GR HW 3

Due Dec 5 at 11am, in class

Goals: Scalar gravity waves, Small angle scattering

1. In Einstein gravity, test particles moving in the field of a spherically symmetric star of mass M follow the geodisics of the Schwarzschild metric:

$$ds^2 = -(1 - \frac{r_g}{r})dt^2 + \frac{dr^2}{1 - \frac{r_g}{r}} + r^2 d\Omega^2 \tag{1}$$

   where $r_g = 2GM/c^2$.

   Calculate the small angle scattering (scattering angle χ in lowest order in $1/b$, b is the impact parameter) for a non-relativistic and an ultrarelativistic particles.

   In particular, calculate light deviation by the Sun, for a light ray just grazing the surface of the Sun (solar mass $M = 1.99 \times 10^{33}$ gr, radius $b = 700,000$ km, give your answer in arcminutes).

2. Do you emit gravitational waves when you move your arm? (Hint: this problem does require a certain numerical estimate before you can tell.)

3. A body of mass $m \ll M$ passes by a body of mass M. In the small angle scattering approximation, estimate total radiated energy. Assume non-relativistic velocity.

4. Estimate how long it would take the Earth to fall down on the Sun due to emission of gravitational waves (in scalar gravity).

5. Is there any dilaton radiation if a photon of energy E is trapped inside a cavity of mass $M \gg E$ and size R? How much does the cavity plus photon weigh on average? (Hint: Use stress-energy conservation to determine the time-average $\left\langle \int d^3x T_{\mu\nu} \right\rangle_t$ for $(\mu, \nu) = (0, 0)$ as well as $(\mu, \nu) = (i, j)$ in a compact periodic system.)

   [Bonus] If there is radiation, calculate the spectrum $d\langle L \rangle / d\omega$ for $\omega \ll 1/\tau$ and estimate the total luminosity. $\tau \ll R$ is the microscopic time it takes for the photon to bounce from the walls of the cavity.