On electronic transport and many-body localization in the Bose glass

Markus Müller

Ann. Phys. (Berlin) **18**, 849 (2009).

Discussions with

M. Feigel'man, M.P.A. Fisher, L. Ioffe, V. Kravtsov,

Experiments: B. Sacépé, D. Shahar

Abdus Salam International Center of Theoretical Physics

Stanford University, 7th January, 2010

Outline

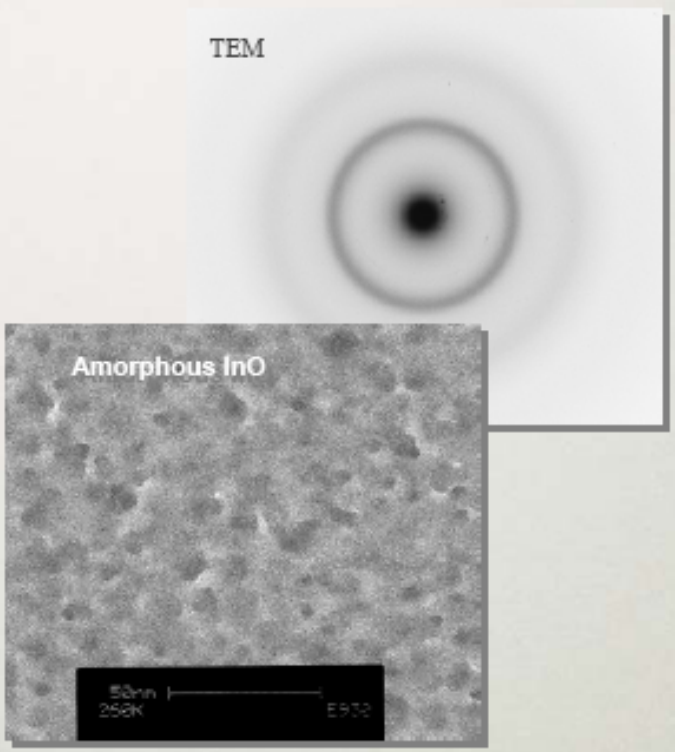
- The disordered superconductor-insulator transition (SIT) – dirty bosons
- Review of various puzzling transport experiments in the Bose glass phase
- Proposed resolution based on: Characterization of insulators via spectral properties
  - Consequences for transport: R(T)
  - ”Many-body localization” and its precursors

Indium-oxide (InO_x)

Indium-oxide: One of the materials used in the experiments discussed here

- Strong disorder
- Tunabile disorder

Similar experiments in TiN films



SI transition in thin films

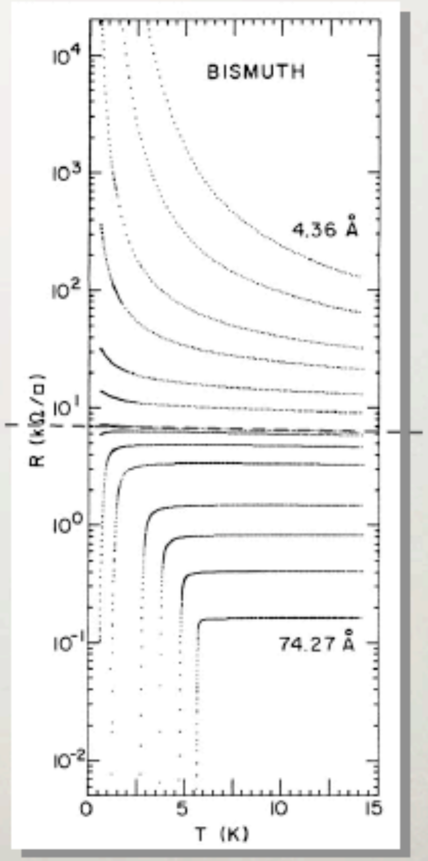
M. Strongin, et. al., Phys. Rev. **B1**, 1078 (1970).

D. B. Haviland, Y. Liu, and A. M. Goldman, Phys. Rev. Lett. 62, 2180 (1989)...

2D

Thickness tuned transition

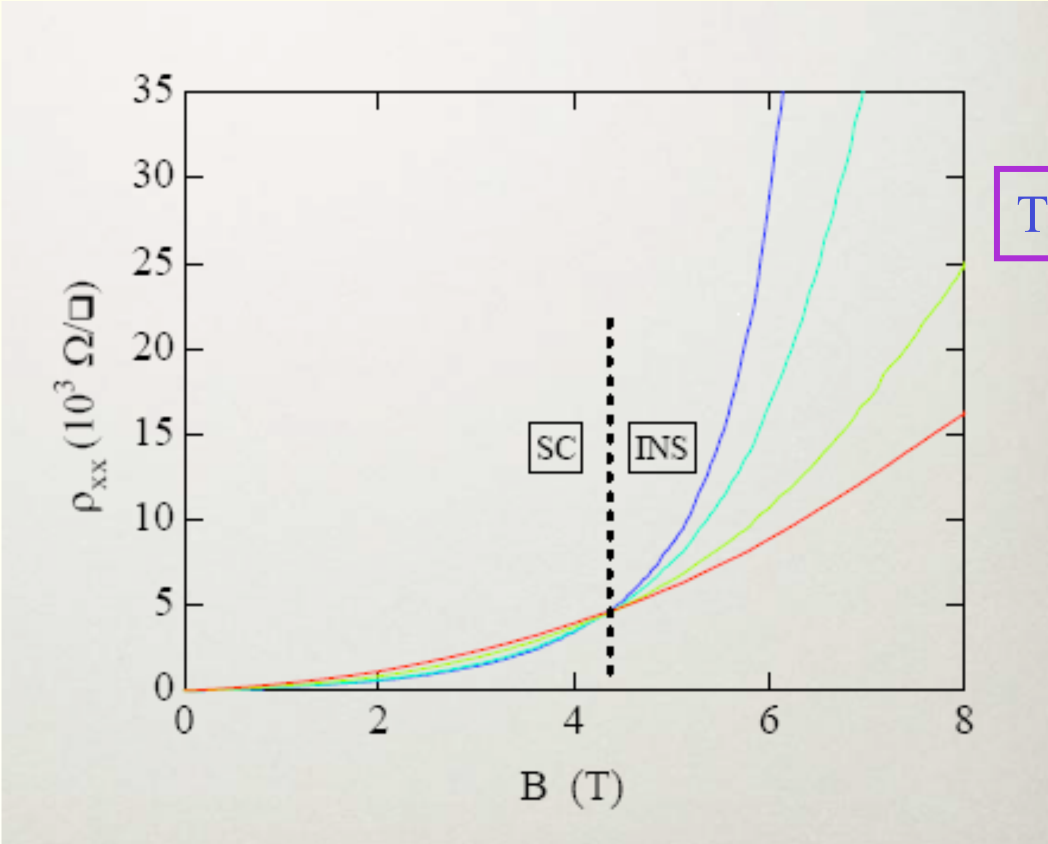
T = 0 transition



Review: Finkl'stein ('94), Markovic and Goldman ('98).

Field driven transition

Magnetic field destroys SC!

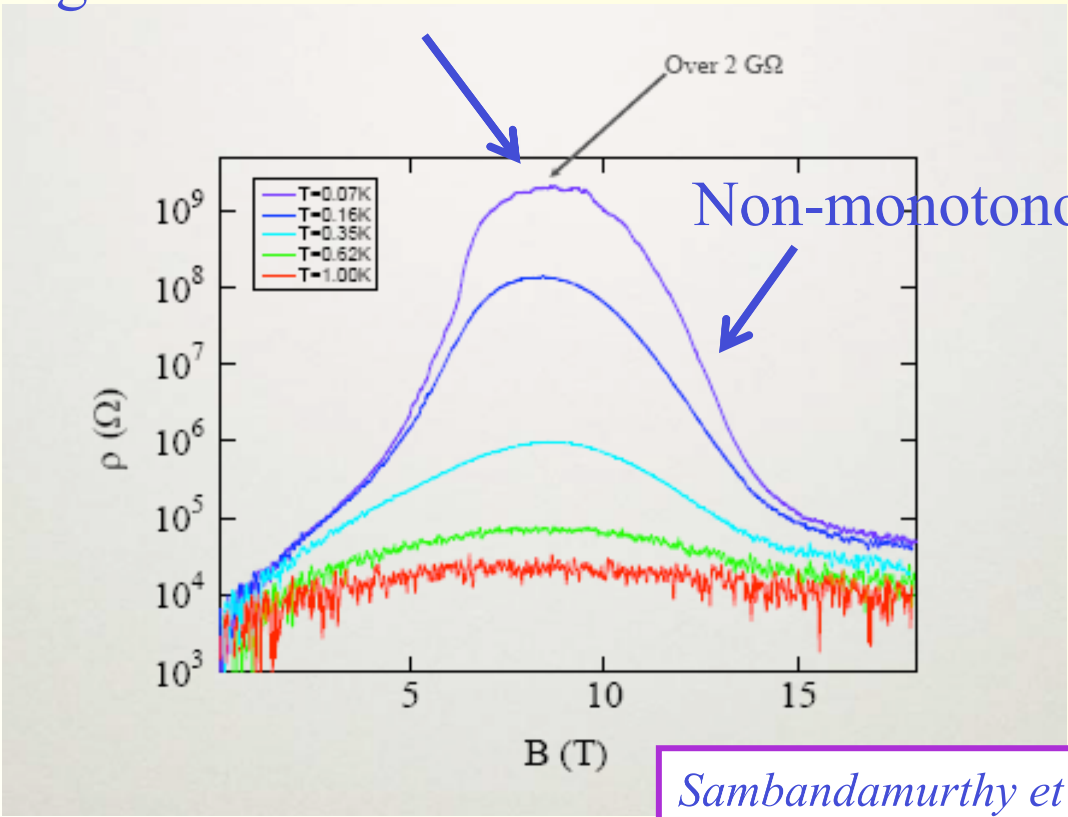


T = const.

Gantmakher, Shahar, Kapitulnik, Goldman, Baturina

Insulator: Giant magnetoresistance

Giant magnetoresistance



Non-monotonous R(B)!

Sambandamurthy et al. (PRL 2005)

Insulating behavior **enhanced** by local superconductivity!

Insulator: Giant magnetoresistance

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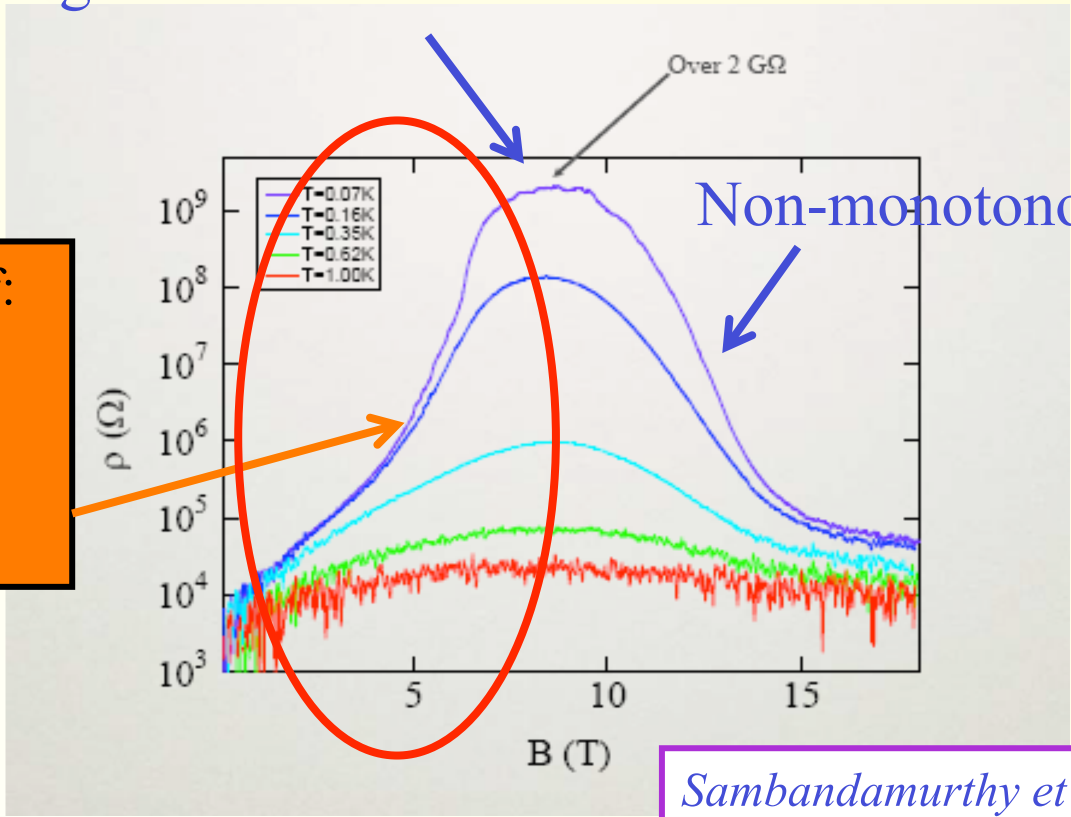


Non-monotonous R(B)!

Common belief:
Pairs (bosons) survive in the insulator:
Bose glass

Sambandamurthy et al. (PRL 2005)

Insulating behavior **enhanced** by local superconductivity!

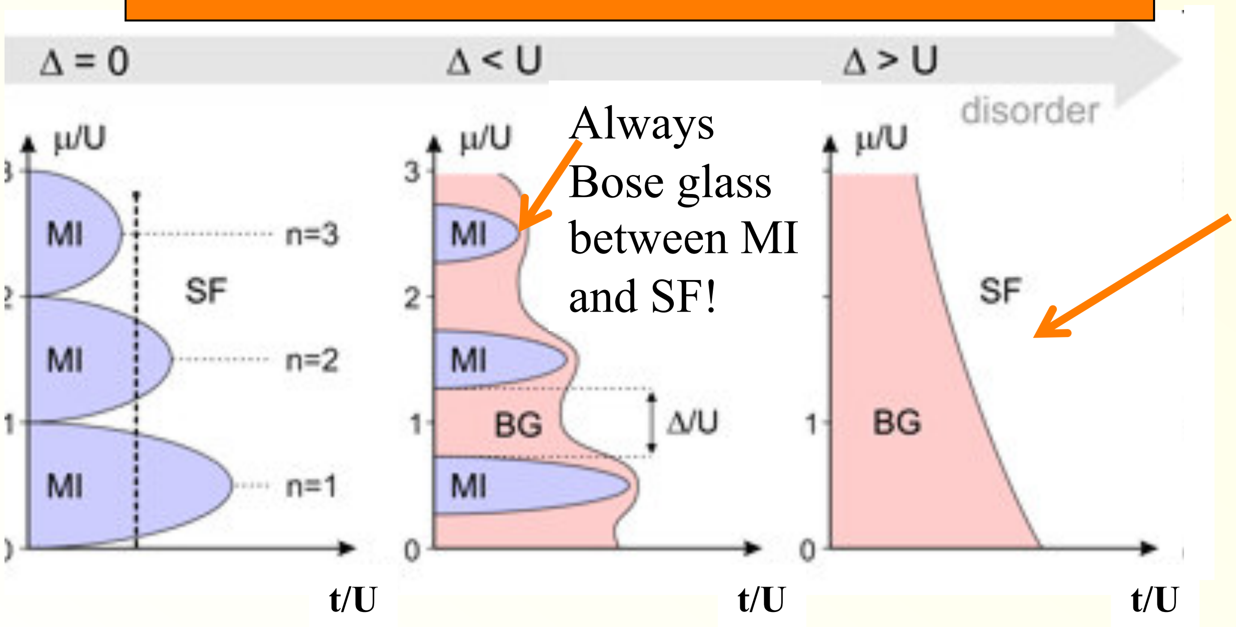
Bose-Hubbard model and Bose glass

Fisher et al., Phys. Rev. B 40, 546 (1989) --- Altland et al, Gurarie et al. (2009)

- Assume "preformed Cooper pairs": bosons without global superconductivity
- Dirty boson model (Bose-Hubbard model with disorder):

$$H = t\sum_{\langle i,j\rangle} b_i^+ b_j + U\sum_i n_i(n_i - 1) + \sum_i (\varepsilon_i - \mu)n_i$$

Disorder: $\varepsilon_i \in [-\Delta, \Delta]$



Most likely scenario for experiments: Strong disorder, no Mott gap!

Two puzzling features in transport in strongly disordered samples

1. Simple activation in R(T)

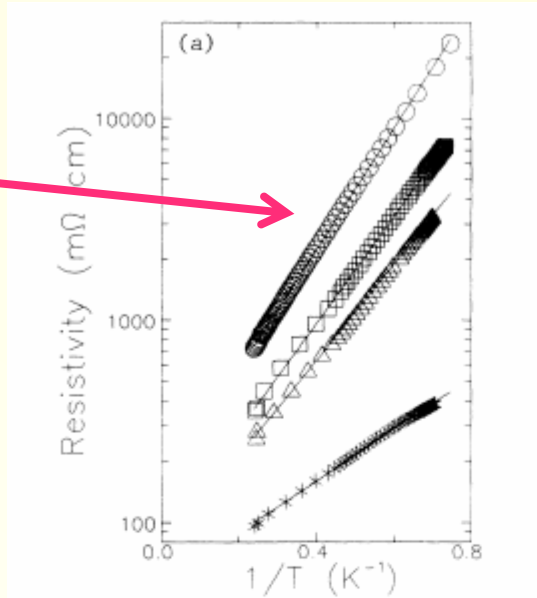
2. Evidence for purely electronic mechanism

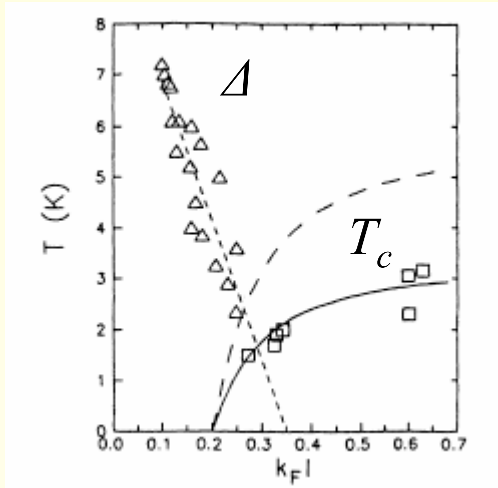
Activated transport near the SIT

*D. Shahar, Z. Ovadyahu, PRB **46**, 10971 (1992).*

Insulating InO_x

Simple activation!





$$R(T) = R_0 \exp\left[-\frac{\Delta}{T}\right]$$

Activation energy increases with distance to SIT

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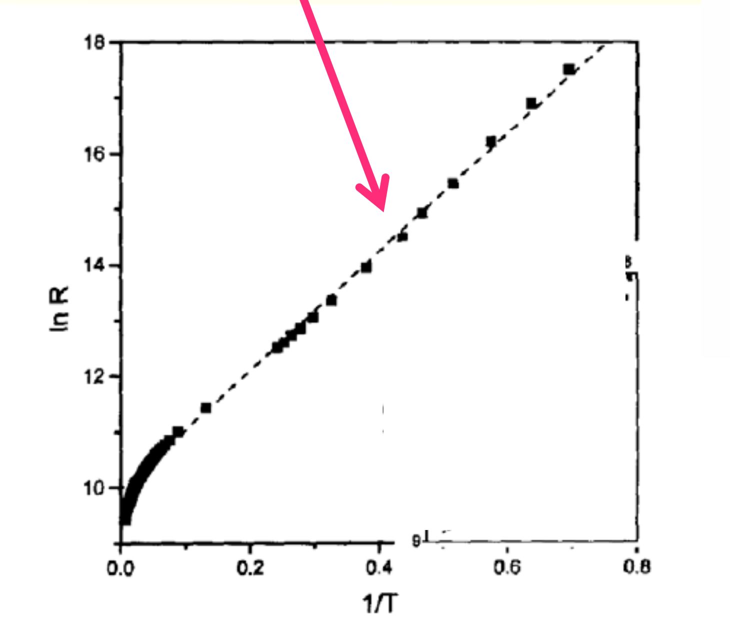

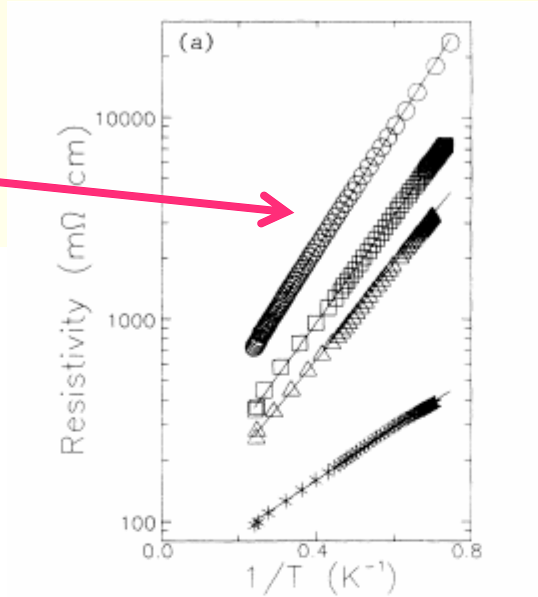

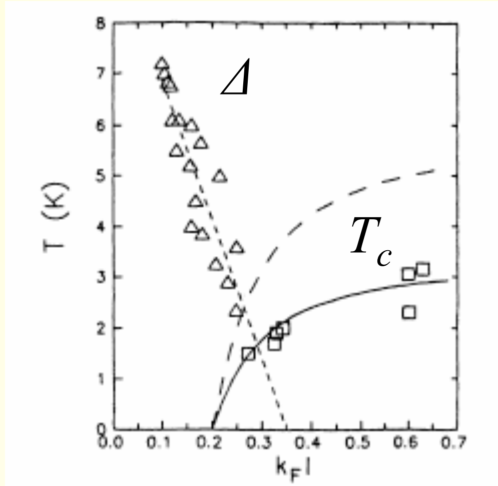

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*D. Kowal and Z. Ovadyahu, Sol. St. Comm. **90**, 783 (1994).*

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*V. F. Gantmakher, M. V. Golubkov, J. Lok, A. K. Geim, Sov. Phys. JETP, **82**, 951 (1996).*

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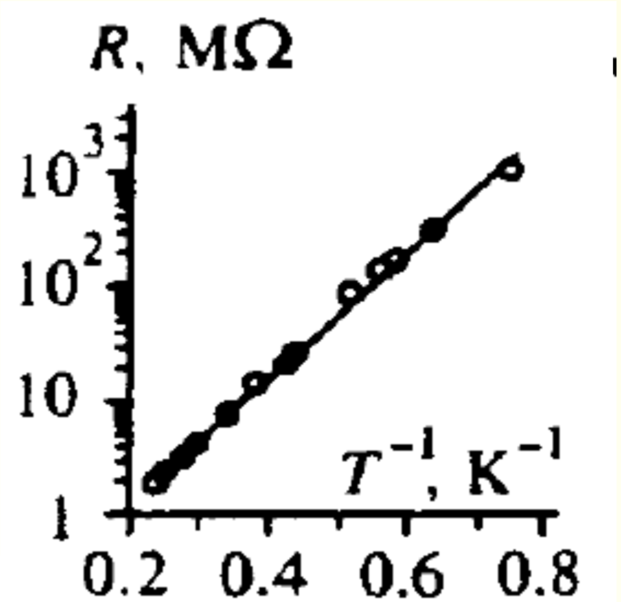

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Origin of simple activation?

• Gap in the density of states?

A: NO! Too disordered systems!

No Mott gap!

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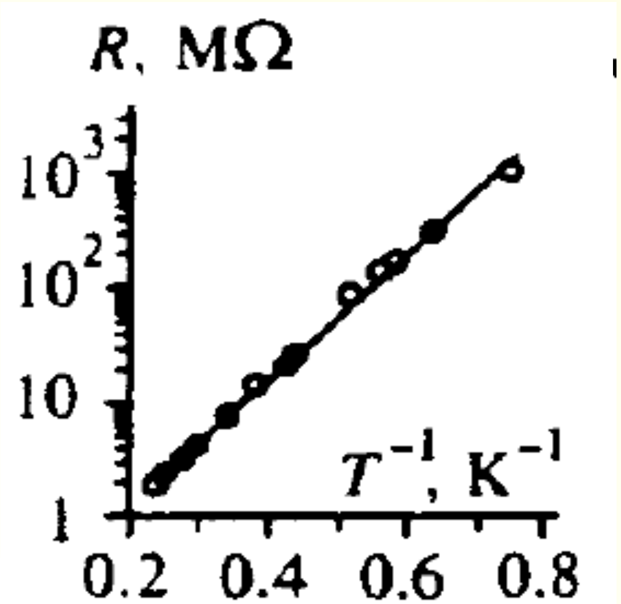

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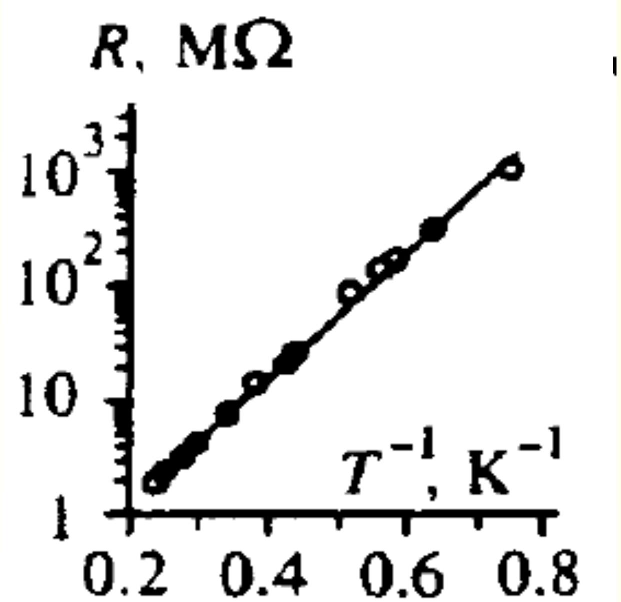

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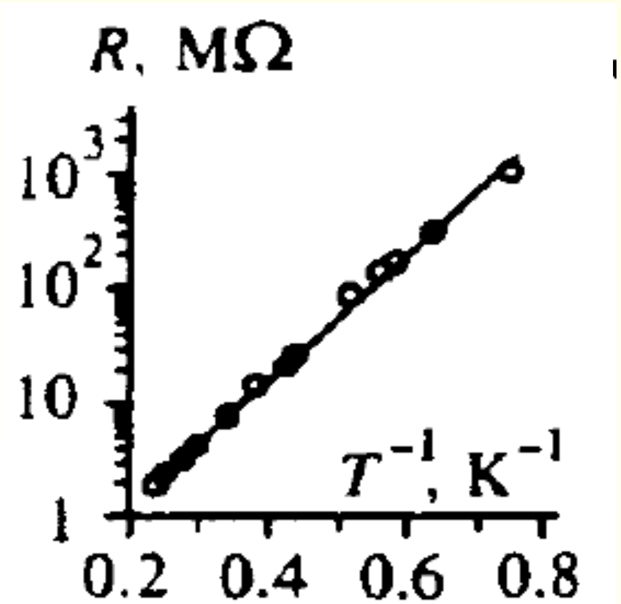

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• No depairing of bosons (positive MR!) [Feigel'man et al.]

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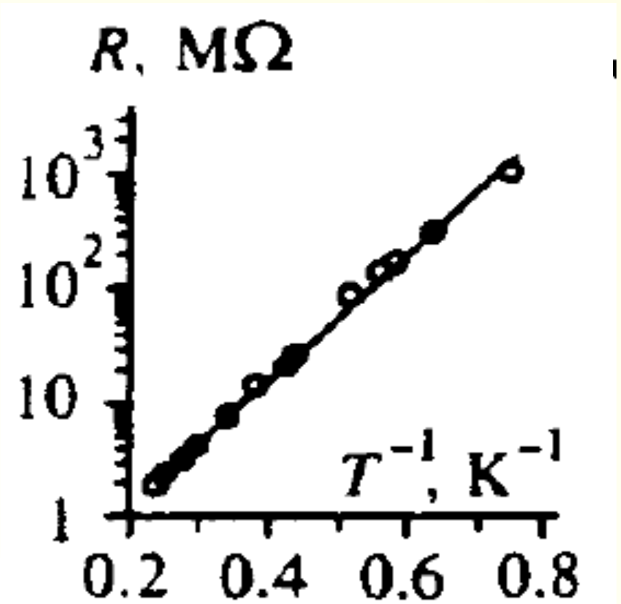

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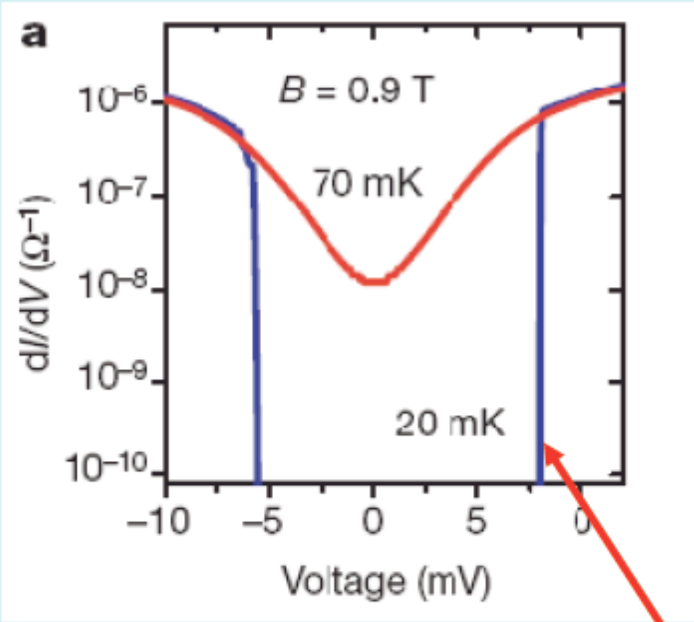
- No depairing of bosons (positive MR!)
- But: Boson mobility edge !
  (Similar to Anderson localization)

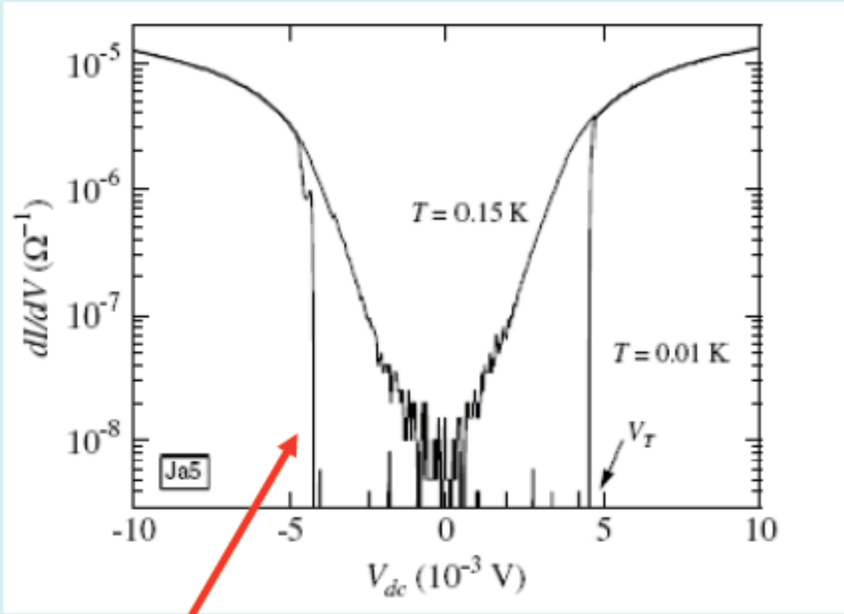
Purely electronic transport mechanism!

TiN films



Baturina, Mironov, Vinokur, Baklanov, Strunk,'07

InO films



Sambandamurthy, Engel, Johansson, Peled, Shahar, '05

Giant jumps in resistance from kΩ to GΩ regime

Non-Ohmic resistance in the insulator!

Purely electronic transport mechanism!

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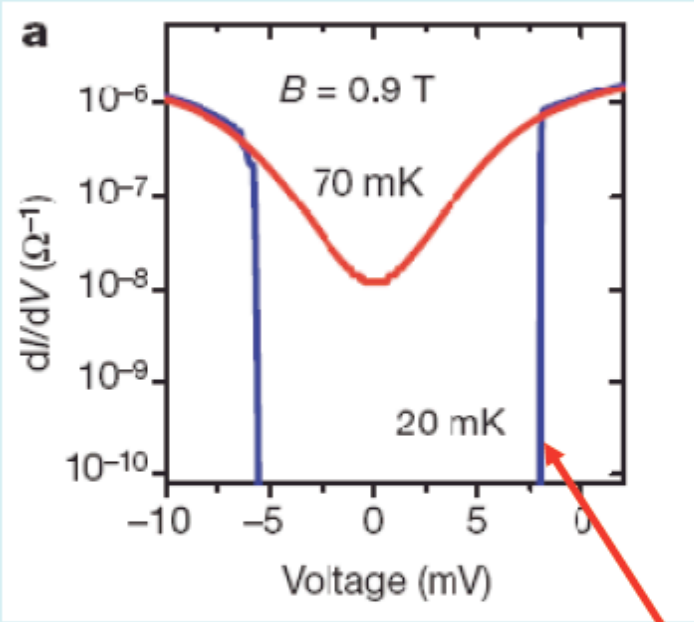

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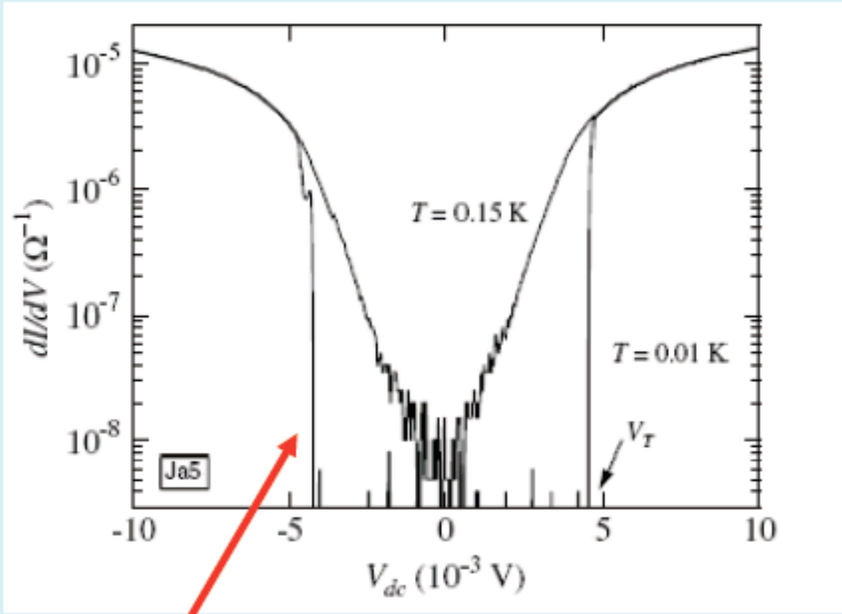

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Simple but effective explanation: bistability from low T to overheated state.

Altshuler, Kravtsov, Lerner, Aleiner (09)

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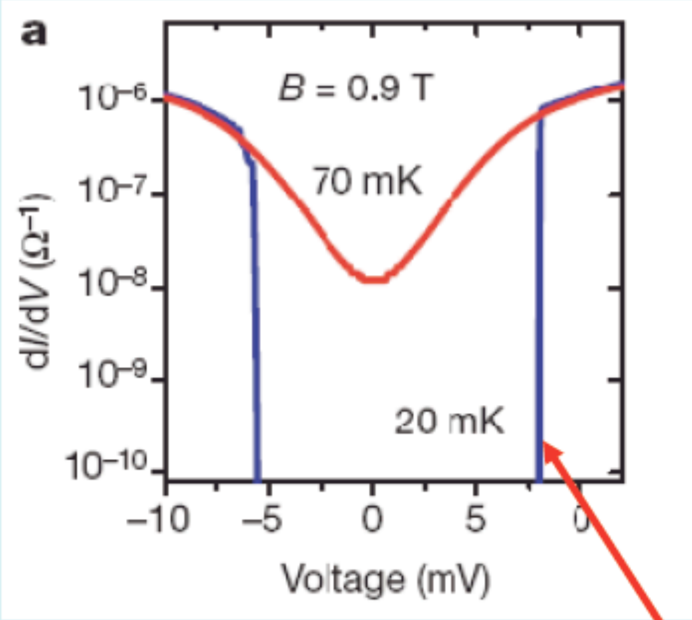

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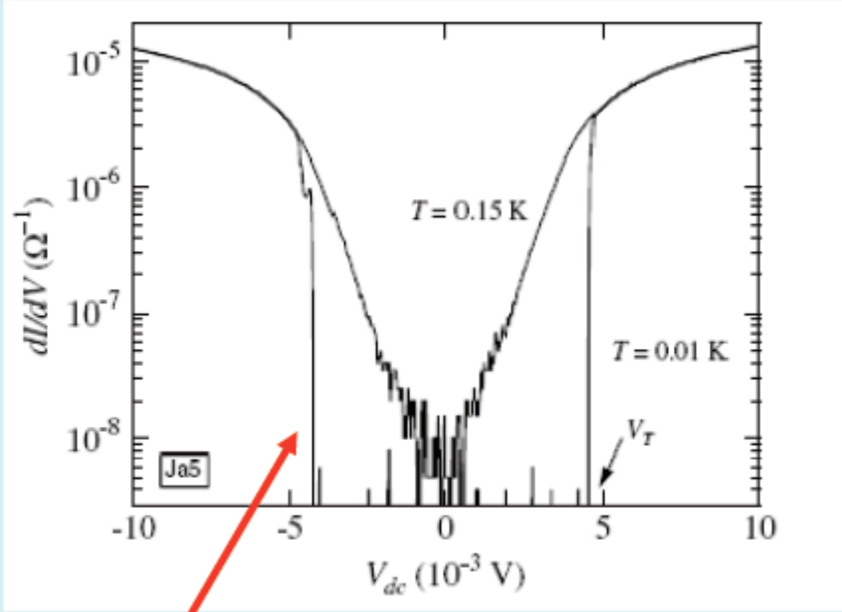

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Crucial ingredient: transport is not phonon- but electron-activated! - Mechanism???

Summary

1. **Close to the SI transition the transport is essentially simply activated (Arrhenius):**

   How come?

2. **Evidence for purely electronic transport from heating instability in non-Ohmic regime.**
   First direct evidence of electronic transport mechanism in insulators

   What is its origin?

From dirty superconductor to Bose glass

Models

$$H = t\sum_{\langle i,j\rangle} b_i^+ b_j + U\sum_i n_i(n_i - 1) + \sum_i (\varepsilon_i - \mu) n_i$$

Disorder: $\varepsilon_i \in [-\Delta, \Delta]$

Easier to think about: U = ∞ limit, i.e., hard core bosons
→ bosons equivalent to pseudospins (s=1/2)

Interactions (e.g. Coulomb)

(Anderson, Ma+Lee, Kapitulnik+Kotliar)

$$H = t\sum_{\langle i,j\rangle} s_i^+ s_j^- + \sum_i (\varepsilon_i - \mu) s_i^z + \sum_{\langle i,j\rangle} J_{ij} s_i^z s_j^z$$

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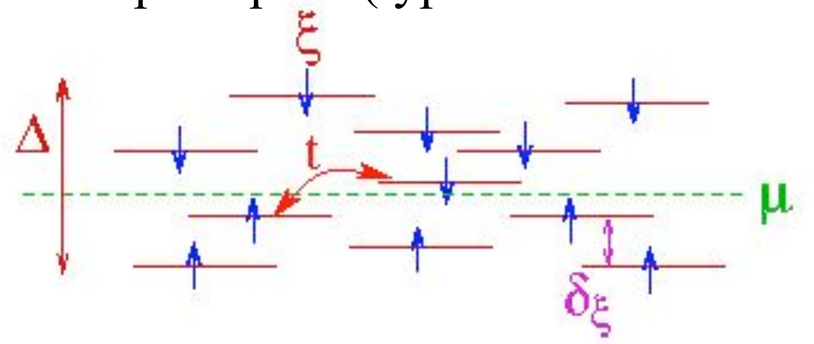
• "Sites" i: states for bosons to occupy. May overlap in space (typical size of a state: ξ)

•Relevant scale characterizing disorder:
Level spacing δ_ξ between close levels
Disorder strength: $g \equiv \delta_\xi / t$



From dirty superconductor to Bose glass

• Superconducting phase: Bose condensation into delocalized mode in the presence of self-consistently screened disorder
→ finite phase stiffness
→ infinite conductivity for $T < T_c$

• Bose glass: No delocalized bosonic mode anymore (otherwise condensation would occur)
- role of disorder: no homogeneous gap, still compressible phase

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- **Note:** "Bose glass": **unfrustrated,** but disordered, Bose insulator
- but: it is an **insulator**, i.e. $\sigma(T \rightarrow 0) = 0$ [no Bose metal!]

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Nature of transport in the Bose glass?

From dirty superconductor to Bose glass

Localization of the bosons?

Look at evolution of the full manybody spectrum!

Berkovits and Shklovskii
Basko, Aleiner, Altshuler
Huse, Oganesyan

Warm up: Clean case

- Superconductor: gapless excitations (phonons)

- Mott insulator of bosons: finite gap
  Spectrum:
  No discrete spectrum!
  All excitations are delocalized and
  disperse with well-defined momenta **k**

↔ With disorder: much more complex!

From dirty superconductor to Bose glass

Local spectrum at T = 0 $\rho_{\mathrm{O}}(\omega)=\int_0^\infty \langle \mathrm{O}(x,t)\mathrm{O}(x,0)\rangle_{GS}\, e^{i\omega t}$

2 possibilities:

- continuous spectrum
- point spectrum: “locally discrete”
  (bunch of delta functions in local
correlation functions)

From dirty superconductor to Bose glass

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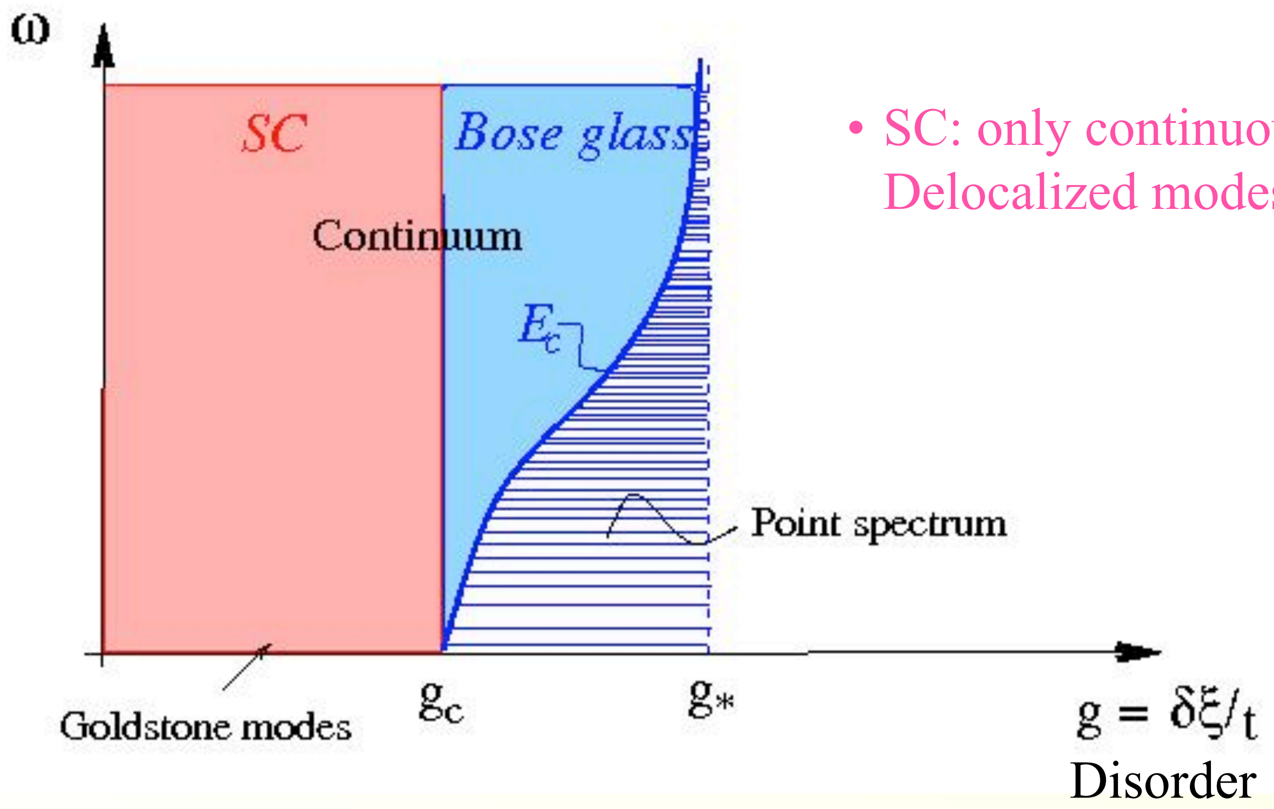

- SC: only continuous spectrum! Delocalized modes down to ω=0

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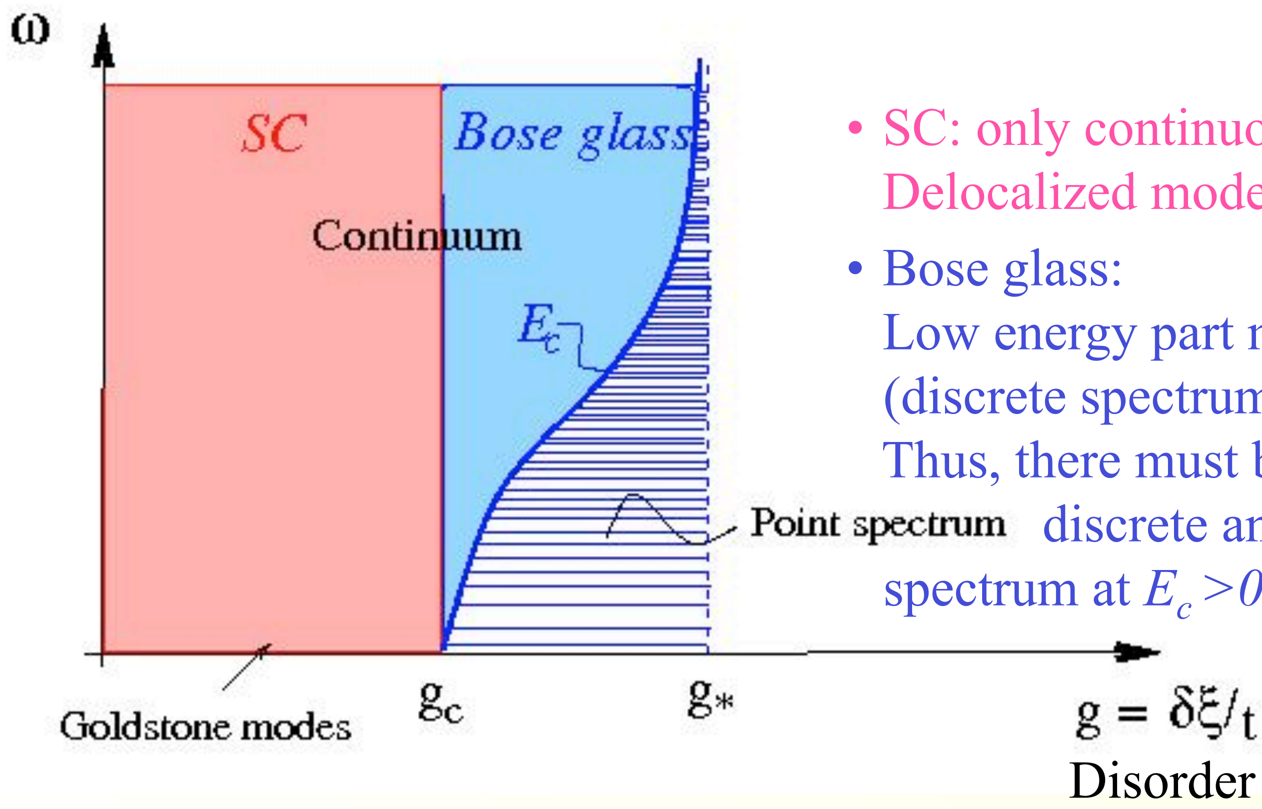
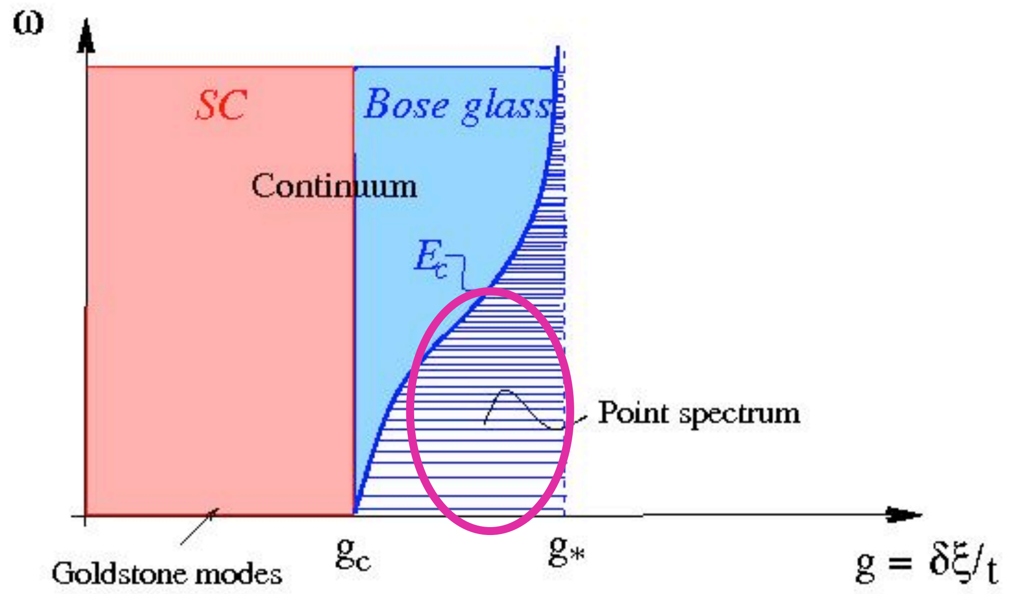


- SC: only continuous spectrum!
  Delocalized modes down to ω=0
- Bose glass:
  Low energy part must be localized (discrete spectrum).
  Thus, there must be a border between discrete and continuous spectrum at $E_c > 0$

Spectrum at T = 0

The point spectrum at low energies



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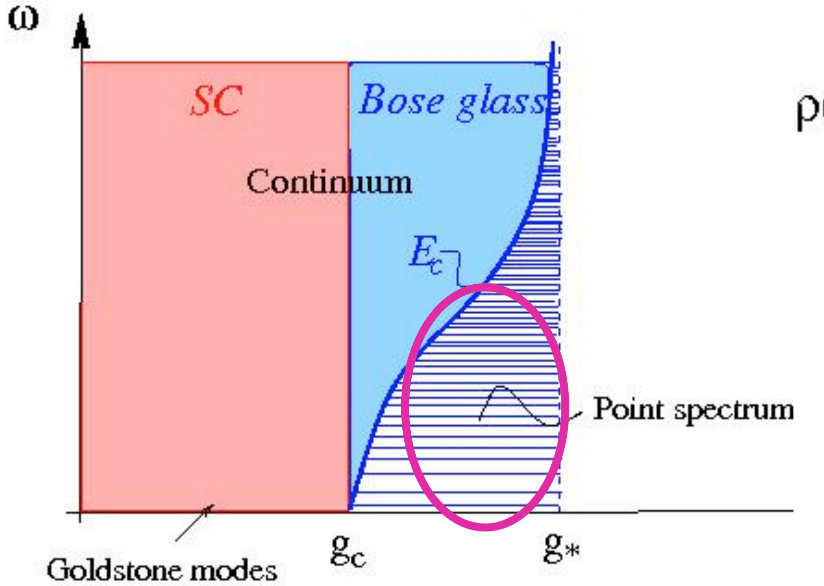

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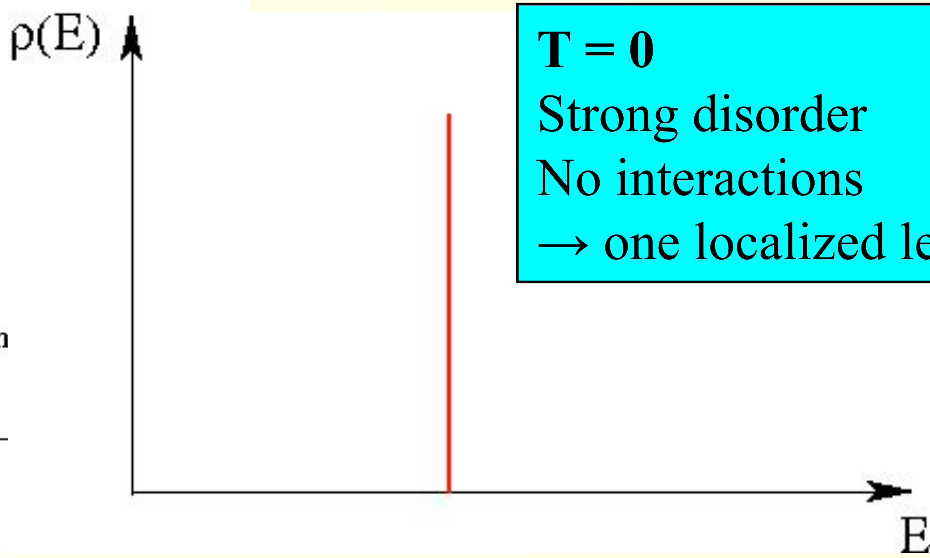
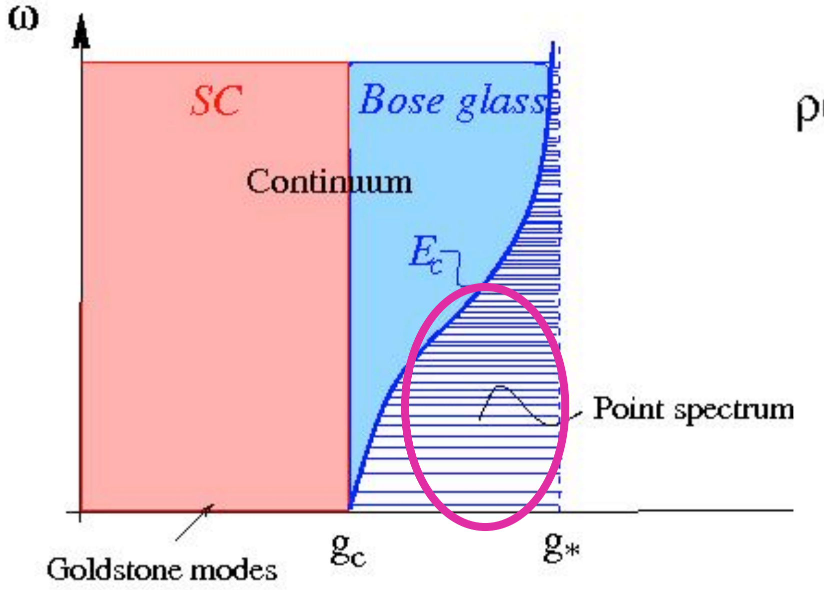


T = 0
Strong disorder
No interactions
→ one localized level

Spectrum at T = 0

The point spectrum at low energies



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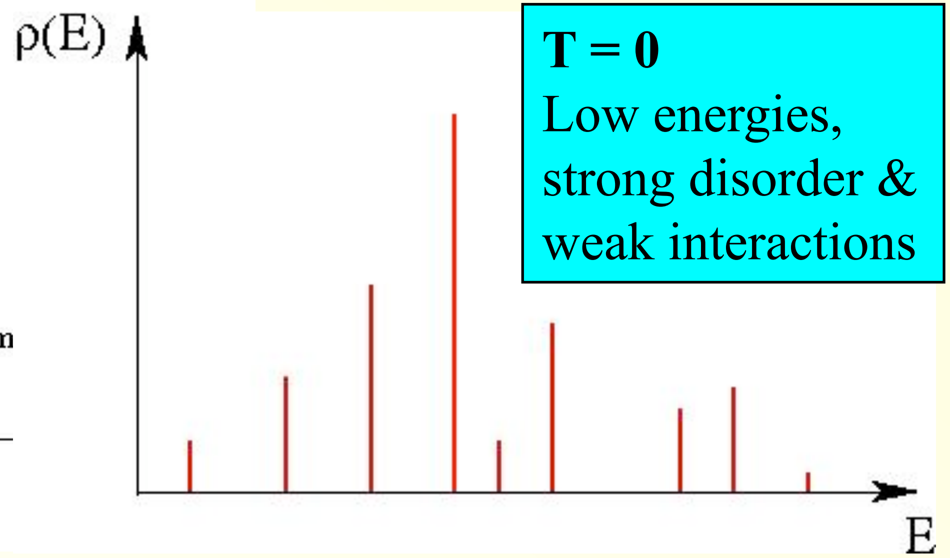


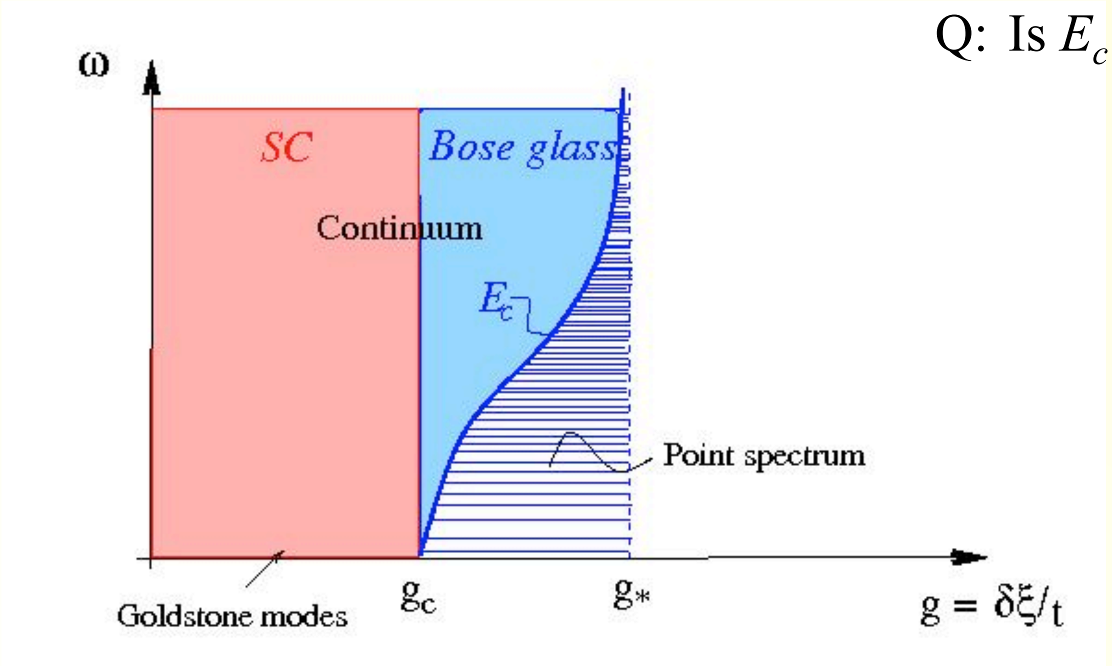
T = 0
Low energies, strong disorder & weak interactions

- Discrete levels: no transport, no current!
  $\sigma(T=0) = 0$
- Genuine glass at T=0: perturbations don't relax

Reason: Transition probabilities are zero because energy conservation can never be satisfied!

Mobility edge

Many-body “mobility edge” in the Bose glass



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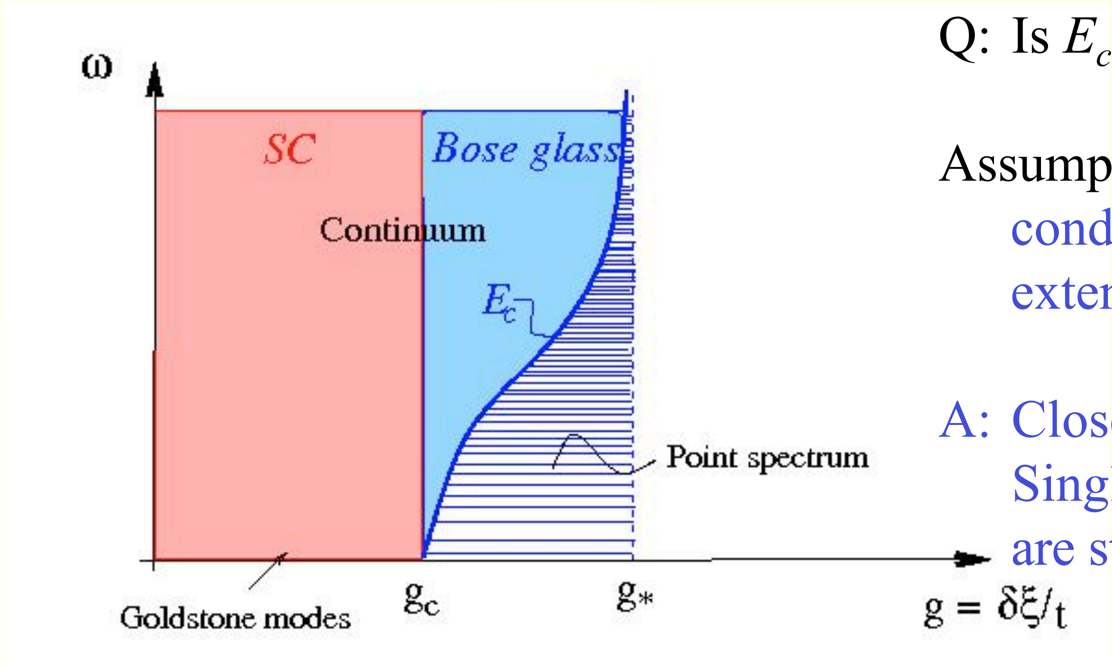

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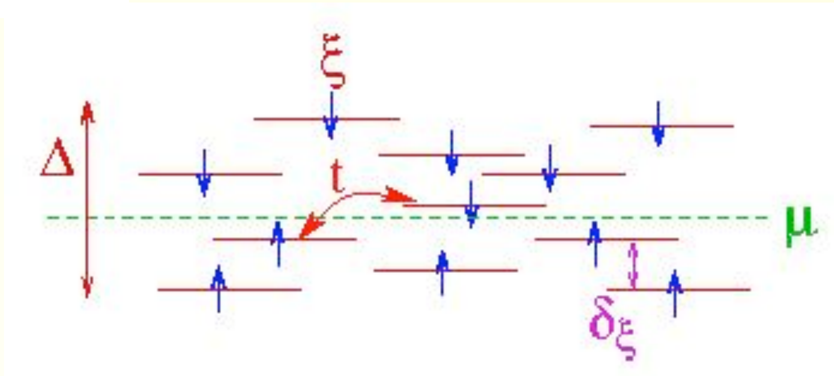
Assumption: The SIT is essentially a condensation of a self-consistent, extended single boson mode.

A: Close to the SIT ($g = g_c$) E_c is bounded: Single boson excitations at $E\text{-}\mu >> t$ are still delocalized (for d > 2) → $E_c < \infty$



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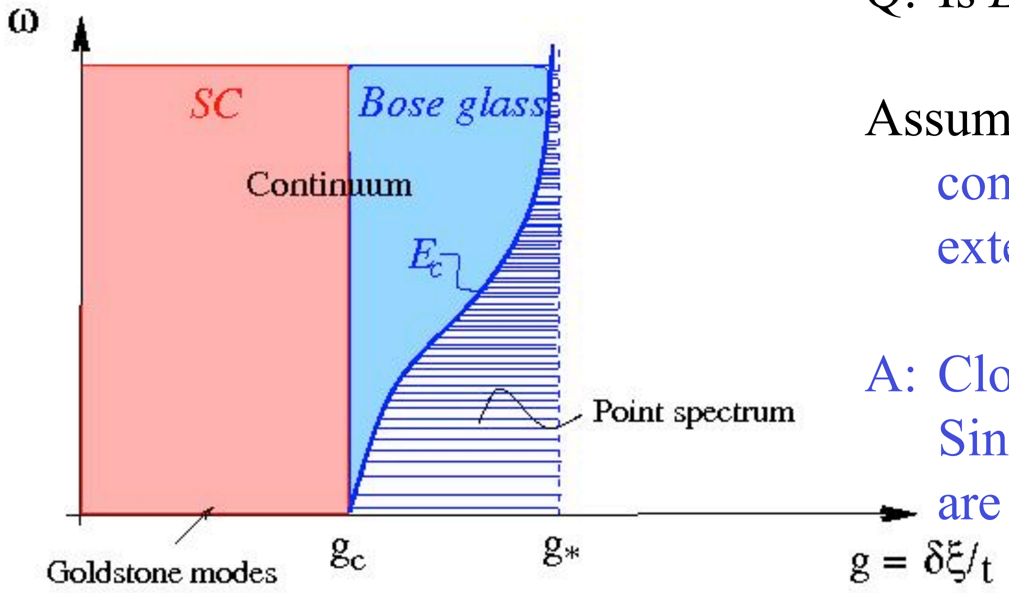

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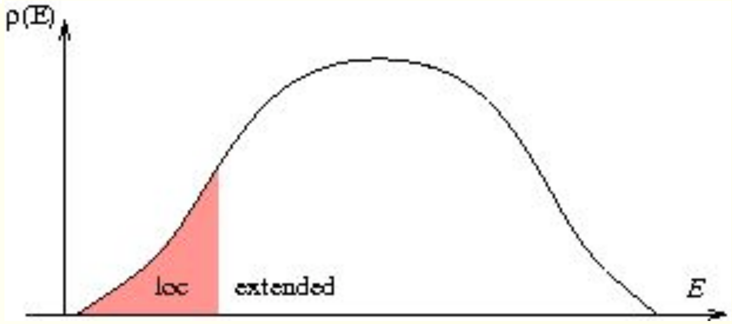
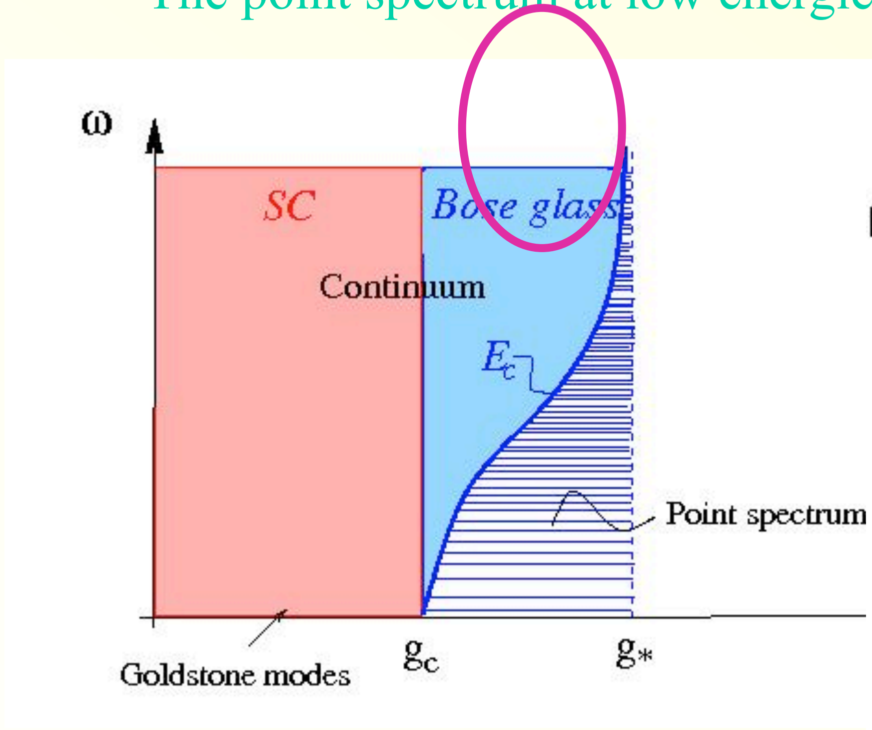
Analogon:
Localization at band edge (Anderson model)

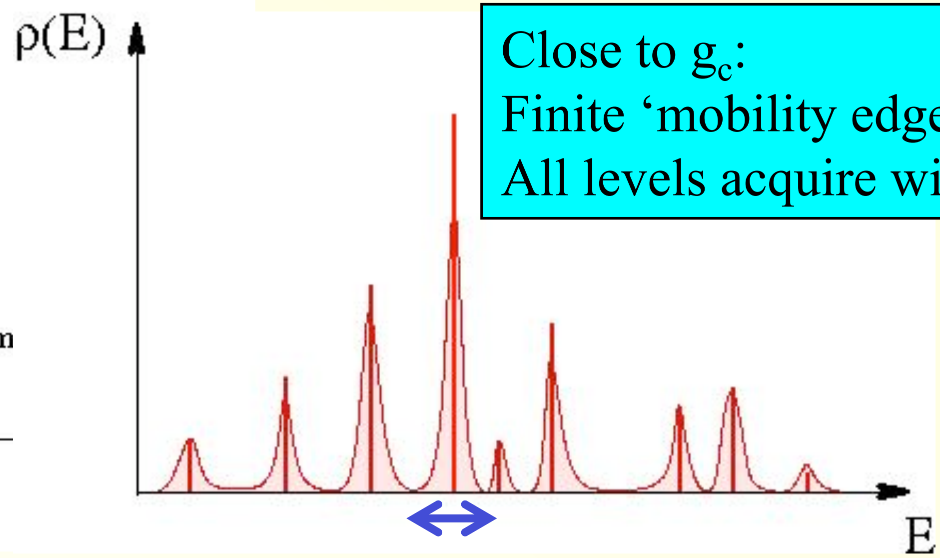


Finite T

The point spectrum at low energies



Local spectrum at T > 0

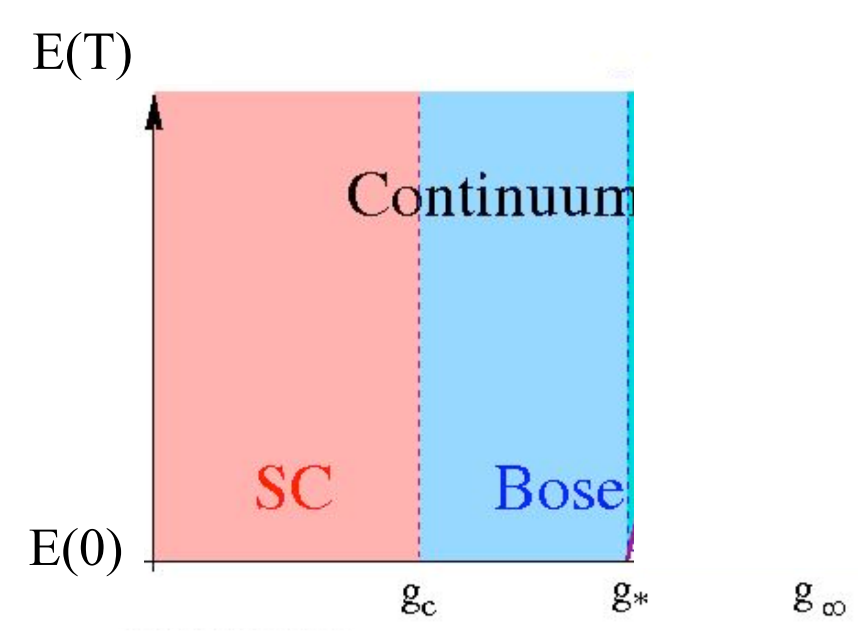


Close to g_c:
Finite 'mobility edge'
All levels acquire width

$\Gamma \sim \exp[-E_c/T]$

Finite T

The point spectrum at low energies



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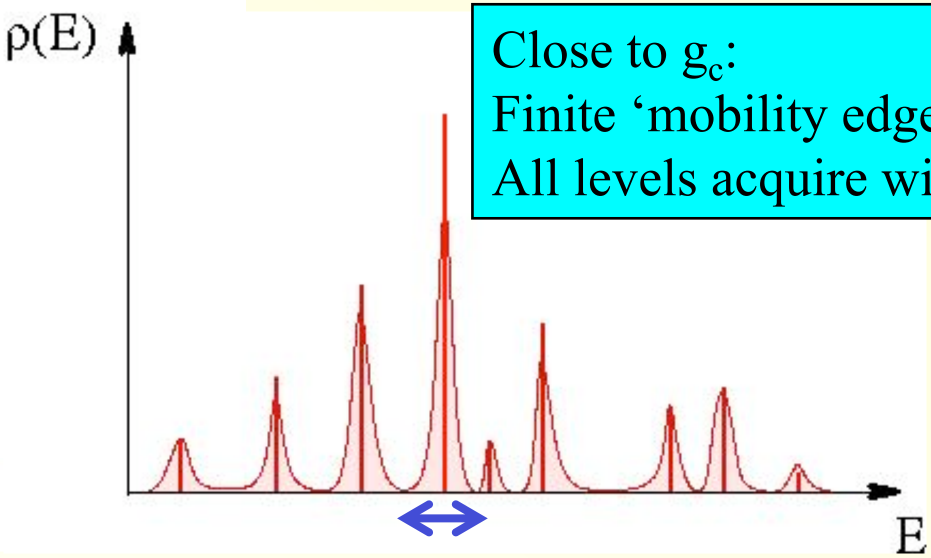

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All levels acquire width

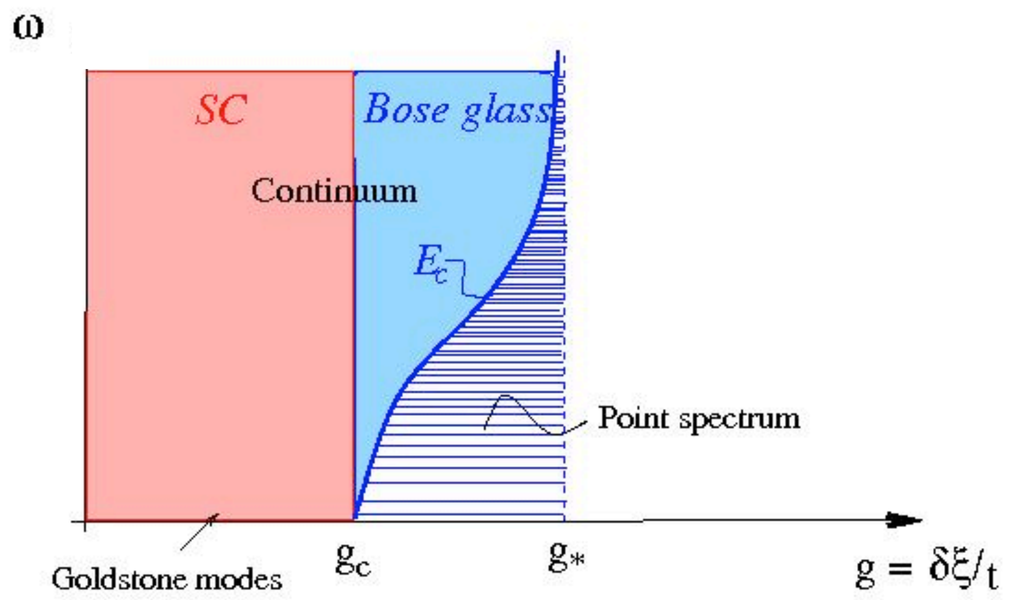
$\Gamma \sim \exp[-E_c/T]$

→ • Continuum everywhere! $\sigma(T > 0) \neq 0$ for $g < g_*$ where $E_c(g) < \infty$

Electronic activated conduction

$g < g_* \;:\; E_c(g) < \infty$

- Continuum at finite T! ⟶ $\sigma(T>0) \neq 0$



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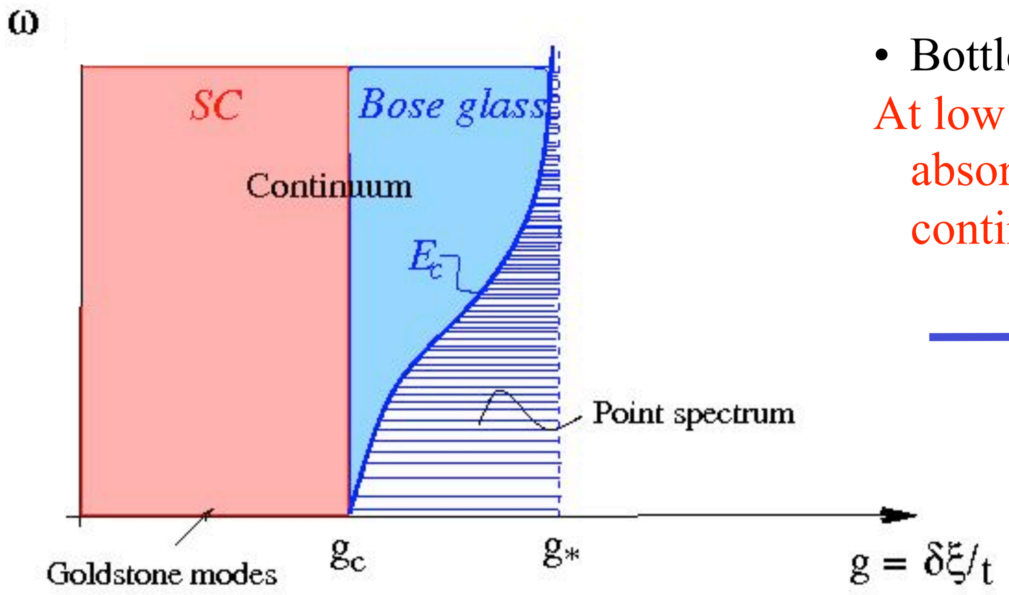

- Continuum at finite T! → $\sigma(T>0) \neq 0$
- Bottle neck for conduction:

At low T: Transitions allowed only due to the absorption of modes from the (T=0) continuum or diffusion above E_c

→ $\sigma(T) \sim \sigma_0 \exp[-E_c/T]$

Simple activation (Arrhenius) law in a compressible, gapless system!
No variable range hopping $e^{1/T^{\alpha}}$!

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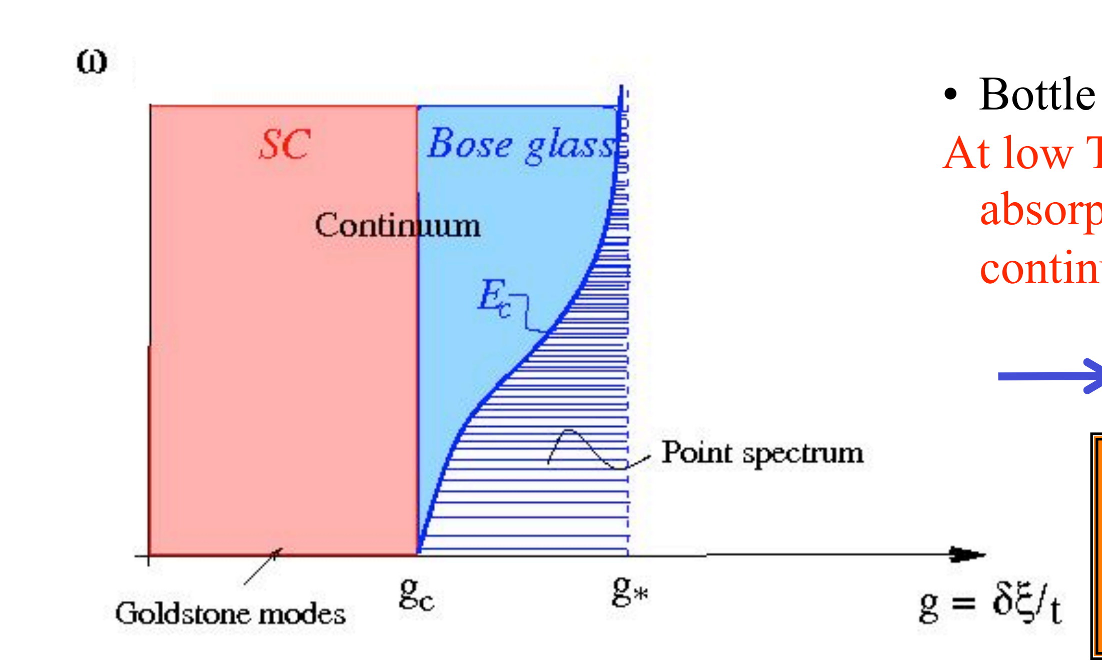

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!

- No phonons needed! (they are anyway very inefficient at low T)
- Purely electronic transport mechanism
  → crucial ingredient to explain the overheating in the non-Ohmic regime
- Prefactor: $\sigma_0 \sim e^2/h\xi^{d-2}$ nearly universal in quasi 2d, similar to experiment!

!

Electronic activated conduction

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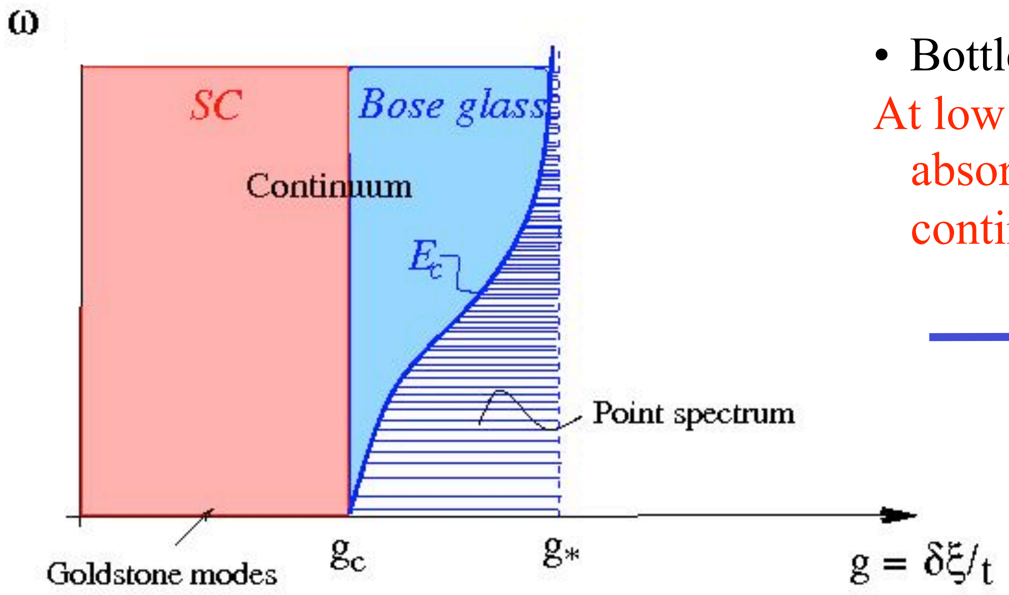

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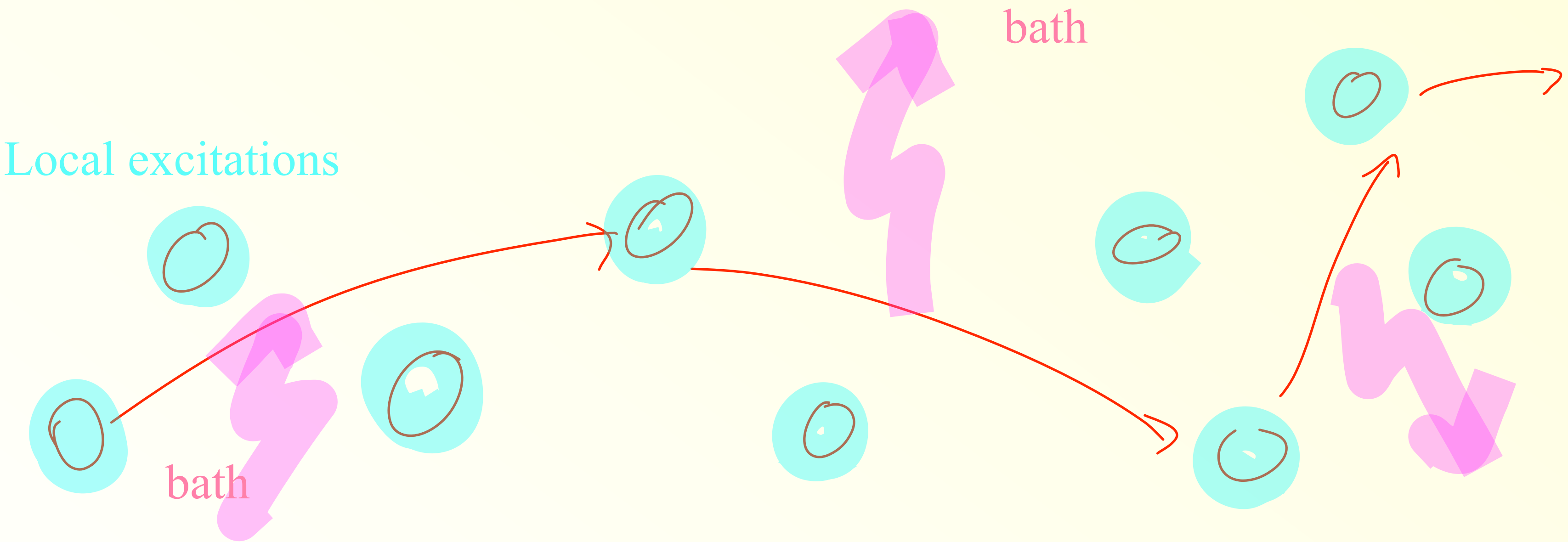
! Arrhenius law is only asymptotic at lowest T : Finite inelastic scattering rate at T > 0 lowers the activation energy needed to get diffusion! → $E_{act} = E_c - \Delta E(T)$! → superactivation! !

What about the standard variable range hopping transport of disordered inslators?

? Transport and thermalization in insulators ?

Essential ingredient into variable range hopping:
Continuous bath which activates the hops!



Candidates for the bath:

• Phonons: at low T for pair hopping are very inefficient!

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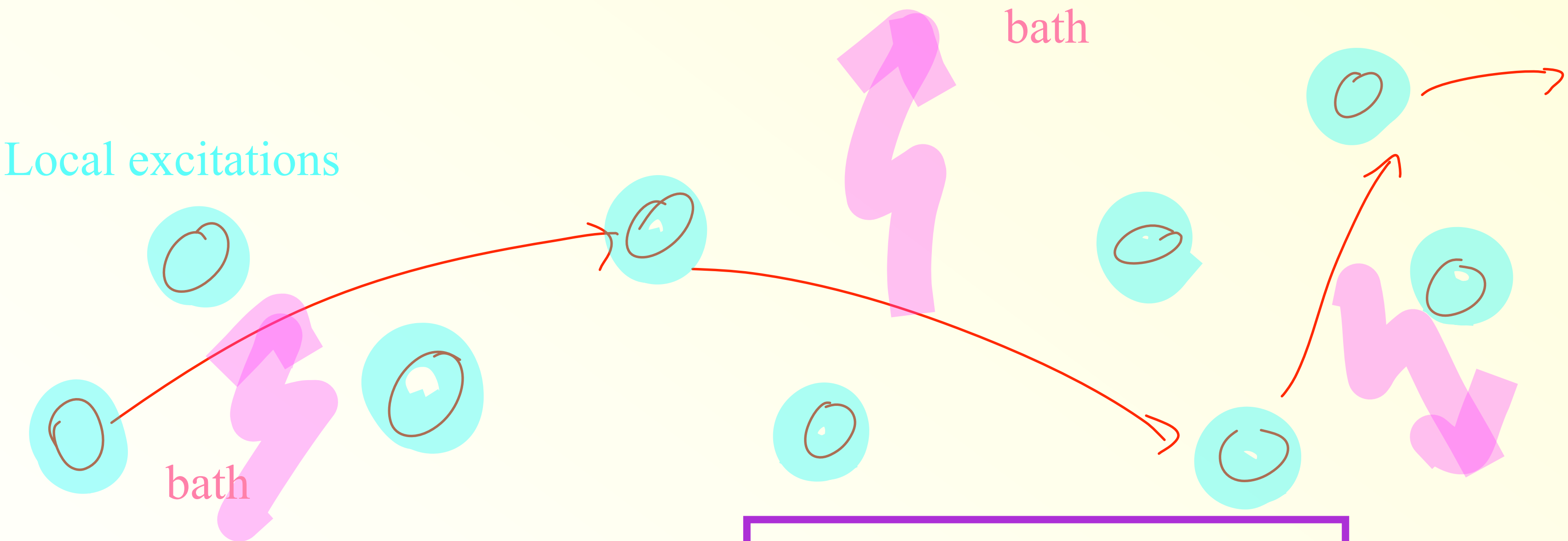

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Too weak → not considered

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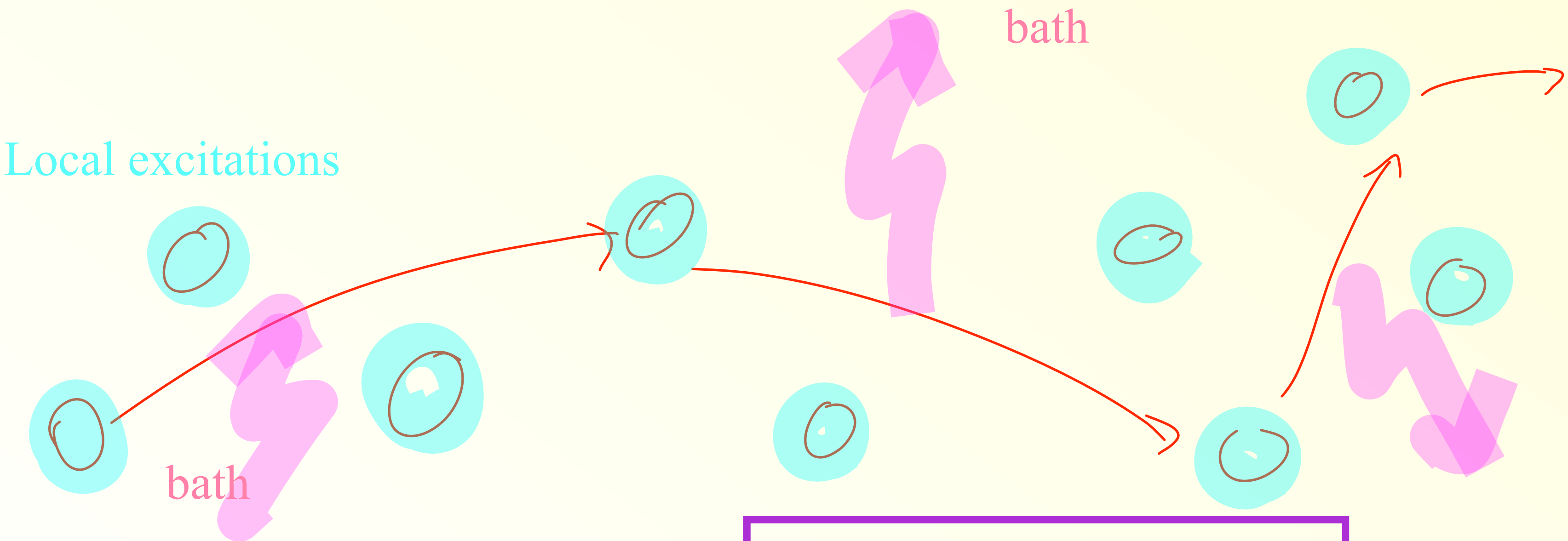

Candidates for the bath:

Too weak → not considered

- ~~Phonons: at low T for pair hopping are very inefficient!~~
- (possibly collective) boson excitations above the mobility edge

What if there is no bath whatsoever?

Strong disorder

$g > g_* \; : \; E_c(g) = \infty$ (~ Volume)

- If disorder is strong ($g = \delta_\xi / t > g_*$) *all* single boson excitations above the GS (at $T = 0$) are localized: $E_c \to \infty$

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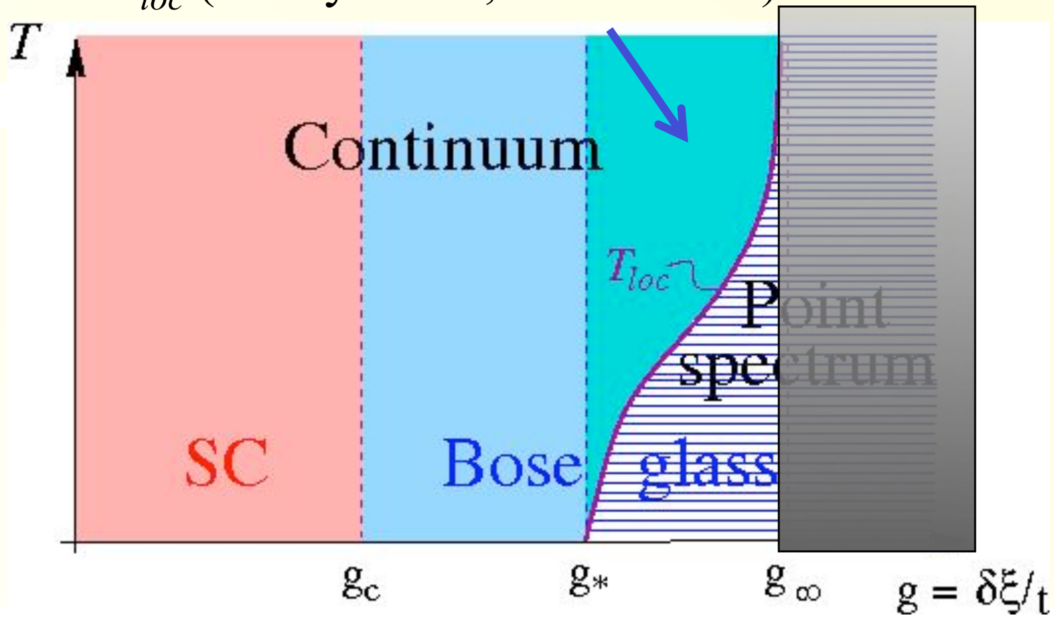
- ⟷ finite T: finite density of excited bosons → increased inelastic scattering → localization tendency reduced
  Once inelastic rate ~ level spacing δ_ξ →
  self-consistent level broadening
  delocalization in Fock space at $T = T_{loc}$ (Gornyi et al.; Basko et al.)
  → Finite T transition from
  $\sigma = 0$ to $\sigma > 0$ state!



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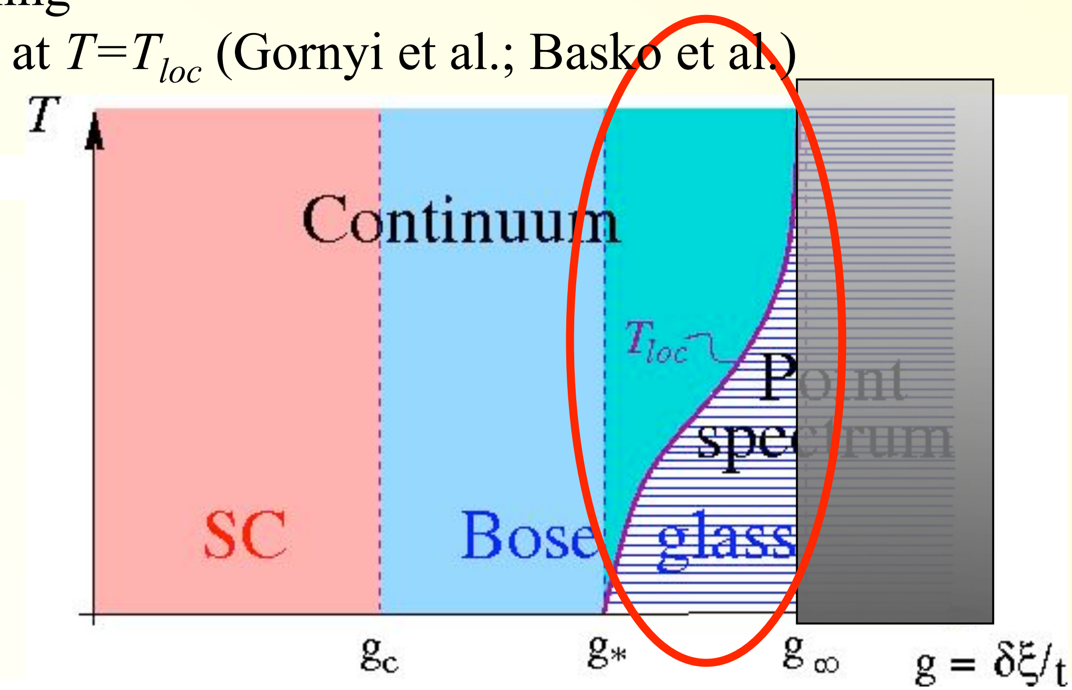

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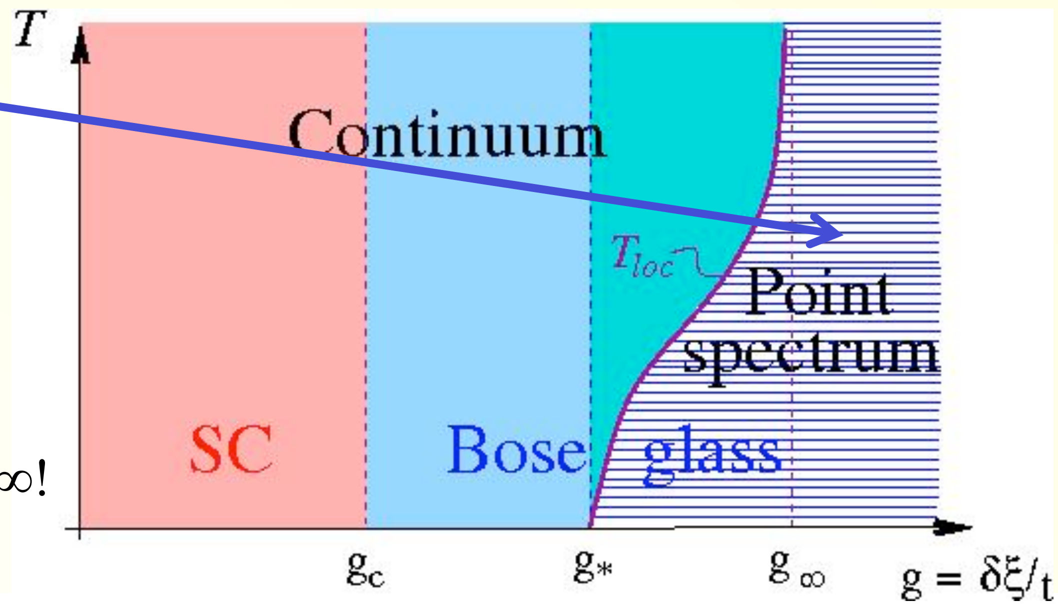
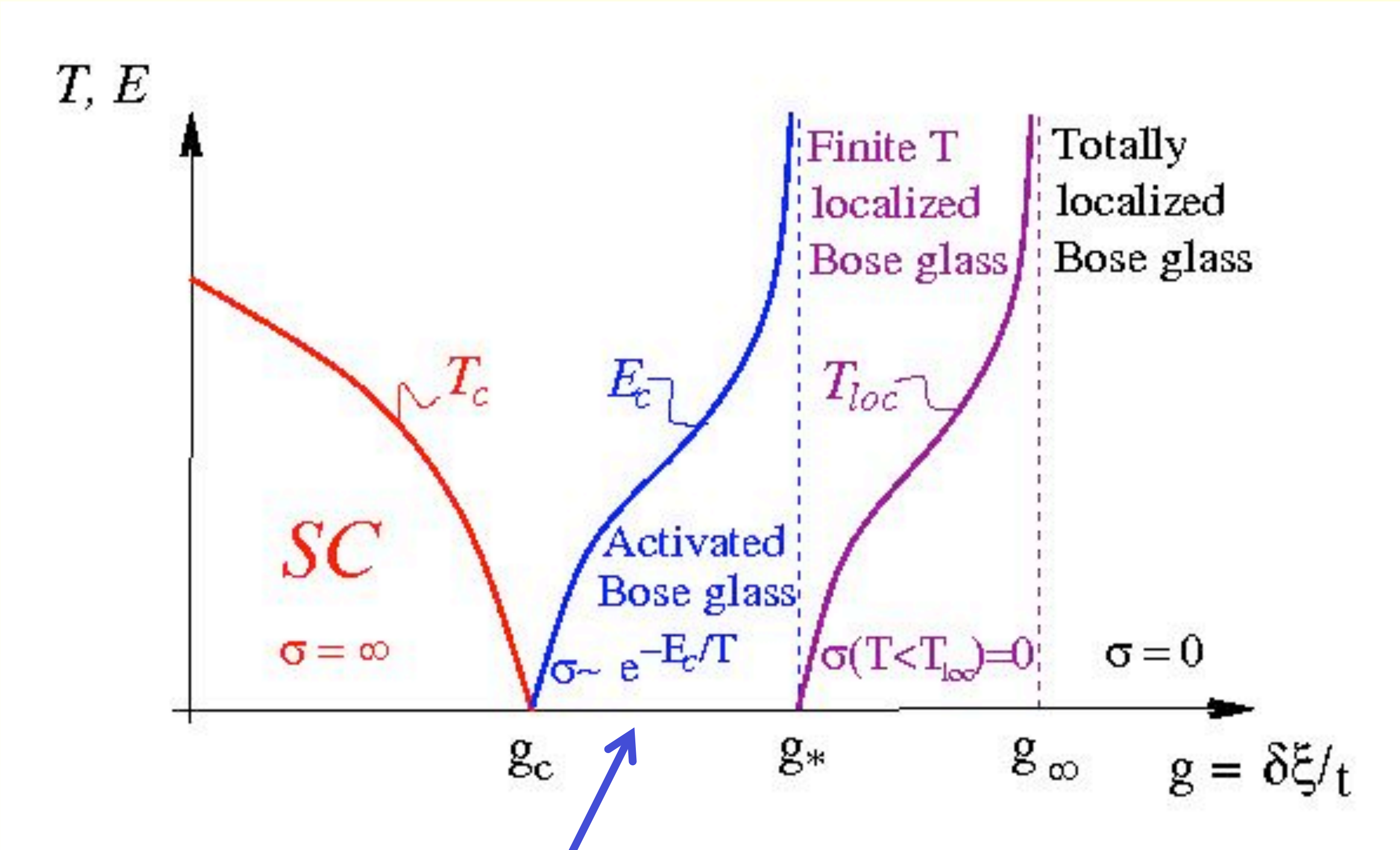
- At biggest $g > g_\infty$:
  finite bandwidth →
  scattering rate limited →
  complete localization in very
  strong disorder when $T_{loc} \rightarrow \infty$!



Summary: Bose-Hubbard model and Bose glass



Purely electronic transport at low T: **Asymptotically** Arrhenius law!

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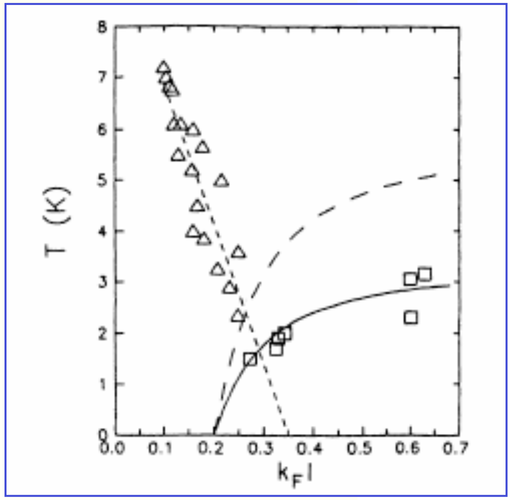
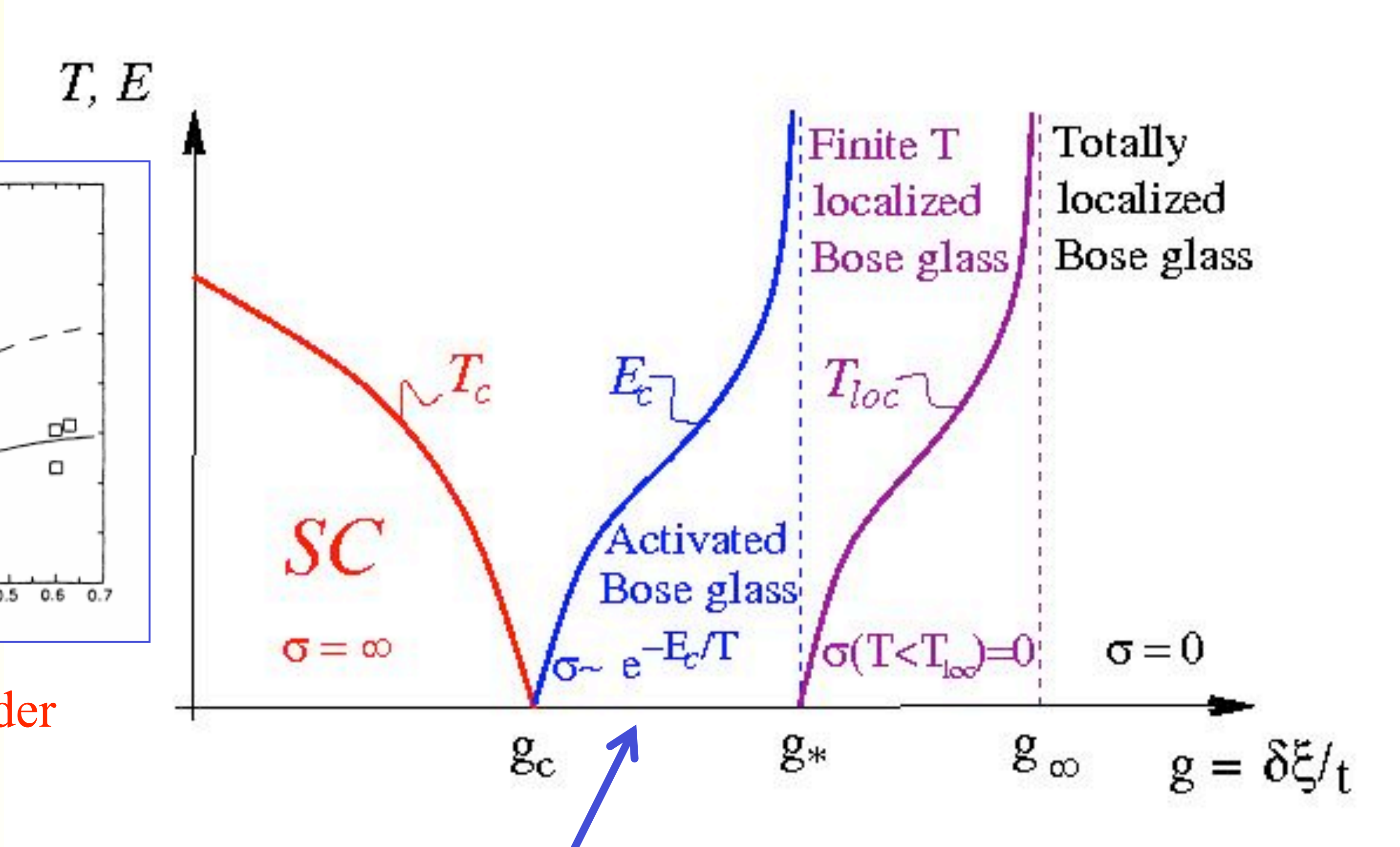


1/Disorder



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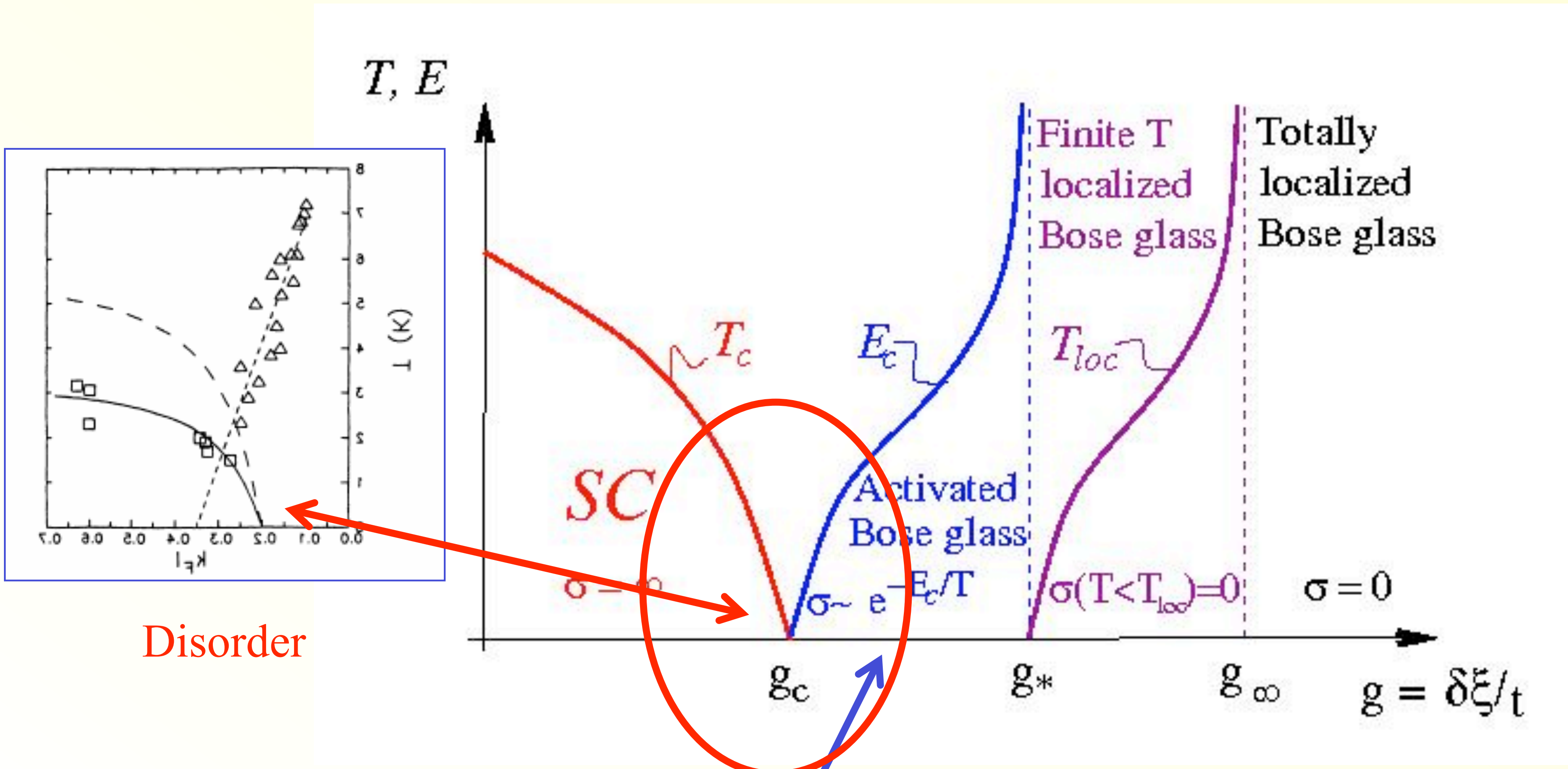

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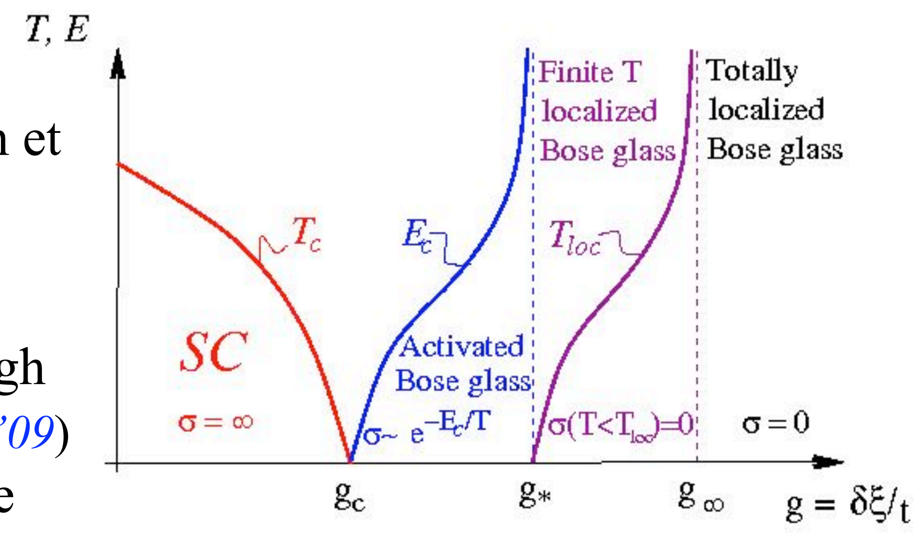
Can this scenario be proved?



- T_{loc}& total localization: similar to Mirlin et al. and Basko et al. (*Aleiner et al., '09*)

- Finite mobility edge: Controlled approximation for hard core bosons on high connectivity Bethe lattice (*Ioffe & Mézard '09*) Study of propagation of level width on the Cayley tree confirms the phase diagram, $g^*>g_c$

- Is the scenario true in d < 3 ?

Caveats concerning the intermediate phase

- SIT viewed as condensation of single bosons is probably OK for **hard core bosons, but not for sufficiently soft cores** (e.g. Josephson junction arrays)

- Our *upper bound* for a finite mobility edge E_c relied on the delocalization of single bosons in weak enough disorder. This strictly holds for $d > 2$. The argument thus needs to be refined to **include interaction effects** in $d < 3$.

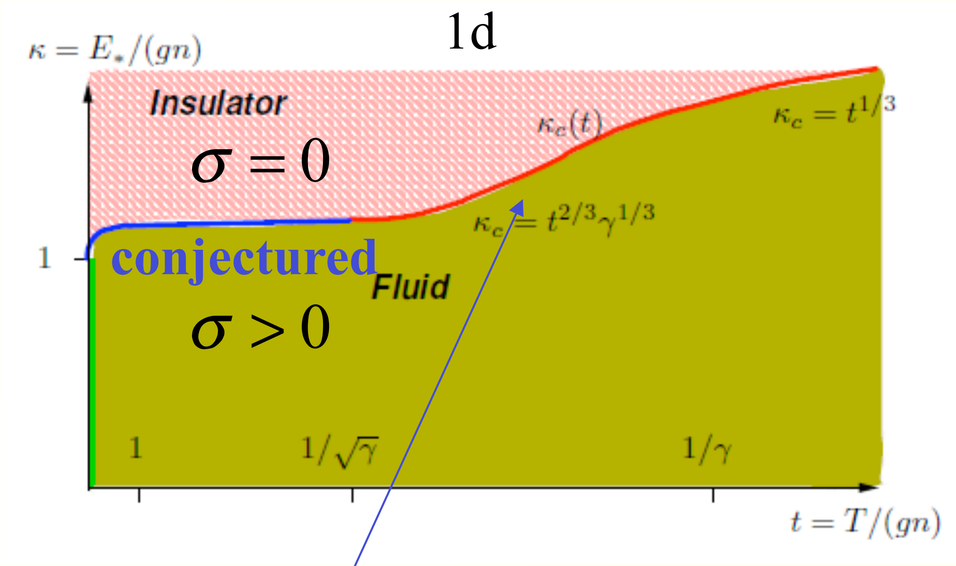
Conjecture: hard core bosons in 2d have an intermediate insulator.

Quantitative studies are in progress.

1d and 2d case

(Aleiner, Altshuler, Shlyapnikov, arXiv:2009)

Calculations and conjectures about the phase diagram of
soft core bosons in 1d and 2d:



Genuine finite T phase transition in 1d!
No order parameter → No Mermin-Wagner theorem

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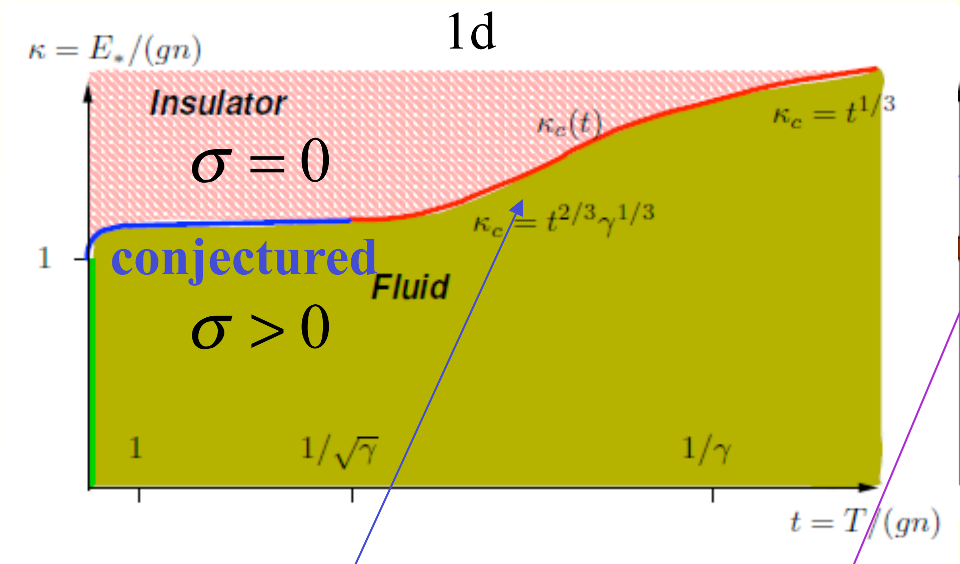

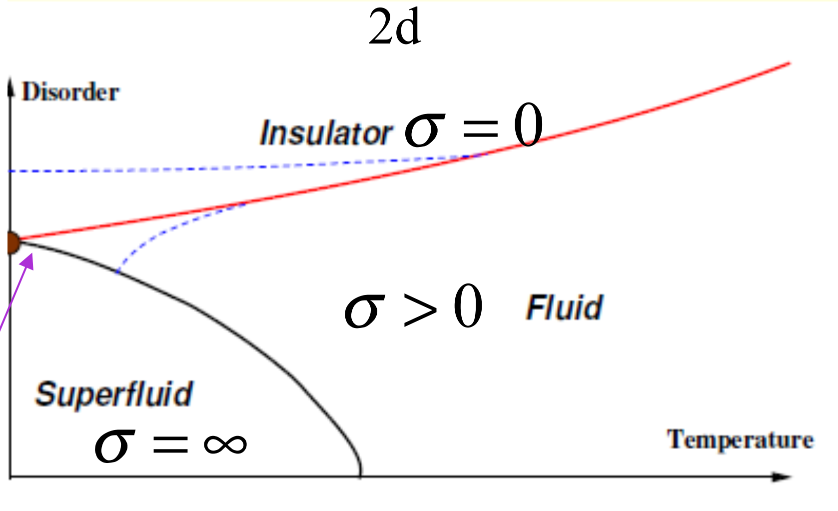

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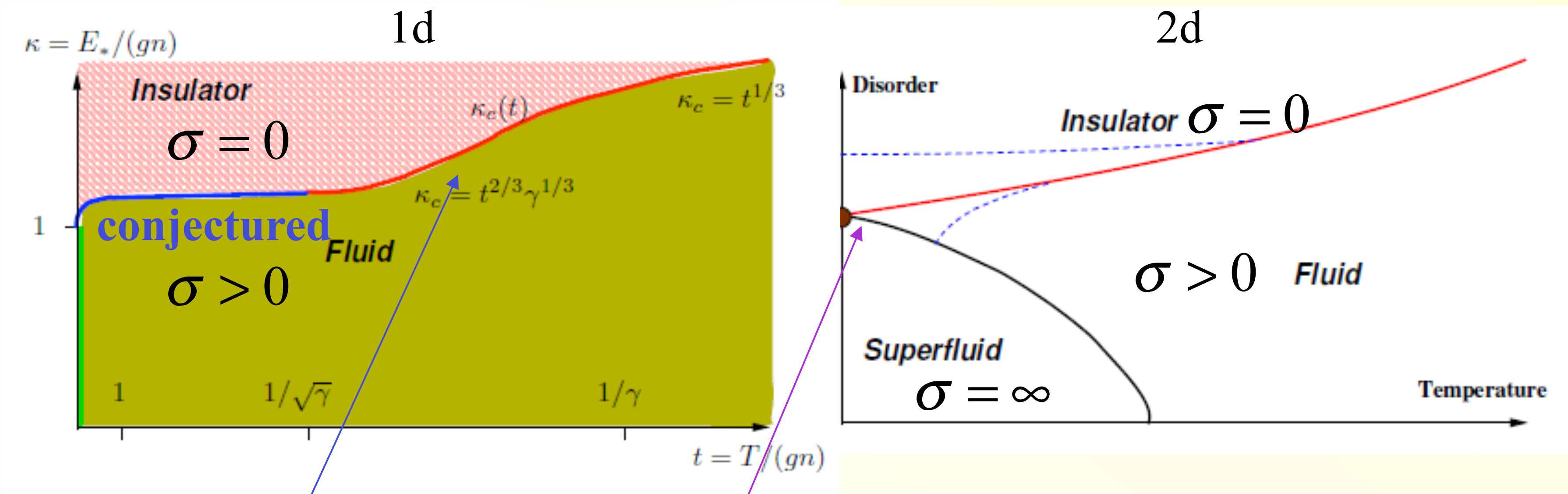

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Further studies of the dependence on U/δ_{ξ} are needed!
Is there a quantum tricritical point between SITs with/without intermediate phase?

Conclusion

• Transport in the Bose glass is a rich problem due to manybody localization phenomena

• The SIT seems most promising to observe and study manybody localization with its precursors experimentally

• To exploit further: Classification of different classes of insulators according to their local spectrum