Volatility and the emergence of socio-economic networks

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- Background (empirical studies) and motivation
- Simple models: searching/coordinating with/learning from others in volatile networks
- General conclusion

Collective social phenomena: anecdotical evidence

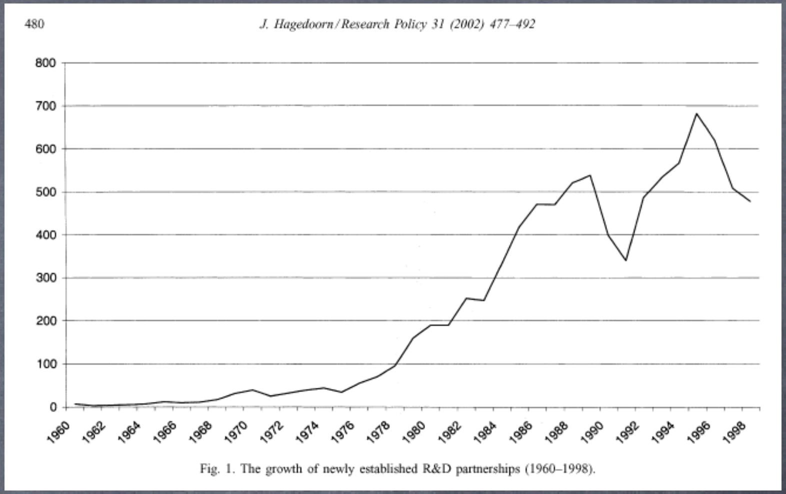
- Many bars in a central area of Trieste
- a crowd of hundreds teenagers forms every tuesday evening in front of bar Costa
- No crowd on different days and in different bars, no such effect two years ago
- How did such dense "social network" arise?
- Why tuesday? Why bar Costa? How did they coordinate?

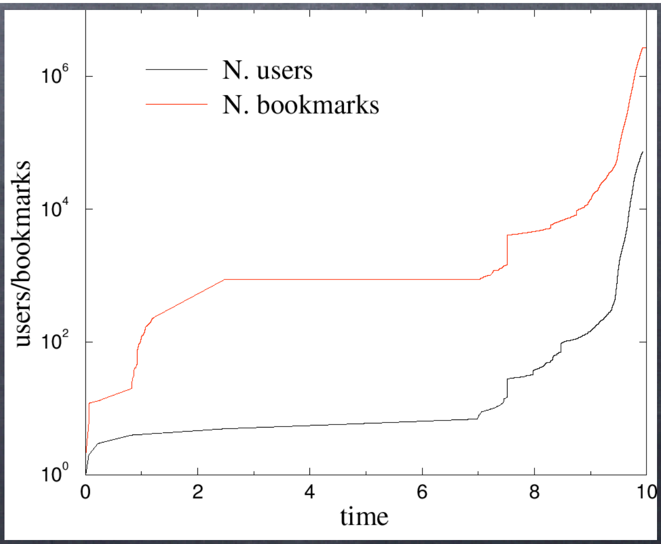
Networks everywhere

- Labor markets (Granovetter, Topa, Calvó-A. & Jackson)
- Crime/social pathologies (Crane, Glaeser et al, Harding)
- R&D partnerships (Gulati et al, Hagerdoorn)
- Scientific collaboration (Newman, Goyal et al)
- Patterns of trade (Kranton & Minehart, Rauch, Greif)
- Organizational performance (Radner, Garicano, Cabrales et al)
- Industrial districts (Jacobs, Saxenian)

The rise of networks

- R&D partnerships, joint ventures (Hagendoorn)
- Scientific collaboration networks (Goyal, Newman)
- Web communities (del.icio.us)
- ...





Networks → Economics

- Economic performance correlates with social capital (Putnam)
- Finding jobs (Granovetter, Topa, Calvó-A. & Jackson)
- Resilience of industrial districts (silicon valley vs route 128: Jacobs, Saxenian)
- Diffusion of ideas and technological progress (Diamond)

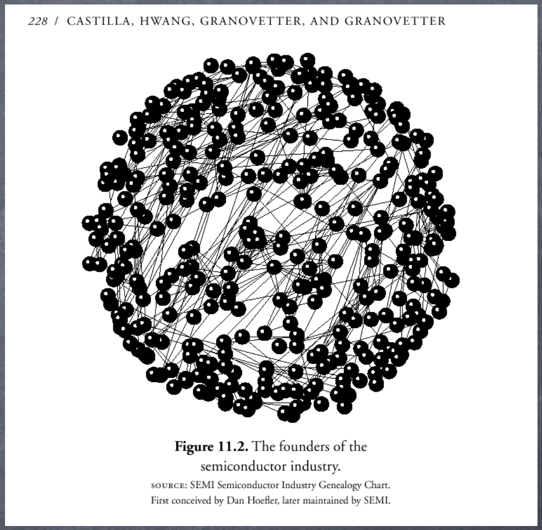
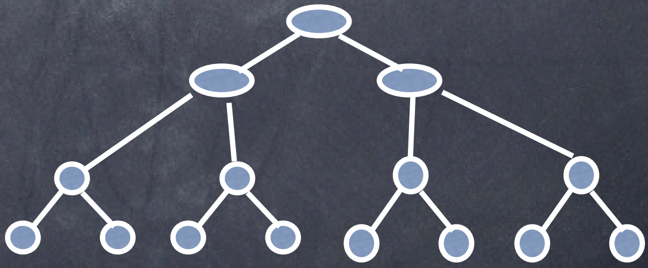

228 / CASTILLA, HWANG, GRANOVETTER, AND GRANOVETTER

Figure 11.2. The founders of the semiconductor industry.

SOURCE: SEMI Semiconductor Industry Genealogy Chart. First conceived by Dan Hoefler, later maintained by SEMI.



Economics → Networks

- Link formation limited by:
  - reputation/trust
  - coordination
  - similarity/proximity
  - information diffusion

- Links are purposefully chosen by agents, costs and benefits
  → Game theory (Jackson, Goyal, Vega-Redondo, ...)

$$\max_s U_i(s) \qquad s = \text{strategy} = (\text{action,links})$$

but choices alone can only explain simple structures (e.g. star, complete/empty graph)

Chance and necessity

(Monod)

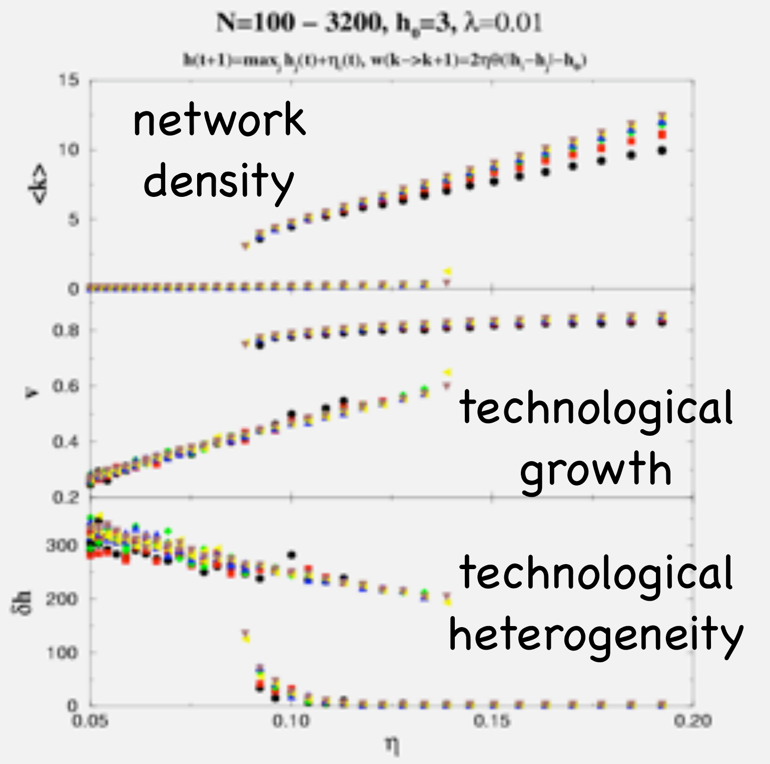
- Necessity: economic incentives
- Chance: environmental volatility
  - Both the links and the agents themselves change as a result of several (often unobservable) factors (e.g. partnership may turn unprofitable)
  - Opportunities of new connections are affected by factors beyond agents' control (e.g. searching partners).
- → The Red Queen effect: "It need all the running you can do to keep in the same place" (Carrol)

Under what conditions do dense networks emerge?

Minimal models of volatile networks

- Links decay at a constant rate
- Links formation limited by
  - similar technological levels
  - similar opinions
  - coordination
  - reputation
  - search through friends
- Dense network promotes similarity/proximity/coordination/information diffusion/searchability/..

e.g. R&D network



Node and link volatility

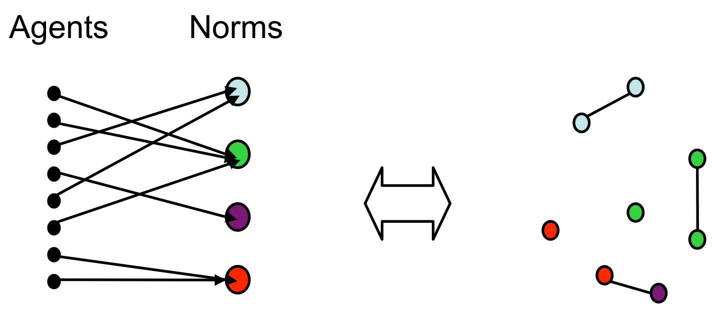
- Both links (relationships) and nodes (individuals) are not permanent in general
- Under what conditions do dense and efficient social networks emerge?
  How stable should the composition of a society be to speak meaningfully of a social network?
- A simple model:
  Learning to coordinate in a volatile world
  (efficiency = coordination)

is this statistical physics?

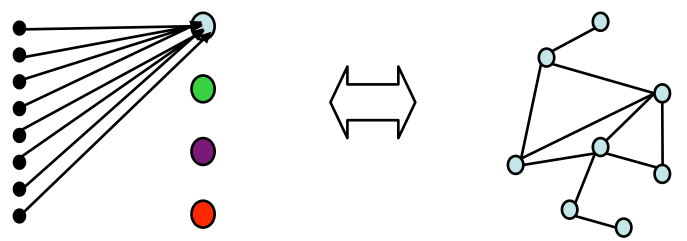
- Topology
  ↓
- Interaction + noise
  ↓
- collective behavior (phase transitions, order/disorder, growth, synchronization, ...)

A stylized model of a society:

- A society of N agents
  Each agent adopts one of q possible norms: $s_i=1,\dots,q$
- Norm revision
  At a rate ν each agent updates his norm to a random norm if isolated (experimentation) the norm of one his neighbors (e.g. voter)
- Link formation
  At a rate η agent i meets an agent j drawn at random. If $s_i=s_j$ they establish a link
- Environment volatility
  1- A profitable cooperation may turn unprofitable: each link decays at a rate 1
  2- Agent turnover:
  each node loses all links at rate α



disorder – low link density



order – high link density

No agent turnover
(no node volatility)

$$\alpha = 0$$

The Master equation

$$\frac{\partial P(\omega,t)}{\partial t} = \sum_{\omega'\in\Omega} [P(\omega',t)W(\omega'\to\omega) - P(\omega,t)W(\omega\to\omega')]$$

- Microscopic state
  - Network + norms: $\omega = \{a_{i,j}, s_i\}$,
    $a_{i,j}$= 0 (no link i-j) or 1 (i-j linked)
    s_i=1,…,q
- Link creation

  $\omega \rightarrow \omega' = \{\omega_{-i,j}, a_{i,j}=1\}$, $W[\omega \rightarrow \omega'] = 2\eta(1-a_{i,j})/(N-1)$
- Link removal

  $\omega \rightarrow \omega' = \{\omega_{-i,j}, a_{i,j}=0\}$, $W[\omega \rightarrow \omega'] = \lambda a_{i,j}$
- Norm revision

  $\omega \rightarrow \omega' = \{\omega_{-i}, r_i = r'\}$, $W[\omega \rightarrow \omega'] = \nu$, r' majority norm

The stationary state I $\text{finite } N$

$t \to \infty$

- Let $\Omega_{=}\{\omega \in \Omega : \; s_i = s_j \; \forall (i,j) : \; a_{i,j} = 1\}$
- All states in $\Omega_=$ are ergodic, all states in $\Omega/\Omega_=$ are transient
  - Proof:
    links between agents with different s are never created
    all states in $\Omega_=$ can be reached passing from the empty network
- The invariant measure is

$$P_s(\omega) = P_0 \begin{cases} \prod_{i<j} z^{a_{i,j}} & \omega \in \Omega_= \\ 0 & \omega \notin \Omega_= \end{cases} \qquad z = \frac{2\eta}{N-1}$$

  - Proof: detailed balance

$$P(\omega', t)W(\omega' \to \omega) = P(\omega, t)W(\omega \to \omega')$$

The stationary state II

- The distribution of the fraction n_s of agents with s_i=s is given by

$$P_s(n_1,\ldots,n_q) = P_0 e^{-Nf(n_1,\ldots,n_q)}, \qquad n_1+\ldots+n_q=1$$

- For N large, $\{n_s\}$ is a.s. given by the minima of

$$f(n_1,\ldots,n_q) = -\sum_s \left[n_s \log n_s - \frac{z}{2}n_s^2\right], \qquad z=2\eta$$

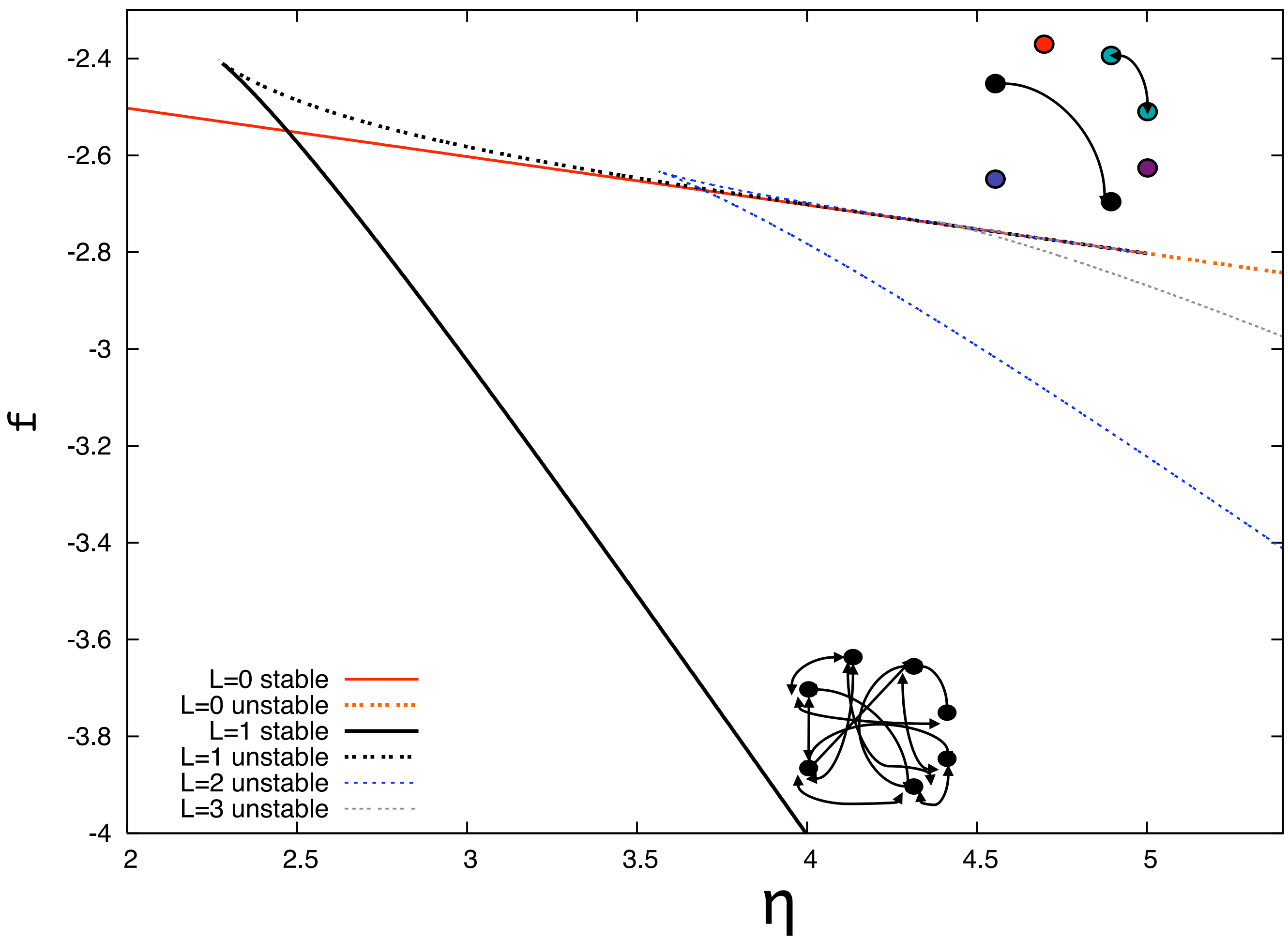
- The solution can be characterized by the number L_+ of $n_s=n_+$ where n_+ (n_-) is the largest (smallest) solution of

$$xe^{-zx} = \frac{n_0}{q}$$

n_0=fraction of isolated nodes (k=0)

- The L_+=0 solution exists and is a minimum for all $z \leq 1$
  $L_+>1$ solutions are saddle points
  L_+=1 solution is a minimum iif $n_+ \nearrow z$

the "free-energy" $f(\mathbf{n}) = \frac{1}{N} \log P\{\mathbf{n}\}$



The dynamics (t finite, N➔∞)

- Mean field dynamics

$$\begin{aligned}\dot{n}_{k,s} &= 2\eta n_s n_{k-1,s} + \lambda(k+1)n_{k+1,s} - 2\eta n_s n_{k,s} - \lambda k n_{k,s}, \qquad k>0 \\ \dot{n}_{0,s} &= \lambda n_{1,s} - 2\eta n_s n_{0,s} + \nu \sum_r [n_{0,r} - n_{0,s}]\end{aligned}$$

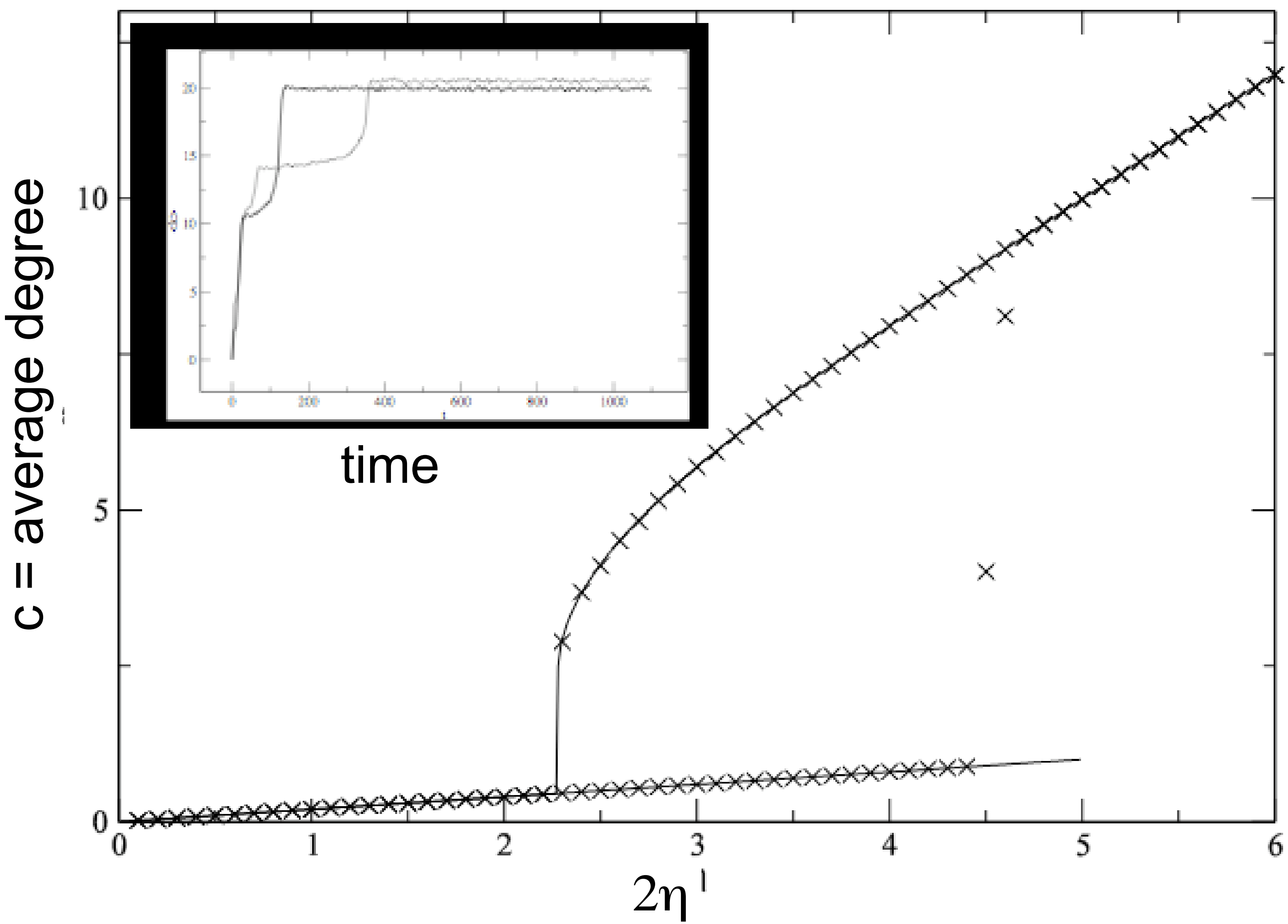
- If n_s➔n_s^* then

$$\lim_{t\to\infty} n_{k,s}(t) = n_s^* \frac{(z n_s^*)^k}{k!} e^{-z n_*}$$

- The stationary states n_s^* are the same as those found above (min f ←➔ stability)
  - Proof: The Poisson transformation

$$n_{k,s} = \int_0^\infty dx \frac{x^k}{k!} e^{-x} g_s(x,t), \qquad \Rightarrow \qquad \partial_t g_s = \lambda \partial_x (x - z n_s) g_s$$

Finite t and N: theory and simulations



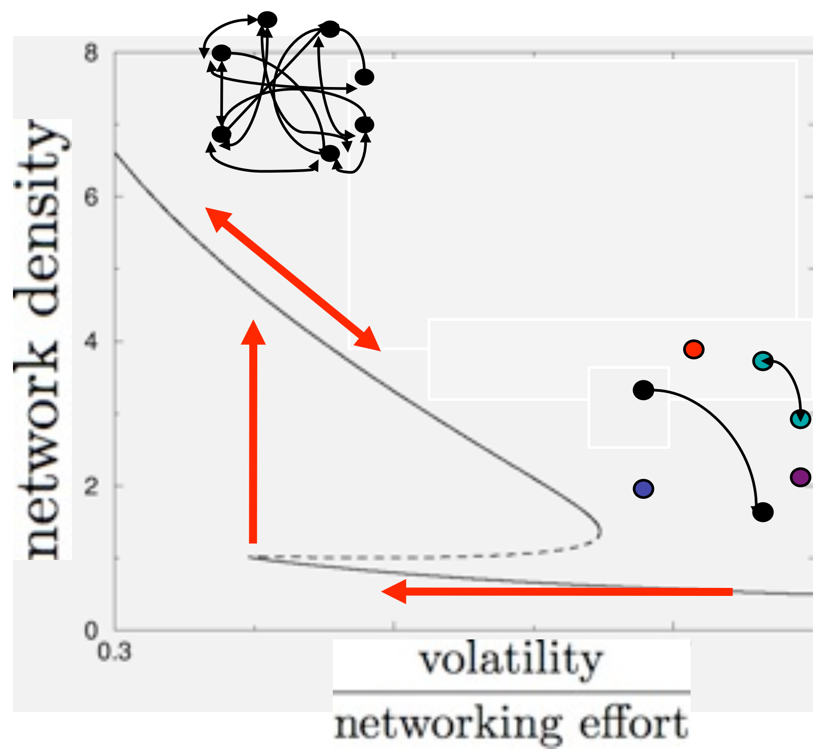
Summary: if volatility affects links

As a consequence of the feedback between networking efforts of individuals and the benefits the network provides in terms of coordination, information and innovation diffusion, social cohesion, …

Sharp transitions: socio-economic networks are expected to emerge in an abrupt manner

Resilience: once dense networks form, they are robust to deterioration of external conditions

Coexistence: for the same environmental parameters, the network can either be dense or very sparse, depending on the history



What about node volatility (agents' turnover)?

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- Environment volatility
  1- A profitable cooperation may turn unprofitable: each link decays at a rate 1
  2- Agent turnover:
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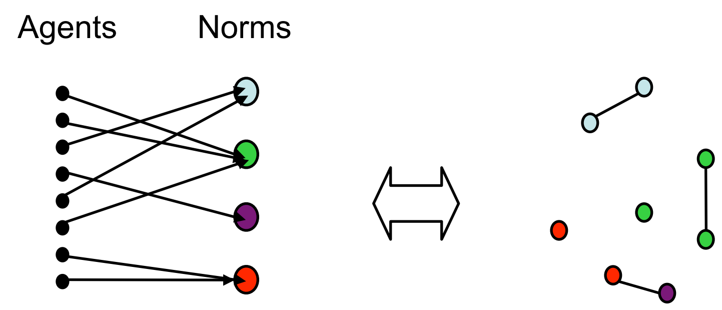
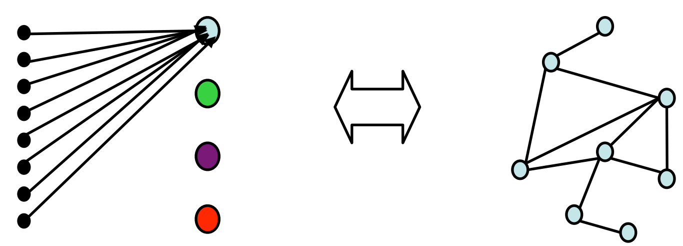


disorder – low link density



order – high link density

Node volatility: α>0

- The dynamics:

$$\dot{n}_{k,\sigma} = (k+1)n_{k+1,\sigma} - kn_{k,\sigma} - \alpha n_{k,\sigma} + x_\sigma(n_{k-1,\sigma} - n_{k,\sigma}) \quad k > 0$$

$$\dot{n}_{0,\sigma} = \alpha \sum_{k>0} n_{k,\sigma} + n_{1,\sigma} - x_\sigma n_{0,\sigma} + \frac{\nu}{q} \sum_{\sigma'=1}^{q} (n_{0,\sigma'} - n_{0,\sigma})$$

$$x_\sigma = \eta \sum_{k=0}^{\infty} n_{k,\sigma}$$

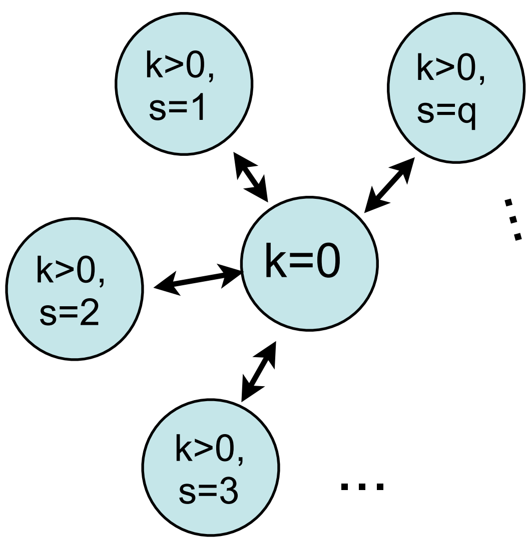
- The network:

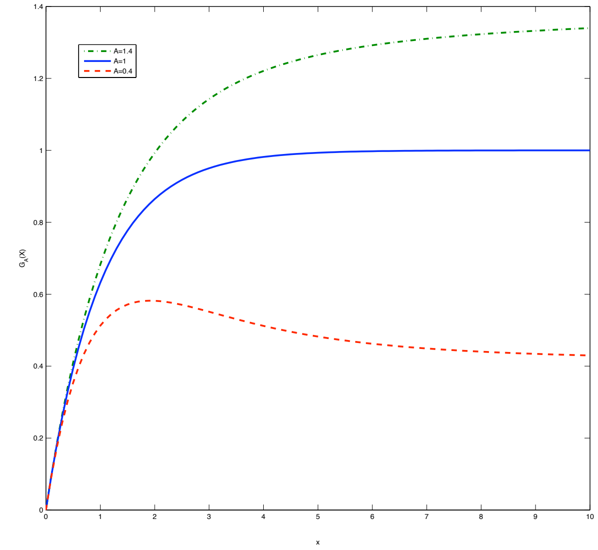
each component has average degree $x_\sigma/(1+\alpha), \quad \sigma = 1, \ldots, q$
degree distribution interpolates between Poisson (**α** =0) and exponential (**α** →∞)

- The distribution of component sizes:

$$\eta n_{0,\sigma} = \frac{\eta n_0}{q} = \alpha x_\sigma \int_0^1 du u^{\alpha-1} e^{x_\sigma(u-1)} \equiv G_\alpha(x_\sigma)$$

$$+ \text{ normalization} \quad \sum_{\sigma=1}^{q} x_\sigma = \eta$$





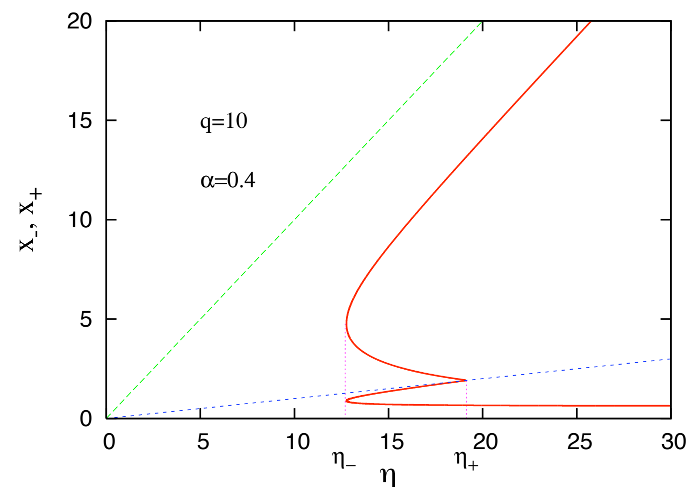
- The symmetric solution:

$$n_0 = \frac{q}{\eta} G_\alpha\left(\eta/q\right)$$

stable only if $G'_\alpha\left(\eta/q\right) > 0$ (i.e. if $\langle k\rangle \nearrow \eta$)

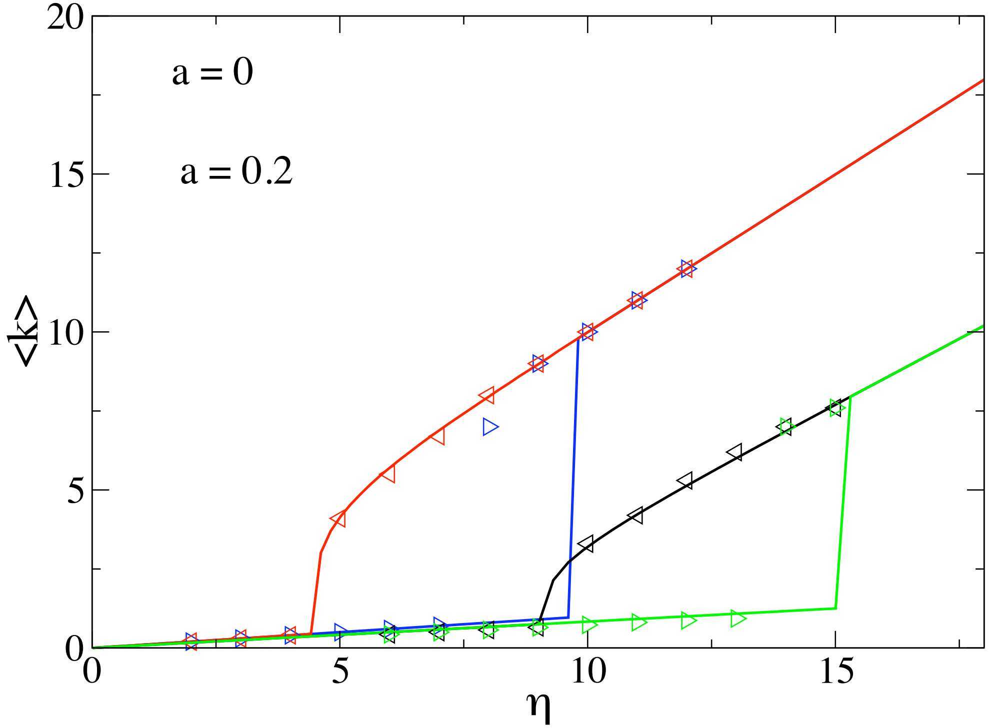
- The asymmetric solution α<1 only

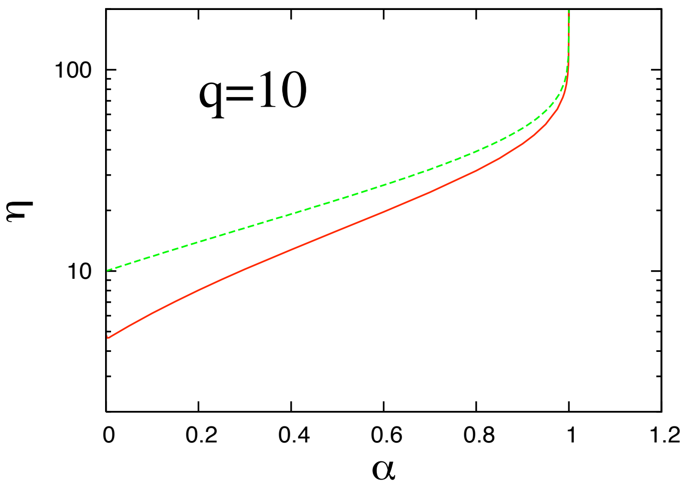
$$\begin{aligned} G_\alpha(x_+) &= G_\alpha(x_-) \\ x_+ + (q-1)x_- &= \eta \end{aligned}$$



solutions with more than one large component unstable

Results and phase diagram

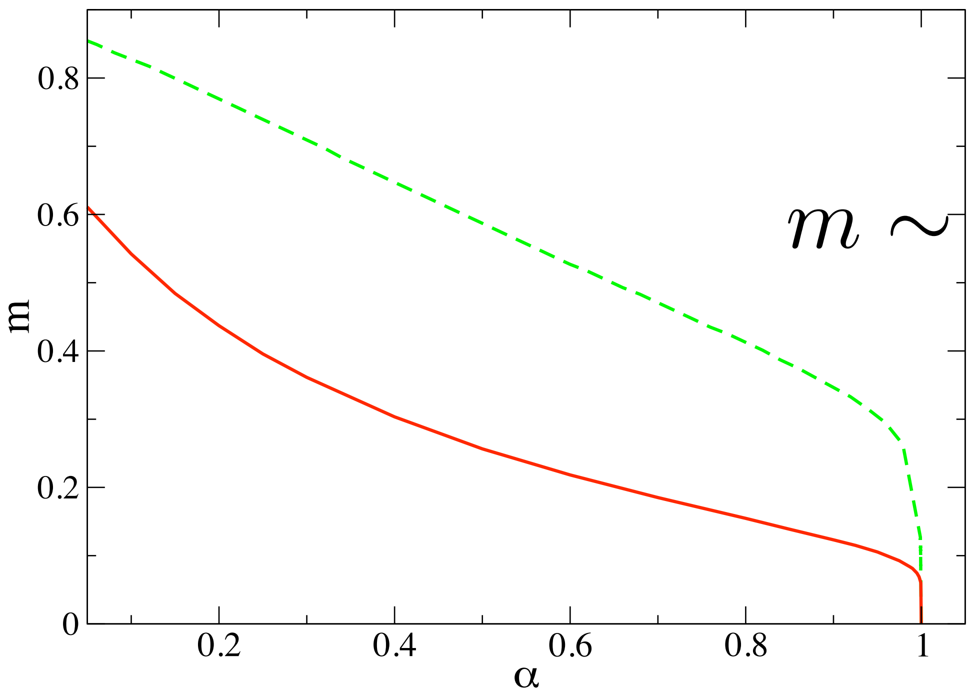




As node volatility increases, it gets harder and harder to achieve coordination. For α>1 there is no coordination at all

Critical behavior

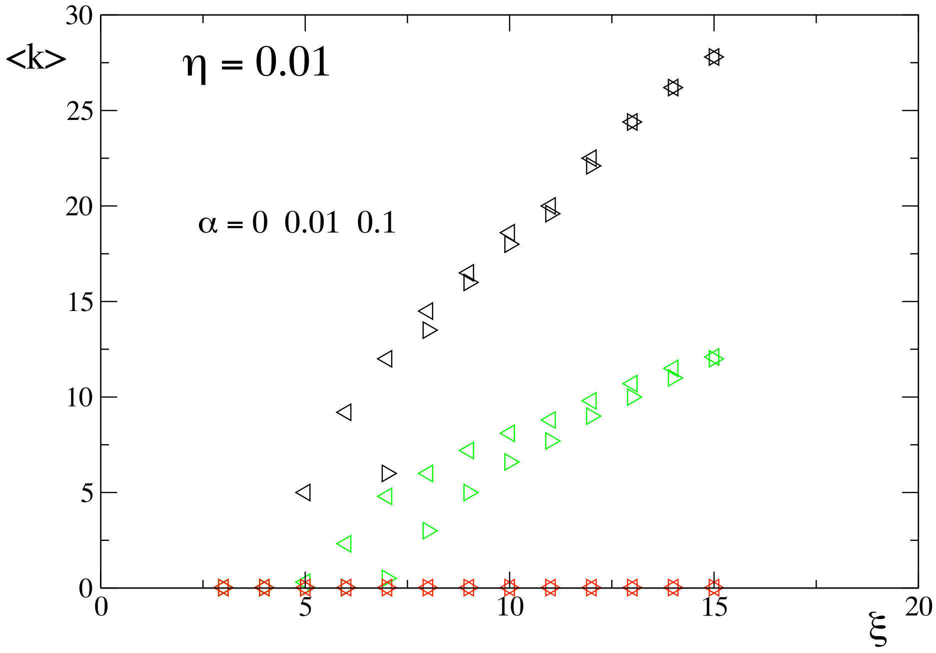
- Order parameter $m = \frac{x_+ - x_-}{\eta}$



$$m \sim |\log(1-\alpha)|^{-1}.$$

Similar effect in other models

- E.g. searching partners on the network in a volatile world

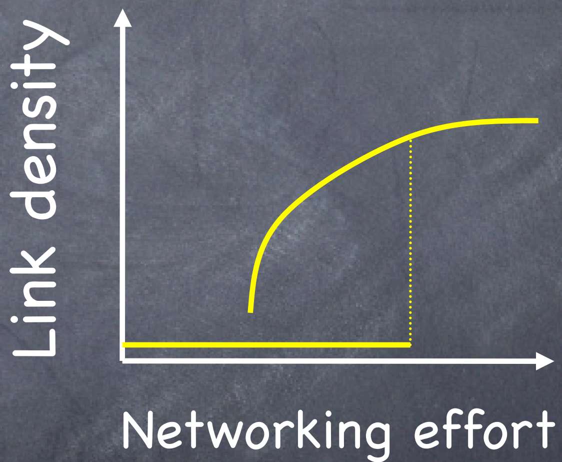


MM, F. Slanina, F. Vega-Redondo, PNAS 2004

Summary:

- Links formation is enhanced by coordination, similarity or proximity
- Link volatility: Links decay when no more useful (i.e. at a constant rate)

→ Discontinuous phase transitions + coexistence, hysteresis/resilience



- when node volatility (agents' turnover) dominates, the transition becomes continuous and no system wide coordination takes place

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Summary

- Generic class of models
  - easier to establish interaction with similar/close agents
  - linked agents become more similar/closer

  ↓

  sharp transition, coexistence, hysteresis **if agents' turnover is weak**
- Empirical evidence?
  Rise of networks and type of volatility
- Spatial models?

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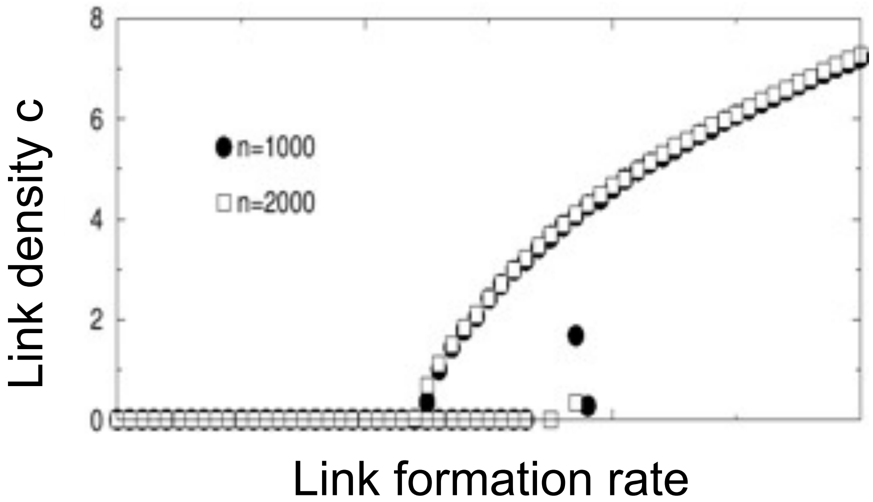

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GE, MM, FVR PRE 2006, IJGT 2006
DD, MM, EPJB 2008

Knowledge/technology level $h_i(t)$

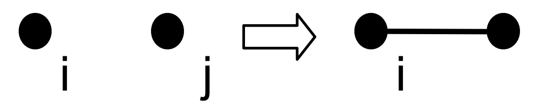
- linked agents tend to become similar

$$h_i(t) \rightarrow h_i(t^+) = \begin{cases} \max_{j \in N_i} h_j(t) & \text{techology adoption} \\ \frac{1}{|N_i|} \sum_{j \in N_i} h_j(t) & \text{knowledge diffusion} \end{cases}$$

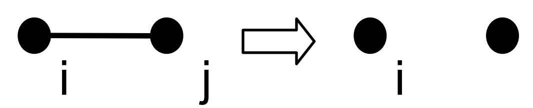
- interaction is easier between similar nodes/agents

- Link formation at rate 1 if $|h_i - h_j| < \delta h$

  η otherwise

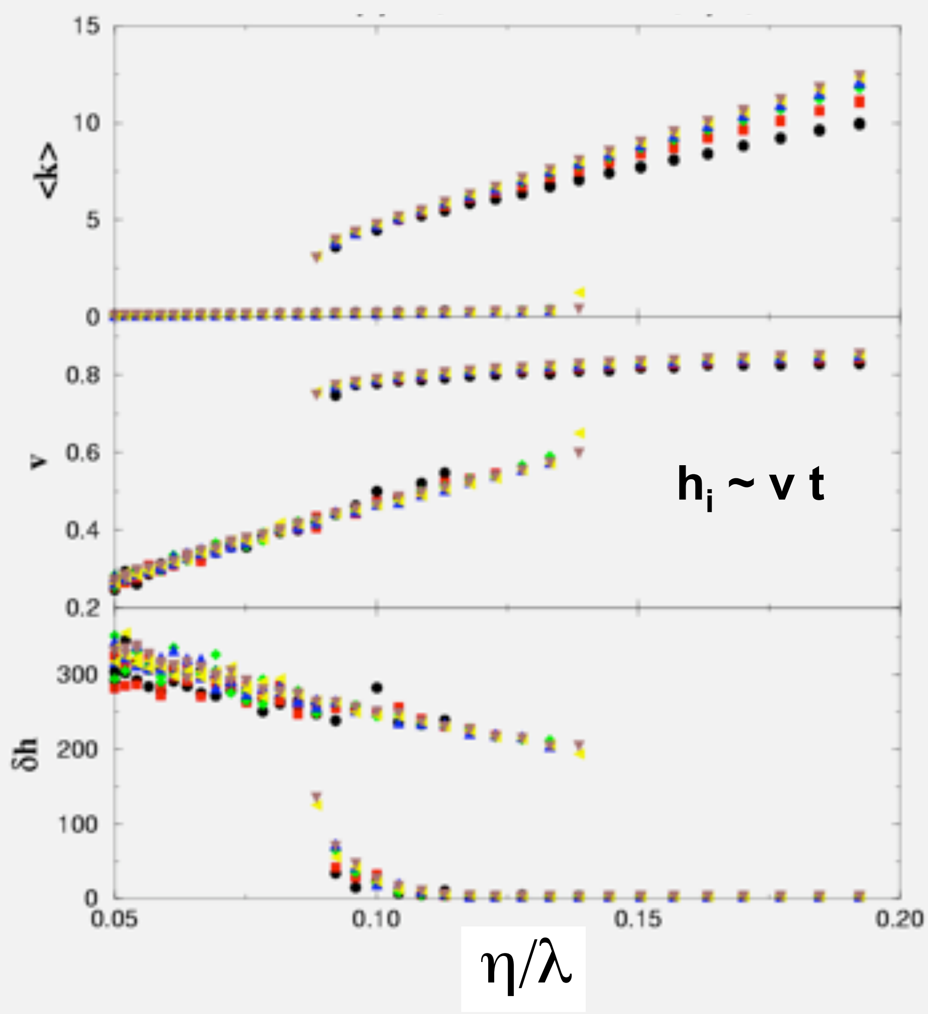


- Volatility λ



Technology adoption:

- Spread of $h_i \downarrow c$
  → link formation rate ↑ c
- Phase with slow growth, sparse network and large fluctuations of h
- Phase with fast growth, dense network and small fluctuations of h
- Sharp transition, coexistence and hysteresis



Knowledge diffusion

- Distribution of $h_i(t)$ from spectral density of Laplacian on random graphs (Dorogotsev et al., Rodgers & Bray, ...)

$$\langle (h_i - \langle h_i \rangle)^2 \rangle = \sum_{\mu > 0} \frac{\nu \Delta}{2\mu} = \frac{\nu \Delta}{2} \int \frac{d\mu}{\mu} \rho(\mu)$$

- average degree

- $P\{|h_i - h_j| < \delta h\}$