GR HW 3

Due Nov 28 at 11am

Goals: Forces mediated by particles, Redshift, Scalar gravity waves, Small angle scattering

1. Find the number of degrees of freedom (polarizations) of dilaton (massless scalar), photon (massless spin 1), and graviton (massless spin 2) in $d = 5$ spacetime dimensions. (Are there photons or gravitons in $d = 2$ or $d = 3$?)

2. [*Optional*]

   (a) Calculate the differential scattering cross-section in attractive $1/r$ potential with strength C. Use Born approximation.

   (b) Calculate the differential scattering cross-section for two non-relativistic particles of mass m, M ($m \ll M$), due to the tree-level exchange of a scalar field φ_c coupled universally to the trace of the stress-energy tensor, i.e. $S_{\text{int}} = \sqrt{4\pi G}\int d^4x \varphi_c T^\mu_\mu$. Is there a choice of C in the previous problem that gives the same answer as what you find here in the rest frame of the mass-M particle?

3. Twin brothers are separated at birth: one stays at sea level the other lives in mountains at altitude 2km. Which one will outlive his brother (will see him die) and by how much?

4. What's the temperature difference (in thermodynamic equilibrium at room temperature) between the floor and the ceiling (3m high room, due to gravity)?

5. A body of mass $m \ll M$ passes by a body of mass M. In the small angle scattering approximation, estimate total radiated energy. Assume non-relativistic velocity.

6. Is there any dilaton radiation if a photon of energy E is trapped inside a cavity of mass $M \gg E$ and size R? How much does the cavity plus photon weigh on average? (Hint: Use stress-energy conservation to determine the time-average $\left\langle \int d^3x T_{\mu\nu} \right\rangle_t$ for $(\mu, \nu) = (0, 0)$ as well as $(\mu, \nu) = (i, j)$ in a compact periodic system.)

   [Bonus] If there is radiation, calculate the spectrum $d\langle L \rangle / d\omega$ for $\omega \ll 1/\tau$ and estimate the total luminosity. $\tau \ll R$ is the microscopic time it takes for the photon to bounce from the walls of the cavity.