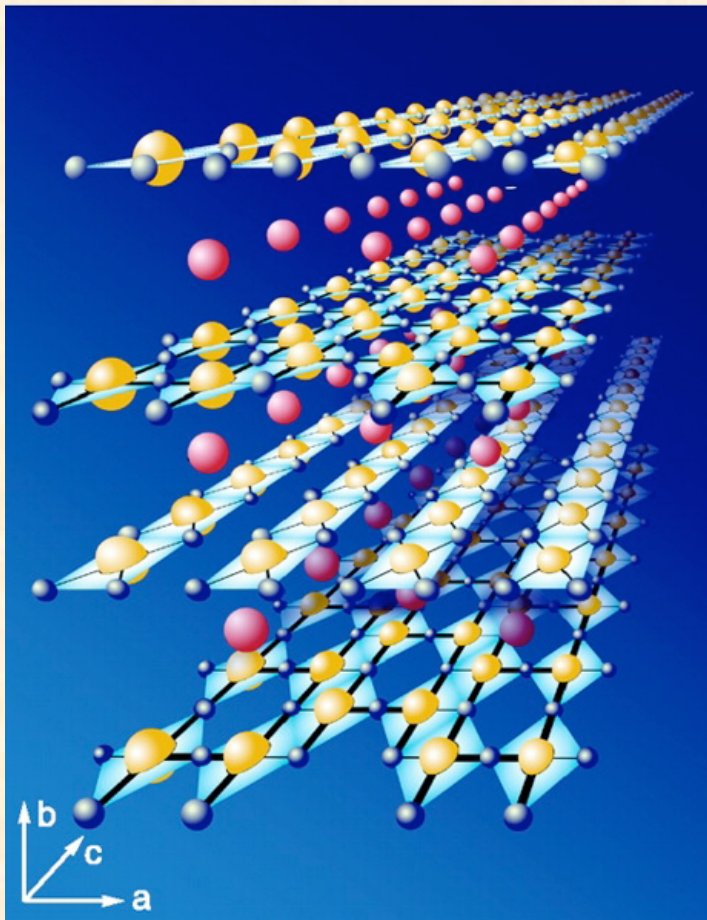


One free electron  
spin on each  
vertex of a  
triangular lattice

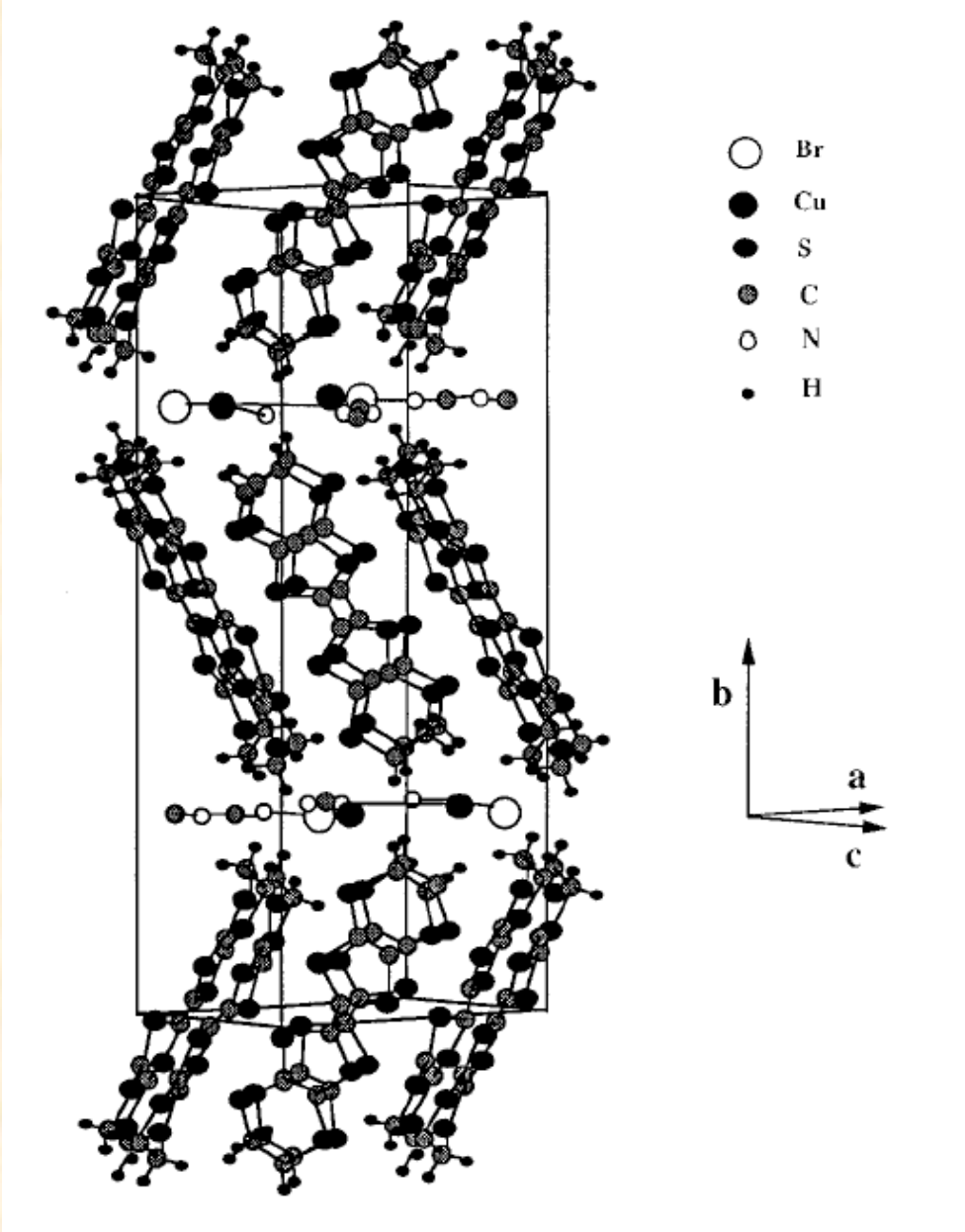




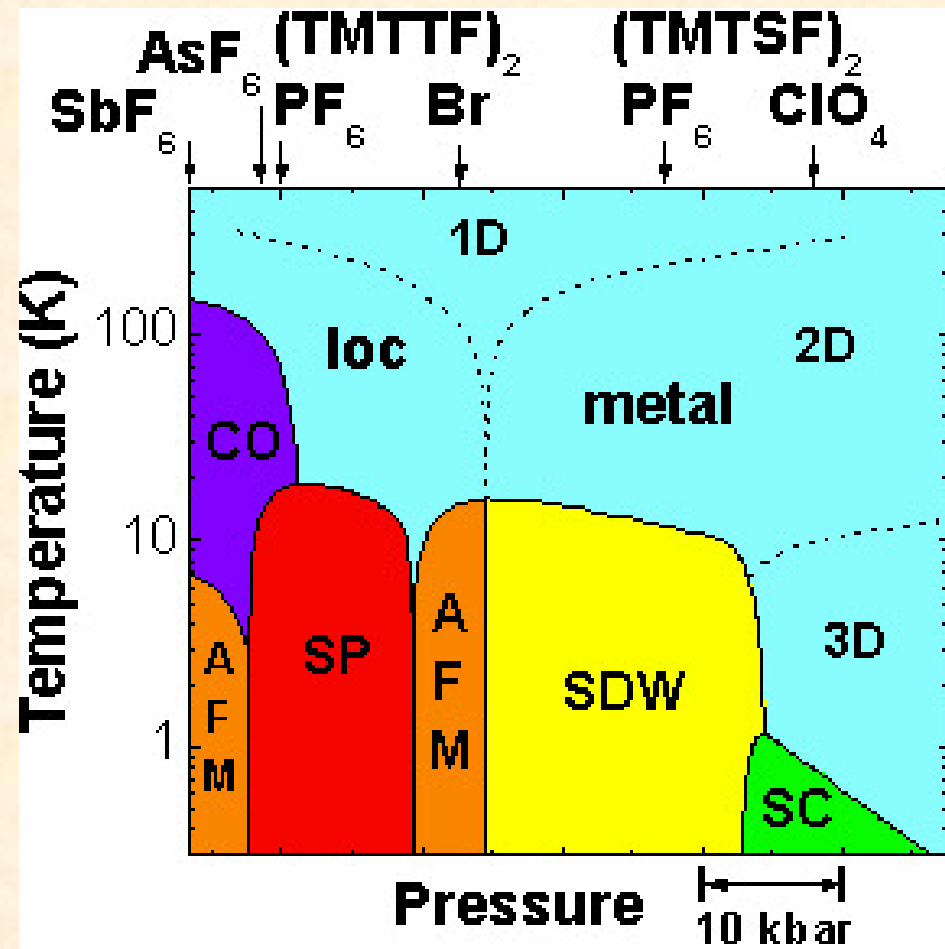
# Spin Ladders



- O
- Cu
- Sr



# Phase Diagram



**and Nanostructures ...**