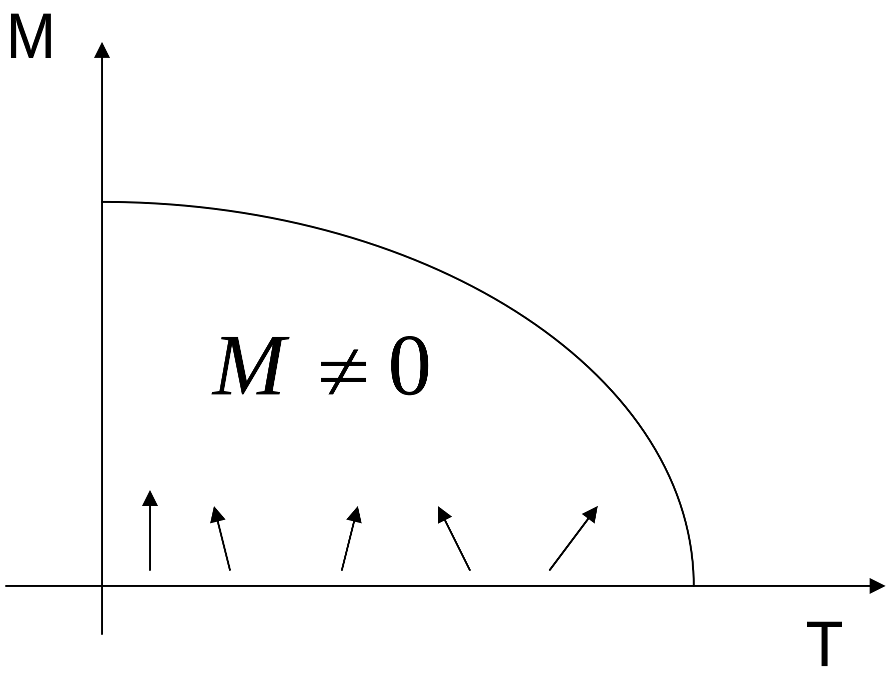
Quantum Phase Transitions

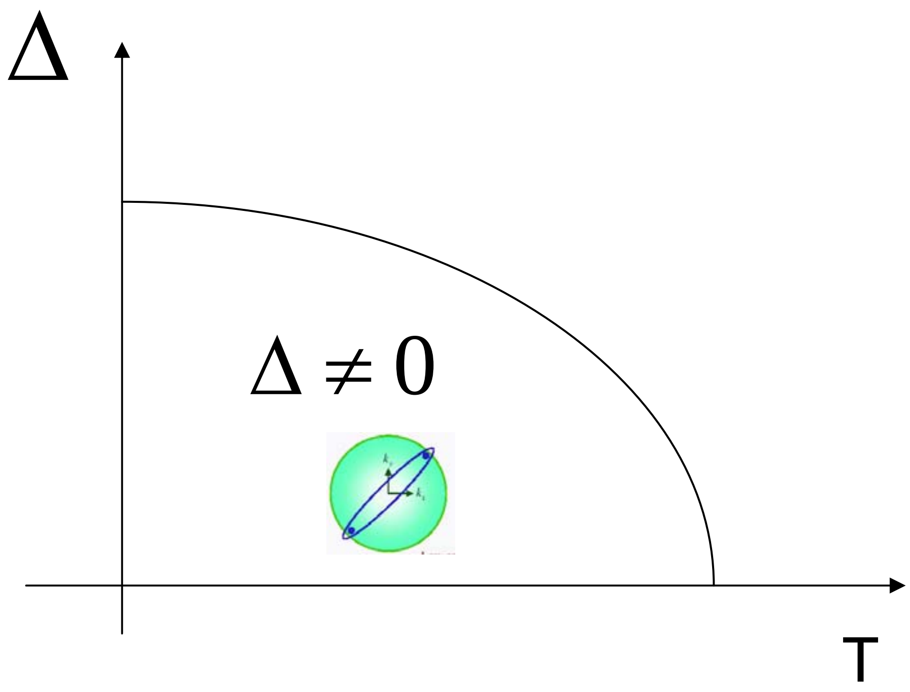
M.N.Kiselev

Ordinary phase transitions

Ferromagnets

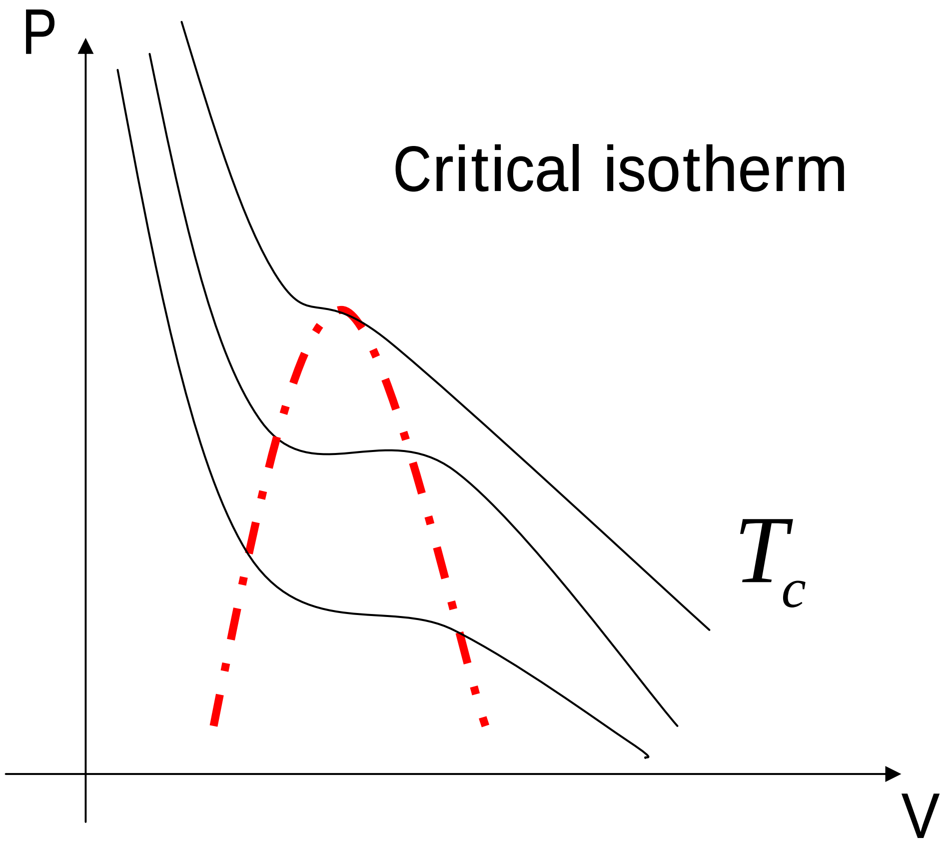


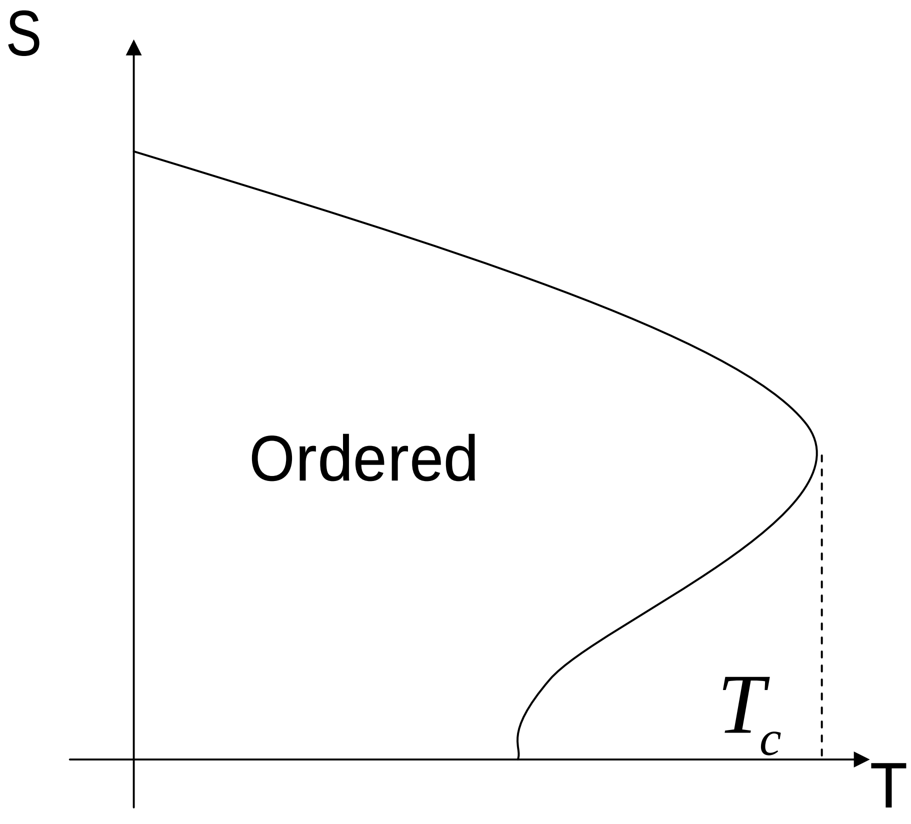
Superconductors



Classical Phase TRansitions

I^{st} order Phase Transition



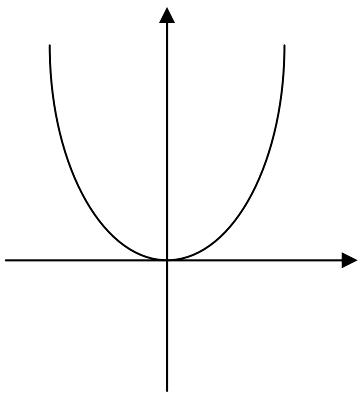


Landau theory of II order phase transitions

$$\Phi[\varphi] = \Phi_0 + \Phi_1\varphi + \Phi_2\varphi^2 + \Phi_3\varphi^3 + \Phi_4\varphi^4 + \dots$$

$$\Phi_1 = 0 \qquad \Phi_3 = 0$$

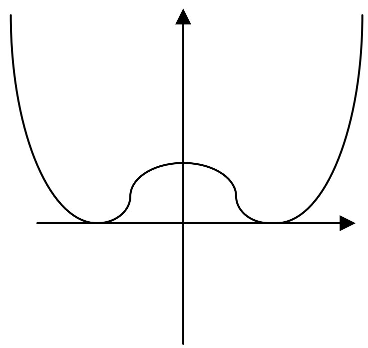
$T > T_c$



$$\Phi_2 = a(T - T_c)$$

$$\Phi_4 = const > 0$$

$T < T_c$



Critical exponents

	exp	def	conditions
Specific heat	α	$\|\tau\|^{-\alpha}$	$\tau \to 0, B = 0$
Order parameter	β	$\|\tau\|^{-\beta}$	$\tau \to 0^-, B = 0$
Susceptibility	γ	$\|\tau\|^{-\gamma}$	$\tau \to 0, B = 0$
Critical isotherm	δ	$\|m\|^{-1/\delta}$	$\tau = 0, B \to 0$
Correlation length	ν	$\|\tau\|^{-\nu}$	$\tau \to 0, B = 0$
Correlation function	η	$\|r\|^{-d+2-\eta}$	$\tau = 0, B = 0$
Dynamics	z	$\|\xi\|^{z}$	$\tau \to 0, B = 0$

Landau theory of II order phase transitions

Scaling Equations

$$\alpha + 2\beta + \gamma = 2 \qquad \beta + \gamma = \beta\delta$$

$$(2-\eta)\nu = \gamma \qquad 2 - \alpha = d\nu$$

	α	β	γ	δ	ν	η
Landau	0	1/2	1	3	1/2	0
Scaling	0	1/3	4/3	5	2/3	0

What is a quantum phase transition?

Hamiltonian $H = H(g)$

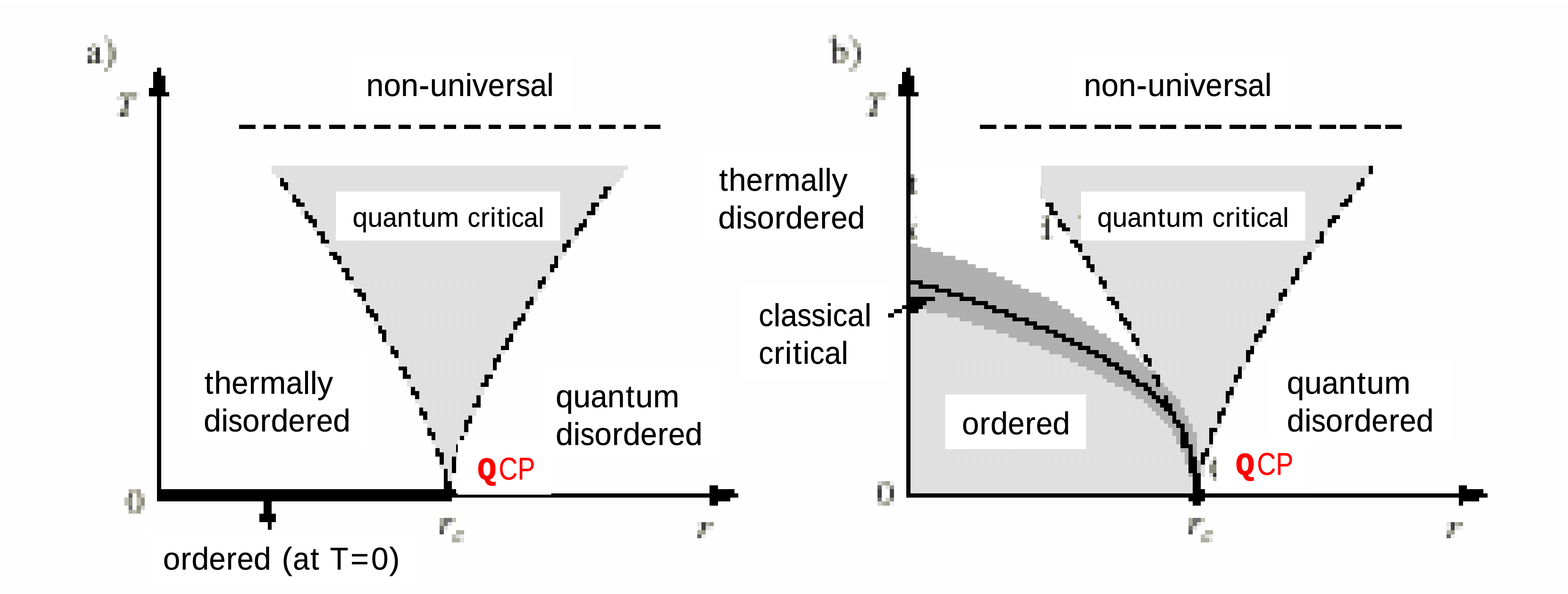
Gap $\Delta = J\,|g - g_c|^{z\nu}$

Correlation length $\xi = |g - g_c|^{\nu}$

z - dynamic critical exponent

Quantum vs classical phase transition

Two possible phase diagrams



Quantum fluctuations driven by the Heisenberg uncertainty principle

One simple example of quantum phase transition:

1D Ising chain in a transverse magnetic field

$$H = -J\sum_i (g\sigma_i^x + \sigma_i^z\sigma_{i+1}^z) = -Jg\sum_i (\sigma_i^x + \frac{1}{g}\sigma_i^z\sigma_{i+1}^z)$$

a) Weak coupling $g \gg 1$

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right) \qquad |\leftarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle - |\downarrow\rangle\right)$$

Ground State $G = |\rightarrow \ldots \rightarrow\rightarrow \ldots \rightarrow\rangle - \frac{1}{2g}|\rightarrow \ldots \leftarrow\leftarrow \ldots \rightarrow\rangle$

Lowest excited state $L = |\rightarrow \ldots \rightarrow \ldots \leftarrow \ldots \rightarrow\rangle$

flipped spin

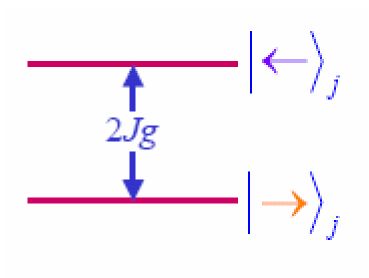
Weak coupling $g \gg 1$

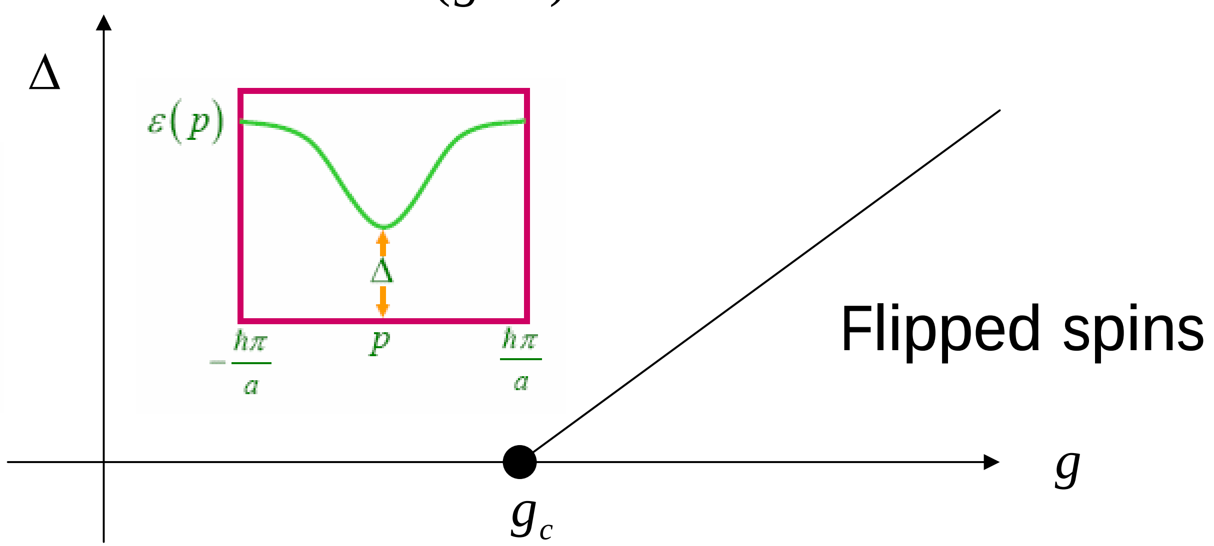
$$|k\rangle = \frac{1}{\sqrt{N}} \sum_j e^{ikx_j} |j\rangle$$

Excitation spectrum $\varepsilon(k) = \langle k | H(g) | k \rangle = 2Jg - 2J\cos(ka)$

Gap

$$\Delta = 2J(g-1)$$





Strong coupling $1/g \gg 1$

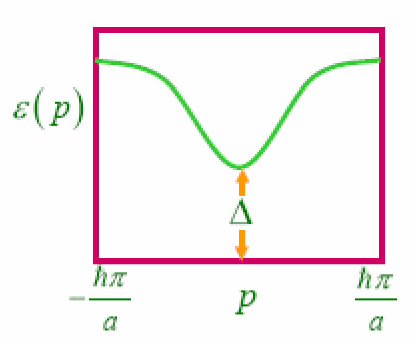
$$G_{\uparrow} = |\uparrow\uparrow \ldots \uparrow \ldots \uparrow\rangle$$

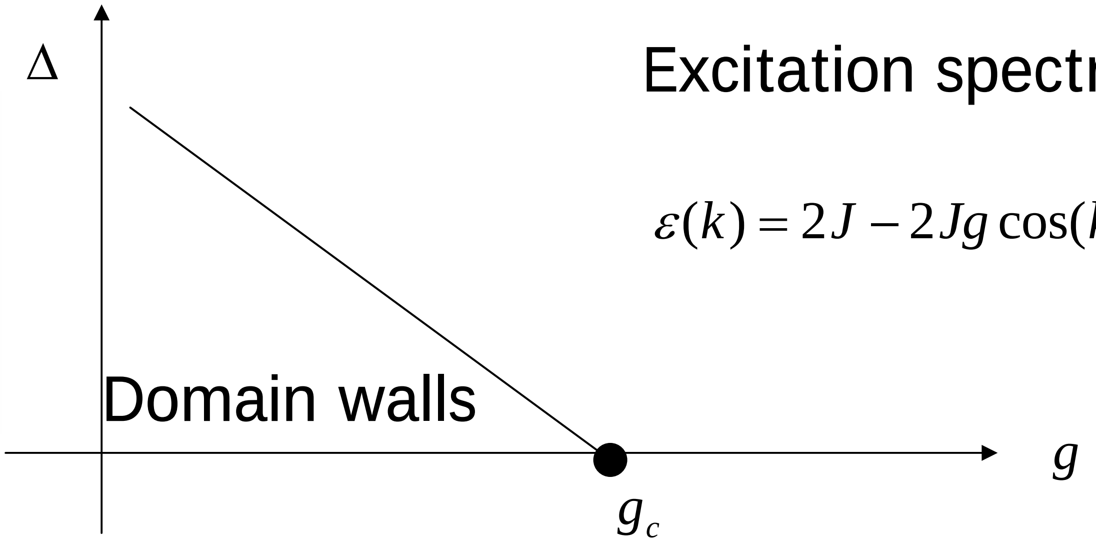
Two-fold degenerate ground state

$$G_{\downarrow} = |\downarrow\downarrow \ldots \downarrow \ldots \downarrow\rangle$$

Excitations-domain walls

$$|d\rangle = |\uparrow \ldots \uparrow\downarrow \ldots \downarrow\rangle$$

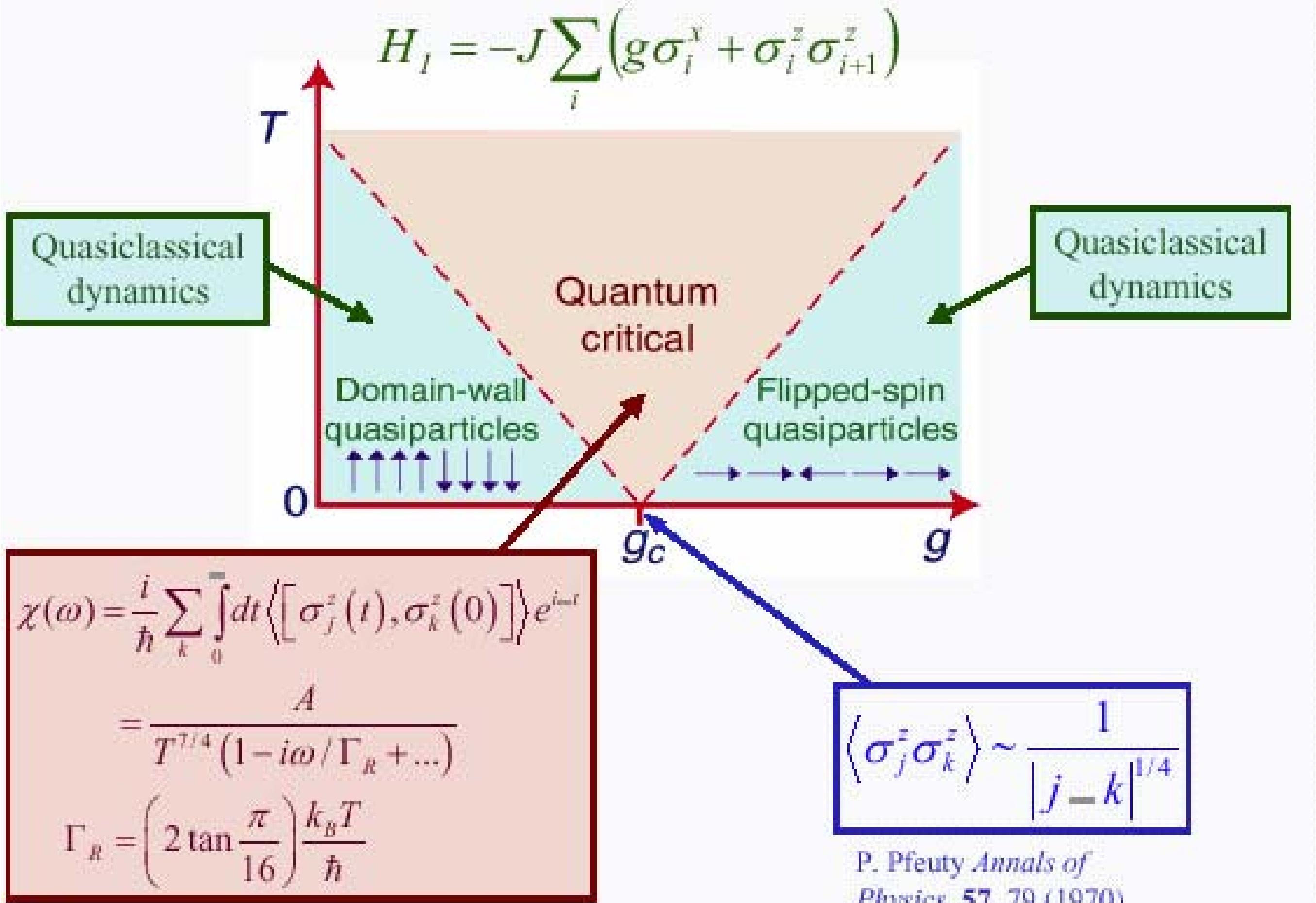




Excitation spectrum

$$\varepsilon(k) = 2J - 2Jg\cos(ka)$$

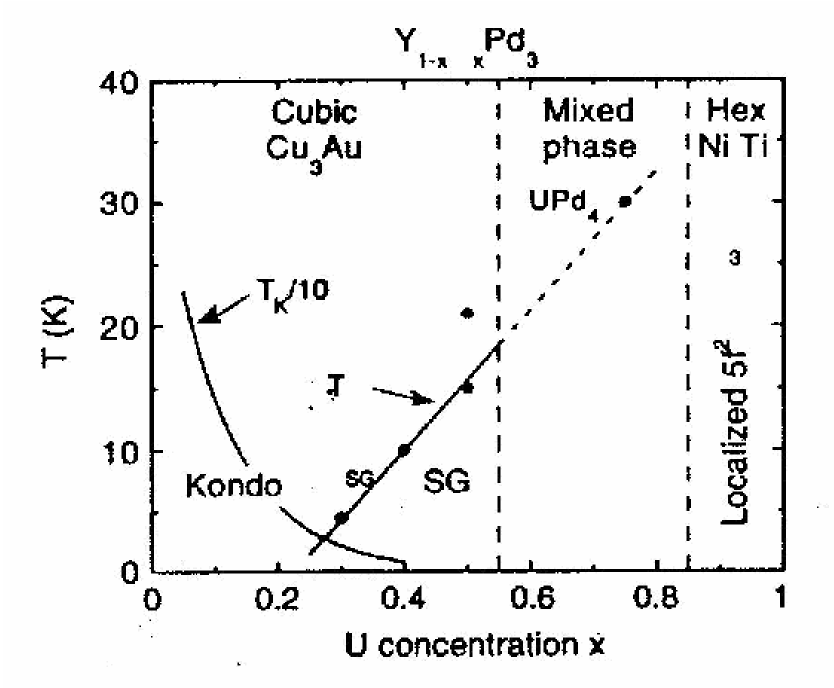
Exact solution of the problem

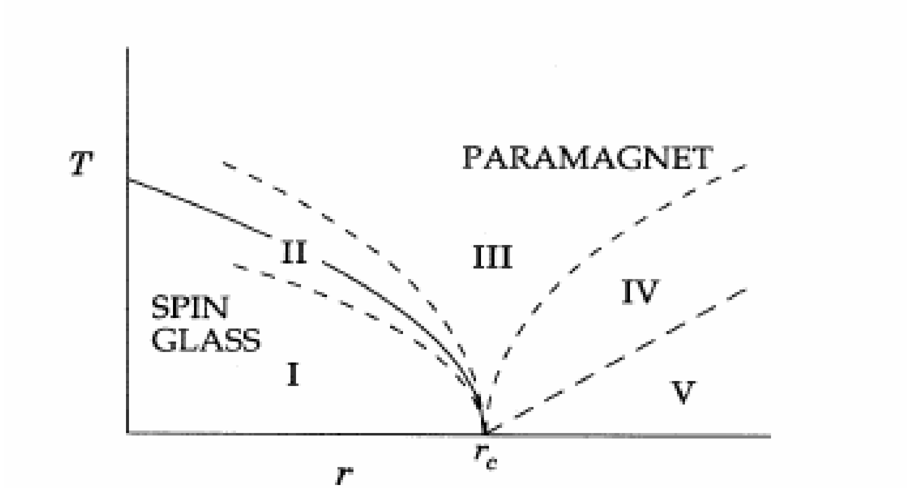


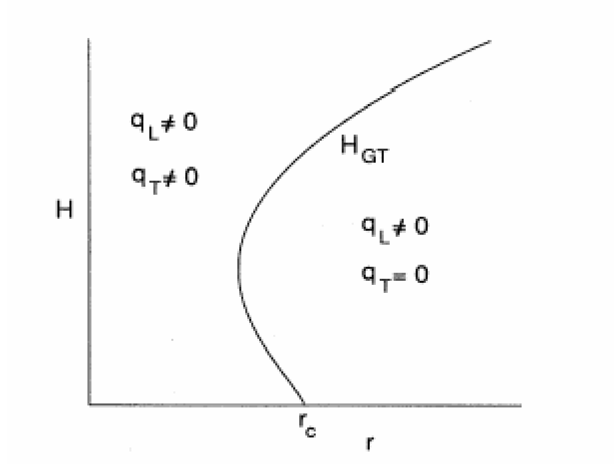
P. Pfeuty *Annals of Physics*, **57**, 79 (1970)

S. Sachdev and J. Ye, Phys. Rev. Lett. **69**, 2411 (1992).
S. Sachdev and A.P. Young, Phys. Rev. Lett. **78**, 2220 (1997).

Application of QPT: Spin Glasses

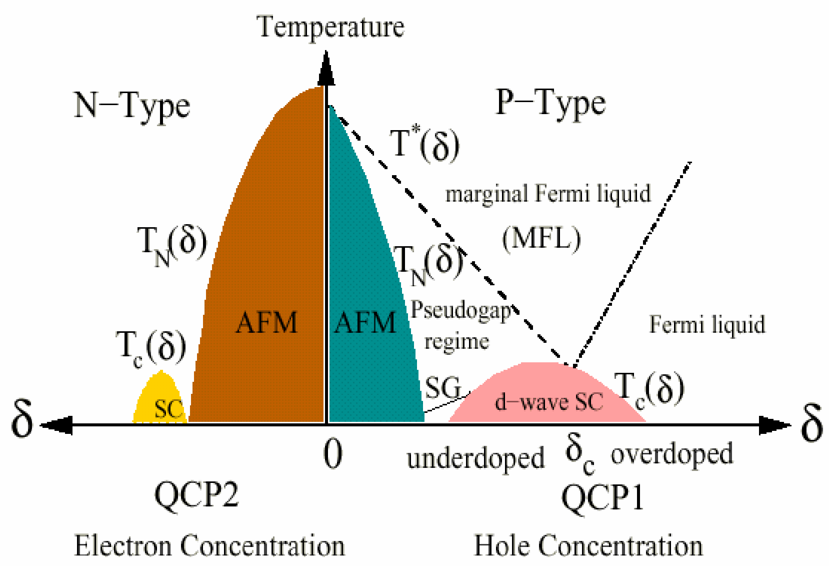




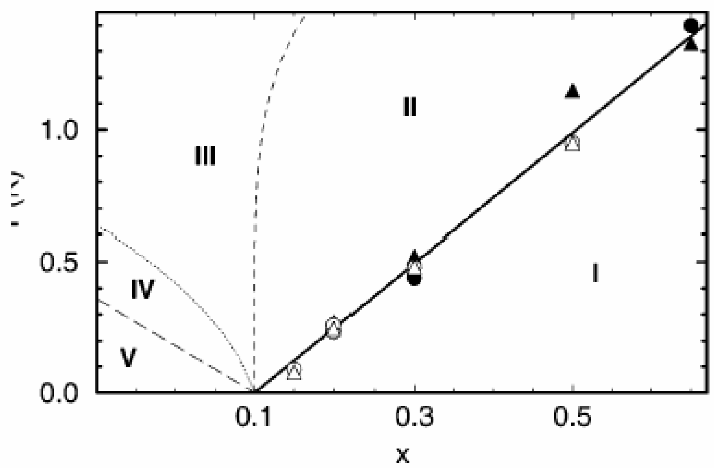


Application of QPT: heavy fermions and HTSC

High-temperature superconductors



Heavy-fermions



$CeCu_{6-x}Au_x$