# GR HW 6

Due Jan 9 at 11am, in class

*Goals: Coordinate systems, Curvature, Geodesic deviation*

1. Calculate components of $d$-dimensional Euclidean space metric in spherical coordinates $r, \theta_1, \cdots, \theta_{d-2}, \phi$. Calculate the corresponding integration measure $\sqrt{g} d^d x$. (Hint: first do it for $d = 3$.)

2. Consider a two-dimensional sphere of radius $R$ embedded in the Euclidean space,

$$x^2 + y^2 + z^2 = R^2. \tag{1}$$

As we discussed, one can construct a coordinate patch $(x, y)$ covering all of the sphere, but the north pole, by considering a stereographic projection on the plane

$$z = -R. \tag{2}$$

Calculate components of the induced metric on the sphere in stereographic coordinates.

3. Trieste latitude is 45°. A vector is applied in Trieste, pointing towards the north pole, and then is parallel transported along the 45° latitude, moving west, back to Trieste. Which way will the vector turn, and by what angle?

4. Calculate the extrinsic curvature (the proper acceleration $\sqrt{a_\mu a^\mu}$) of lines of constant $\phi$ and lines of constant $\theta$ in $S^2$, and of lines of constant $r$ and lines of constant $\phi$ in $\mathbb{R}_2$.

5. Lobachevski plane is defined by the metric

$$ds^2 = \frac{dx^2 + dy^2}{x^2}, \quad x > 0, \tag{3}$$

a) Find the geodesics by direct minimization of the length. Draw the geodesics on the $xy$ plane.

b) Show that $R_{\mu\nu\alpha\beta} \propto x^{-4}, R_{\mu\nu} \propto x^{-2}$ and $R = \text{const}$. Using this, find all these quantities by doing parallel transport along a convenient contour.

6. Solve problem 1.(a) of section 14 of the notes.