Interaction effects in graphene

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collaborations with
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Pavel Kovtun (Victoria)

The Abdus Salam
ICTP Trieste

Taiwan National University, 23rd December, 2009

Outline

- Relativistic physics in graphene, quantum critical systems and conformal field theories

- Strong coupling features in collision-dominated transport – as inspired by AdS-CFT results

- Comparison with strongly coupled fluids (via AdS-CFT)

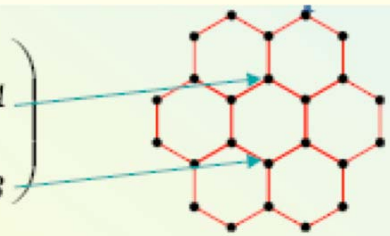
- Graphene: an almost perfect quantum liquid

Dirac fermions in graphene

(Semenoff '84, Haldane '88)

Honeycomb lattice of C atoms

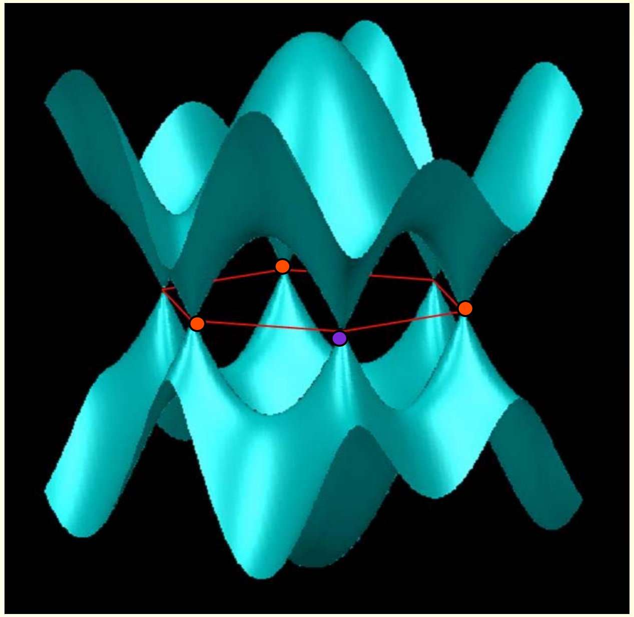
$$\psi = \begin{pmatrix} \varphi_A \\ \varphi_B \end{pmatrix}$$



$$\hat{H} = v\begin{pmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{pmatrix} = v\vec{\sigma}\cdot\vec{p}$$

$$\mathbf{p} = \mathbf{k} - \mathbf{K} \quad \rightarrow \quad E_{\mathbf{p}} = \mathrm{v}_F |\mathbf{p}|$$

Tight binding dispersion



2 massless Dirac cones in the Brillouin zone:
(Sublattice degree of freedom ↔ pseudospin)

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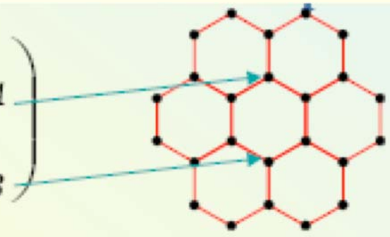

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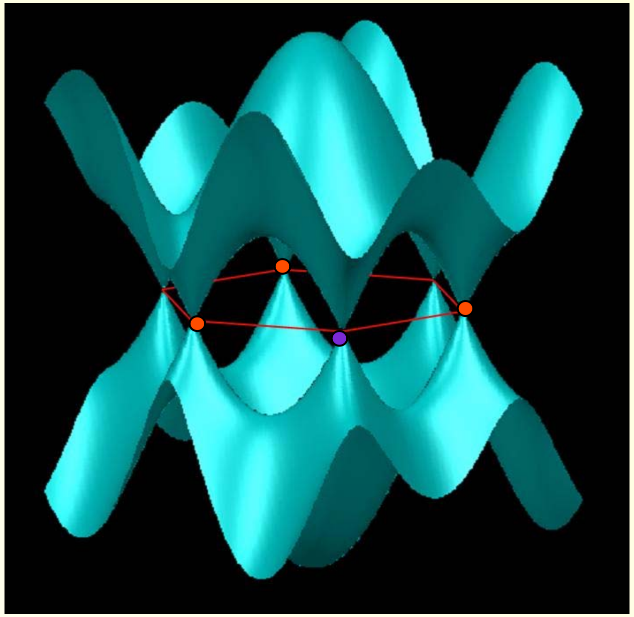

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Fermi velocity (speed of light")

$$v_F \approx 1.1\cdot 10^6 \text{ m/s} \approx \frac{c}{300}$$

Coulomb interactions: Fine structure constant

$$\alpha \equiv \frac{e^2}{\varepsilon\,\hbar v_F} = \frac{2.2}{\varepsilon}$$

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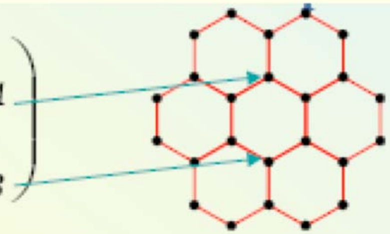

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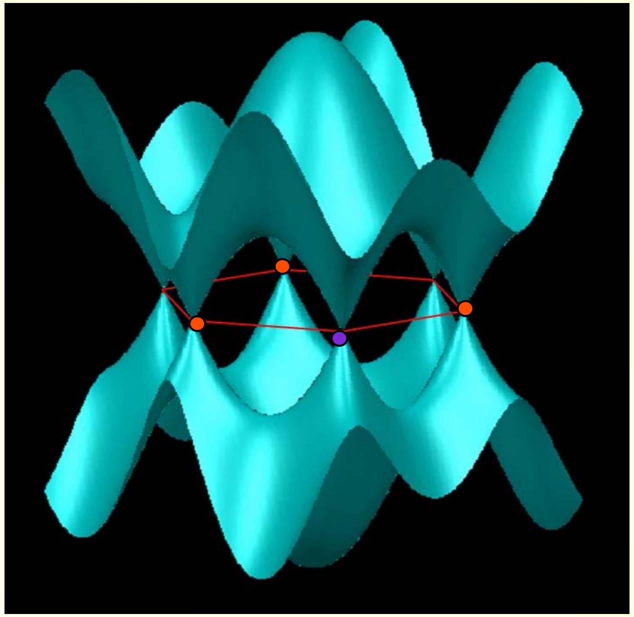

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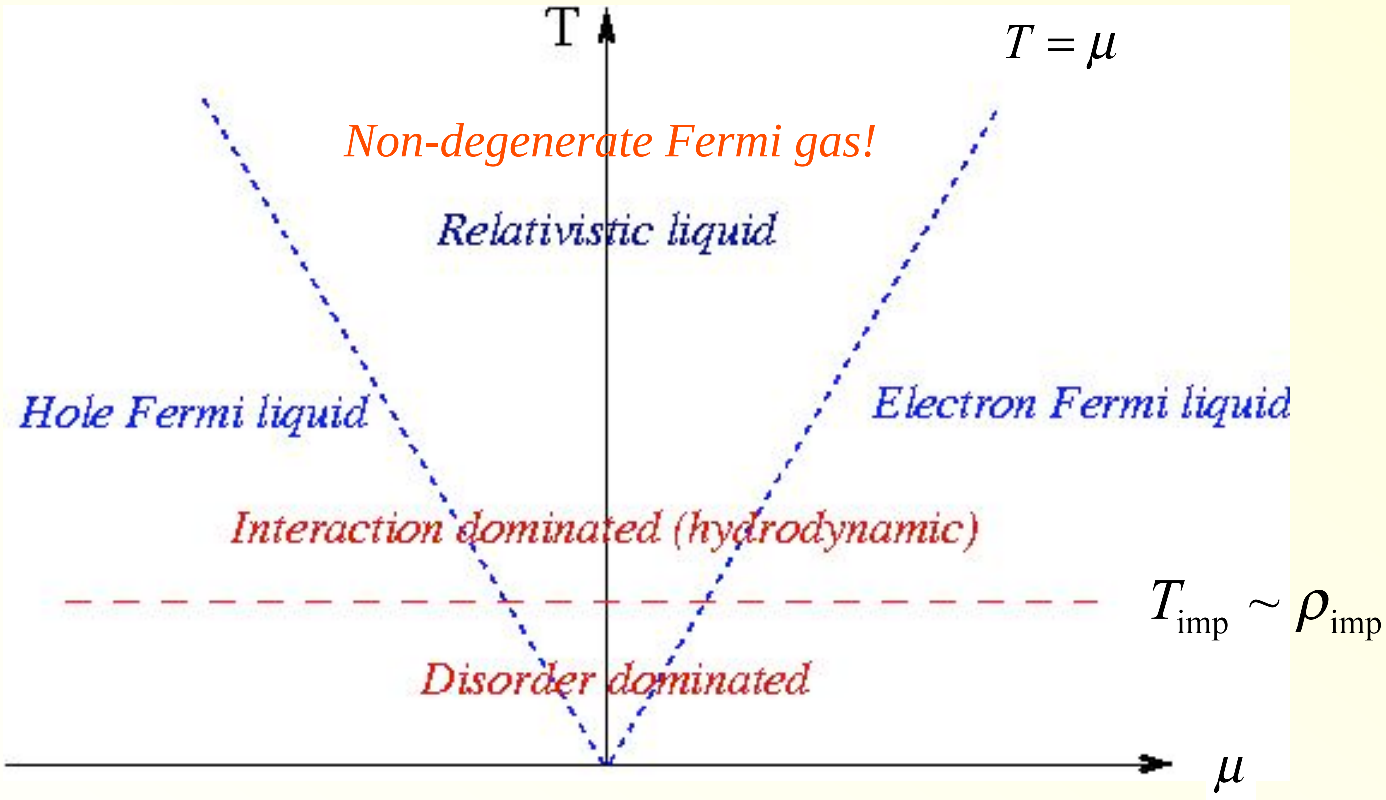
D. Sheehy, J. Schmalian, Phys. Rev. Lett. ***99****, 226803 (2007).*

- Relativistic plasma physics of interacting particles and holes!



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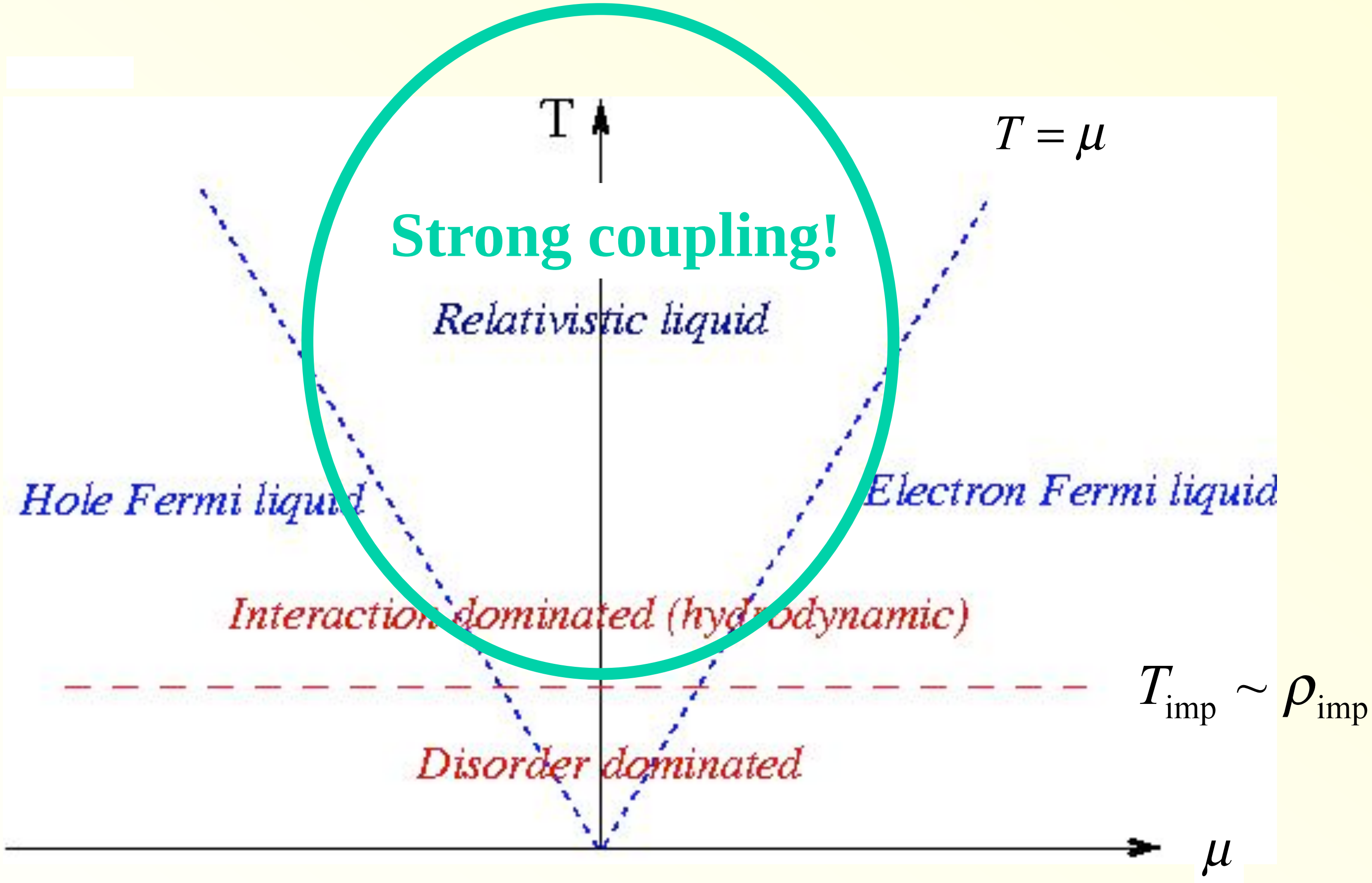
- Strongly coupled, nearly quantum critical fluid at $\mu = 0$



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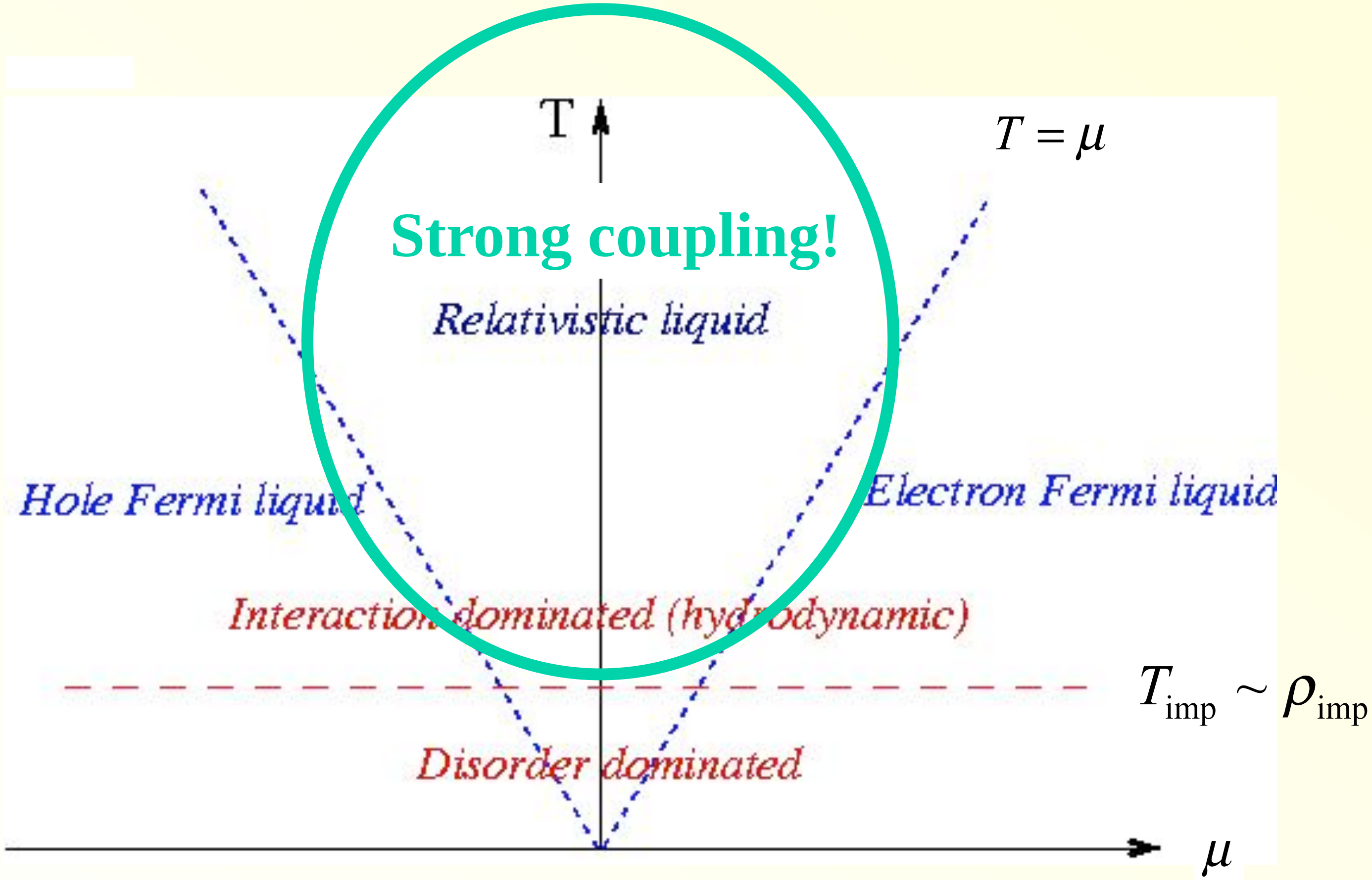

Very similar as for quantum criticality (e.g. SIT) and in their associated CFT's

Other relativistic fluids:

- Bismuth (3d Dirac fermions with very small mass)

- Systems close to quantum criticality (with z = 1)

Example: Superconductor-insulator transition (Bose-Hubbard model)

Maximal possible relaxation rate!

$$\tau_{rel}^{-1} \approx \frac{\hbar}{k_B T}$$

Damle, Sachdev (1996)
Bhaseen, Green, Sondhi (2007).
Hartnoll, Kovtun, MM, Sachdev (2007)

- Conformal field theories (critical points)

E.g.: strongly coupled Non-Abelian gauge theories (akin to QCD):
→ Exact treatment via AdS-CFT correspondence!

C. P. Herzog, P. Kovtun, S. Sachdev, and D. T. Son (2007)
Hartnoll, Kovtun, MM, Sachdev (2007)

Are Coulomb interactions strong?

Fine structure constant (QED concept)

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r_s (Wigner crystal concept) $$r_s \equiv \frac{E_{Cb}(n)}{E_F(n)} = \frac{\sqrt{n}\, e^2/\varepsilon}{\hbar \mathrm{v}_F \sqrt{\pi n}} = \frac{\alpha}{\sqrt{\pi}}$$ Small!?

n-independent!

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Recall QED/QCD:

- The coupling strength α depends on the scale.
- Different theories have different scale behavior!

α is the high energy limit of the coupling.
But we care about α(T)!

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Coulomb interactions:
Unexpectedly strong!
$\longrightarrow$ nearly quantum critical!

$$V(\mathbf{q}) = \frac{2\pi e^2}{\varepsilon|\mathbf{q}|}$$

$$H_1 = \frac{1}{2}\int \frac{d^2k_1}{(2\pi)^2}\frac{d^2k_2}{(2\pi)^2}\frac{d^2q}{(2\pi)^2}\Psi_a^\dagger(\mathbf{k}_2-\mathbf{q})\Psi_a(\mathbf{k}_2)V(\mathbf{q})\Psi_b^\dagger(\mathbf{k}_1+\mathbf{q})\Psi_b(\mathbf{k}_1)$$

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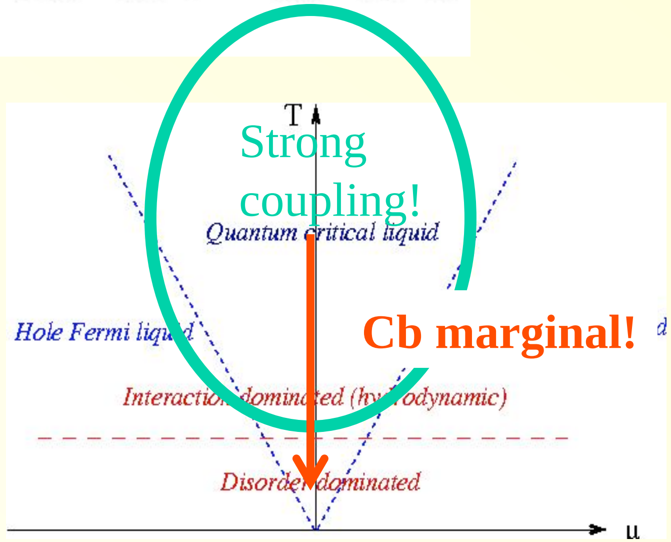
RG:
($\mu = 0$)



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RG:
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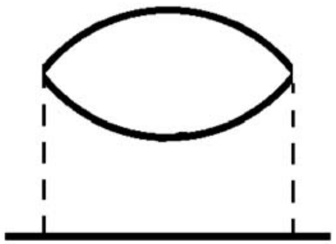
$$\frac{d\alpha}{d\ell} = -\frac{\alpha^2}{4} + \mathcal{O}(\alpha^3)$$

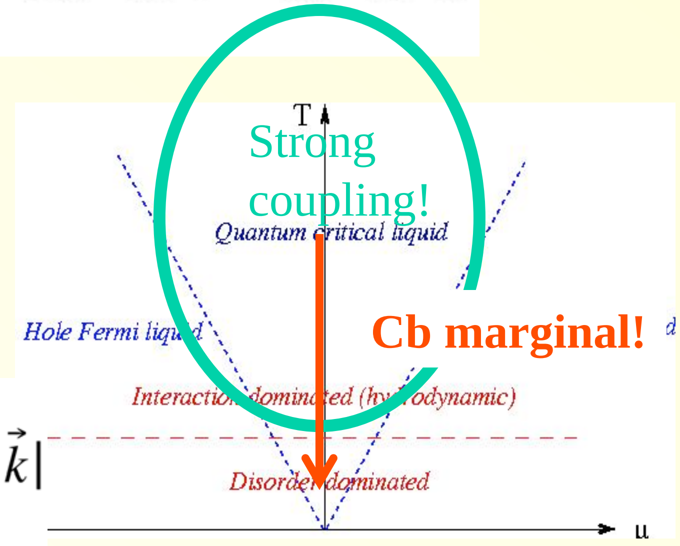
$$\alpha \equiv \frac{e^2}{\varepsilon\hbar v_F} = O(1)$$

$$\alpha(T) = \frac{\alpha^0}{1+(\alpha^0/4)\ln(\Lambda/T)} \overset{T\to 0}{\sim} \frac{4}{\ln(\Lambda/T)}$$



$$\mathrm{Im}\,\Sigma(\omega,\vec{k}) = \frac{1}{48}\left(\frac{e^2}{\epsilon_0\hbar v_F}\right)^2 \hbar v_F|\vec{k}|$$

RG flow of v_F ↔ RG flow of α



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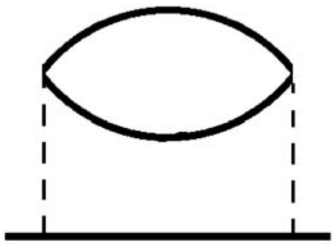

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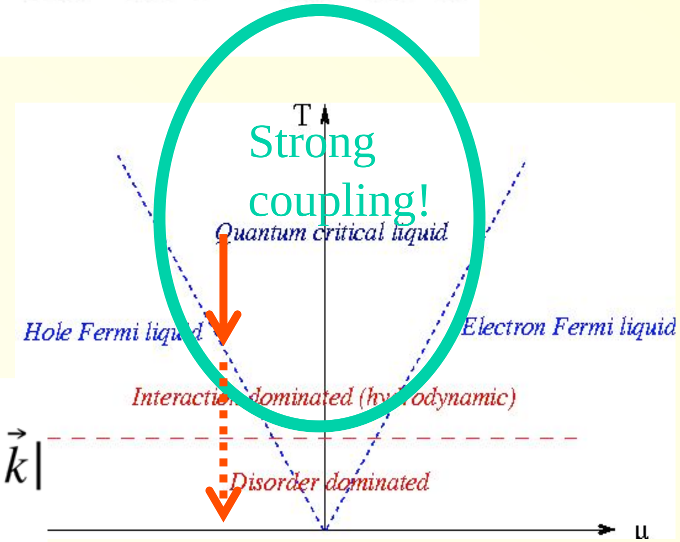

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But: ($\mu > 0$)

For $T < \mu$: screening kicks in, short ranged Cb irrelevant

Consequences for transport

1. Collision-limited conductivity σ
in clean undoped graphene

2. Emergent relativistic invariance
at low frequencies!

3. Graphene is a perfect quantum
liquid: very small viscosity η!

Hydrodynamic approach to transport

Strong coupling in undoped graphene

MM, L. Fritz, and S. Sachdev, PRB '08.

1. Inelastic scattering rate
(Electron-electron interactions)

μ >> T: standard 2d Fermi liquid

$$\tau_{ee}^{-1} = C \frac{(k_B T)^2}{\hbar \mu}$$

C: Independent of the Coulomb coupling strength!

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Relaxation rate ~ T,
like in quantum critical systems!
Fastest possible rate!

$\mu <$ T: strongly coupled relativistic liquid

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"Heisenberg uncertainty principle for well-defined quasiparticles"

$$E_{qp} (\sim k_B T) \geq \Delta E_{\mathrm{int}} = \hbar \tau_{\mathrm{ee}}^{-1} \sim \alpha^2 k_B T$$

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"Heisenberg uncertainty principle for well-defined quasiparticles"

$$E_{qp}(\sim k_B T) \geq \Delta E_{int} = \hbar\tau_{ee}^{-1} \sim \alpha^2 k_B T$$

As long as α(T) ~ 1, energy uncertainty is saturated, scattering is maximal
→ Nearly universal strong coupling features in transport,
similarly as at the 2d superfluid-insulator transition [Damle, Sachdev (1996, 1997)]

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(Scattering from charged impurities)
Subdominant at high T, low disorder

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(Cyclotron frequency of non-interacting particles with thermal energy)

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Hydrodynamic regime:
(collision-dominated)

$$\tau_{\rm ee}^{-1} >> \tau_{\rm imp}^{-1}, \tau_{\rm B}^{-1}, \omega$$

Hydrodynamics

Hydrodynamic collision-dominated regime $\tau_{\mathrm{ee}}^{-1} >> \tau_{\mathrm{imp}}^{-1}, \tau_{\mathrm{B}}^{-1}, \omega$

Long times,
Large scales $t >> \tau_{\mathrm{ee}}$

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Long times,
Large scales $t >> \tau_{\mathrm{ee}}$

- Local equilibrium established: $T_{\mathrm{loc}}(r)\ ,\ \mu_{\mathrm{loc}}(r)\ ;\ \vec{u}_{\mathrm{loc}}(r)$

- Study relaxation towards global equilibrium

- Slow modes: Diffusion of the density of conserved quantities:
  - Charge
  - Momentum
  - Energy

Relativistic Hydrodynamics

*S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B **76**, 144502 (2007).*

Energy-momentum tensor $T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$ $\begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{pmatrix}$

Current 3-vector $J^\mu = \rho u^\mu + \nu^\mu$ $\begin{pmatrix} \rho \\ \rho u_x + \nu_x \\ \rho u_y + \nu_y \end{pmatrix}$

u^μ : 3-velocity: $u^\mu = (1,0,0) \rightarrow$ No energy current

ν^μ : Dissipative current

$\tau^{\mu\nu}$: Viscous stress tensor (Reynold's tensor)

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\+ Thermodynamic relations

$$\varepsilon + P = Ts + \mu\rho, \quad d\varepsilon = Tds + \mu d\rho,$$

Gibbs-Duheme 1st law of thermodynamics

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Conservation laws (equations of motion):

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Energy/momentum conservation

$$\partial_\nu T^{\mu\nu} = F^{\mu\nu} J_\nu$$

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$$\vec{E} = -i\vec{k}\frac{2\pi}{|k|}\rho_{\vec{k}}$$ Coulomb interaction

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Weak disorder → momentum relaxation

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Dissipative current and viscous tensor?

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Dissipative current and viscous tensor?

Heat current $Q^\mu = (\varepsilon + P)u^\mu - \mu J^\mu$

$\rightarrow$ Entropy current $S^\mu = Q^\mu / T$

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Positivity of entropy production (Second law):

$$\partial_\mu S^\mu \equiv A_\alpha\left(\partial T, \partial\mu, F^{\mu\nu}\right)\nu^\alpha + B_{\alpha\beta}\left(\partial T, \partial\mu, F^{\mu\nu}\right)\tau^{\alpha\beta} \geq 0$$

$$\Rightarrow \nu^\mu = \text{const.} \times A^\mu\left(\partial T, \partial\mu, \partial u; F^{\mu\nu}\right)$$

$$\tau^{\mu\nu} = \text{const.} \times B^{\mu\nu} + \text{const.} \times \delta^{\mu\nu} B_\alpha^\alpha$$

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$\rightarrow$

$$\nu^\mu = \sigma_Q(g^{\mu\nu} + u^\mu u^\nu)\left[(-\partial_\nu\mu + F_{\nu\lambda}u^\lambda) + \mu\frac{\partial_\mu T}{T}\right]$$

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Relativistic Hydrodynamics

*S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B **76**, 144502 (2007).*

Dissipative current and viscous tensor?

Heat current $Q^\mu = (\varepsilon + P)u^\mu - \mu J^\mu$

$\rightarrow$ Entropy current $S^\mu = Q^\mu / T$

Positivity of entropy production (Second law):

$$\partial_\mu S^\mu \equiv A_\alpha(\partial T, \partial\mu, F^{\mu\nu})\nu^\alpha + B_{\alpha\beta}(\partial T, \partial\mu, F^{\mu\nu})\tau^{\alpha\beta} \geq 0$$

$$\Rightarrow \nu^\mu = \text{const.} \times A^\mu(\partial T, \partial\mu, \partial u; F^{\mu\nu})$$

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Irrelevant for response at k $\rightarrow$ 0

One single transport coefficient (instead of two)!

Meaning of σ_Q ?

- At zero doping (particle-hole symmetry):

$$\sigma_Q = \sigma_{xx}\left(\rho_{\text{imp}} = 0\right) < \infty \quad !$$

→ Interaction-limited conductivity of the pure system!

How is it possible that $\sigma_{xx}\left(\rho_{\text{imp}} = 0\right)$ is finite ??

Collision-limited conductivity

Damle, Sachdev, (1996).
Fritz et al. (2008), Kashuba (2008)

Finite conductivity in a pure system at particle-hole symmetry ($\rho = 0$)!

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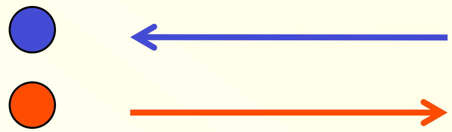
- Key: Charge current without momentum!

(particle)

(hole)



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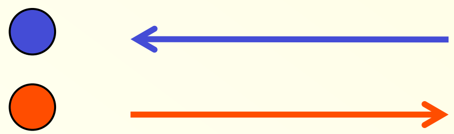

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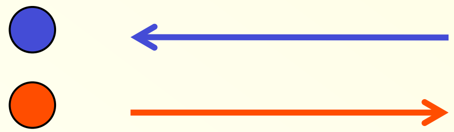

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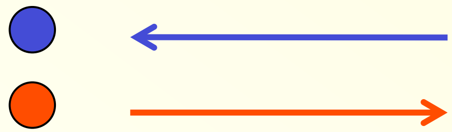

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$$\sigma_{Drude} = \frac{e}{m}\rho\tau \rightarrow \sigma(\mu = 0) \sim \frac{e}{k_B T / \mathrm{v}^2}\left(e\frac{(k_B T)^2}{(\hbar \mathrm{v})^2}\right)\frac{\hbar}{\alpha^2 k_B T} \sim \frac{1}{\alpha^2}\frac{e^2}{h}$$

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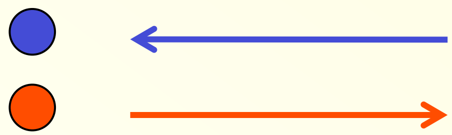

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Marginal irrelevance of Coulomb: $\alpha \approx \frac{4}{\log(\Lambda/T)}$

Back to Hydrodynamics

S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B ***76****, 144502 (2007).*

Elements discussed so far:

Conservation laws (equations of motion):

$\partial_\mu J^\mu = 0$ Charge conservation

$$\partial_\nu T^{\mu\nu} = F^{\mu\nu} J_\nu + \frac{1}{\tau_{\text{imp}}} T^{0\nu}$$

Energy/momentum conservation

Dissipative current (relating electrical and energy current)

$$\nu^\mu = \sigma_Q (g^{\mu\nu} + u^\mu u^\nu)\left[(-\partial_\nu \mu + F_{\nu\lambda} u^\lambda) + \mu \frac{\partial_\mu T}{T} \right]$$

Thermoelectric response

S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B ***76****, 144502 (2007).*

Charge and heat current:

$$J^\mu = \rho\, u^\mu - \nu^\mu$$

$$Q^\mu = (\varepsilon + P) u^\mu - \mu\, J^\mu$$

Thermo-electric response

$$\begin{pmatrix} \vec{J} \\ \vec{Q} \end{pmatrix} = \begin{pmatrix} \hat{\sigma} & \hat{\alpha} \\ T\hat{\alpha} & \hat{\bar{\kappa}} \end{pmatrix} \begin{pmatrix} \vec{E} \\ -\vec{\nabla} T \end{pmatrix}$$

$$\hat{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix}$$

etc.

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etc.

i) Solve linearized conservation laws

ii) Read off the response functions from the dynamic response to initial conditions! *(see Kadanoff & Martin, 1960)*

Results from Hydrodynamics

Response functions at B=0

Symmetry z ⟶ -z : $\sigma_{xy} = \alpha_{xy} = \kappa_{xy} = 0$

Longitudinal conductivity:

$$\sigma_{xx}(\omega, k; B=0) = \left(\sigma_Q + \frac{\rho^2}{P+\varepsilon}\frac{\tau}{1-i\omega\tau}\right)$$

Collision-limited conductivity at the quantum critical point r = 0

Drude-like conductivity, divergent for $\tau \to \infty, \omega \to 0, \rho \neq 0$

Momentum conservation (r≠0)!

Response functions at B=0

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Longitudinal conductivity:

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Thermal conductivity:

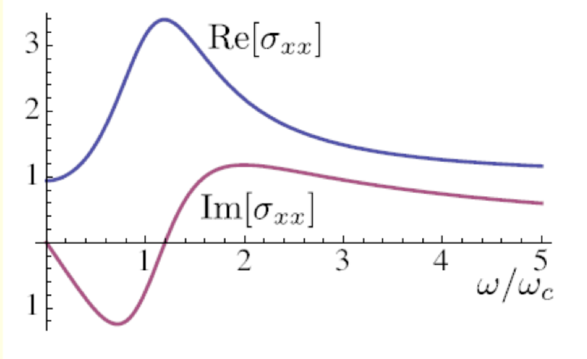
$$\kappa_{xx}(\omega, k; B=0) = \sigma_Q \frac{\mu^2}{T} + \frac{s^2 T}{P+\varepsilon}\frac{\tau}{1-i\omega\tau} + \mathcal{O}(k^2).$$

Relativistic Wiedemann-Franz-like relations between σ and κ in the quantum critical window!

B > 0 : Cyclotron resonance

E.g.: Longitudinal conductivity

$$\sigma_{xx}(\omega) = \sigma_Q \frac{\omega\left(\omega + i\gamma + i\omega_c^2/\gamma\right)}{(\omega + i\gamma)^2 - \omega_c^2}$$

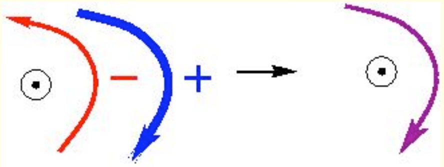


Pole in the response

$$\omega = \pm\omega_c^{\mathrm{QC}} - i\gamma$$

Collective cyclotron frequency of the relativistic plasma

$$\omega_c^{\mathrm{QC}} = \frac{\rho\, B/c}{(\varepsilon + P)/\mathrm{v}_{\mathrm{F}}^2} \leftrightarrow \omega_c^{\mathrm{FL}} = \frac{e\,B/c}{m}$$



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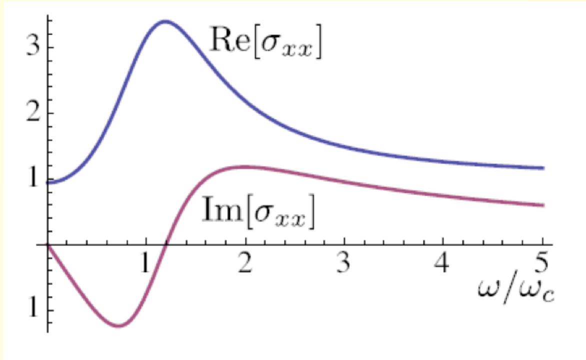

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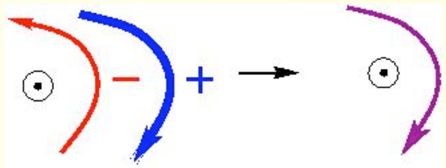


Intrinsic, interaction-induced broadening
(↔ Galilean invariant systems:
No broadening due to Kohn's theorem)

$$\gamma = \sigma_Q \frac{(B/c)^2}{(\varepsilon + P)/\mathrm{v}_{\mathrm{F}}^2}$$

Observable at room temperature in the GHz regime!

Boltzmann Approach

MM, L. Fritz, and S. Sachdev, PRB 2008

→ Recover and refine the hydrodynamic description

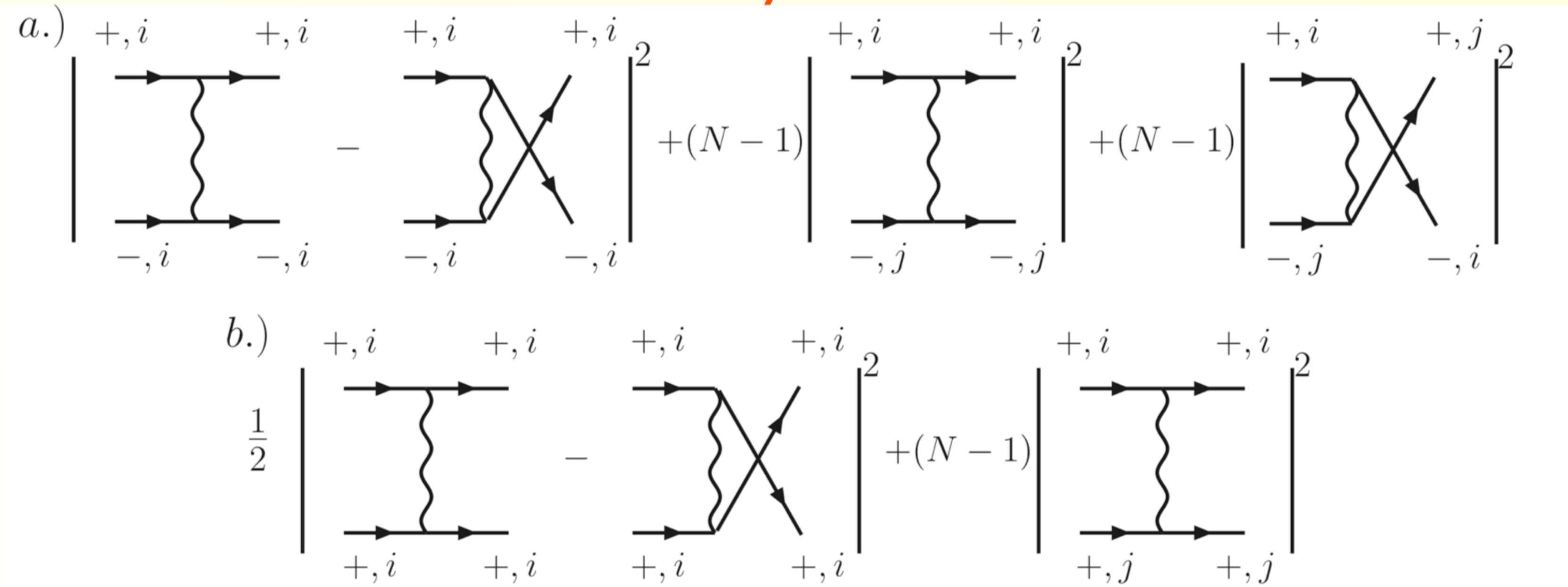
→ Describe relativistic-to-Fermi-liquid crossover

Boltzmann approach

L. Fritz, J. Schmalian, MM, and S. Sachdev, PRB 2008

Boltzmann equation in Born approximation

$$\left(\partial_t + e\mathbf{E}\cdot\frac{\partial}{\partial \mathbf{k}}\right) f_{\pm}(\mathbf{k},t) = I_{\text{coll}}^{Cb}\left[\mathbf{k},t \mid \{f_{\pm}(\mathbf{k}',t)\}\right] \propto \alpha^2(T)$$



→ **e.g.: collision-limited conductivity:**

$$\sigma(\mu = 0) = \frac{0.76}{\alpha^2(T)}\frac{e^2}{h}$$

Relativistic hydrodynamics from microscopics

Does relativistic hydro really apply to graphene even though Coulomb interactions break relativistic invariance?

Yes! Within weak-coupling theory:

Key point: There is a zero (“momentum”) mode of the collision integral due to translational invariance of the interactions

The dynamics of the zero mode under an AC driving field reproduces relativistic hydrodynamics at low frequencies.

What to do beyond the weak coupling (Boltzmann) approximation [$\alpha(T) \rightarrow 0$] ??

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Newest progress from string theory:

1) **Look at "similar" theories which are very strongly coupled, but can be solved exactly**

2) **Try to extract the "generally valid, universal" part of the result and use it as a guide**

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Warning: There is no AdS-CFT mapping for graphene itself!

Compare graphene to: Strongly coupled relativistic liquids

S. Hartnoll, P. Kovtun, MM, S. Sachdev (2007)

Obtain exact results via string theoretical AdS–CFT correspondence

$\longrightarrow$ Response functions in particular strongly coupled relativistic fluids
(for maximally supersymmetric Yang Mills theories with $N \rightarrow \infty$ colors):

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Obtain exact results via string theoretical AdS–CFT correspondence

→ Response functions in particular strongly coupled relativistic fluids
(for maximally supersymmetric Yang Mills theories with $N \to \infty$ colors):

- Confirm the structure of the hydrodynamic response functions such as s(w).
- Calculate the transport coefficients for a strongly coupled theory!

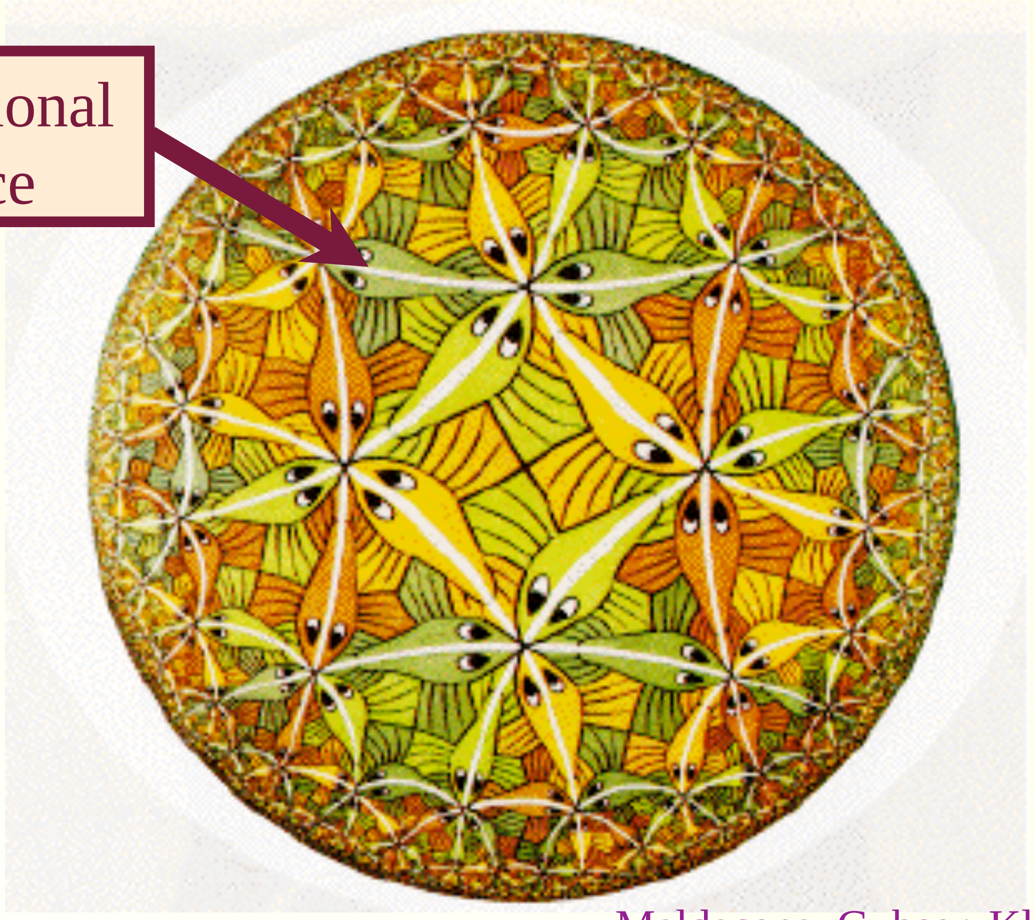
SUSY - SU(N): $\sigma(\mu = 0) = \sqrt{\frac{2}{9}} N^{3/2} \frac{e^2}{h}$; $\frac{\eta_{shear}}{s}(\mu = 0) = \frac{1}{4\pi} \frac{\hbar}{k_B}$

Interpretation: $N^{3/2}$ **effective degrees of freedom, strongly coupled: τT = O(1)**

AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions

3+1 dimensional AdS space



Maldacena, Gubser, Klebanov, Polyakov, Witten

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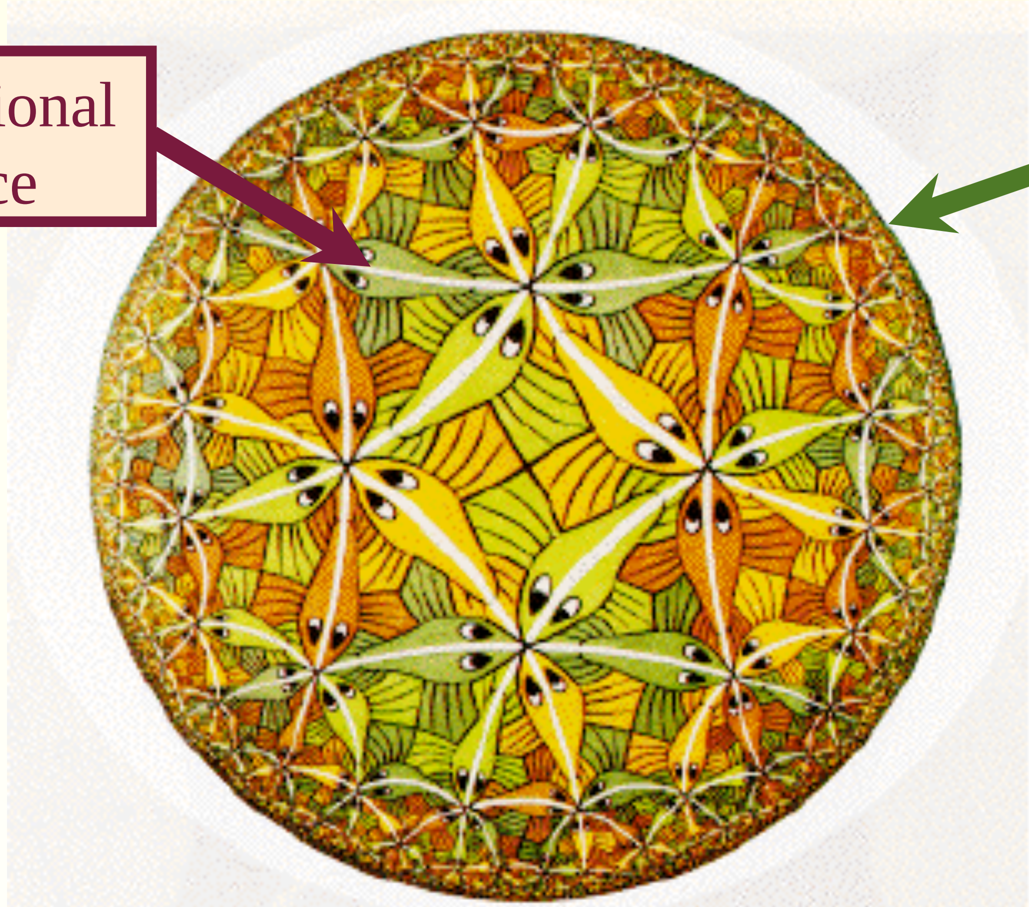

A 2+1 dimensional system at its quantum critical point

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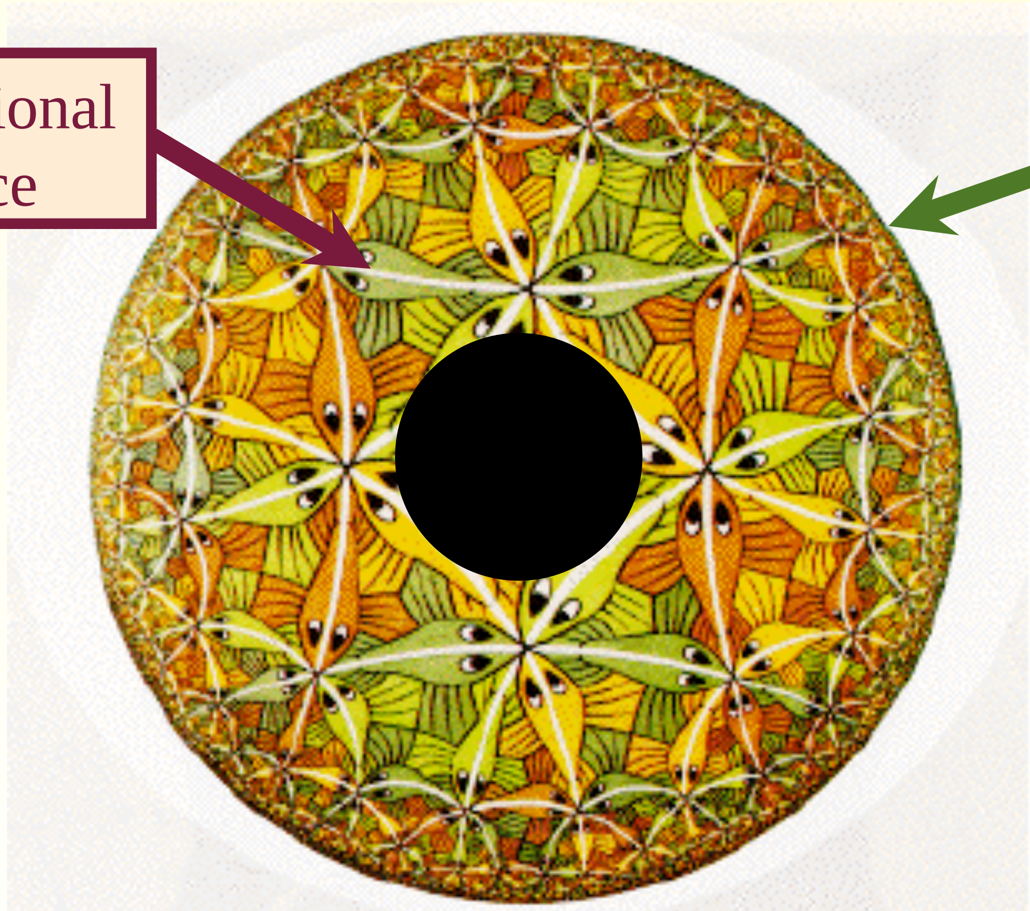

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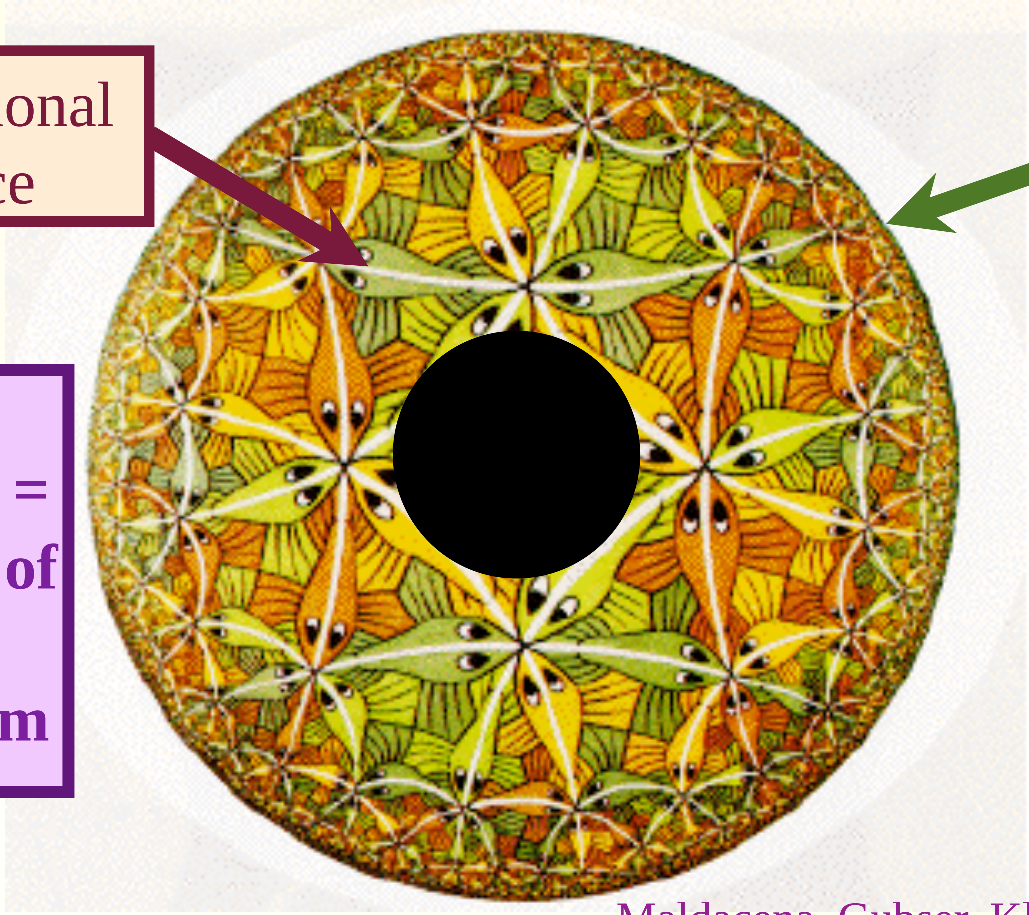

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Black hole temperature = temperature of quantum critical system

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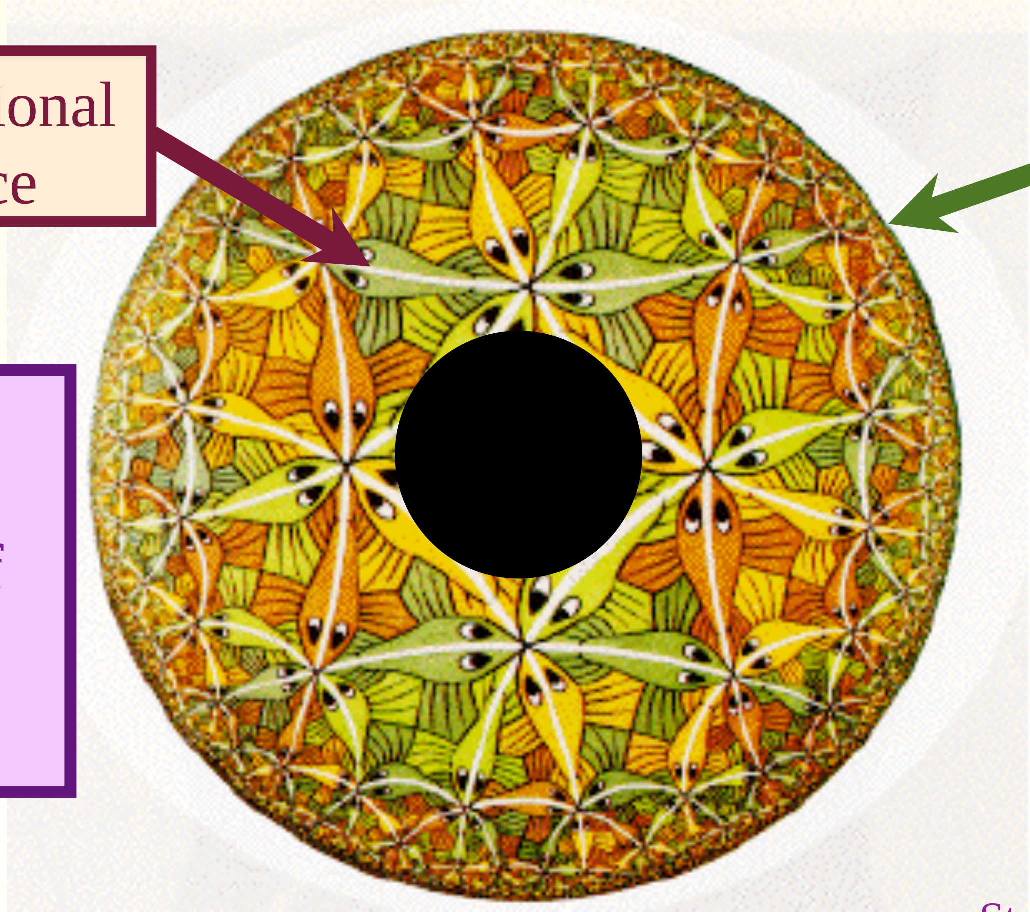

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Black hole entropy = Entropy of quantum criticality

Strominger, Vafa

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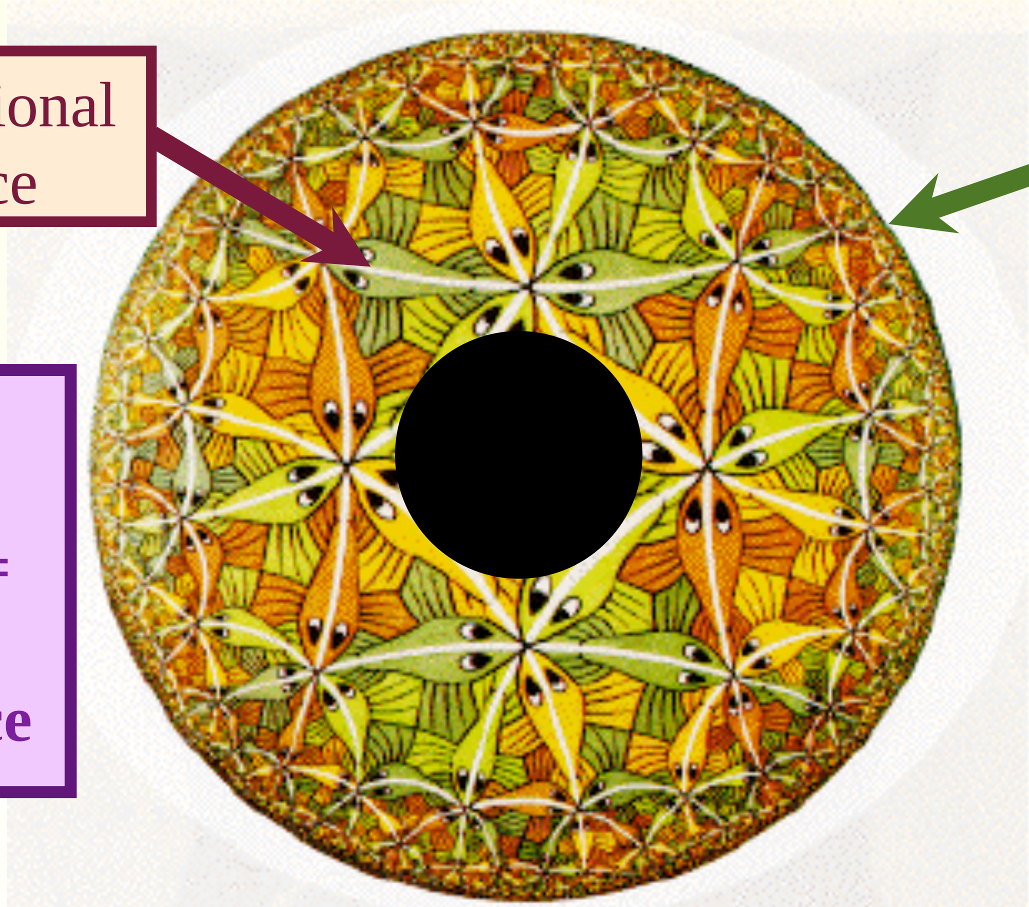

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Quantum critical dynamics = waves in curved space

Maldacena, Gubser, Klebanov, Polyakov, Witten

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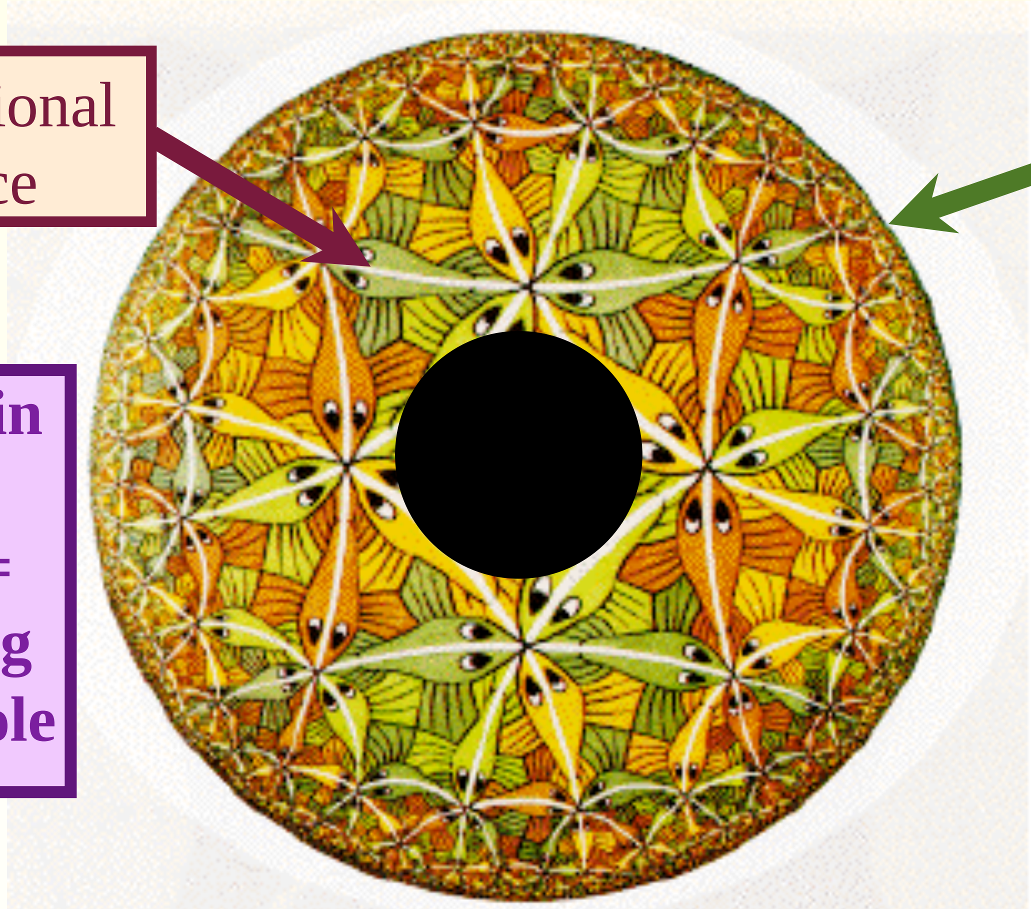
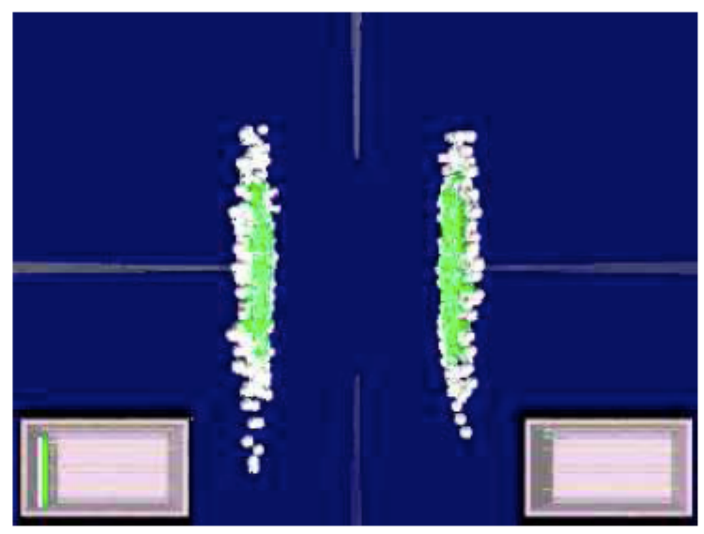


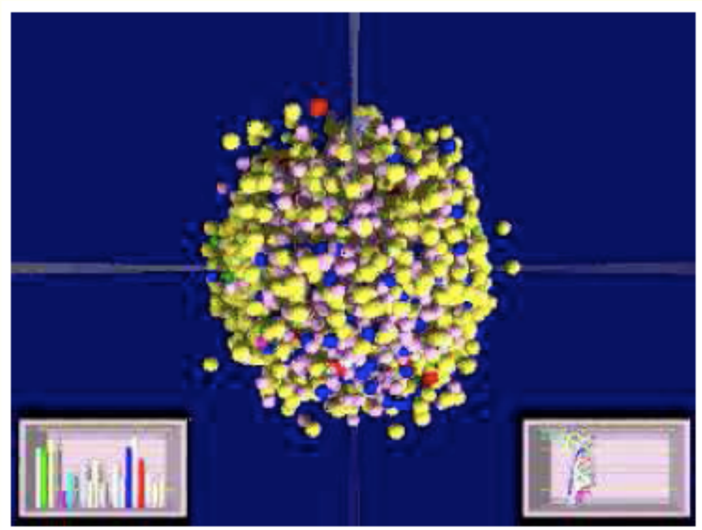
Kovtun, Policastro, Son

Au+Au collisions at RHIC



Quark-gluon plasma can be described by QCD (nearly conformal, critical theory)

Extremely low viscosity fluid!



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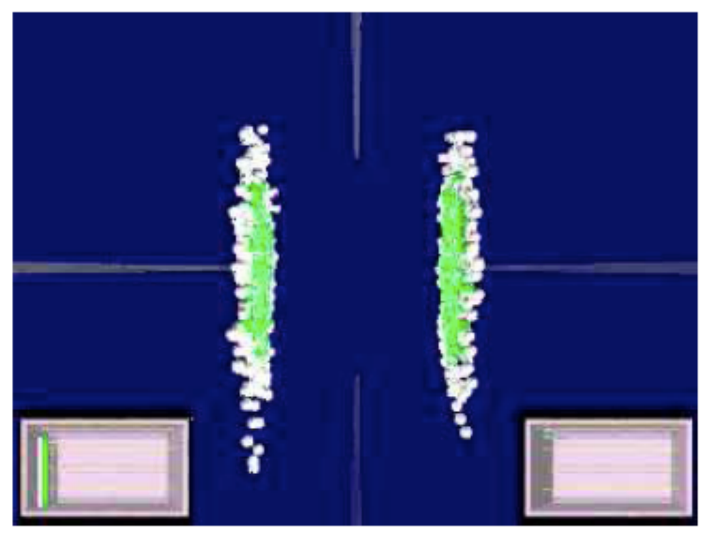

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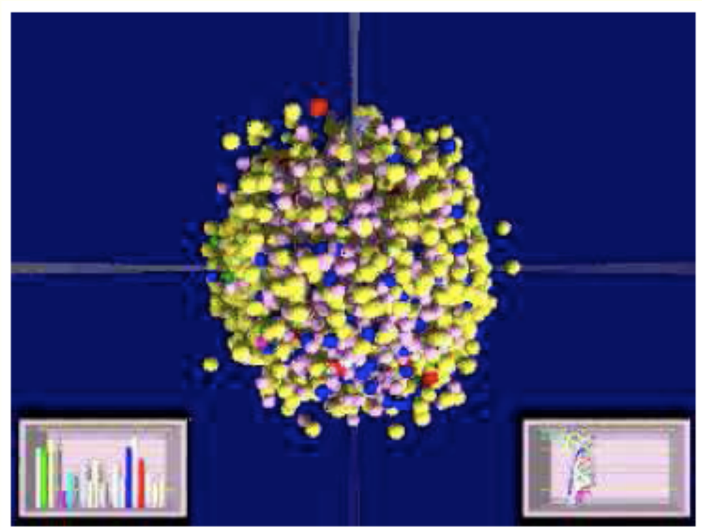

SUSY - SU(N):

$$\frac{\eta_{shear}}{s}(\mu = 0) = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

This IS an extremely low value!
Is there a lowest possible value, or a "most perfect" liquid?

Graphene – a nearly perfect liquid!

MM, J. Schmalian, and L. Fritz, (PRL 2009)

↔

Anomalously low viscosity (like quark-gluon plasma)

"Heisenberg" $$\frac{\eta}{s} \sim E_{qp}\tau \geq 1$$ → Measure of strong coupling:

Conjecture from AdS-CFT:

$$\frac{\text{shear viscosity}}{\text{entropy density}} = \frac{\eta}{s} > \frac{\hbar}{k_B}\frac{1}{4\pi}$$

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Doped Graphene & Fermi liquids:
(Khalatnikov etc)

$\eta \propto n \cdot m\text{v}^2 \cdot \tau \rightarrow n \cdot E_F \cdot \frac{\hbar E_F}{(k_B T)^2}$ $s \propto k_B n \frac{T}{E_F}$ $\frac{\eta}{s} \sim \frac{\hbar}{k_B}\left(\frac{E_F}{T}\right)^3$

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↔

Anomalously low viscosity (like quark-gluon plasma)

"Heisenberg" $\frac{\eta}{s} \sim E_{qp}\tau \geq 1$ → Measure of strong coupling:

Conjecture from AdS-CFT:

$$\frac{\text{shear viscosity}}{\text{entropy density}} = \frac{\eta}{s} > \frac{\hbar}{k_B}\frac{1}{4\pi}$$

Doped Graphene & Fermi liquids: (Khalatnikov etc)

$$\eta \propto n \cdot m\mathrm{v}^2 \cdot \tau \;\rightarrow\; n \cdot E_F \cdot \frac{\hbar E_F}{(k_B T)^2}$$

$$s \propto k_B n \frac{T}{E_F}$$

$$\frac{\eta}{s} \sim \frac{\hbar}{k_B}\left(\frac{E_F}{T}\right)^3$$

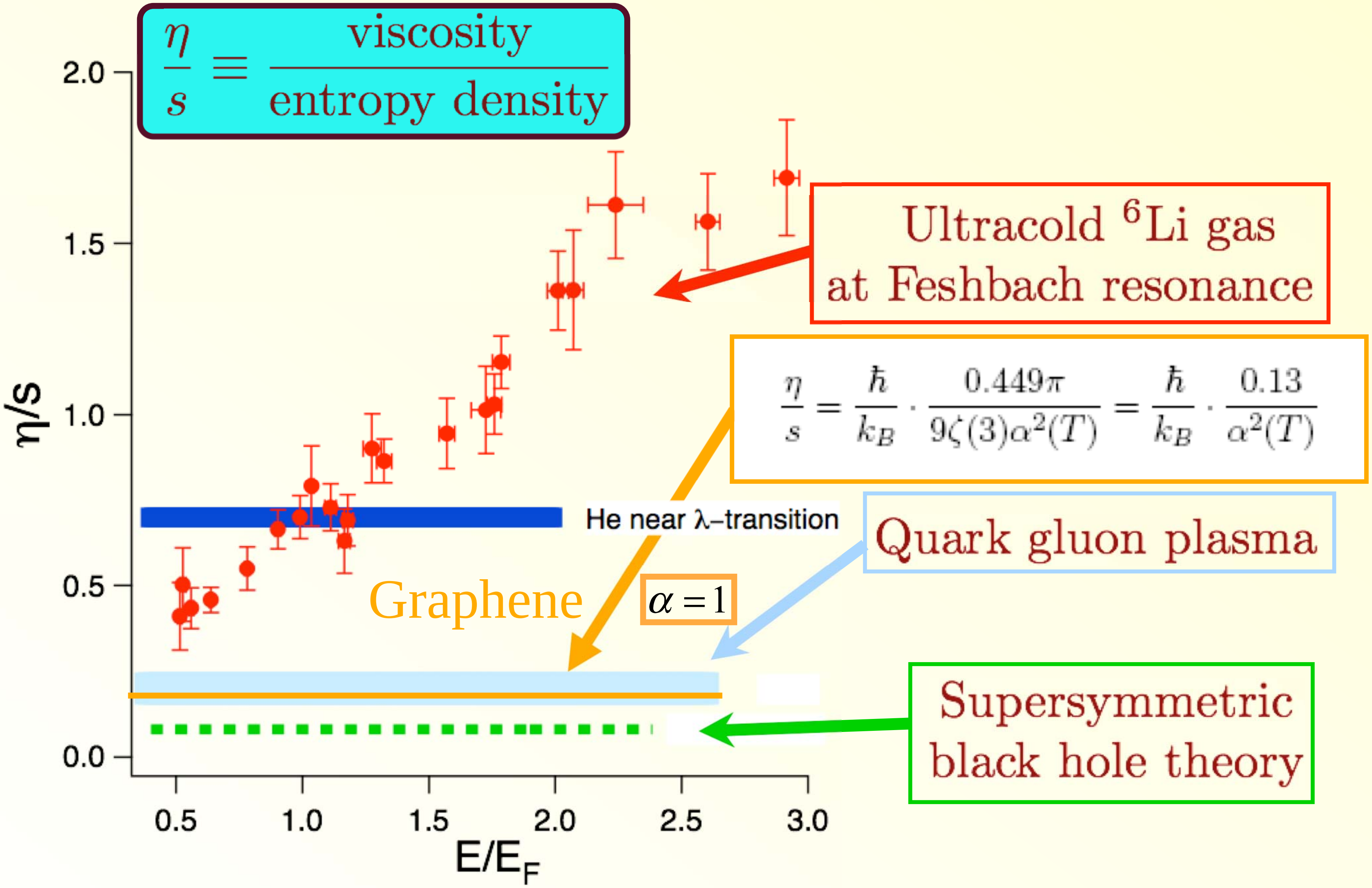
Undoped Graphene:

$$\eta \propto n \cdot m\mathrm{v}^2 \cdot \tau \;\rightarrow\; n_{\text{th}} \cdot k_B T \cdot \frac{\hbar}{\alpha^2 k_B T} = \frac{\hbar}{\alpha^2} n_{\text{th}}$$

$$s \propto k_B n_{\text{th}}$$

Boltzmann-Born Approximation:

$$\frac{\eta}{s} = \frac{\hbar}{k_B} \cdot \frac{0.449\pi}{9\zeta(3)\alpha^2(T)} = \frac{\hbar}{k_B} \cdot \frac{0.13}{\alpha^2(T)}$$



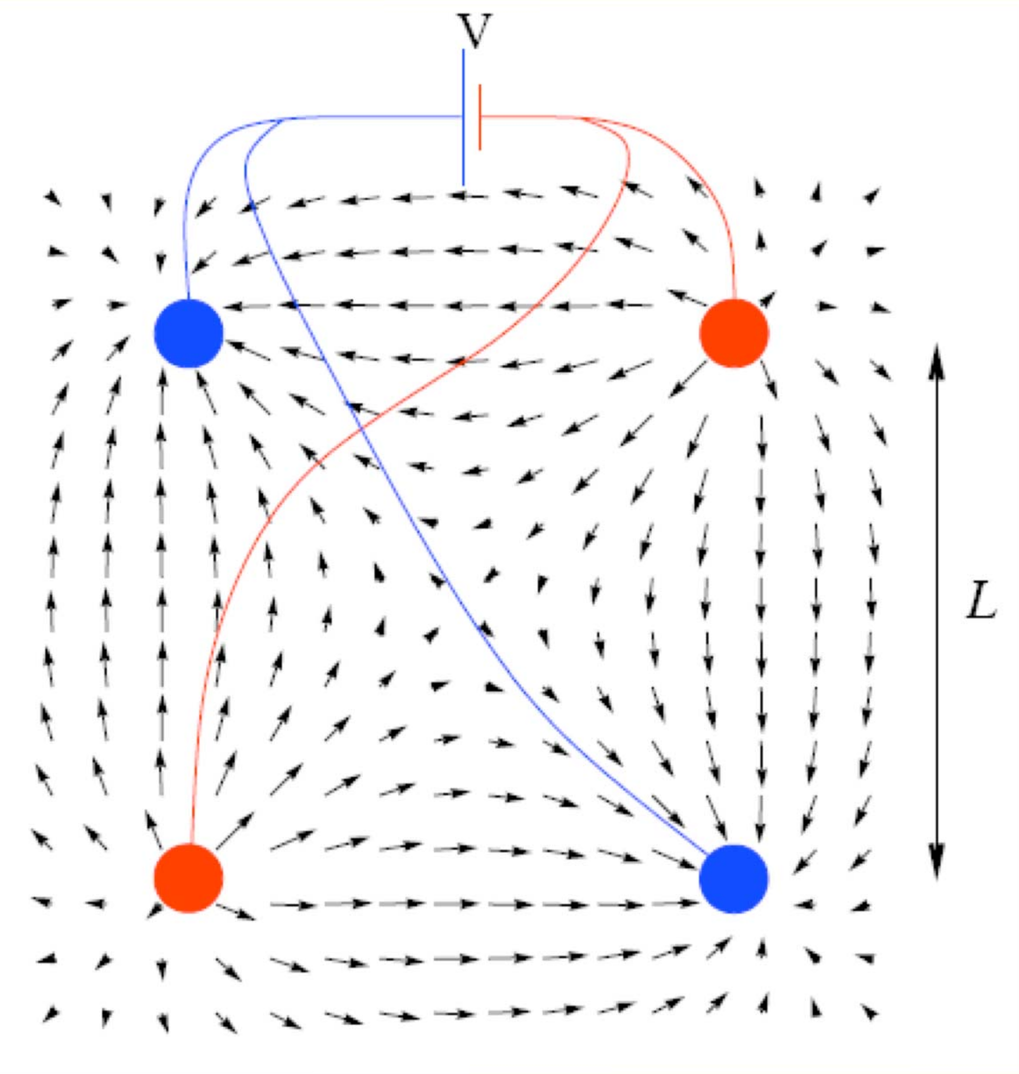
T. Schäfer, Phys. Rev. A 76, 063618 (2007).
A. Turlapov, J. Kinast, B. Clancy, Le Luo, J. Joseph, J. E. Thomas, J. Low Temp. Phys. 150, 567 (2008)

Electronic consequences of low viscosity?

MM, J. Schmalian, L. Fritz, (PRL 2009)

Expected viscous effects on conductance in non-uniform current flow:

Decrease of conductance with length scale L



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Electronic turbulence in clean, strongly coupled graphene?
(or at quantum criticality!)
Reynolds number:

$$\mathrm{Re} = \frac{s/k_B}{\eta/\hbar} \times \frac{k_B T}{\hbar v/L} \times \frac{u_{\rm typ}}{v}$$

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Strongly driven mesoscopic systems: (Kim's group [Columbia])

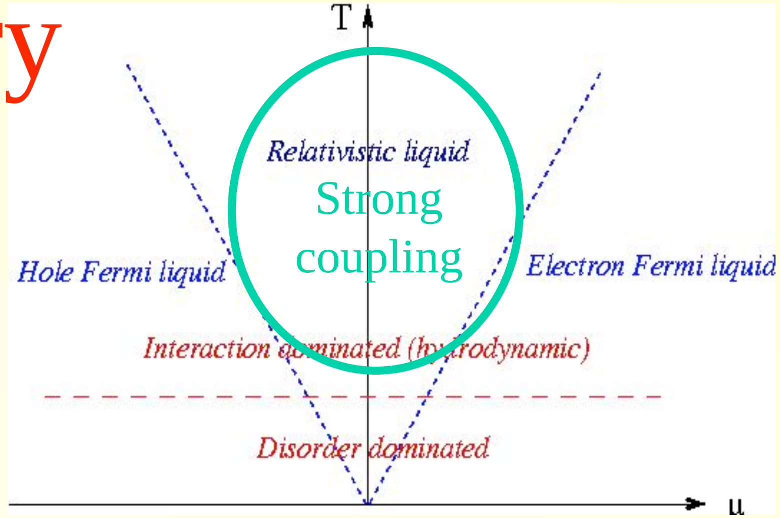
$L = 1\mu m$
$u_{typ} = 0.1\mathrm{v}$
$T = 100K$

→ $\mathrm{Re} \sim 10 - 100$

Complex fluid dynamics!
(pre-turbulent flow)

New phenomenon in an electronic system!

Summary



- Undoped graphene is strongly coupled in a large temperature window!
- Nearly universal strong coupling features in transport; many similarities with strongly coupled critical fluids (described by AdS-CFT)
- Emergent relativistic hydrodynamics at low frequency
- Graphene: Nearly perfect quantum liquid!
  → Possibility of complex (turbulent?) current flow at high bias