GR HW 4

Due Dec 12 at 11am, in class

Goals: Field of a static source, Emission and detection of GWs, Conserved quantities on curved spacetime

1. Find the gravitational field $h_{\mu\nu}$ at large distances from a massive body of mass M. Work in the gauge $h_{0i} = 0$. Can you simultaneously set $h_{ij} \propto \delta_{ij}$?
2. Derive Eqs.(1-4) of `http://arxiv.org/abs/1207.7176`. Calculate the amplitude h_0 for a neutron star with mass 2 solar-mass, eccentricity 0.1, radius 10km, spinning at 100Hz, distance to the Earth 1kpc, and 0 inclination angle.
3. Consider a burst of GWs of the form $h_{yy} = -h_{zz} = A\cos(\omega(t - x))$ shining on LIGO mirrors, perfectly free masses in y direction, initially separated by $\Delta y = L \ll c/\omega$. Calculate the time it takes, measured by the clock attached to one of the mirrors, for the laser light to go to the other mirror and come back ($A = 10^{-20}, \omega = 100\text{Hz}, L = 4\text{km}$).
4. In the above problem, what is the distance between the two masses as measured by good rulers (rulers made of microscopic elements that are strongly bound together)? If the two masses are attached by a soft spring with $2k/m \approx \omega^2$ does it resonate?

   FYI: Joseph Weber proposed a method for detecting GWs based on this idea.
5. We know that $T^{\mu}_{\nu;\mu} = 0$ does not express any conservation law for a generic spacetime. But in a constant gravitational field, there does exist a conserved energy E. Express E in terms of T^{μ}_{ν}, and prove its conservation in a constant gravitational field. (Note: for spacetimes invariant under spatial translations momentum is conserved, rotations give angular momentum conservation.)