Volatility and the emergence of socio-economic networks

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- Background (empirical studies) and motivation
- Simple models: searching/coordinating with/learning from others in volatile networks
- General conclusion
Collective social phenomena: anecdotical evidence

Many bars in a central area of Trieste

A crowd of hundreds teenagers forms every Tuesday evening in front of bar Costa

No crowd on different days and in different bars, no such effect two years ago

How did such dense “social network” arise?

Why Tuesday? Why bar Costa? How did they coordinate?
Networks everywhere

- Labor markets (Granovetter, Topa, Calvó-A. & Jackson)
- Crime/social pathologies (Crane, Glaeser et al, Harding)
- R&D partnerships (Gulati et al, Hagerdoorn)
- Scientific collaboration (Newman, Goyal et al)
- Patterns of trade (Kranton & Minehart, Rauch, Greif)
- Organizational performance (Radner, Garicano, Cabrales et al)
- Industrial districts (Jacobs, Saxenian)
The rise of networks

- R&D partnerships, joint ventures (Hagendoorn)
- Scientific collaboration networks (Goyal, Newman)
- Web communities (del.icio.us)
- ...

Fig. 1. The growth of newly established R&D partnerships (1980-1998).
Networks → Economics

- Economic performance correlates with social capital (Putnam)
- Finding jobs (Granovetter, Topa, Calvó-A. & Jackson)
- Resilience of industrial districts (silicon valley vs route 128: Jacobs, Saxenian)
- Diffusion of ideas and technological progress (Diamond)
Economics → Networks

- Link formation limited by:
  - reputation/trust
  - coordination
  - similarity/proximity
  - information diffusion

- Links are purposefully chosen by agents, costs and benefits
  → Game theory (Jackson, Goyal, Vega-Redondo, ...)

\[
\max_s U_i(s) \quad s = \text{strategy} = (\text{action, links})
\]

but choices alone can only explain simple structures (e.g. star, complete/empty graph)
Chance and necessity

(Monod)

Necessity: economic incentives

Chance: environmental volatility

Both the links and the agents themselves change as a result of several (often unobservable) factors (e.g. partnership may turn unprofitable)

Opportunities of new connections are affected by factors beyond agents’ control (e.g. searching partners).

→ The Red Queen effect: “It need all the running you can do to keep in the same place” (Carrol)

Under what conditions do dense networks emerge?
Minimal models of volatile networks

- Links decay at a constant rate
- Links formation limited by
  - similar technological levels
  - similar opinions
  - coordination
  - reputation
  - search through friends
- Dense network promotes similarity/proximity/coordination/information diffusion/searchability/...

e.g. R&D network

- Network density
- Technological growth
- Technological heterogeneity
Node and link volatility

- Both links (relationships) and nodes (individuals) are not permanent in general.
- Under what conditions do dense and efficient social networks emerge?
  How stable should the composition of a society be to speak meaningfully of a social network?
- A simple model: Learning to coordinate in a volatile world (efficiency = coordination).
is this statistical physics?

- Topology
  - Interaction + noise
    - collective behavior
      - (phase transitions, order/disorder, growth, synchronization, ...)
A stylized model of a society:

- A society of \( N \) agents
  Each agent adopts one of \( q \) possible norms: \( s_i = 1, \ldots, q \)

- Norm revision
  At a rate \( \nu \) each agent updates his norm to a random norm if isolated (experimentation) or the norm of one of his neighbors (e.g. voter)

- Link formation
  At a rate \( \eta \) agent \( i \) meets an agent \( j \) drawn at random. If \( s_i = s_j \) they establish a link

- Environment volatility
  1- A profitable cooperation may turn unprofitable: each link decays at a rate \( \frac{1}{2} \)
  2- Agent turnover:
      each node loses all links at rate \( \alpha \)
No agent turnover
(no node volatility)

\[ \alpha = 0 \]
The Master equation

\[ \frac{\partial P(\omega, t)}{\partial t} = \sum_{\omega' \in \Omega} \left[ P(\omega', t)W(\omega' \rightarrow \omega) - P(\omega, t)W(\omega \rightarrow \omega') \right] \]

- **Microscopic state**
  - Network + norms: \( \omega = \{a_{i,j}, s_i\} \),
    \( a_{i,j} = 0 \) (no link i-j) or 1 (i-j linked)
    \( s_i = 1, \ldots, q \)

- **Link creation**
  \( \omega \rightarrow \omega' = \{\omega_{-i,j}, a_{i,j} = 1\} \)
  \( W[\omega \rightarrow \omega'] = 2\eta(1-a_{i,j})/(N-1) \)

- **Link removal**
  \( \omega \rightarrow \omega' = \{\omega_{-i,j}, a_{i,j} = 0\} \)
  \( W[\omega \rightarrow \omega'] = \lambda a_{i,j} \)

- **Norm revision**
  \( \omega \rightarrow \omega' = \{\omega_{-i}, r_i = r'\} \)
  \( W[\omega \rightarrow \omega'] = \nu, \ r' \) majority norm
The stationary state I

- Let

\[ \Omega = \{ \omega \in \Omega : s_i = s_j \ \forall (i, j) : a_{i,j} = 1 \} \]

- All states in \( \Omega_- \) are ergodic, all states in \( \Omega/\Omega_- \) are transient
  - Proof:
    links between agents with different \( s \) are never created
    all states in \( \Omega_- \) can be reached passing from the empty network

- The invariant measure is

\[ P_s(\omega) = P_0 \left\{ \begin{array}{ll} \prod_{i < j} z_{a_{i,j}}^{i,j} & \omega \in \Omega_- \\ 0 & \omega \notin \Omega_- \end{array} \right. \quad z = \frac{2n}{N - 1} \]

  - Proof: detailed balance

\[ P(\omega', t)W(\omega' \to \omega) = P(\omega, t)W(\omega \to \omega') \]
The stationary state II

• The distribution of the fraction \( n_s \) of agents with \( s_i = s \) is given by

\[
P_s(n_1, \ldots, n_q) = P_0 e^{-Nf(n_1, \ldots, n_q)}, \quad n_1 + \ldots + n_q = 1
\]

• For \( N \) large, \( \{n_s\} \) is a.s. given by the minima of

\[
f(n_1, \ldots, n_q) = -\sum_s \left[ n_s \log n_s - \frac{z}{2} n_s^2 \right], \quad z = 2n_i
\]

• The solution can be characterized by the number \( L_+ \) of \( n_s = n_+ \) where \( n_+ (n_-) \) is the largest (smallest) solution of

\[
x e^{-z x} = \frac{n_0}{q}
\]

\( n_0 = \) fraction of isolated nodes (\( k=0 \))

• The \( L_+ = 0 \) solution exists and is a minimum for all \( z \leq 1 \)

\( L_+ > 1 \) solutions are saddle points

\( L_+ = 1 \) solution is a minimum iif \( n_+ \uparrow z \)
the "free-energy"\[ f(n) = \frac{1}{N} \log P\{n\} \]
The dynamics (t finite, $N \to \infty$)

- Mean field dynamics

\[
\begin{align*}
\dot{n}_{k,s} &= 2\eta s n_{k-1,s} + \lambda (k+1)n_{k+1,s} - 2\eta s n_{k,s} - \lambda k n_{k,s}, \quad k > 0 \\
\dot{n}_{0,s} &= \lambda n_{1,s} - 2\eta s n_{0,s} + \nu \sum_r [n_{0,r} - n_{0,s}]
\end{align*}
\]

- If $n_s \to n_s^*$ then

\[
\lim_{t \to \infty} n_{k,s}(t) = n_s^* \frac{(zn_s^*)^k}{k!} e^{-zn_s^*}
\]

- The stationary states $n_s^*$ are the same as those found above (min $f$ $\leftrightarrow$ stability)
  - Proof: The Poisson transformation

\[
n_{k,s} = \int_0^\infty dx \frac{x^k}{k!} e^{-x} g_s(x,t), \quad \Rightarrow \quad \partial_t g_s = \lambda \partial_x (x - zn_s) g_s
\]

Finite $t$ and $N$: theory and simulations

c = average degree

time

$2\eta^1$
Summary: if volatility affects links

As a consequence of the feedback between networking efforts of individuals and the benefits the network provides in terms of coordination, information and innovation diffusion, social cohesion, …

**Sharp transitions**: socio-economic networks are expected to emerge in an abrupt manner

**Resilience**: once dense networks form, they are robust to deterioration of external conditions

**Coexistence**: for the same environmental parameters, the network can either be dense or very sparse, depending on the history

What about node volatility (agents’ turnover)?
A stylized model of a society:

• A society of $N$ agents
  Each agent adopts one of $q$ possible norms: $s_i=1,\ldots,q$

• Norm revision
  At a rate $\nu$ each agent updates his norm to a random norm if isolated (experimentation) the norm of one his neighbors (e.g. voter)

• Link formation
  At a rate $\eta$ agent $i$ meets an agent $j$ drawn at random. If $s_i=s_j$ they establish a link

• Environment volatility
  1- A profitable cooperation may turn unprofitable: each link decays at a rate $1$
  2- Agent turnover:
      each node loses all links at rate $\alpha$
Node volatility: $\alpha > 0$

- The dynamics:
  \[
  \dot{n}_{k,\sigma} = (k + 1)n_{k+1,\sigma} - kn_{k,\sigma} - \alpha n_{k,\sigma} + x_{\sigma}(n_{k-1,\sigma} - n_{k,\sigma}) \quad k > 0
  \]
  \[
  \dot{n}_{0,\sigma} = \alpha \sum_{k > 0} n_{k,\sigma} + n_{1,\sigma} - x_{\sigma}n_{0,\sigma} + \nu \sum_{\sigma' = 1}^{q} (n_{0,\sigma'} - n_{0,\sigma})
  \]
  \[
  x_{\sigma} = \eta \sum_{k=0}^{\infty} n_{k,\sigma}
  \]

- The network:
  each component has average degree $x_{\sigma}/(1 + \alpha)$, $\sigma = 1, \ldots, q$
  degree distribution interpolates between Poisson ($\alpha = 0$) and exponential ($\alpha \rightarrow \infty$)

- The distribution of component sizes:
  \[
  \eta n_{0,\sigma} = \frac{\eta n_{0}}{q} = \alpha x_{\sigma} \int_{0}^{1} du u^{\alpha-1} e^{x_{\sigma}(u-1)} \equiv G_{\alpha}(x_{\sigma})
  \]
  \[
  + \text{ normalization } \sum_{\sigma = 1}^{q} x_{\sigma} = \eta
  \]
• The symmetric solution:

\[ n_0 = \frac{q}{\eta} G_\alpha(\eta/q) \]

stable only if \( G'_\alpha(\eta/q) > 0 \) (i.e. if \( \langle k \rangle \nearrow \eta \) )

• The asymmetric solution \( \alpha < 1 \) only

\[ G_\alpha(x_+) = G_\alpha(x_-) \]
\[ x_+ + (q - 1)x_- = \eta \]

solutions with more than one large component unstable
Results and phase diagram

As node volatility increases, it gets harder and harder to achieve coordination. For $\alpha > 1$ there is no coordination at all.
Critical behavior

• Order parameter

\[ m = \frac{x_+ - x_-}{\eta} \]

\[ m \sim |\log(1 - \alpha)|^{-1}. \]
Similar effect in other models

- E.g. searching partners on the network in a volatile world

MM, F. Slanina, F. Vega-Redondo, PNAS 2004
Summary:

- Links formation is enhanced by coordination, similarity or proximity.
- Link volatility: Links decay when no more useful (i.e. at a constant rate).
  - Discontinuous phase transitions + coexistence, hysteresis/resilience.
- When node volatility (agents’ turnover) dominates, the transition becomes continuous and no system wide coordination takes place.
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Summary

• Generic class of models
  - easier to establish interaction with similar/close agents
  - linked agents become more similar/closer
  ↓ sharp transition, coexistence, hysteresis if agents’ turnover is weak

• Empirical evidence?
  Rise of networks and type of volatility

• Spatial models?

References:
MM, FS, FVR, PNAS 2004
GE, MM, FVR PRE 2006, IJGT 2006
DD, MM, EPJB 2008
Knowledge/technology level $h_i(t)$

- linked agents tend to become similar

$$h_i(t) \rightarrow h_i(t^+) = \begin{cases} 
\max_{j \in N_i} h_j(t) & \text{technology adoption} \\
\frac{1}{|N_i|} \sum_{j \in N_i} h_j(t) & \text{knowledge diffusion}
\end{cases}$$

- interaction is easier between similar nodes/agents

- Link formation at rate 1 if $|h_i - h_j| < \delta h$

- Volatility $\lambda$
Technology adoption:

• Spread of $h_i \downarrow c$
  $\rightarrow$ link formation rate $\uparrow c$

• Phase with slow growth, sparse network and large fluctuations of $h$

• Phase with fast growth, dense network and small fluctuations of $h$

• Sharp transition, coexistence and hysteresis
Knowledge diffusion

- Distribution of $h_i(t)$ from spectral density of Laplacian on random graphs (Dorogotsev et al., Rodgers & Bray, ...)

$$\langle (h_i - \langle h_i \rangle)^2 \rangle = \sum_{\mu > 0} \frac{\nu \Delta}{2 \mu} = \frac{\nu \Delta}{2} \int \frac{d\mu}{\mu} \rho(\mu)$$

- average degree

- $P\{|h_i - h_j| < \delta h\}$