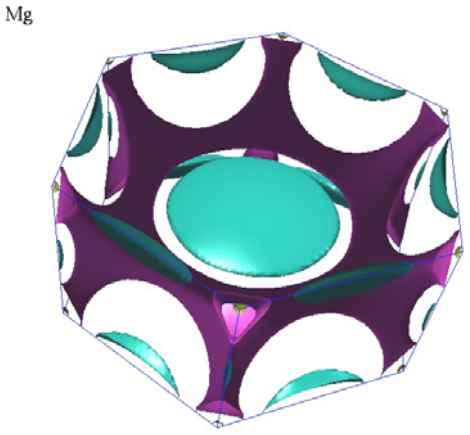
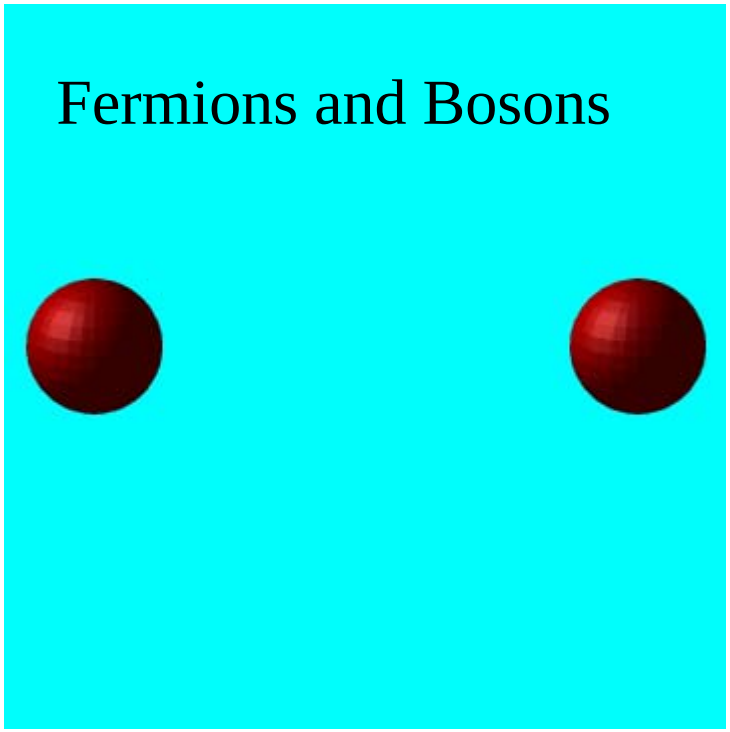
Fermions, Statistics and Fermi Surfaces

M.N.Kiselev



Fermions and Bosons



$$\Psi(\xi_1, \xi_2) = e^{i\alpha}\Psi(\xi_2, \xi_1)$$

$\alpha = 0$

Bosons

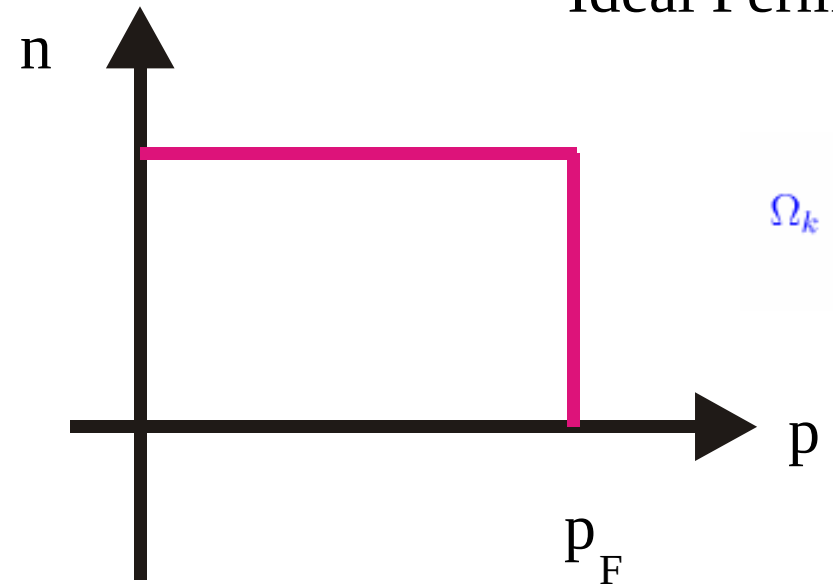
$\alpha = \pi$

Fermions

$$\Psi = \frac{1}{\sqrt{N! n_{p_1}! \ldots}} \sum P \psi_{p_1}(\xi_1)\psi_{p_2}(\xi_2)\ldots\psi_{p_N}(\xi_N)$$

$$\Psi = \frac{1}{\sqrt{N!}} \begin{vmatrix} \psi_{p_1}(\xi_1) & \psi_{p_1}(\xi_2) & \ldots & \psi_{p_1}(\xi_N) \\ \psi_{p_2}(\xi_1) & \psi_{p_2}(\xi_2) & \ldots & \psi_{p_2}(\xi_N) \\ \ldots & \ldots & \ldots & \ldots \\ \psi_{p_N}(\xi_1) & \psi_{p_N}(\xi_2) & \ldots & \psi_{p_N}(\xi_N) \end{vmatrix}$$

Ideal Fermi - gas



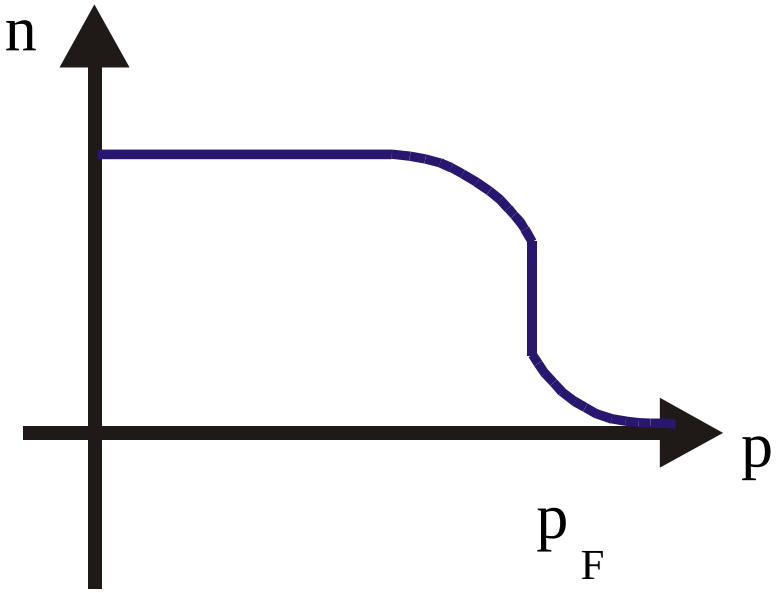
$$\Omega_k = -T \ln\left(\sum_k \left(e^{\frac{\mu-\epsilon_k}{T}}\right)^{n_k}\right) = -T \ln\left(1 + e^{\frac{\mu-\epsilon_k}{T}}\right)$$

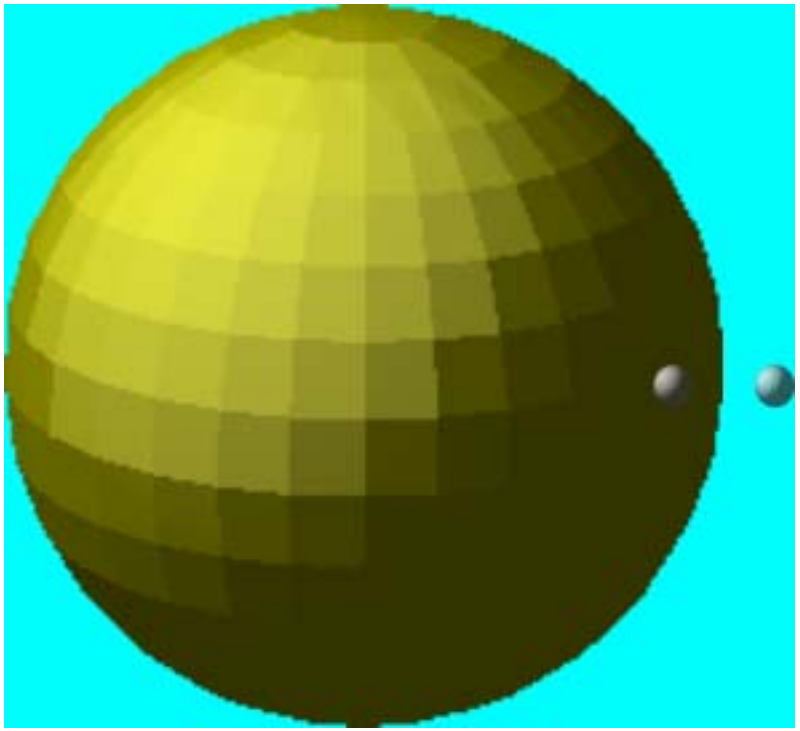
$$n_k = 0, 1$$

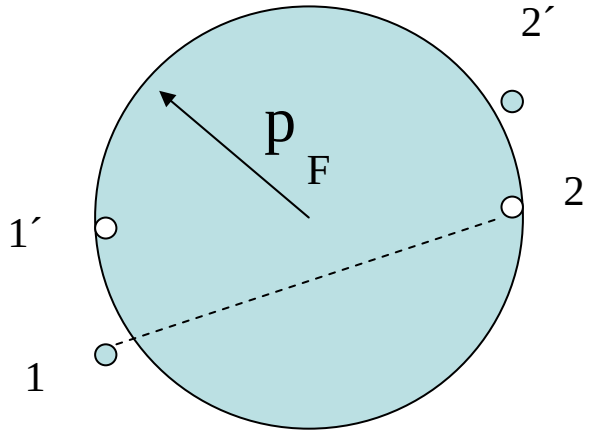
$$n(\epsilon_k) = \frac{1}{e^{\frac{\epsilon_k - \mu}{T}} + 1}$$

$$N = \int 2\frac{4\pi p^2 dp V}{(2\pi\hbar)^3} = \frac{V p_F^3}{(3\pi^2\hbar^3)}$$

Fermi - liquid theory

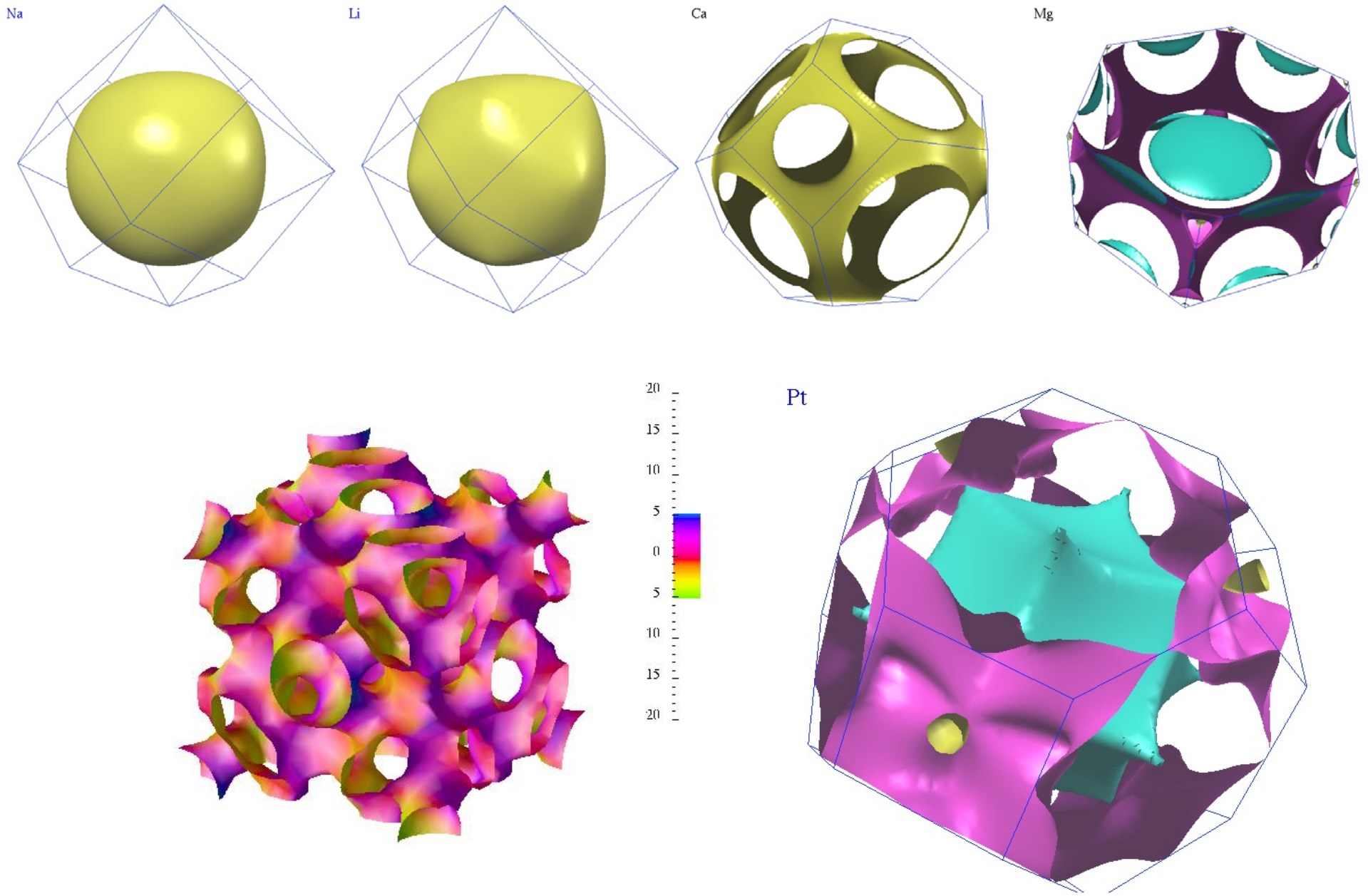


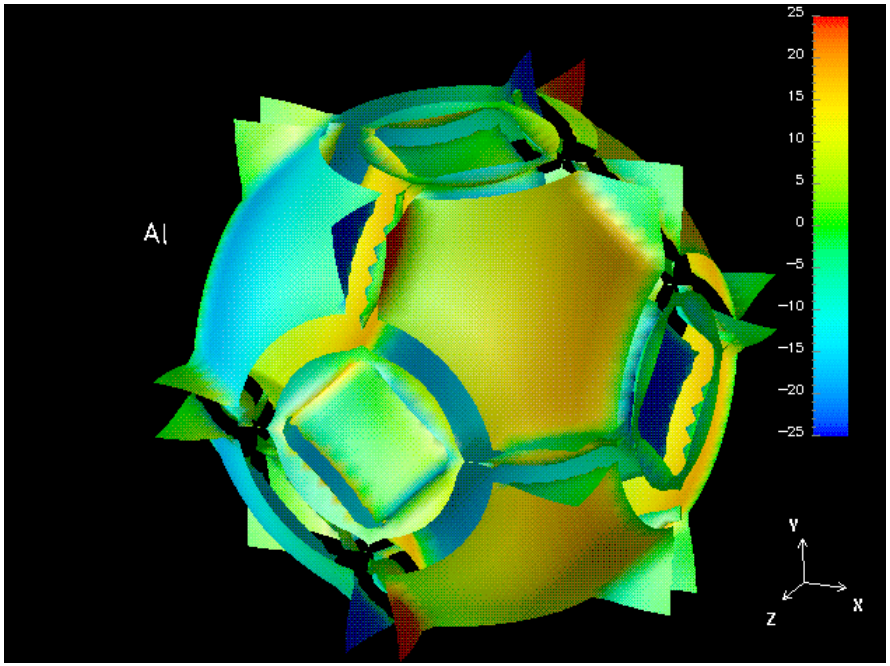


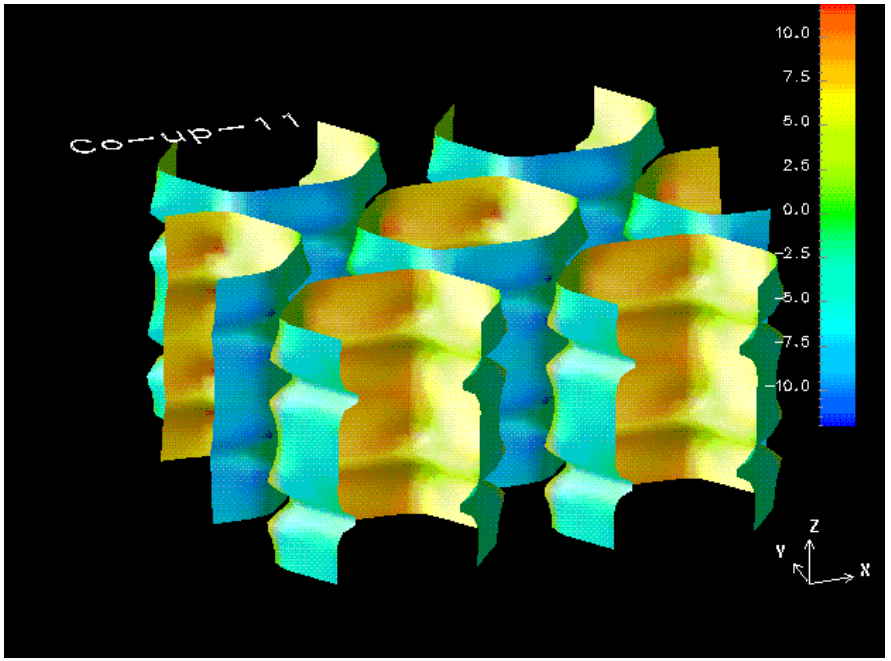


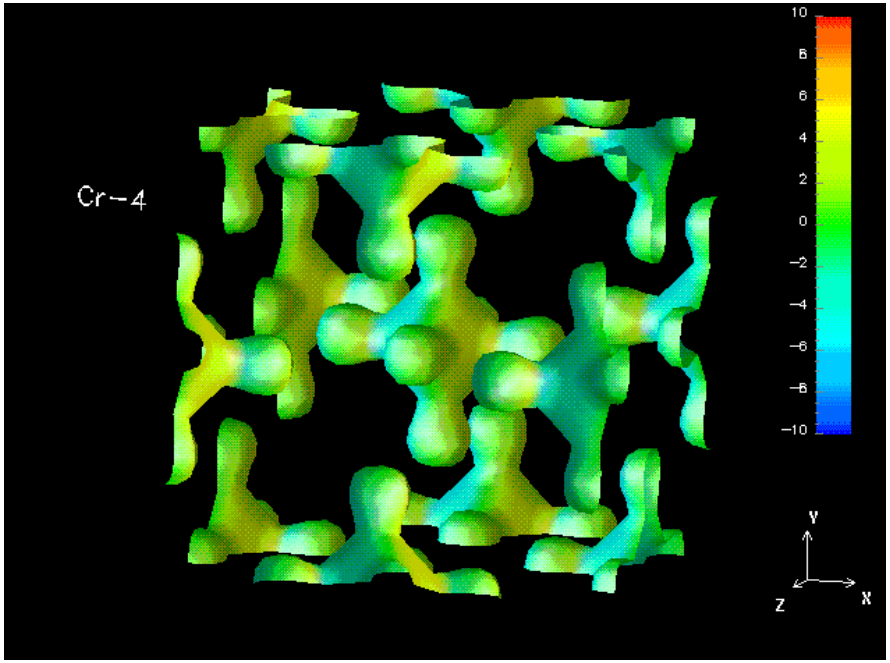
Luttinger theorem

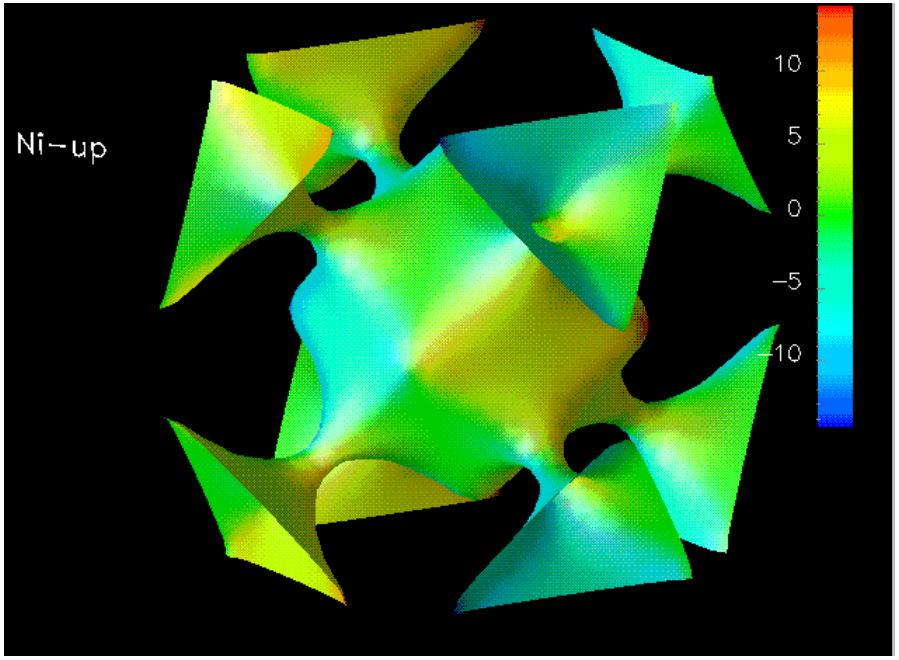
$$\frac{N_e}{V} = 2q\frac{N}{V} + 2\frac{V_F}{(2\pi\hbar)^3}$$


Na
Li
Ca
Mg
20
15
10
5
0
5
10
15
20
Pt


Al
25
20
15
10
5
0
−5
−10
−15
−20
−25
Y
Z
X


Co−up−11
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
Z
Y
X


Cr−4
10
8
6
4
2
0
−2
−4
−6
−8
−10
Y
Z
X


Ni−up
10
5
0
−5
−10