

Puzzles of transport at the Superconductor-Insulator Transition

Markus Müller

Discussions with

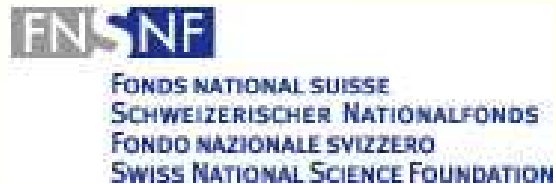
M. Feigel'man

L. Ioffe

B. Shklovskii

B. Sacépé

D. Shahar



Rutgers, Feb 4, 2009

Outline

- The **superconductor-insulator transition (SIT)** – an old, but still interesting quantum phase transition
- Review of **transport** experiments on the insulating side of the SIT
- Discussion where and why many **standard scenarios fail**.
- Proposal for a mechanism explaining **simple activation**, as well as **over-activation** (using ideas of many-body localization)

SI transition in thin films

M. Strongin, et al., Phys. Rev. B1, 1078 (1970).

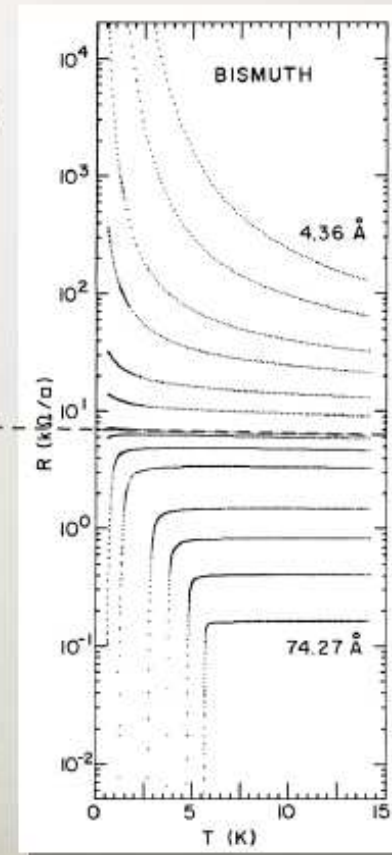
D. B. Haviland, Y. Liu, and A. M. Goldman, Phys. Rev. Lett. 62, 2180 (1989)...

Thickness tuned transition

T = 0 transition

Review: Finkl'stein ('94),
Markovic and Goldman ('98).

2D

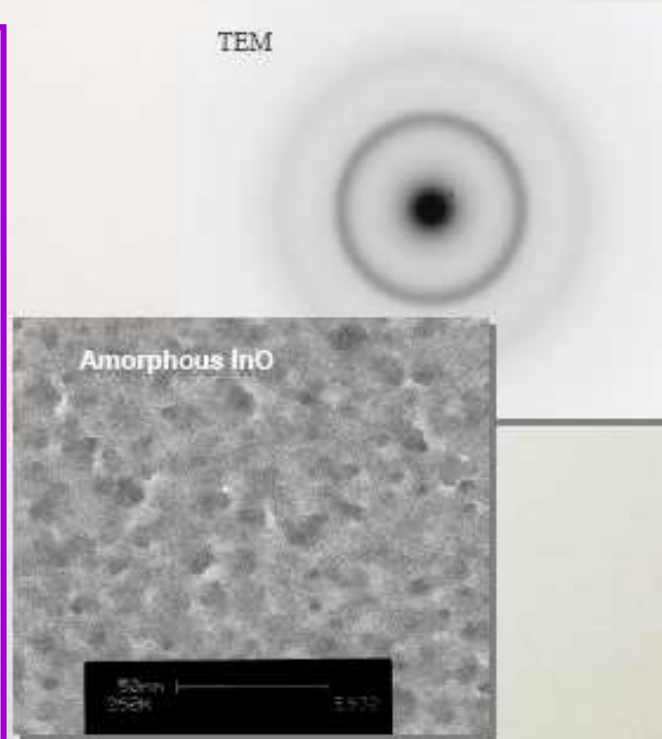


Indium-oxide (InO_x)

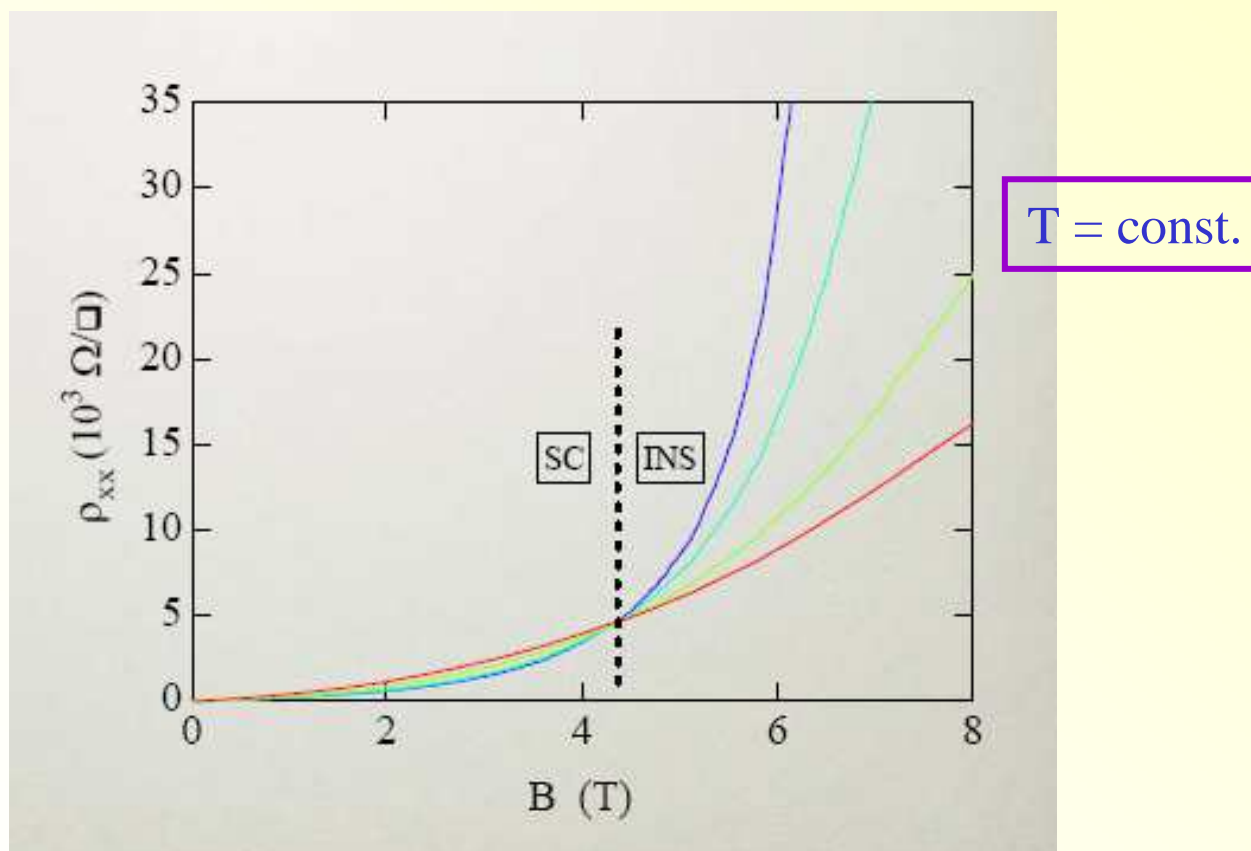
One of the major materials used in experiments:

- Strong disorder,
- High carrier density
- Tunability

Other similar experiments in TiN films



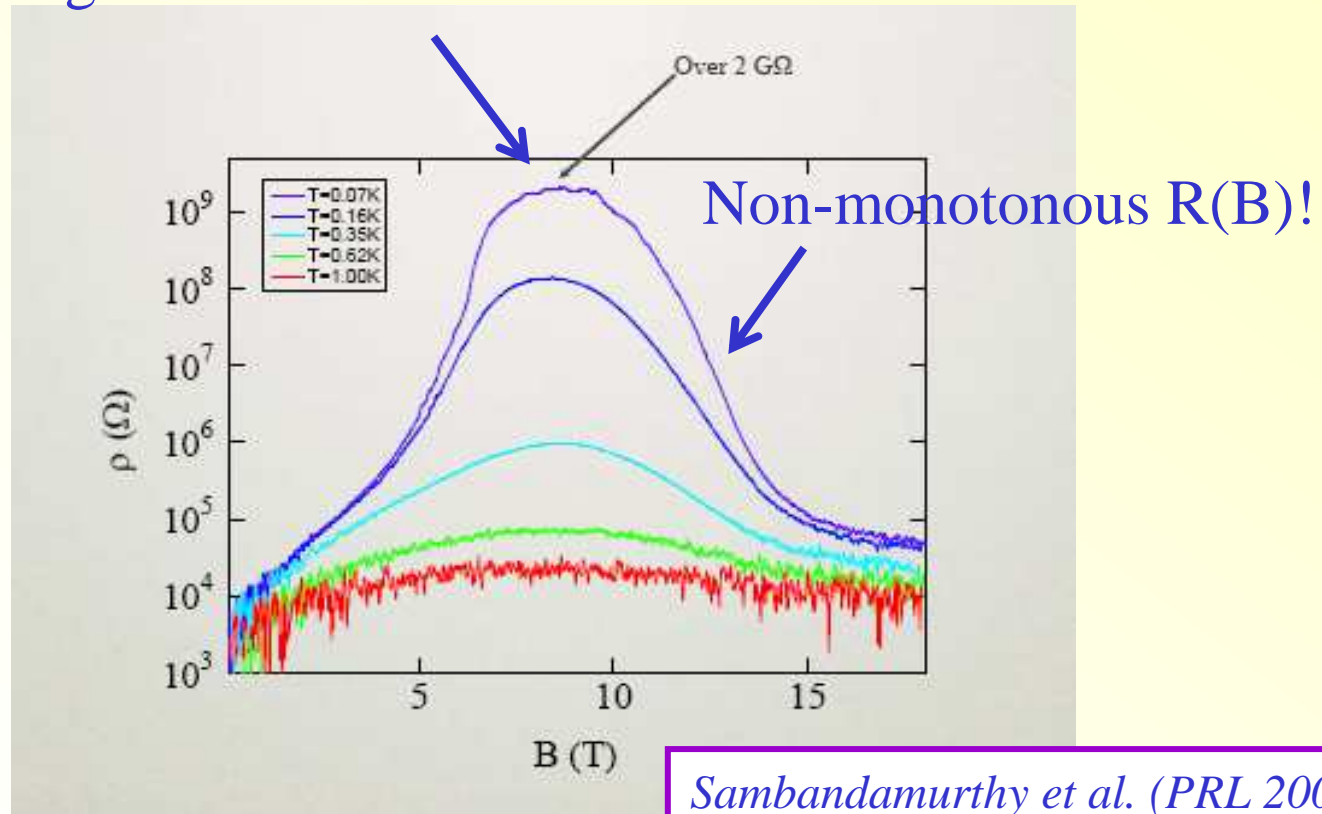
Field driven transition



Gantmakher, Shahar, Kapitulnik, Goldman, Baturina

Insulator: Giant magnetoresistance

Giant magnetoresistance



Insulating behavior **enhanced** by local superconductivity!

What is the nature of the
insulating state?

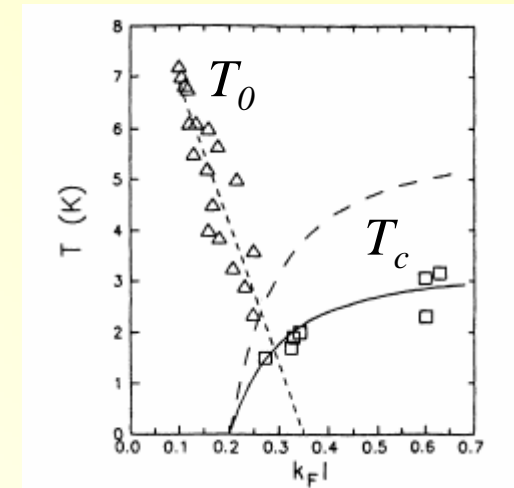
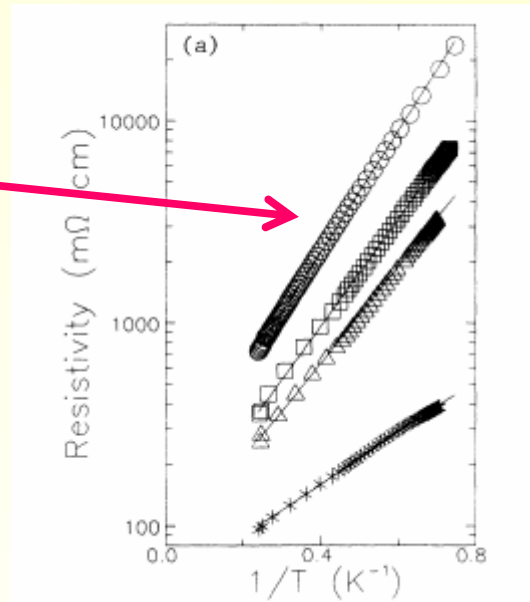
What is the transport
mechanism?

Activated transport near the SIT

D. Shahar, Z. Ovadyahu, PRB 46, 10971 (1992).

Insulating InO_x

Simple activation!



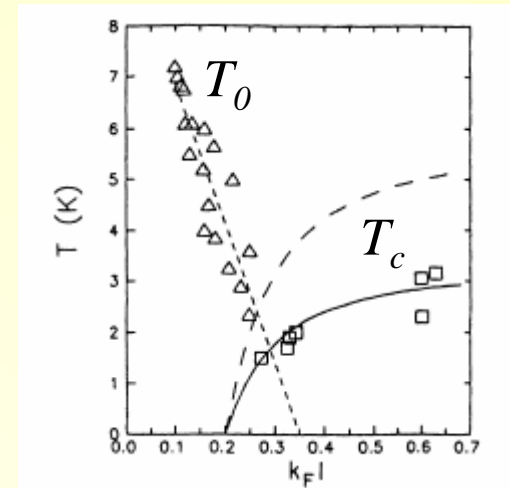
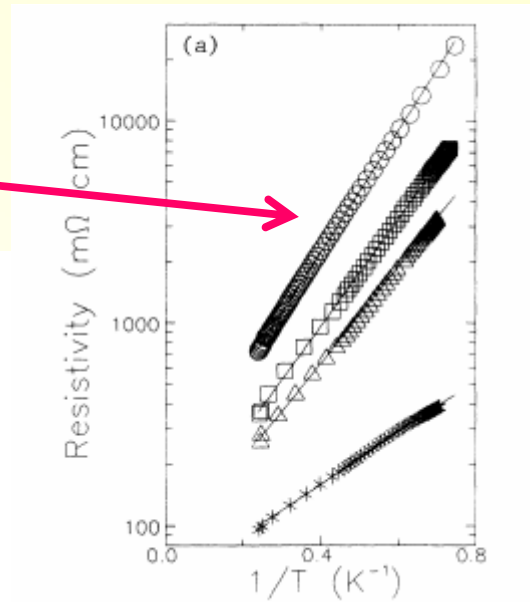
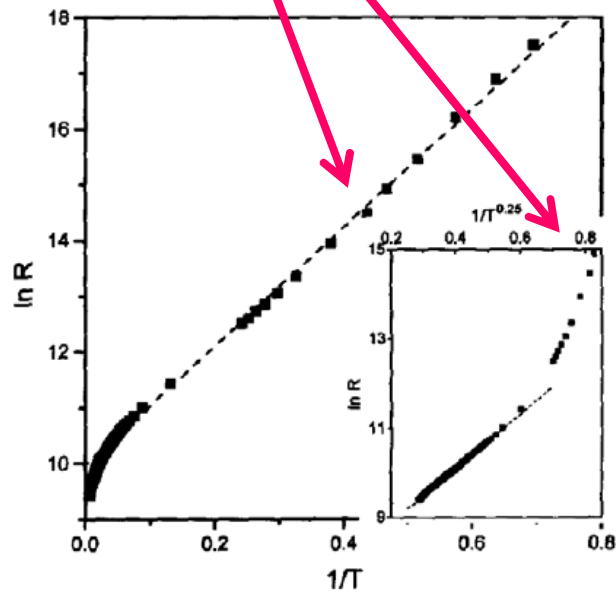
Activation energy
increases with
distance to SIT

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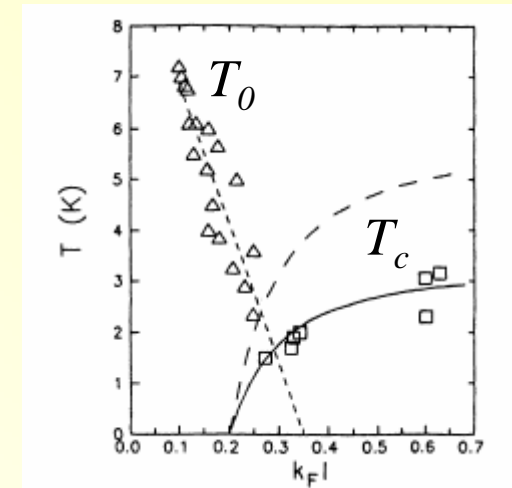
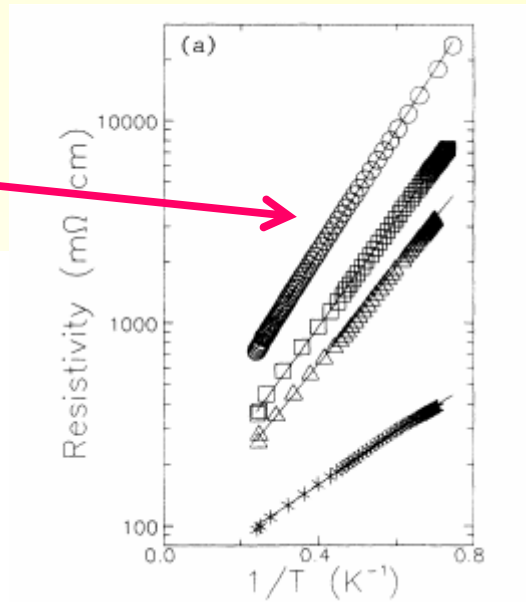
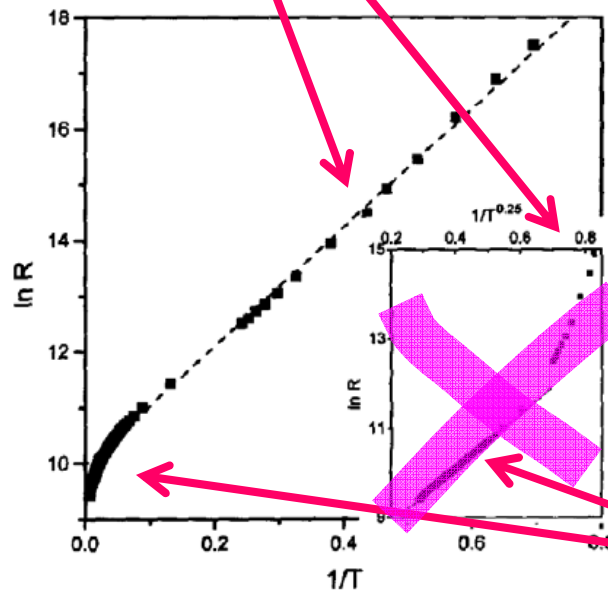
D. Kowal and Z. Ovadyahu, Sol. St. Comm. 90, 783 (1994).

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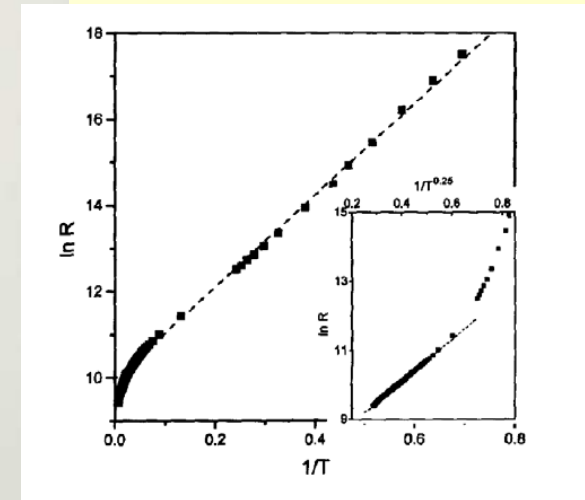
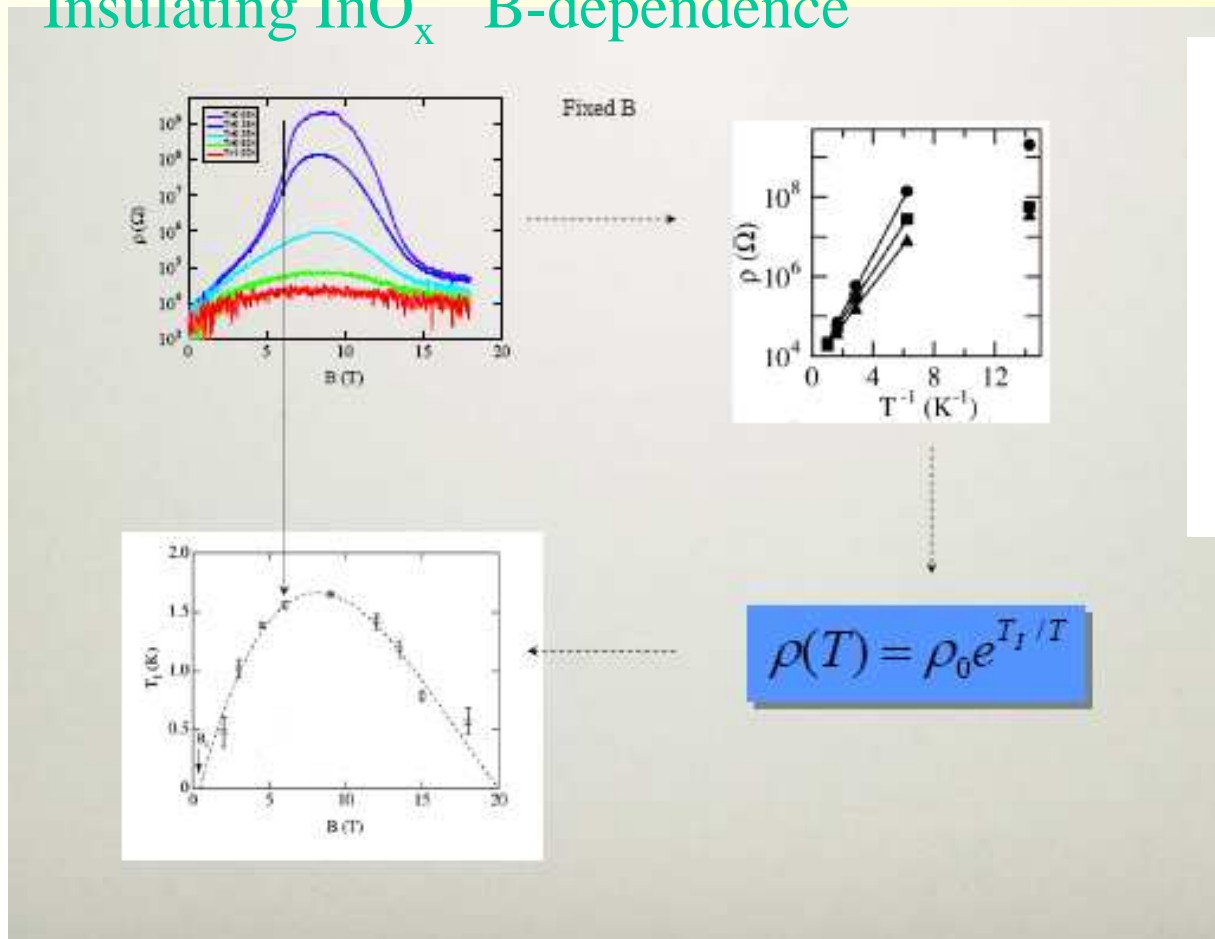
Activation energy increases with distance to SIT

Fit to variable range hopping over small range of high T is unjustified!

D. Kowal and Z. Ovadyahu, Sol. St. Comm. 90, 783 (1994).

Activated transport near the SIT

Insulating InO_x B-dependence



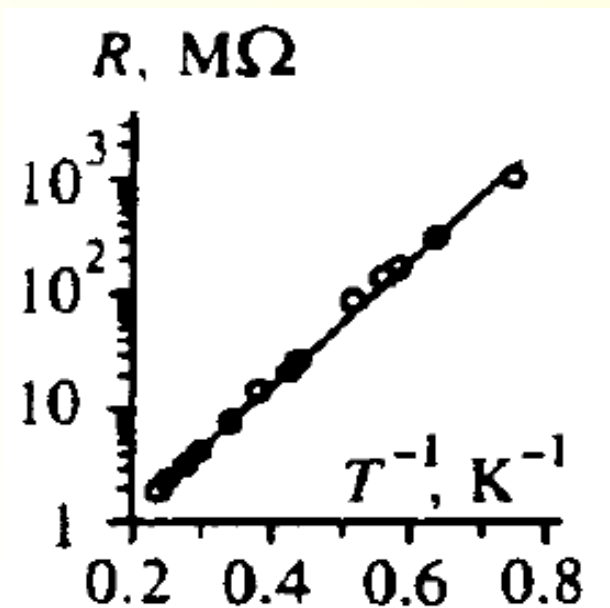
*D. Kowal and
Z. Ovadyahu, (1994).*

G. Sambandamurthy, L.W. Engel, A. Johansson, and D. Shahar, PRL 97, 107005 (2004).

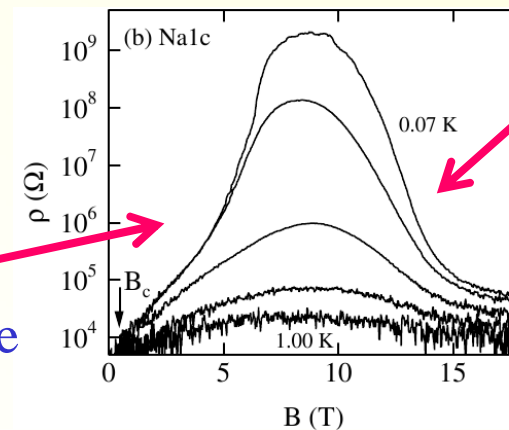
Activated transport near the SIT

V. F. Gantmakher, M. V. Golubkov, J. Lok, A. K. Geim, *Sov. Phys. JETP*, **82**, 951 (1996).

Insulating InO_x



Competing mechanisms:
SC suppressed \leftrightarrow
Single e's easier to liberate

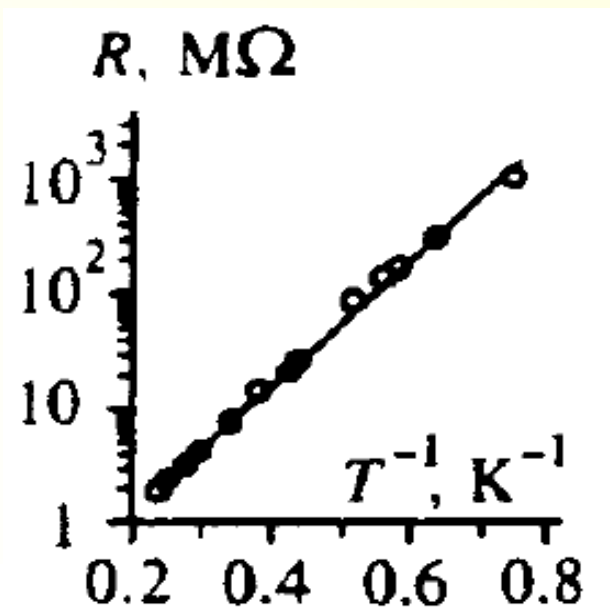


Simpler to understand (simple activation with tendency to VRH)

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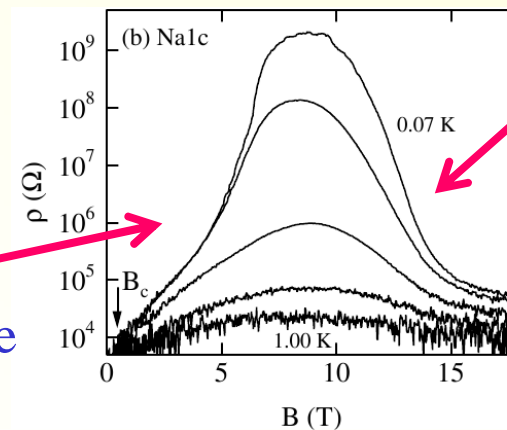
Insulating InO_x



Competing mechanisms:
SC suppressed \leftrightarrow
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Origin of simple activation?

- Gap in the DOS
- Or: mobility edge

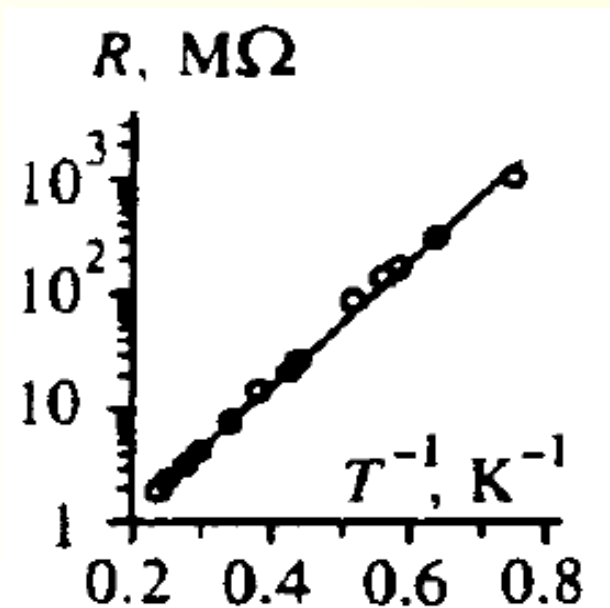


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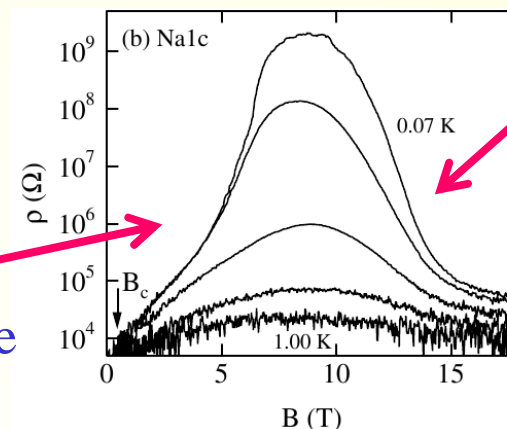


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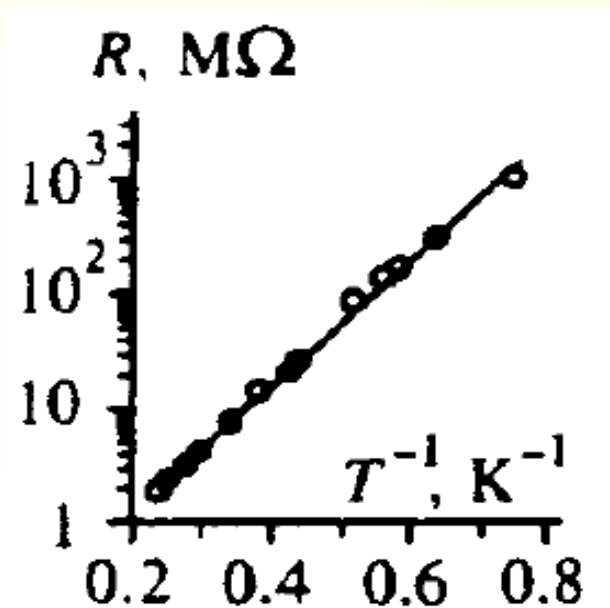


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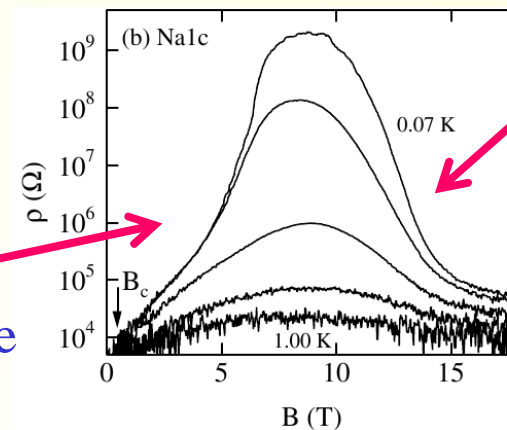
Insulating InO_x



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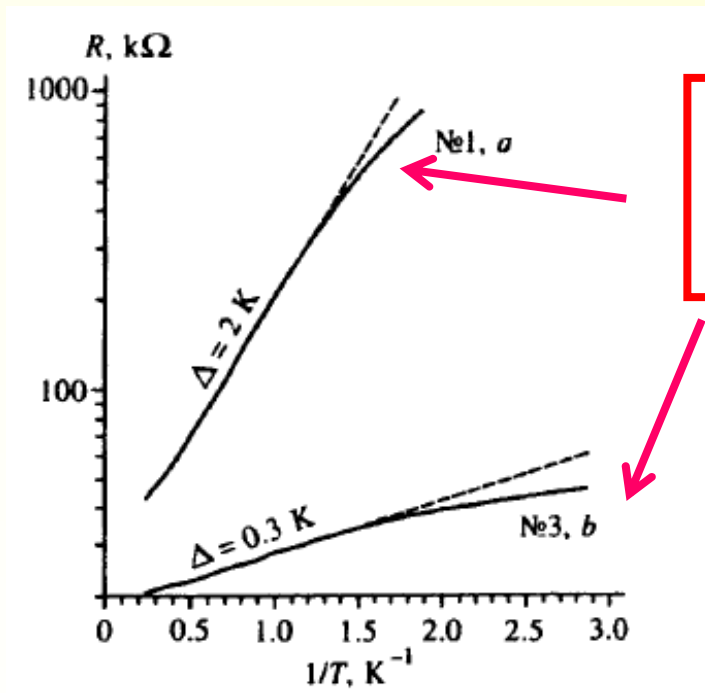
- ~~Gap in the DOS~~
- Or: mobility edge \leftarrow
- Electrons or pairs?
- Nearest neighbor hopping?
- Why no variable range hopping?



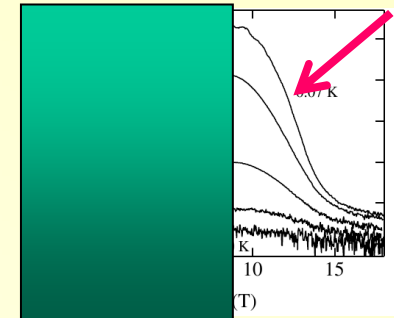
Simpler to understand (simple activation with tendency to VRH)

Remark on high field behavior

V. F. Gantmakher, M. V. Golubkov, J. Lok, A. K. Geim, Sov. Phys. JETP, 82, 951 (1996).

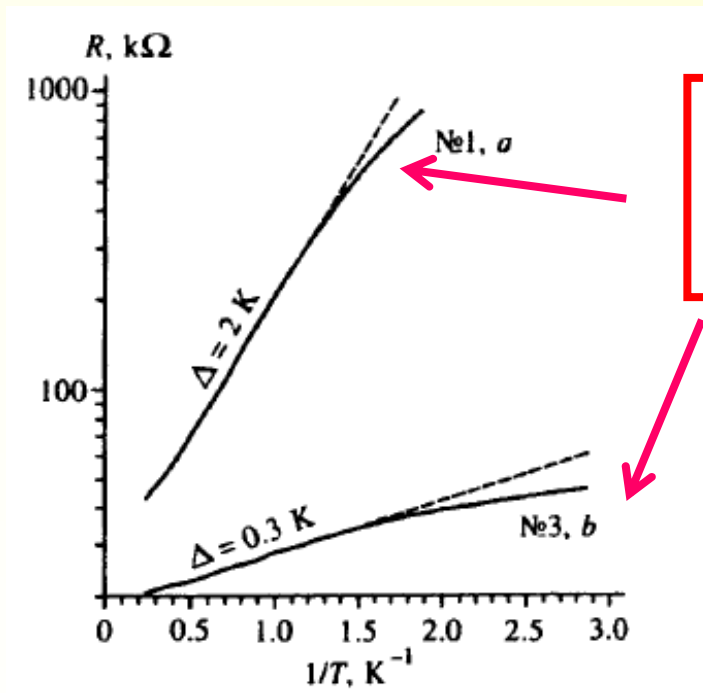


Tendency to subactivation at high B fields, low T

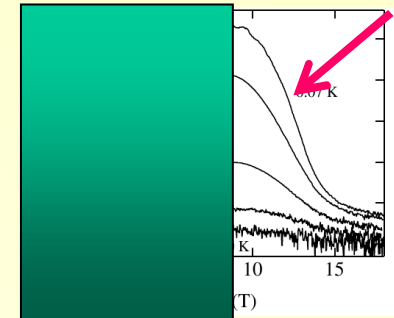


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Tendency to subactivation at high B fields, low T



Most likely interpretation:

Single electron transport:

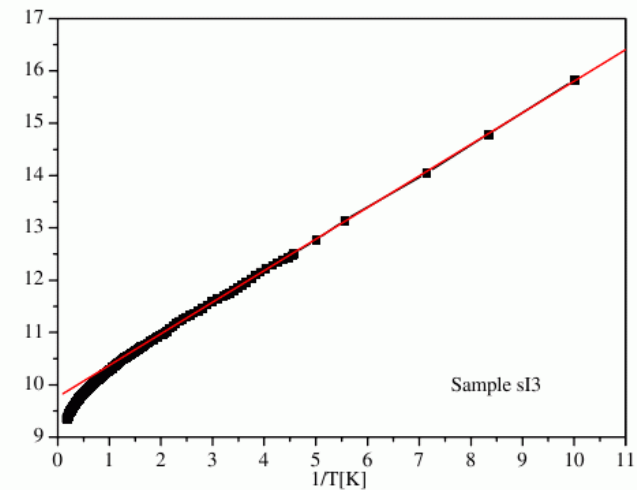
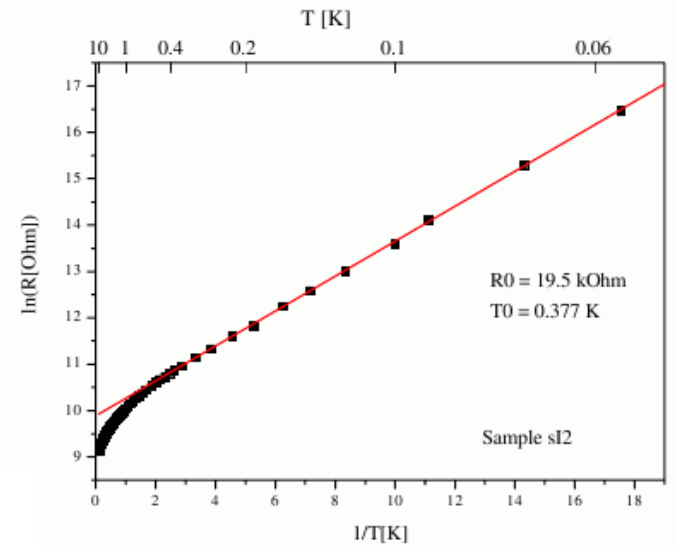
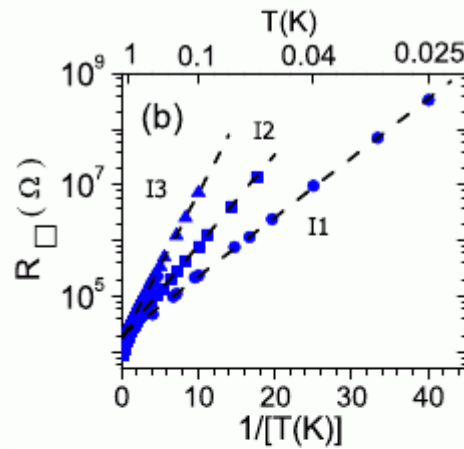
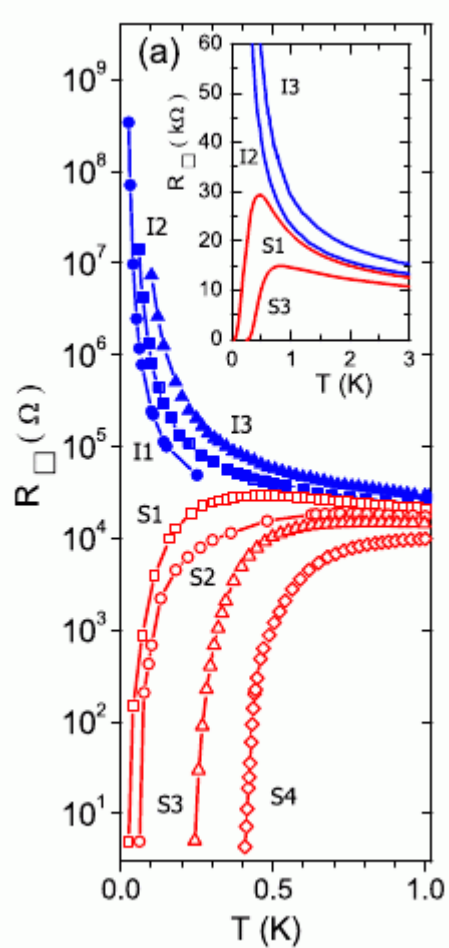
Bottle neck: depairing e's from pairs
(possibly paired in fractal wavefunctions,
Feigelman, Ioffe, Kravtsov Yuzbashyan)

+ variable range hopping of single e's

Activated transport near the SIT

Insulating TiN

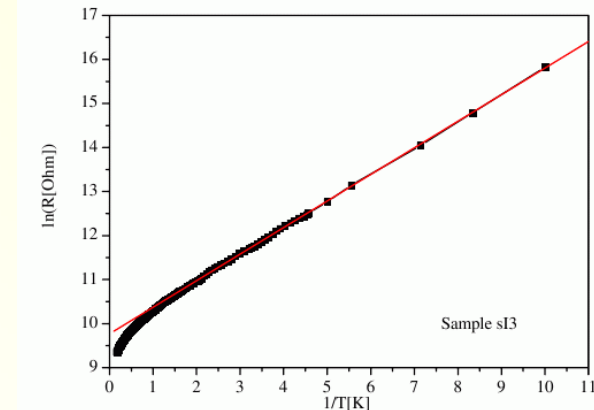
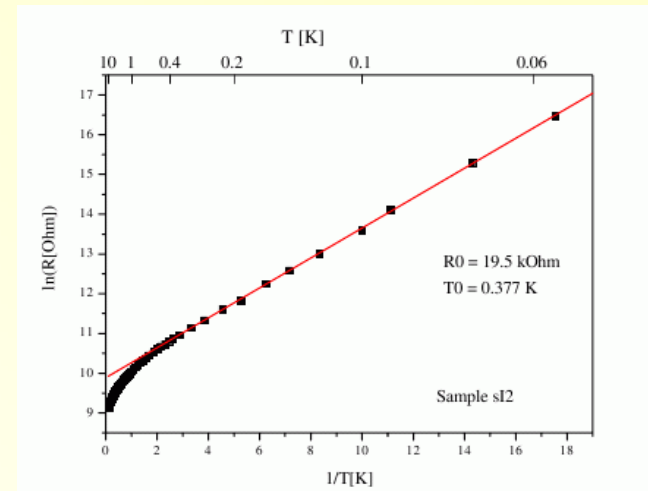
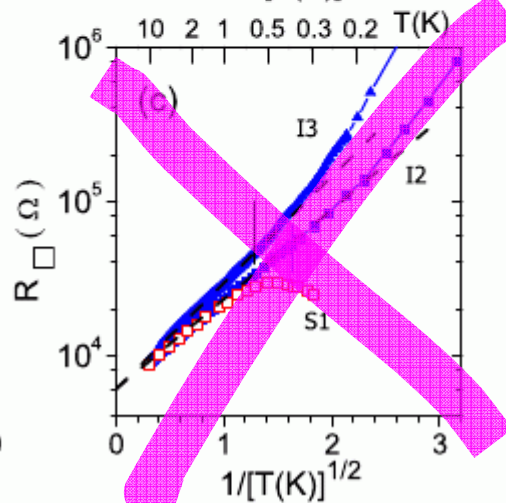
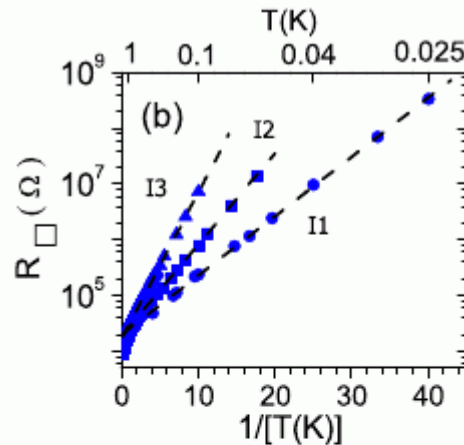
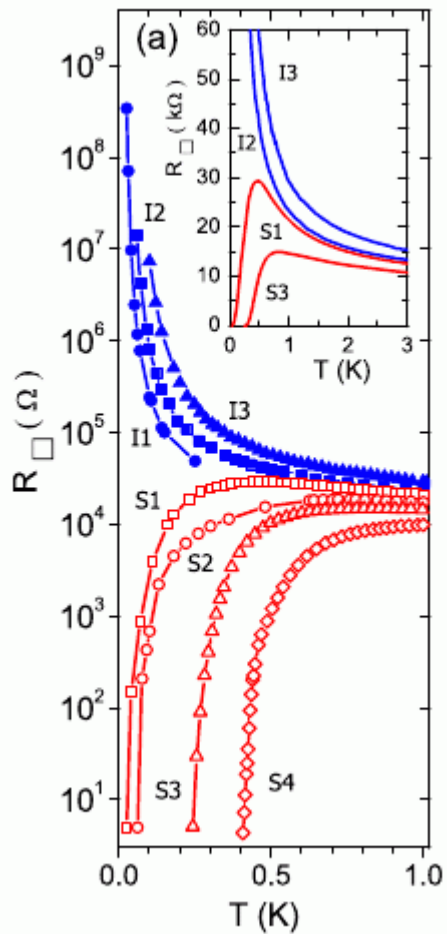
T. I. Baturina et al., PRL 99, 257003 (2007)



Activated transport near the SIT

Insulating TiN

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Summary

1. Close to SIT the transport is essentially simply activated

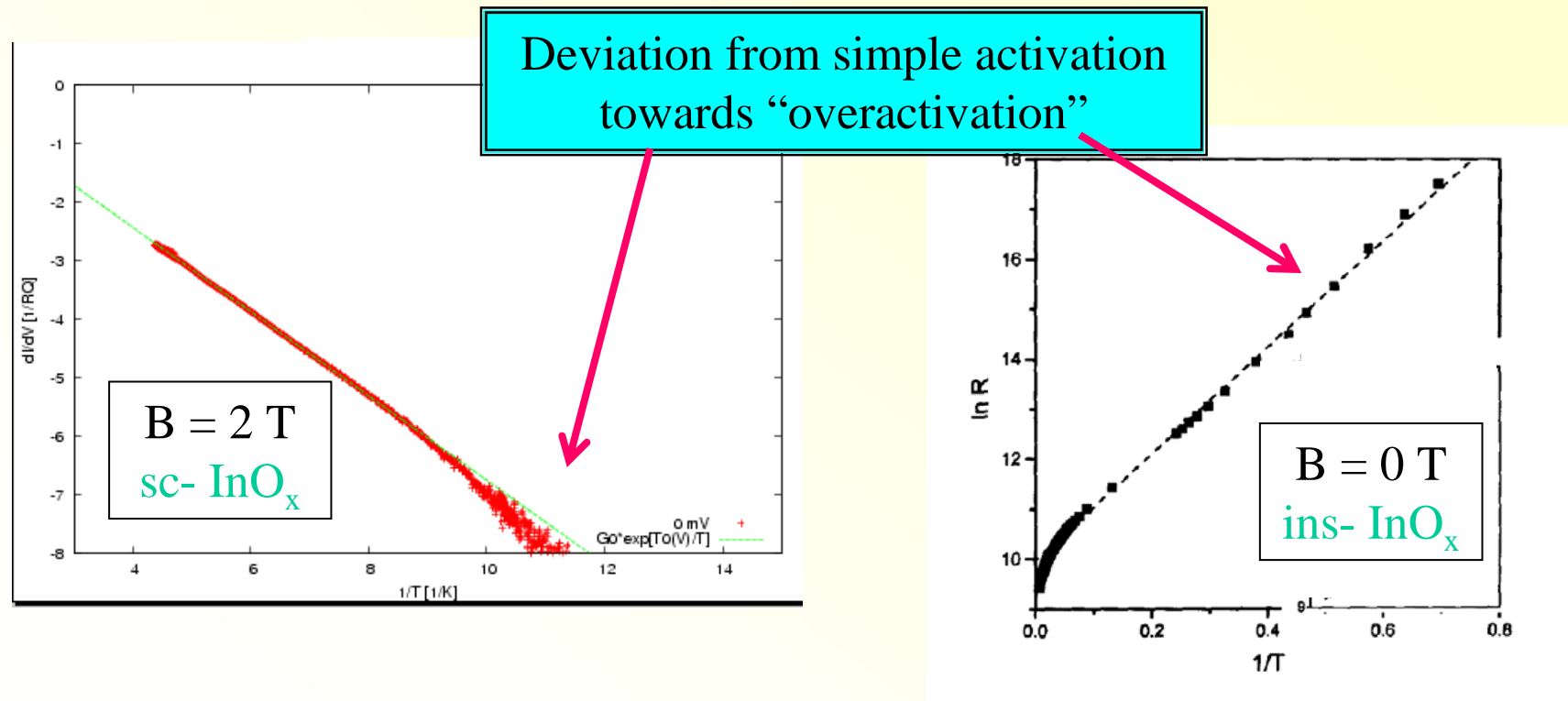
Why?

**2. Beyond the MR peak transport becomes subactivated at
low enough T**

But this is not the whole story yet!

Trend to overactivation

G. Sambandamurthy, L.W. Engel, A. Johansson, and D. Shahar, PRL 97, 107005 (2004).



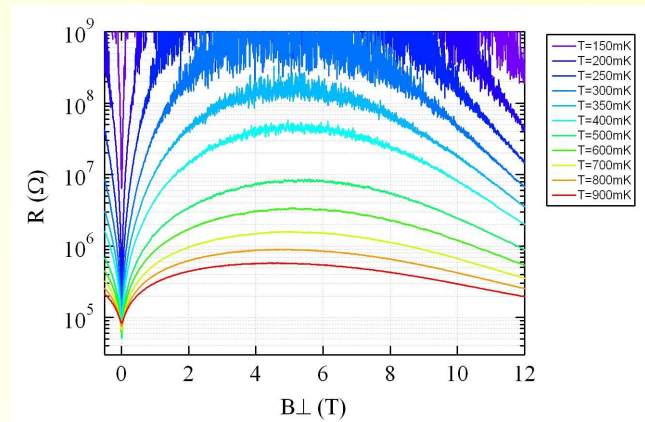
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Trend to overactivation close to SIT

ins- InO_x

B. Sacépé et al. (unpublished - 2008).

Magnetoresistance
(isotherms)

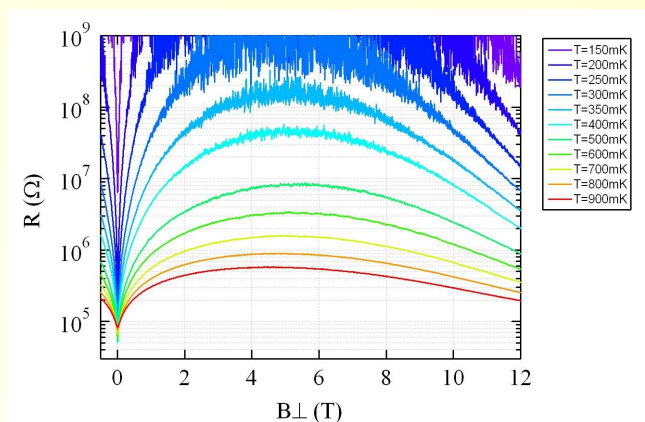


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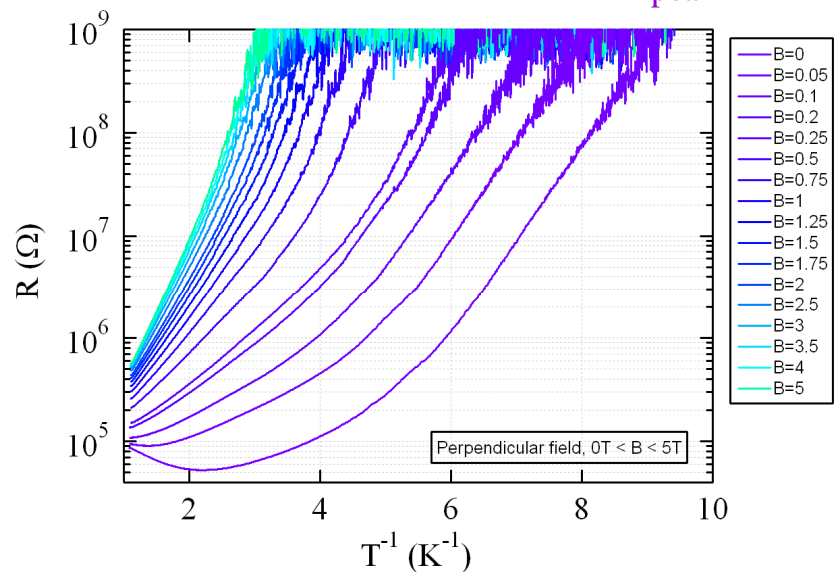
ins- InO_x

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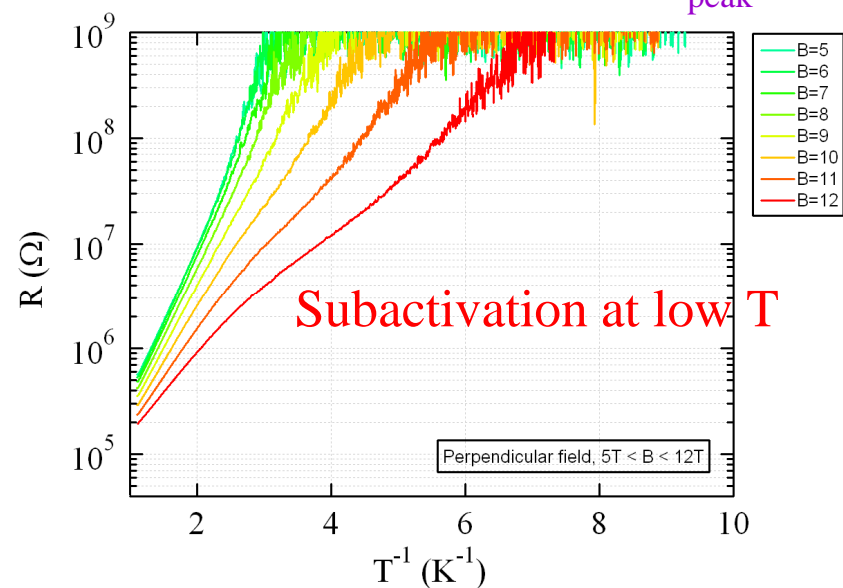
Magnetoresistance
(isotherms)



Resistance at fixed $B < B_{\text{peak}}$



Resistance at fixed $B > B_{\text{peak}}$

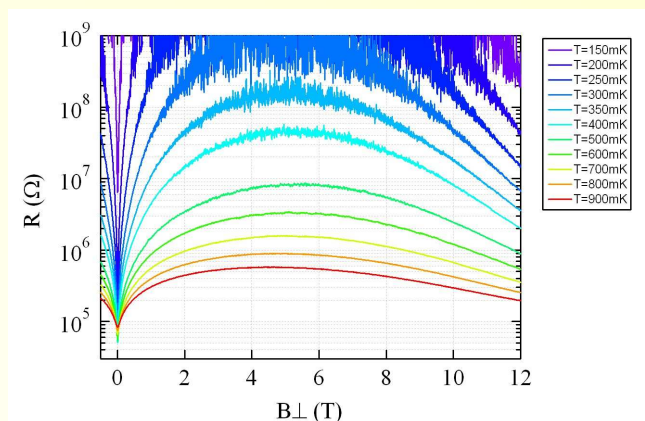


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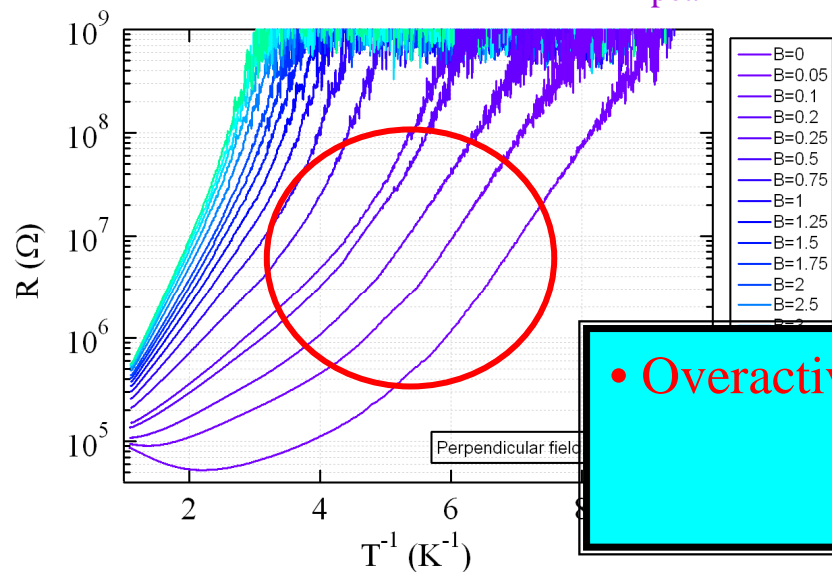
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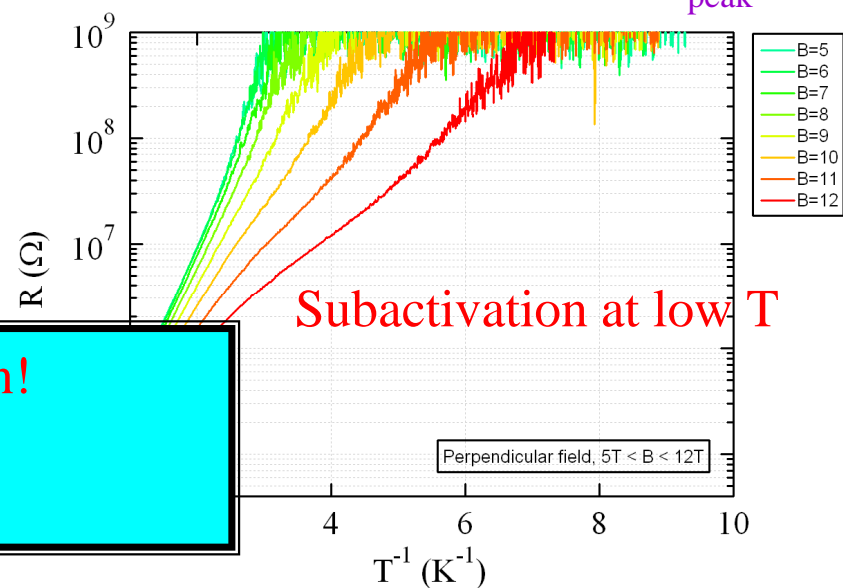
Magnetoresistance
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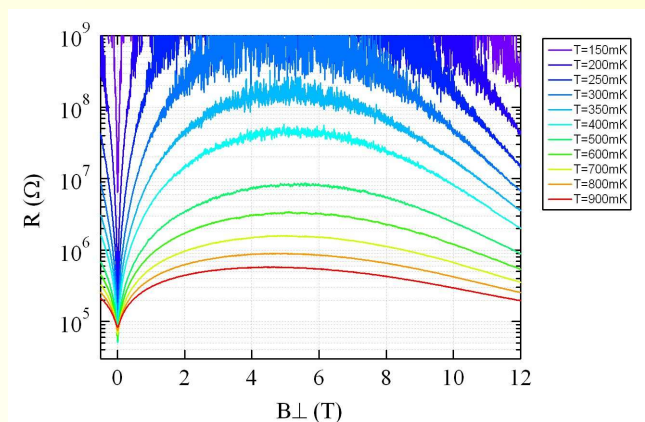
• Overactivation!

Trend to overactivation close to SIT

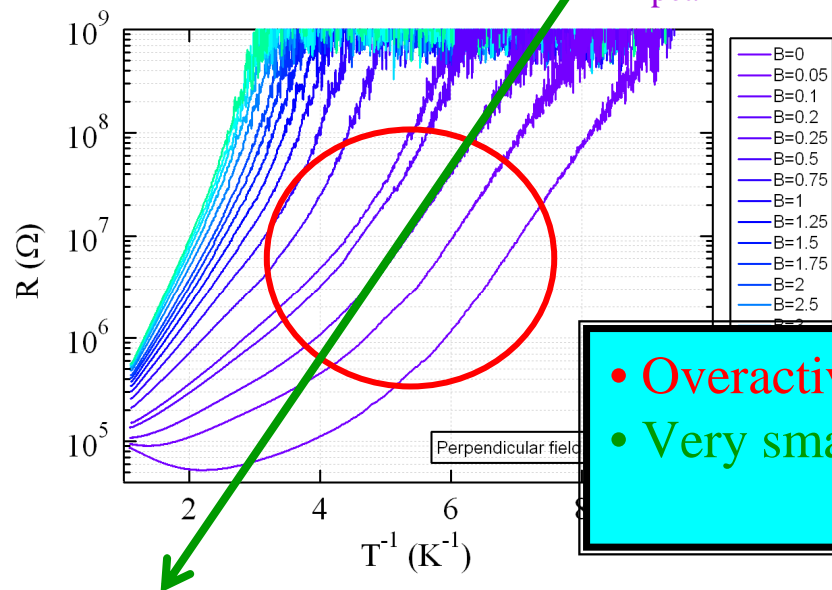
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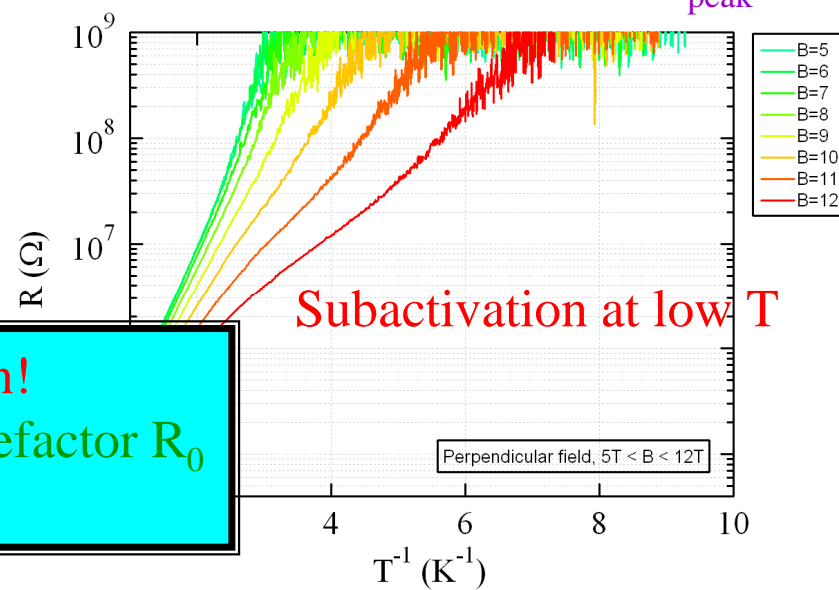
Magnetoresistance
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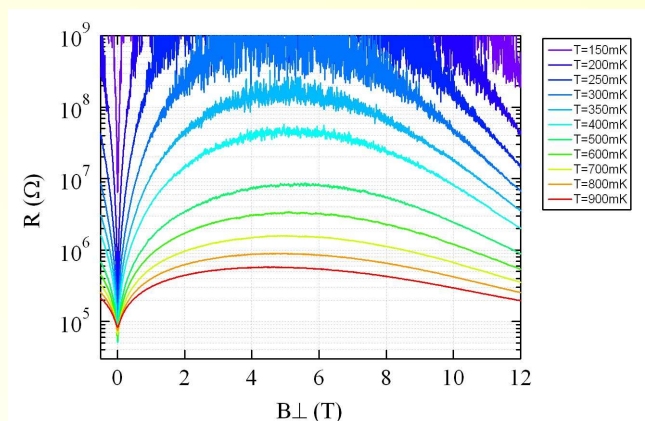
- Overactivation!
- Very small prefactor R_0

Trend to overactivation close to SIT

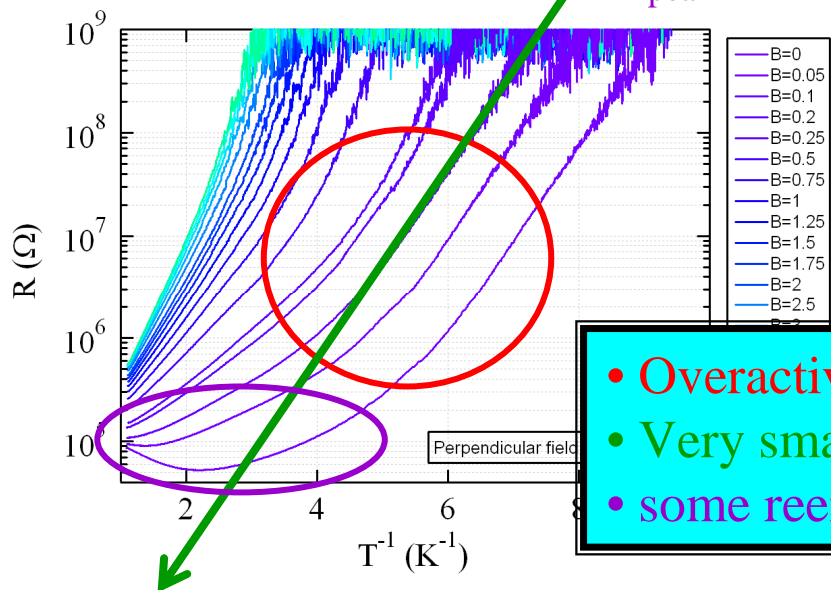
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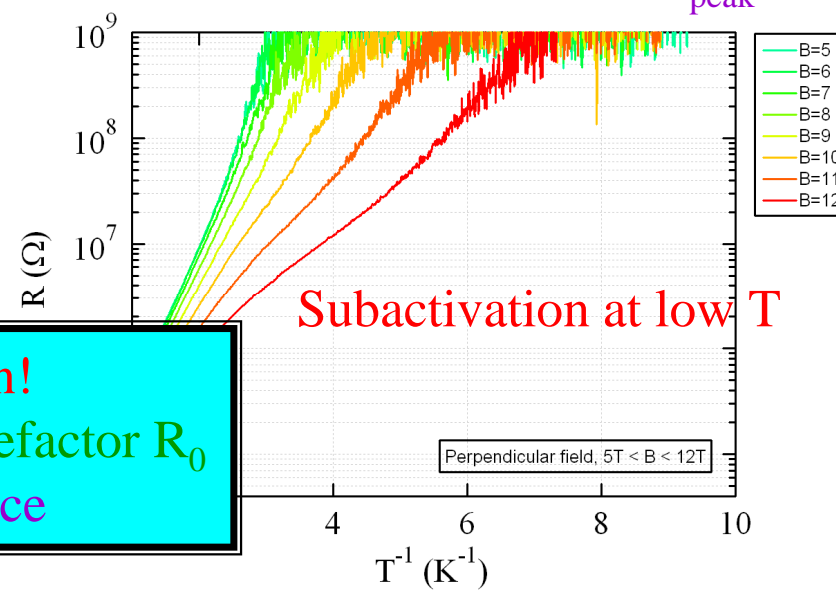
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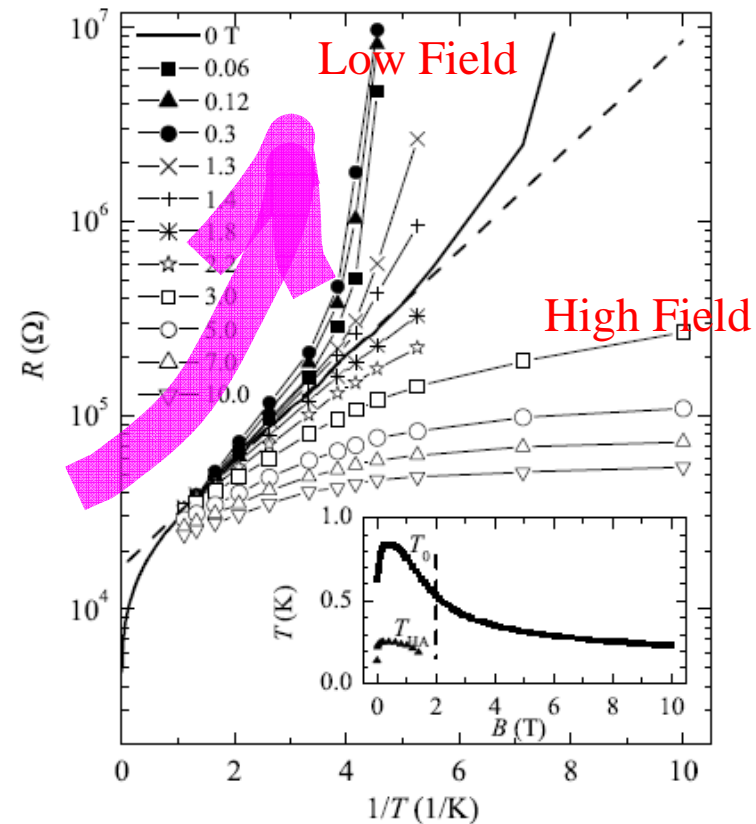
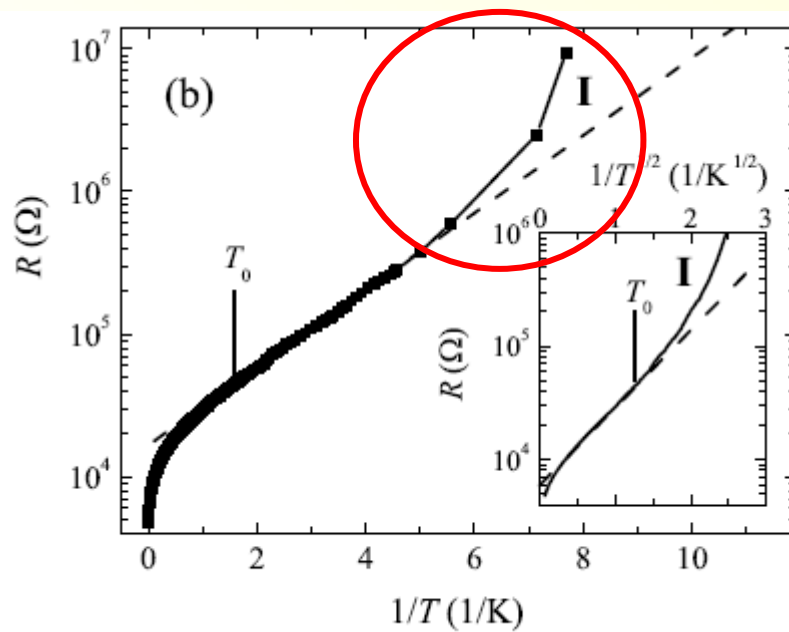


- Overactivation!
- Very small prefactor R_0
- some reentrance

Trend to overactivation close to SIT

T. Baturina et al. (condmat 0810.4351).

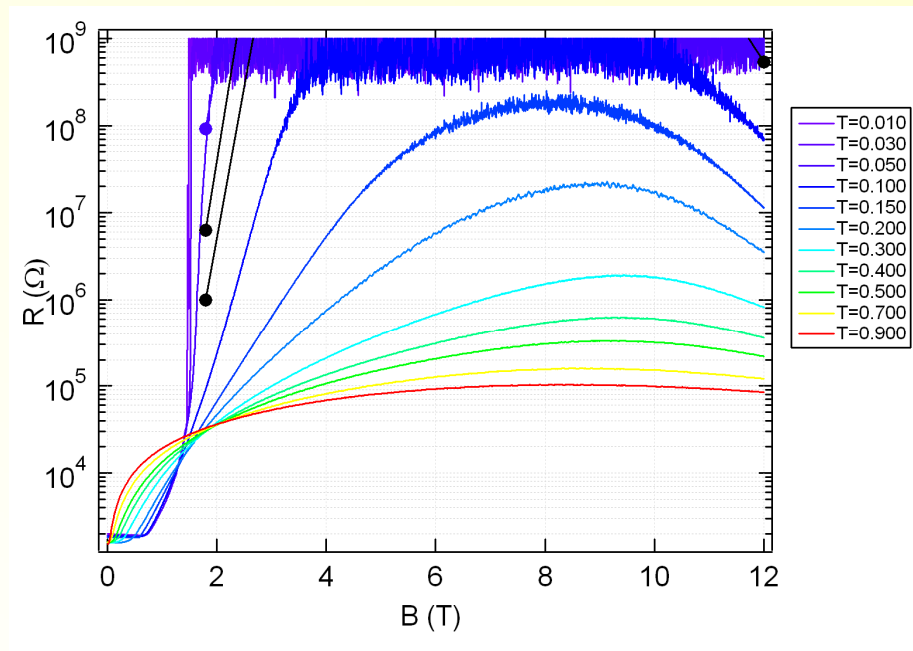
sc- TiN



Summary II

- Transport is **simply activated** at **low T** over several orders of magnitude
- There is a tendency to
 - **overactivation** close to the SIT
(saturating to simple activation at low T)
This is very unusual in a disordered system!
 - **subactivation** beyond the MR peak (at lowest T)

Magnetoresistance near the SIT

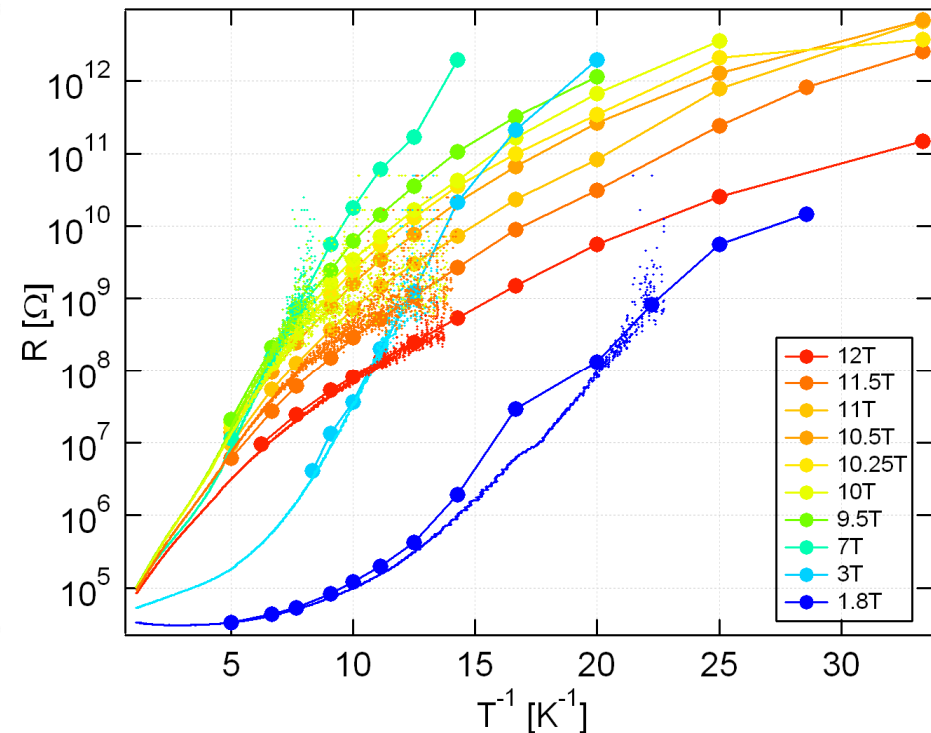
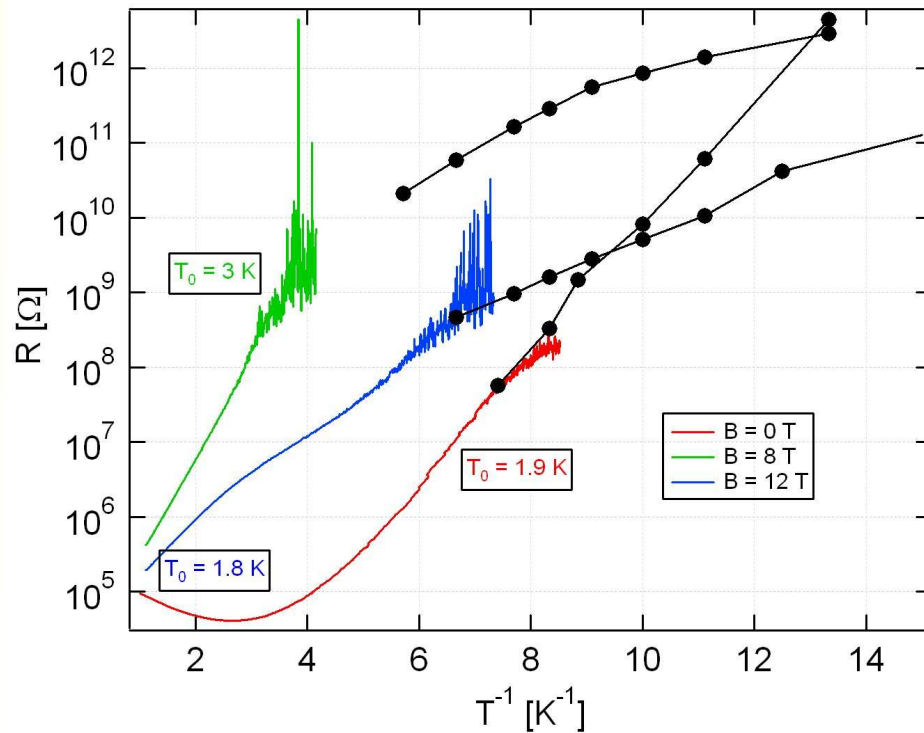


Magnetoresistance near the SIT

(B perpendicular)

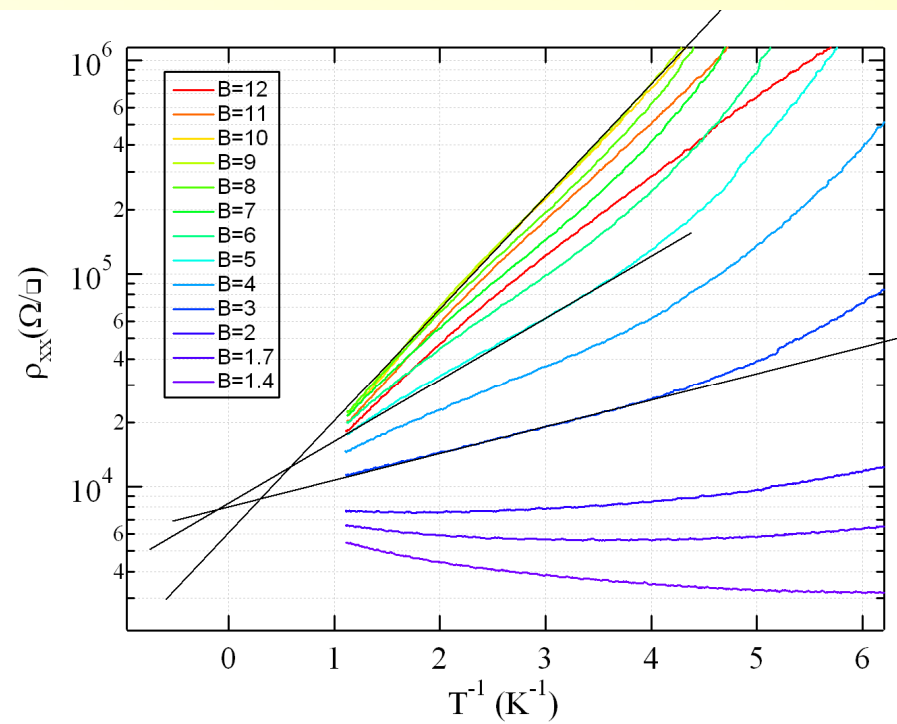
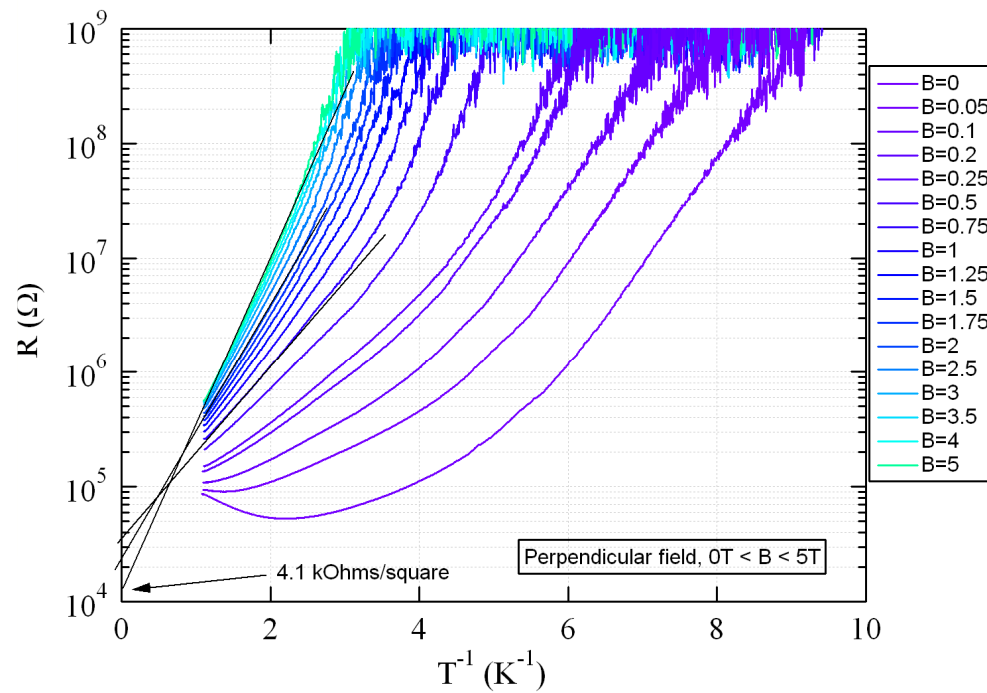
InO-I

InO-S

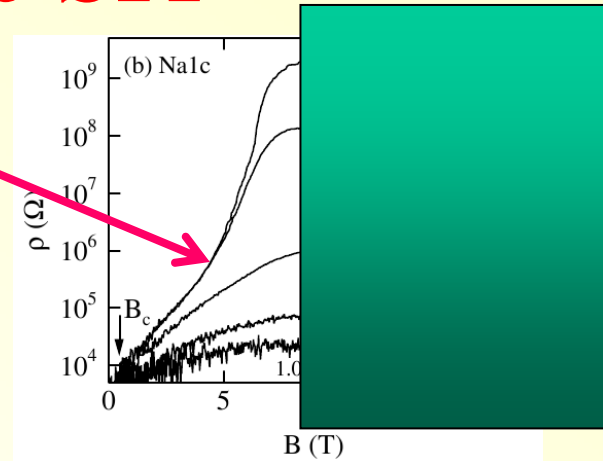


Arrhenius behaviour « seems » to hold at low field ($<$ MR peak)

Magnetoresistance near the SIT

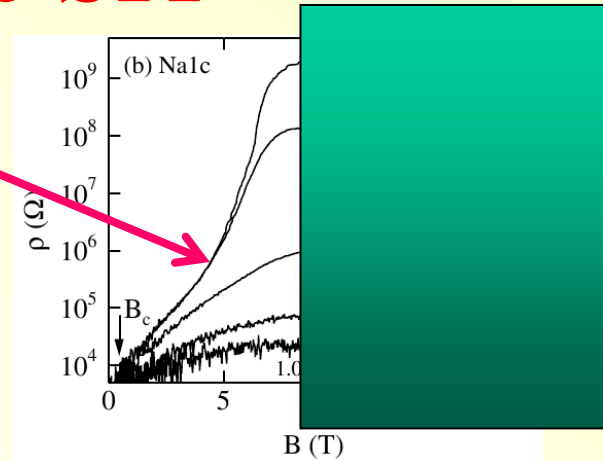


Transport must be by pairs close to the SIT



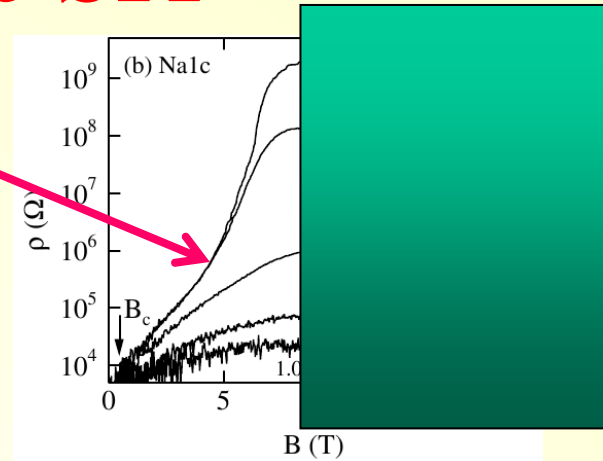
1. $B \uparrow \rightarrow$ Pairs are less delocalized \rightarrow positive MR

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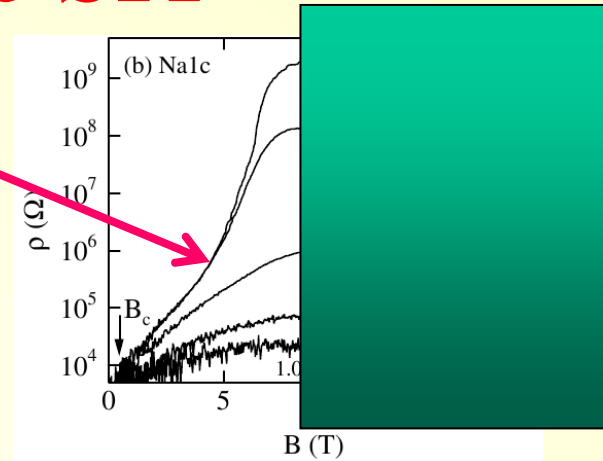
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2. If transport were carried by electrons, MR would be negative:
it becomes easier to depair electrons at higher fields.

Transport must be by pairs close to the SIT



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2. If transport were carried by electrons, MR would be negative: it becomes easier to depair electrons at higher fields.
3. Shrinking wavefunctions (negative MR in the 1 electron channel as in dilute semiconductors) is irrelevant.

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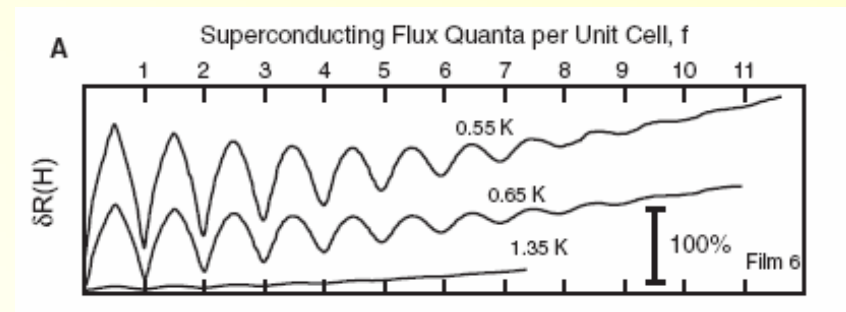
\rightarrow Transport must be by pairs close to the SIT!

Transport must be by pairs close to the SIT

→ Transport must be by pairs close to the SIT!

Further support:

Magnetoresistance in a film
with a regular array of holes
Period $\Phi_0/2 \rightarrow$ pair transport!



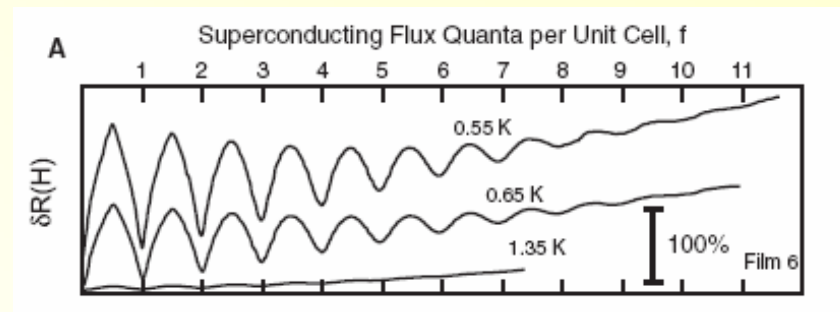
(J. Valles et al. Science 2006)

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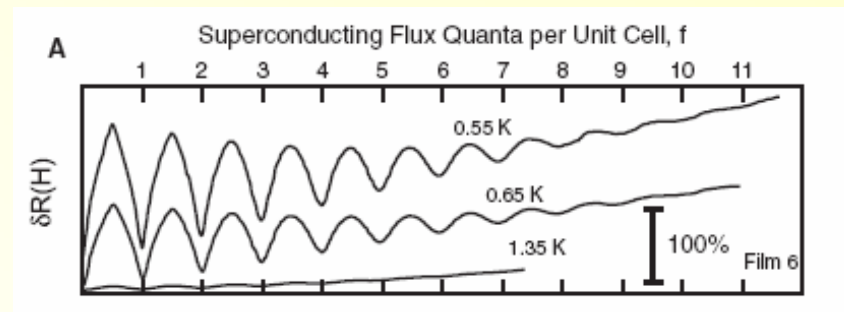
- Why? Pairs survive in the insulator! (Pairing in time-reversed localized wavefunctions (Anderson 1956, Feigelman et al.) – as suggested by STM).

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- Why? Pairs survive in the insulator! (Pairing in time-reversed localized wavefunctions (Anderson 1956, Feigelman et al.) – as suggested by STM).
- As long as $E_{\text{act}} < E_{\text{bind}}$ it **does not** pay to **depair electrons at lowest T.**

Scenarios for simple activation in the positive MR regime



- A. Global charge gap?
 - effectively granular material?
 - Wigner crystal?

B. Nearest neighbor transport?

If none of the above applies:

C. Why is variable range hopping not observed?

→ Proposal: Activation to the pair mobility edge

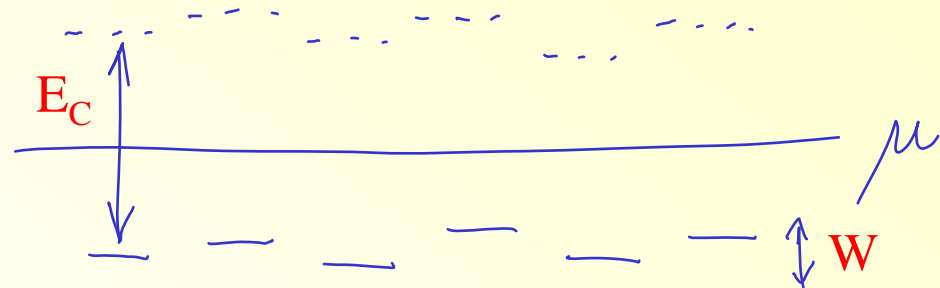
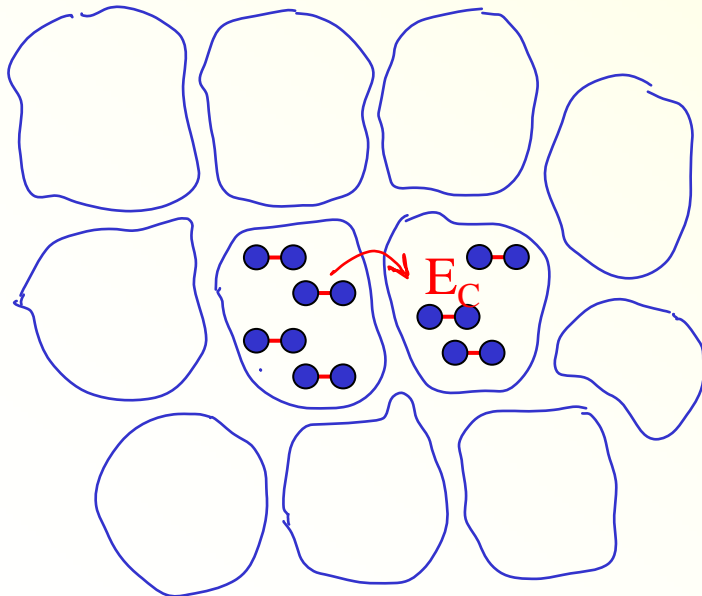
Scenarii for simple activation:

A. Global charge gap I

Vinokur et al. (2007/2008):

Postulates:

- I. Effective granularity:
Superconducting puddles with low transparency tunnel junctions.
- II. Weak disorder $W < E_C$



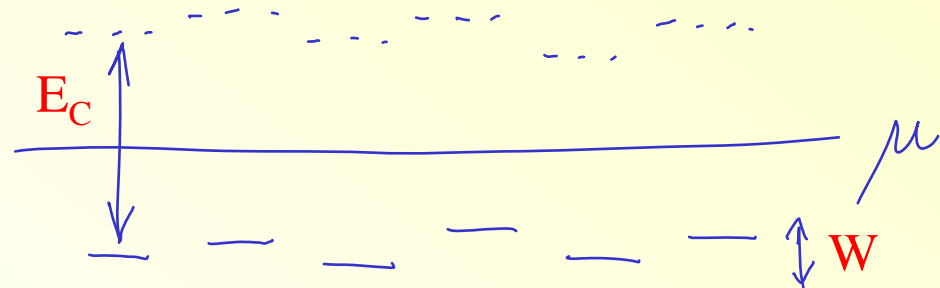
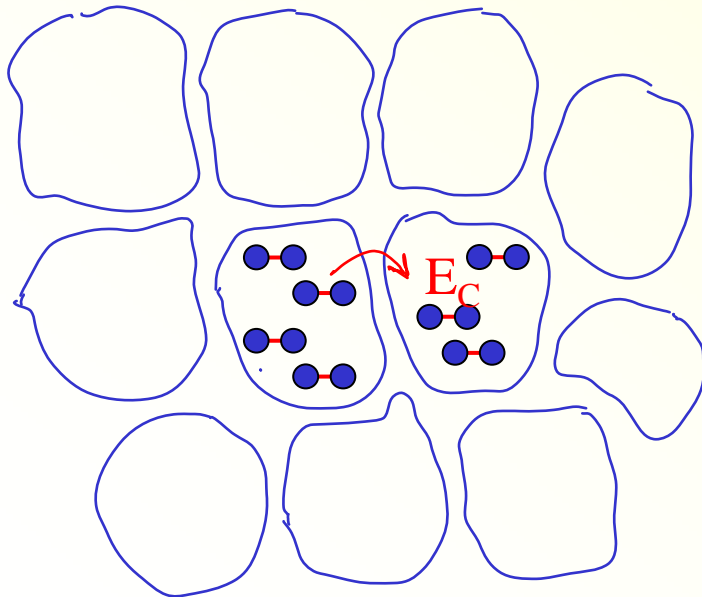
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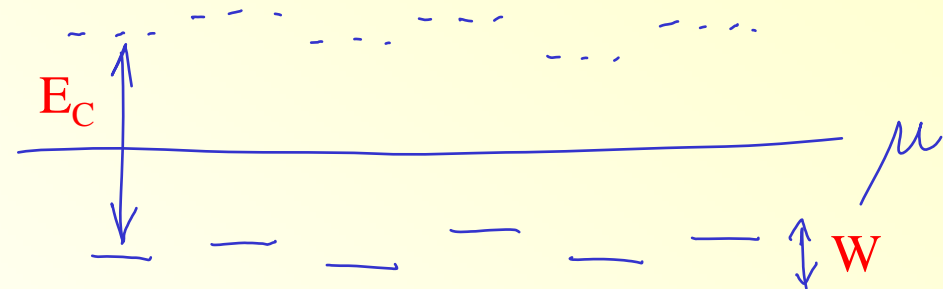
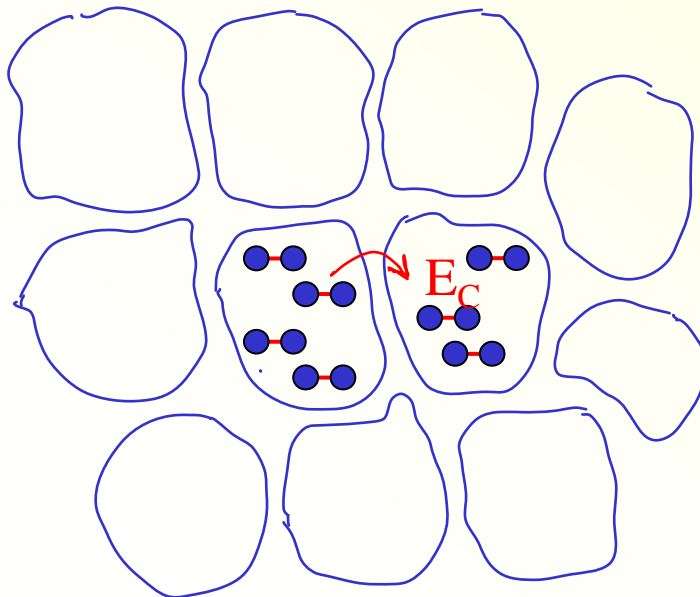
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- Incompressible system
- Simple activation due to charge gap (Coulomb blockade)

(I) can occur in strong disorder (MM and B. Shklovskii, 2008),
but **(II) is very hard to justify** in the absence of clean physical grains.

Scenarii for simple activation:

A. Global charge gap II

Charged pairs (2e) in a weak background potential (white noise)

Intermediate between

Falco, Nattermann, Pokrovsky (2008) [neutral bosons, white noise]

Müller and Shklovskii (2008) [charged bosons, charged impurities]

$$\frac{\hbar^2}{2m} \nabla^2 \psi + (E - U(\mathbf{x})) \psi = 0.$$

$$\langle U(\mathbf{x}) U(\mathbf{x}') \rangle = \kappa^2 \delta(\mathbf{x} - \mathbf{x}')$$

$$\mathcal{L} = \frac{\hbar^4}{m^2 \kappa^2}, \quad \mathcal{E} = \frac{\hbar^2}{m \mathcal{L}^2},$$

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e.g., position fractal pseudospins

IF weak disorder/heavy pair masses:

$$\frac{e^2}{\kappa L} > \mathcal{E} \iff L > a_B$$



Charge correlated state
(distorted pair Wigner crystal)
at low enough density:

$$n a_B^3 < 1 \quad a_B = \frac{\hbar^2 \kappa}{m e^2}$$

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Transport ?

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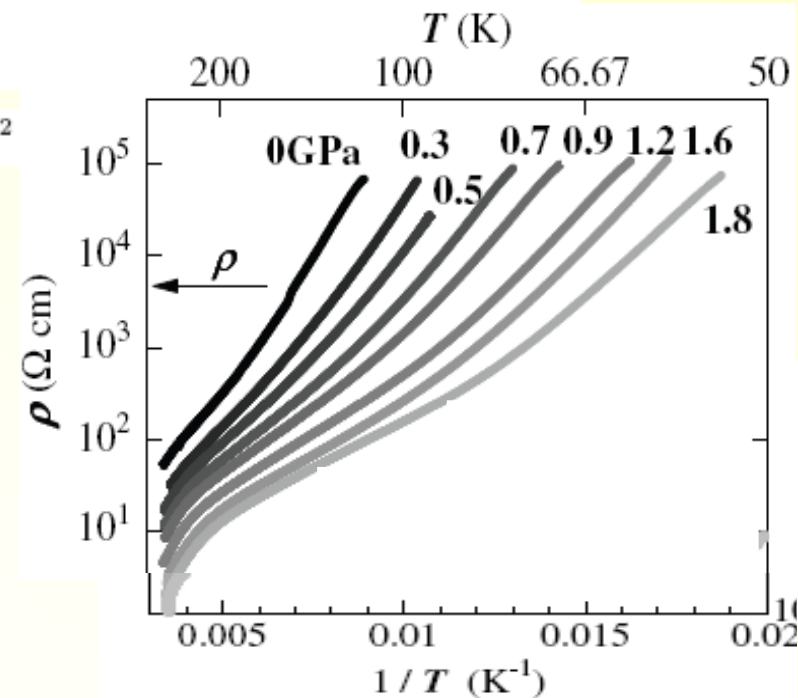
Compare to standard Mott insulator:

Transport properties of an organic Mott insulator

β' -(BEDT-TTF)₂ICl₂

N. TAJIMA^{1(a)}, R. KATO¹ and H. TANIGUCHI²

EPL, 83 (2008) 27008



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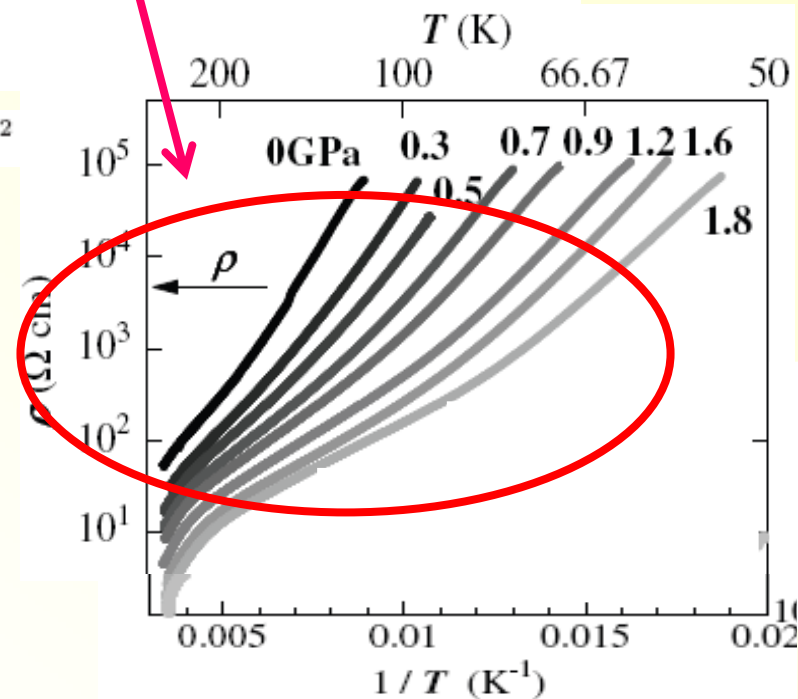
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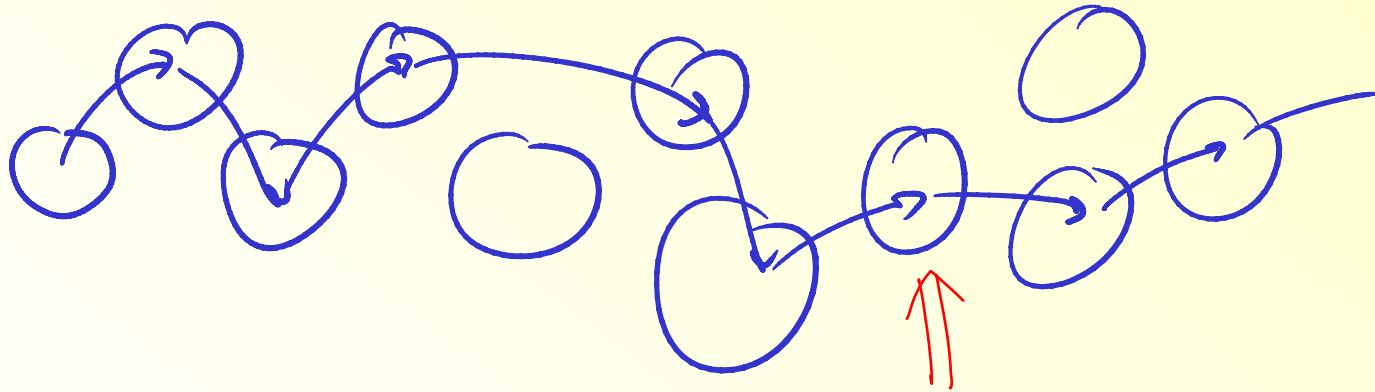


Partial conclusion:

- Global charge gap (incompressibility) seems very unlikely in a non-granular film
- It would require very weak effective disorder
- Should in principle be detectable by pinning frequency of vibration modes of the charge ordered structure (Wigner crystal)

Scenario B: nearest neighbor hopping?

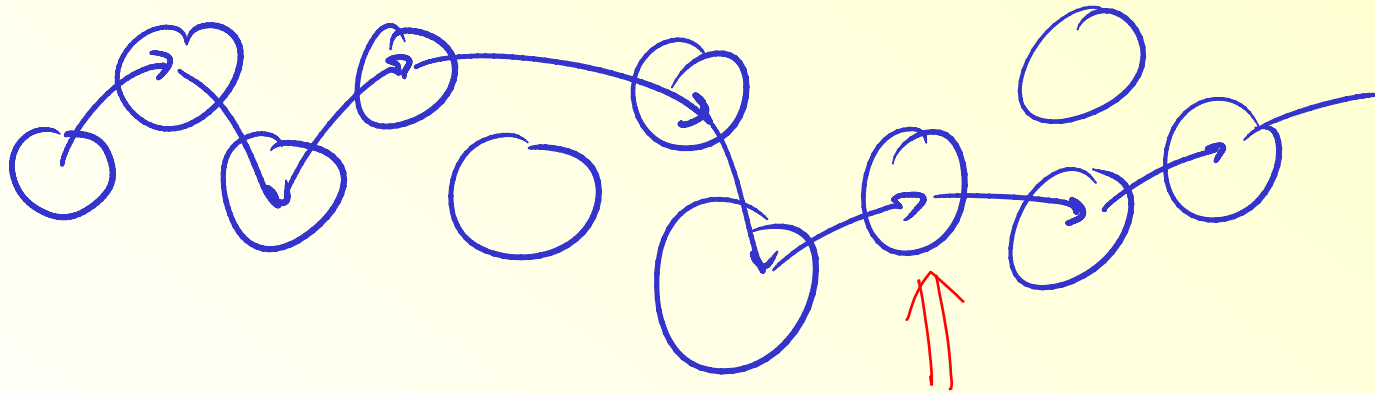
Hopping from puddle to puddle (in strong disorder):



Weakest link (highest barrier along the path)
Determines activation energy

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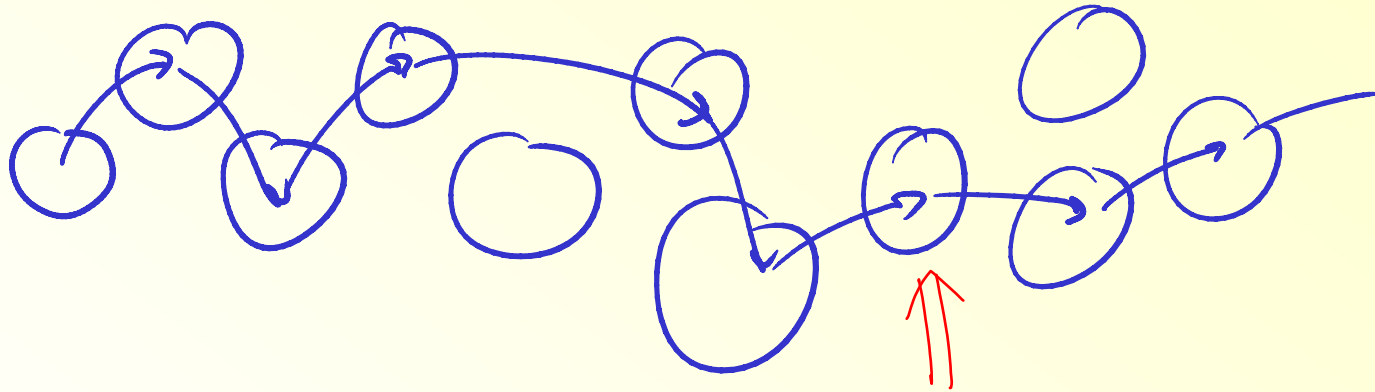
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This ONLY gives simple activation in an appreciable T-window if puddle-puddle resistance is very high, otherwise one obtains VRH!

$$R = R_0 \exp[T_0/T] \quad \text{with} \quad R_0 \gg h/e^2$$

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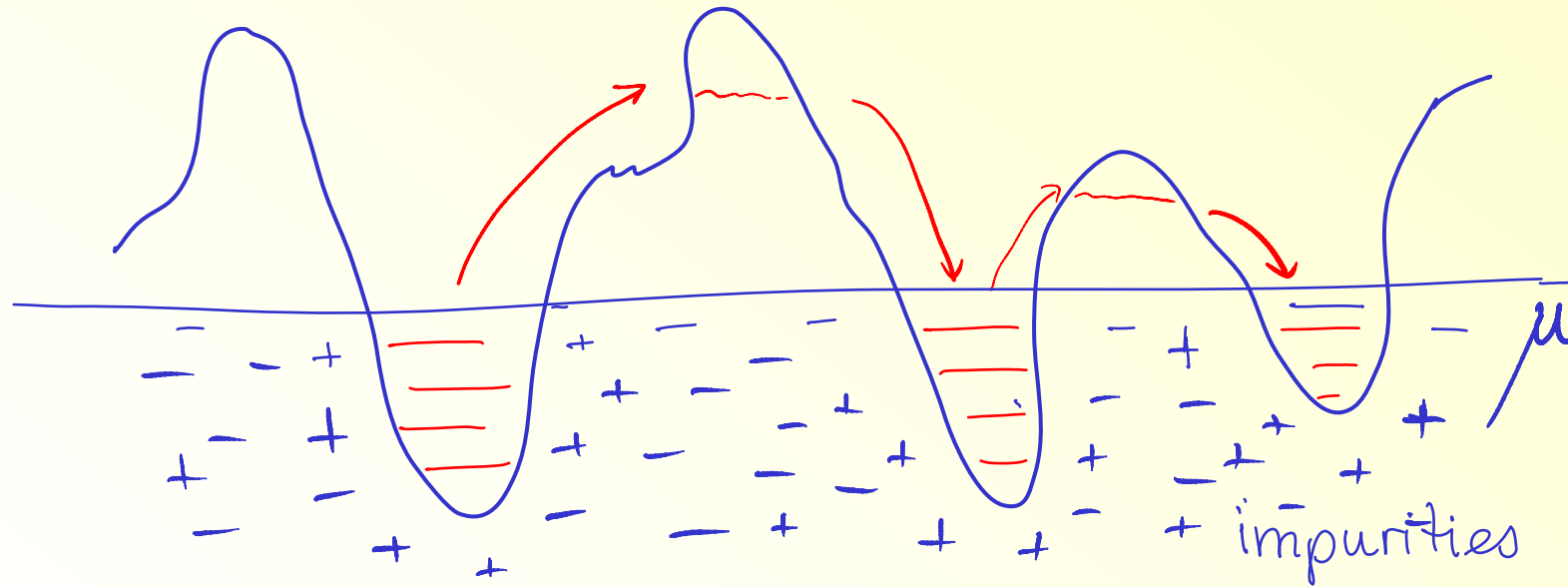
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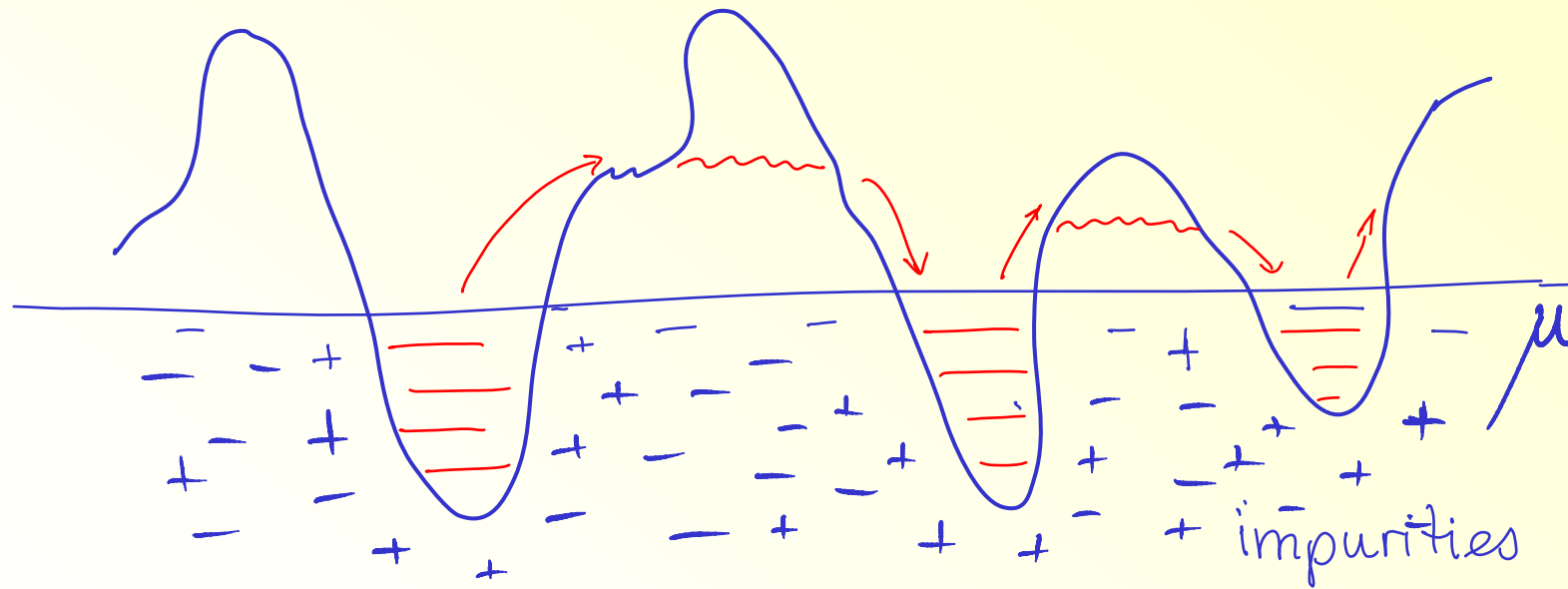
C. Hopping between droplets

High T:



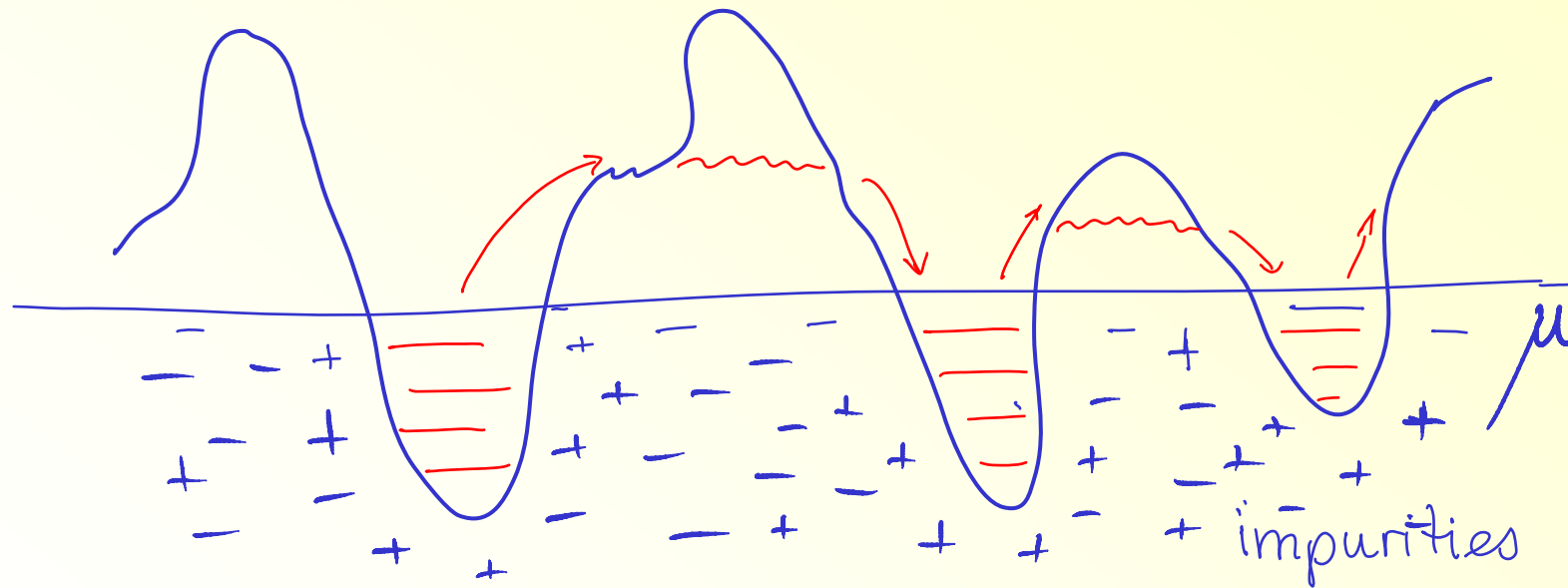
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Lower T:



C. Hopping between droplets

Lower T:



Activation + Tunneling

$\Rightarrow E_{act}(T) \downarrow \text{ as } T \downarrow$

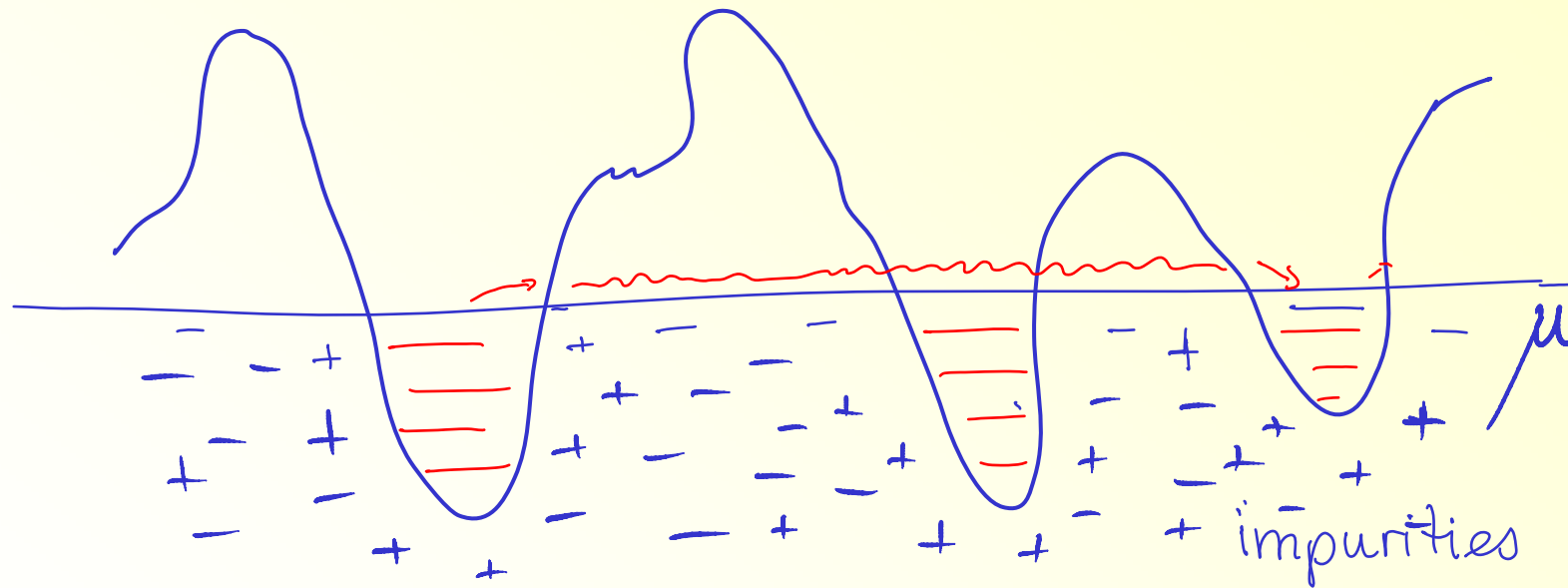
\Rightarrow Subactivation !

$$E_{act} = E_c - AT^{-2/3}$$

(Shklovskii 1973)

C. Hopping between droplets

Lowest T: Variable range hopping



Activation + Tunneling

$$R = R_0 \exp\left[\left(T_0/T\right)^\gamma\right] \quad \gamma \approx \frac{1}{2} \quad (=5/11)$$

Subactivation !

(Shklovskii 1973)

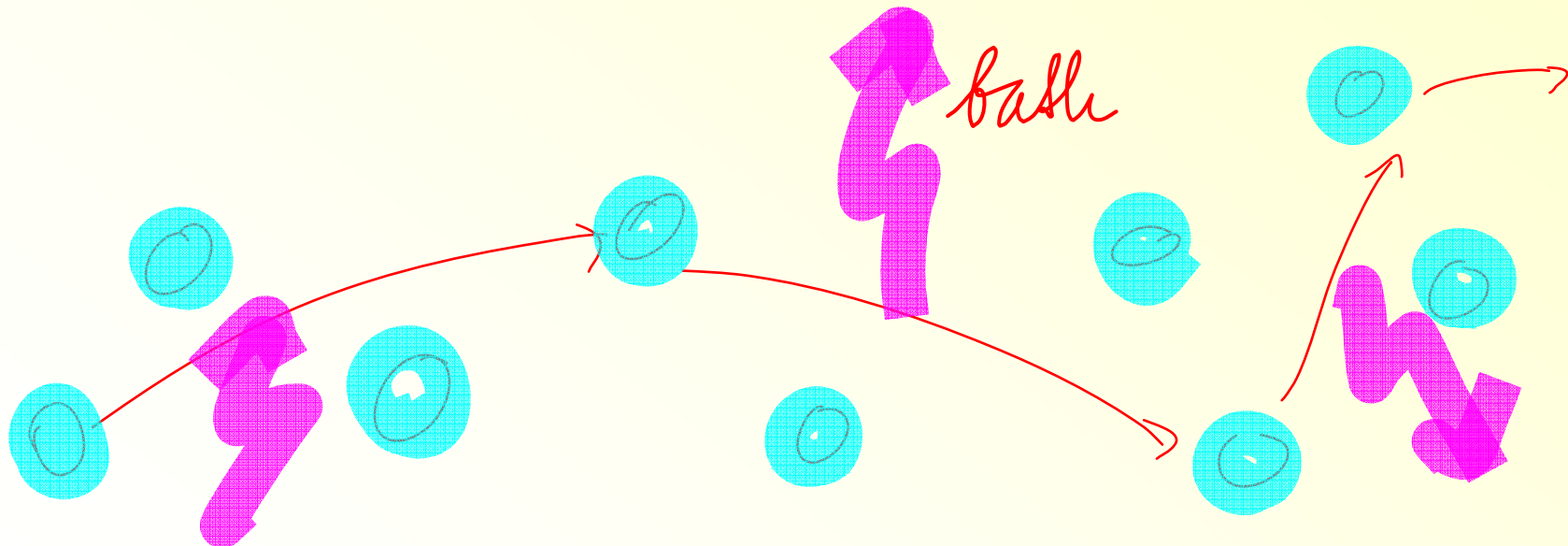
D. Activation to mobility edge –

No variable range hopping,
but overactivation instead ? !

- Review of essentials of VRH
- Necessity of a continuous bath!
- Argue that there is **NO BATH**: get simple and over-activation!

? How to understand that variable range hopping is not seen, but instead overactivation? ?

Essential ingredient into VRH:
Continuous bath which activates the hops!

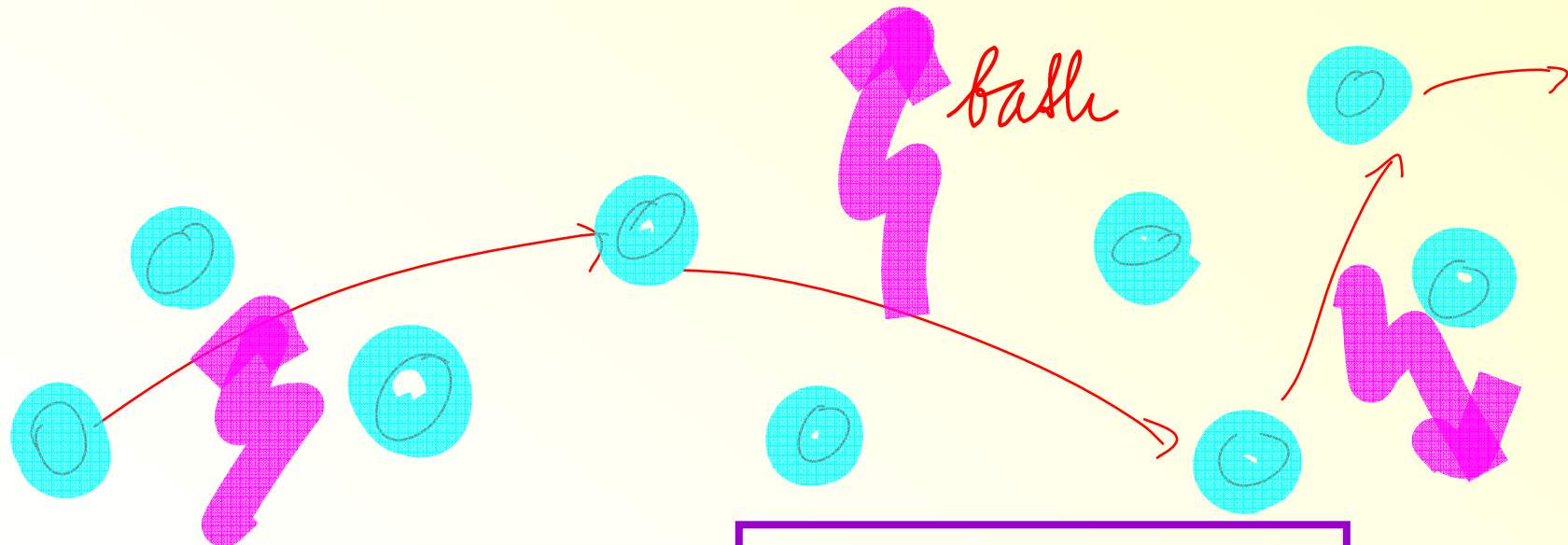


Candidates for the bath:

- Phonons: at low T for pair hopping are very inefficient!

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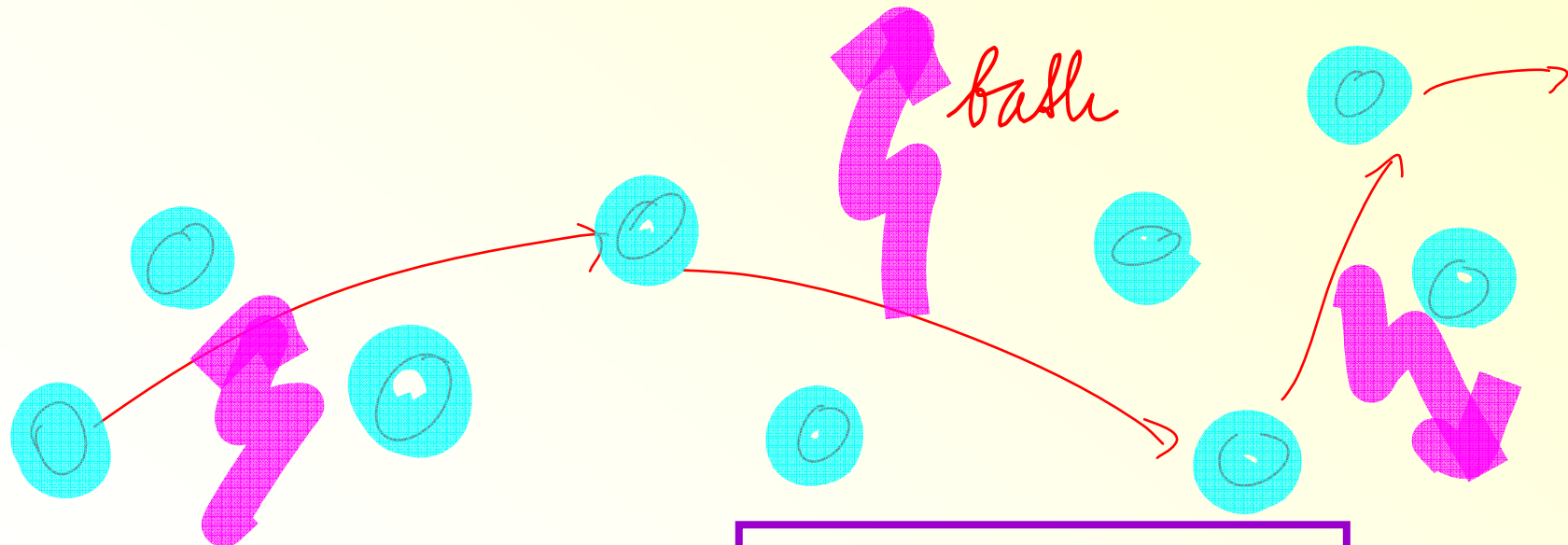
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Too weak → not considered

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Candidates for the bath:

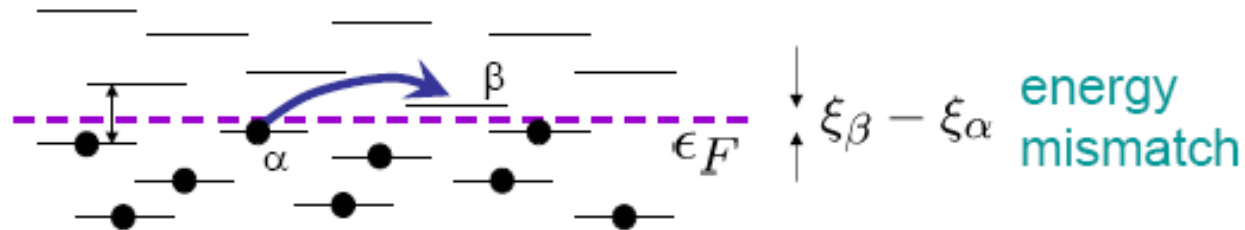
- ~~Phonons: at low T for pair hopping are very inefficient!~~
- Collective electronic (i.e., pair) excitations ?

Localization despite interactions?

Fleishman, Anderson, Licciardello (1980, 1982)

Basko et al., Gornyi et al. (2005, 2006)

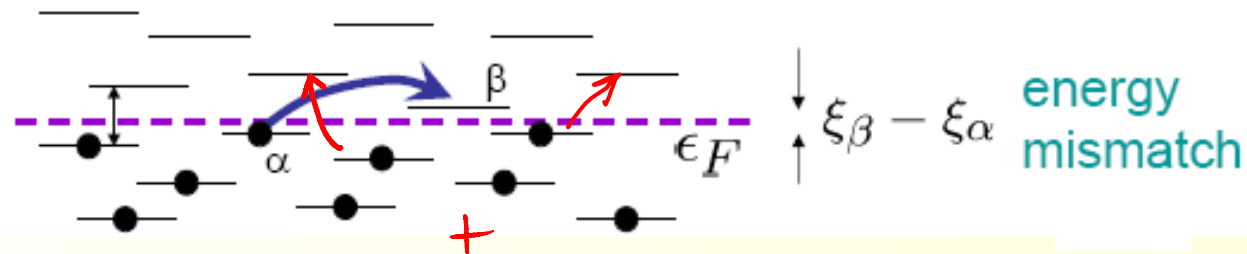
Is there **many-body localization** (localization in Hilbert space) \leftrightarrow **absence of diffusion**; even at finite **T**?



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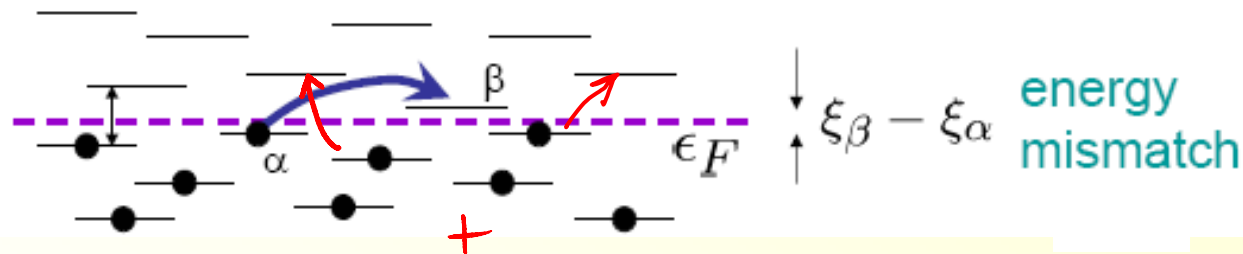


Can multi-particle arrangements
bridge the energy mismatch?

Localization despite interactions?

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Is there **many-body localization** (localization in Hilbert space) \leftrightarrow **absence of diffusion**; even at finite **T**?



Assumptions:

1. Low dimensions \rightarrow all single particle states are localized
2. Weak short range interactions
3. No phonons

Answer: For $T < \delta_\xi / \lambda$ ($\lambda \ll 1$: interaction parameter)

- **Energy conservation impossible**: electrons do not constitute a continuous bath!
- All many body excitations remain **discrete** in energy!
- **Conductivity = 0** even at finite T!

Localization with interaction?

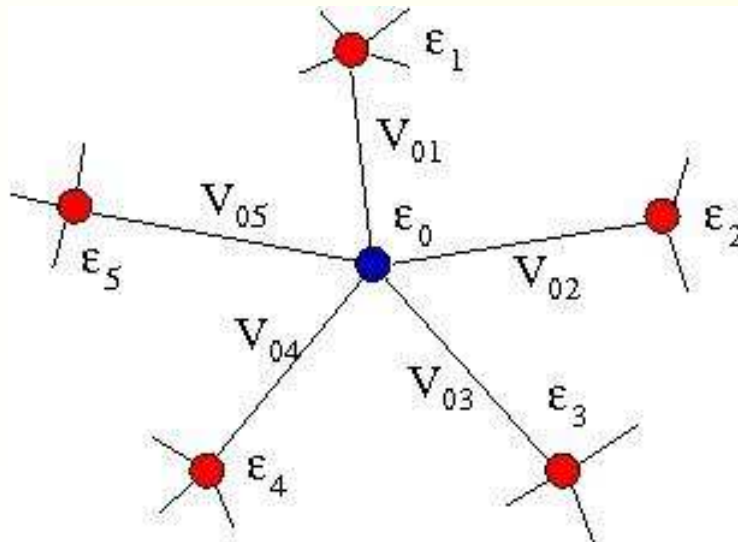
Investigation to all orders in perturbation theory:

I. V. Gornyi, A. D. Mirlin, and D. G. Polyakov, PRL 95, 206603 (2005).

D. M. Basko, I. L. Aleiner, and B. L. Altshuler, Ann. Phys. 321, 1126 (2006).

Assumption: Weak interactions: $V_{int} \ll \delta_\xi$ (level spacing).

Conclusion: An energy crisis (and thus a **metal-insulator transition** in the absence of phonons) occurs at high temperature $T \gg \delta_\xi$ -- due to “localization in Fockspace”.



$$T_{MIT} \approx \frac{\delta_\xi^2}{V_{int}} \gg \delta_\xi \approx T_{\text{Mott-hopping}}$$

Argument:

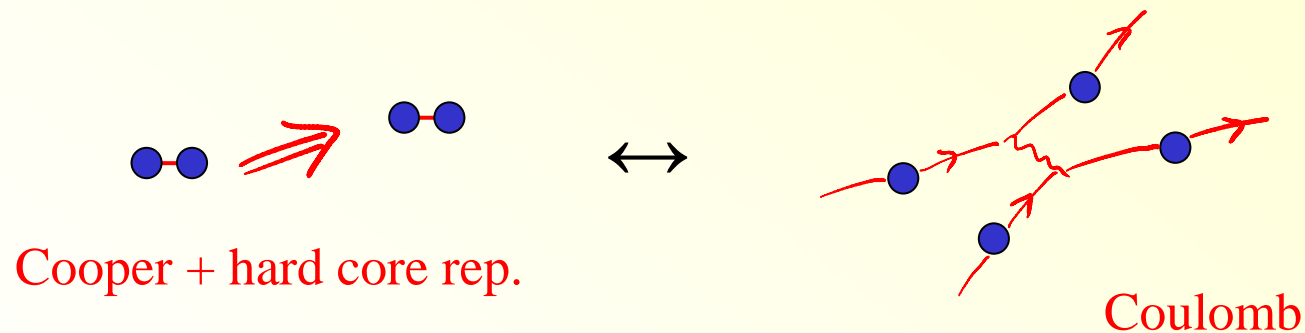
Same as for usual Anderson localization

But: ● Sites: Many-body states

— Links: off-diagonal interactions

Why to expect many body localization at the SIT?

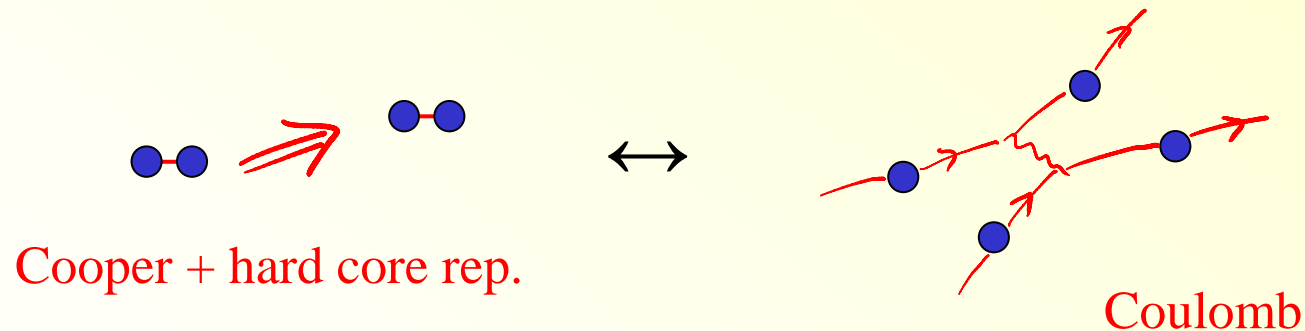
- Electrons are bound in localized pairs
- Phase volume for inelastic processes is strongly reduced as compared to the single electron problem MIT



→ Less fluctuations → stronger tendency to localize
Many body localization is easier at the SIT!
Probably important difference to the MIT!

Why to expect many body localization at the SIT?

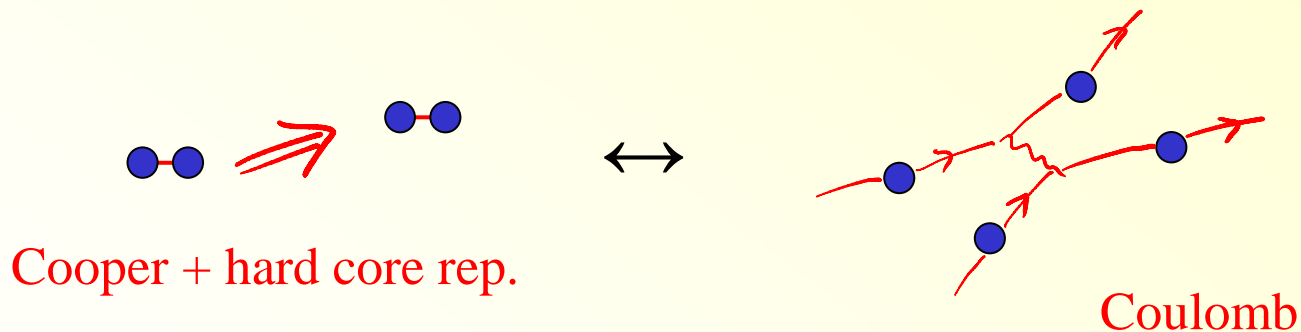
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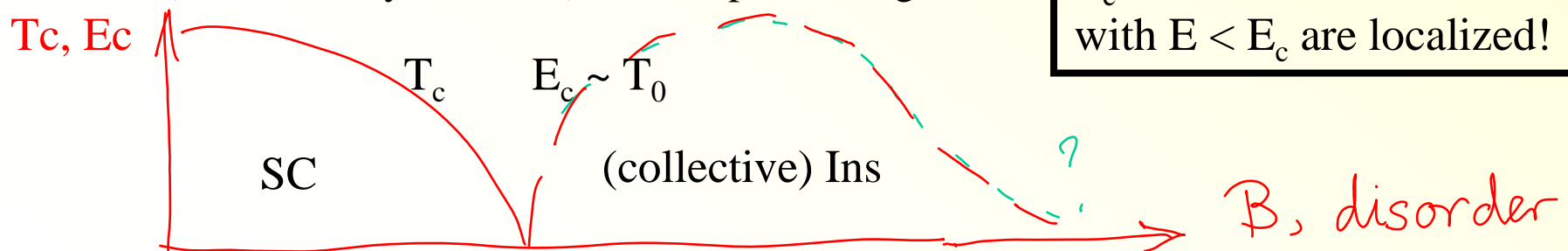
- At strong magnetic field pairs are dissolved
 - Many body localization eventually disappears
 - (electronically activated) VRH is possible again.

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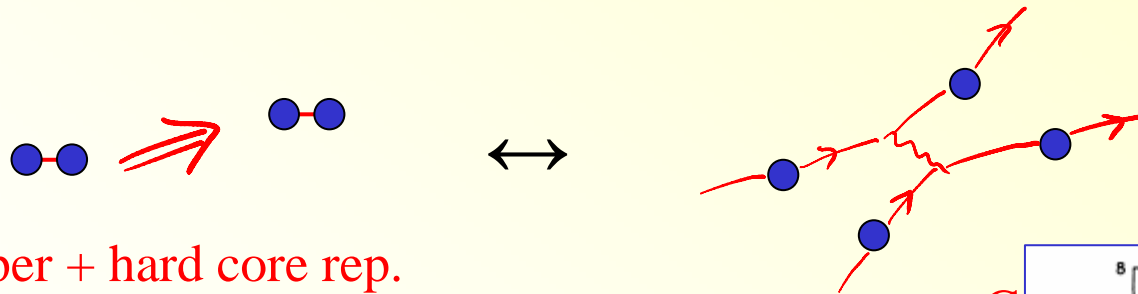


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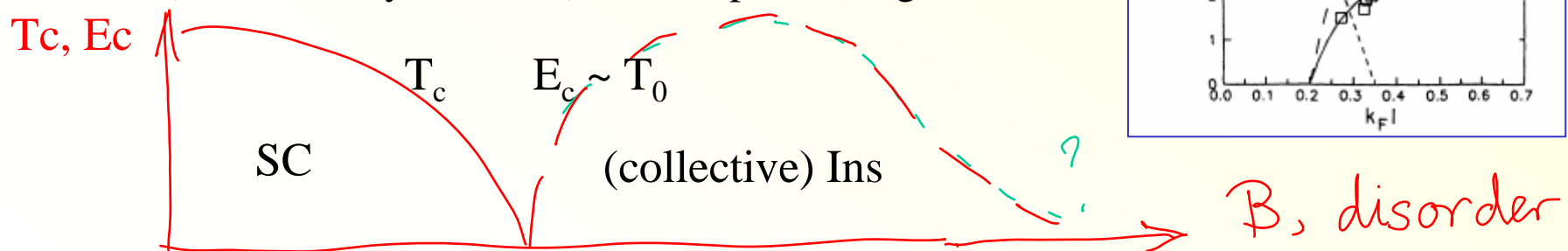
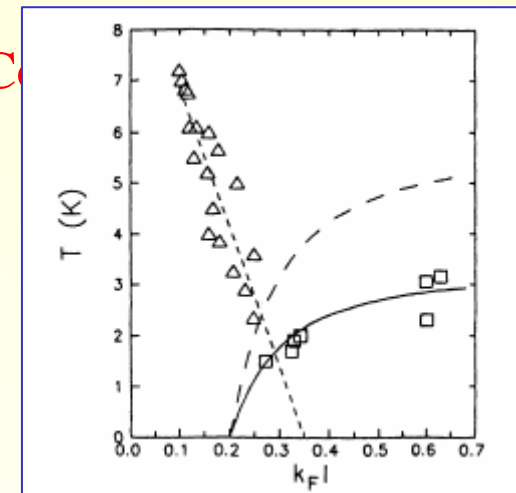
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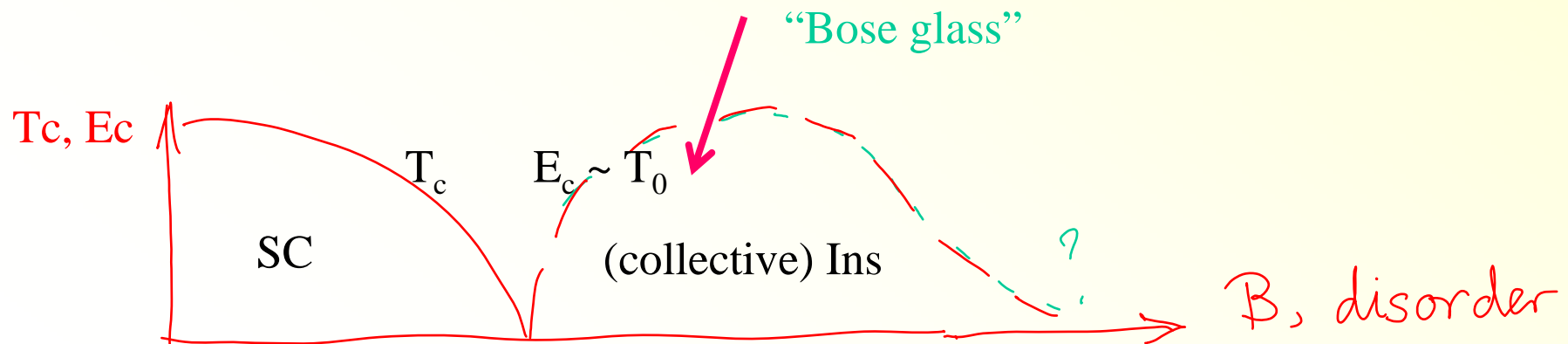


Cooper + hard core rep.

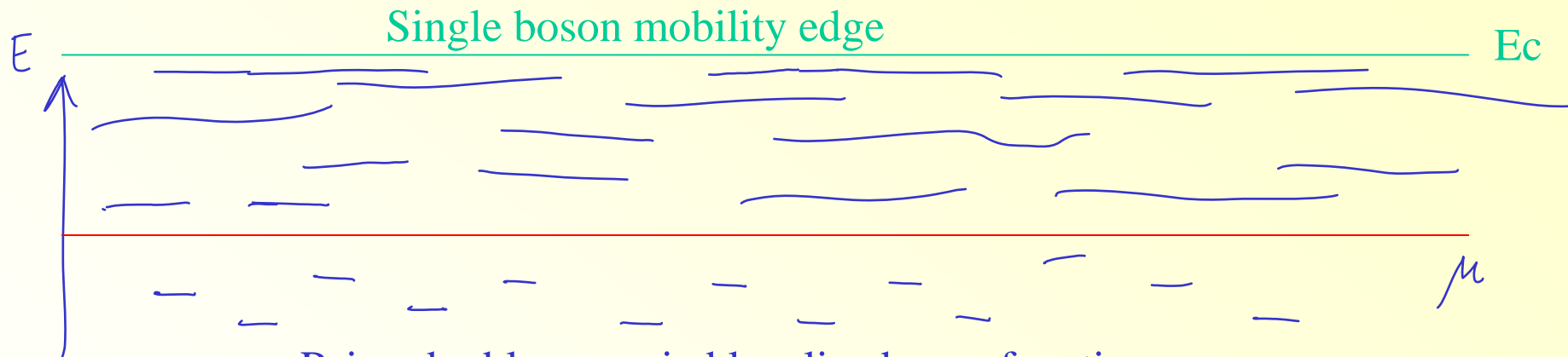
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Transport in the collective insulator



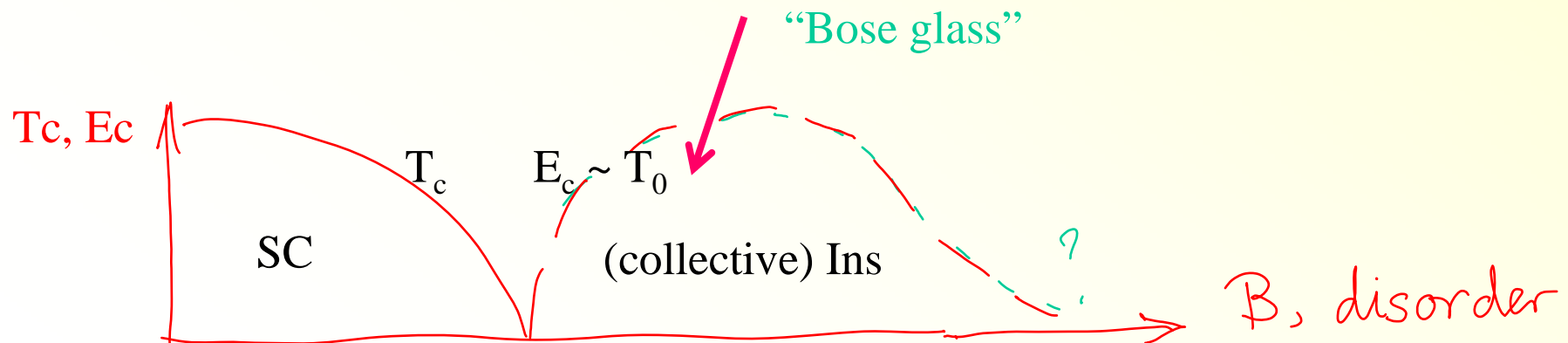
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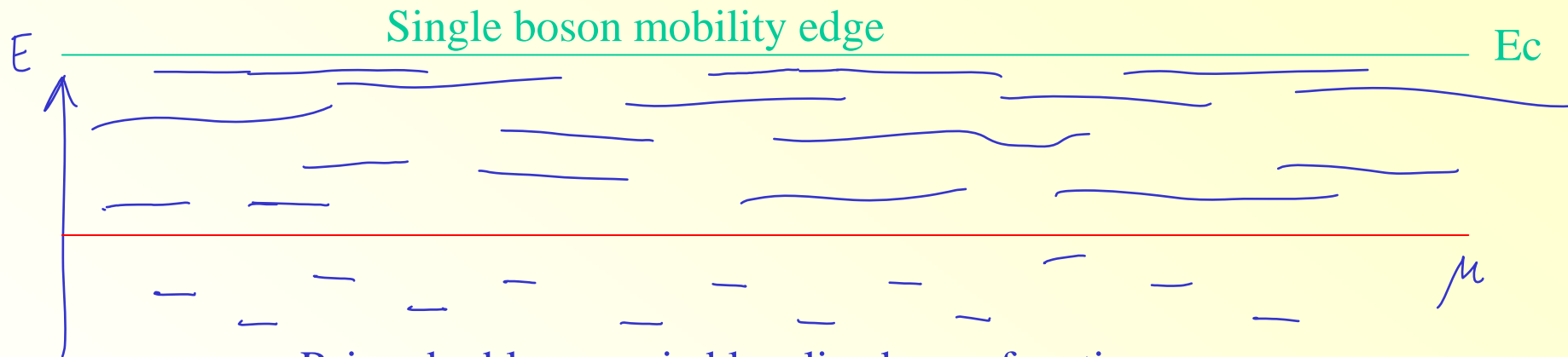
Pairs: doubly occupied localized wavefunctions

$$H_{pair} = \sum_i \varepsilon_i \sigma_i^z + \sum_{ij} t_{ij}(B) \sigma_i^+ \sigma_j^- + \sum_{ij} J_{ij} \sigma_i^z \sigma_j^z$$

(Anderson, Ma+Lee [+Halperin])



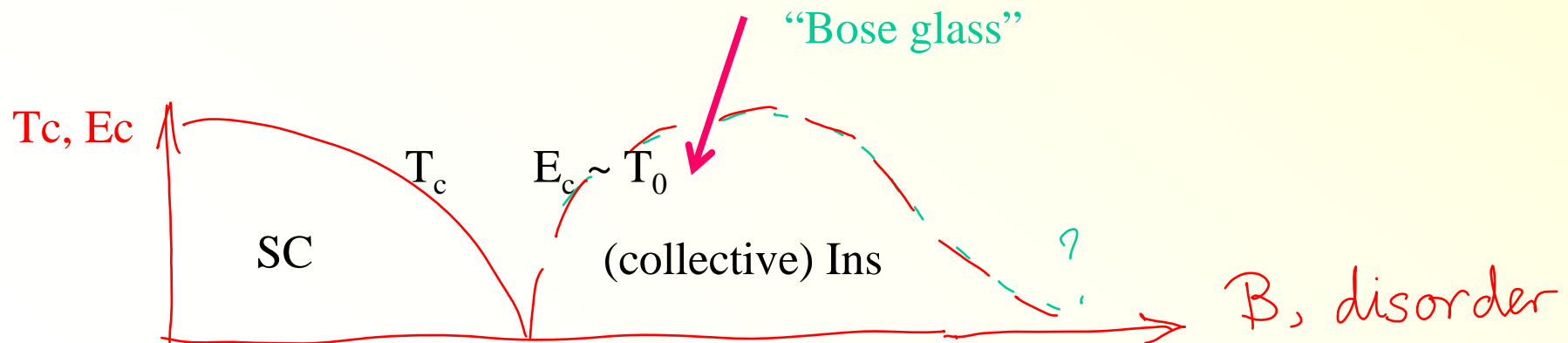
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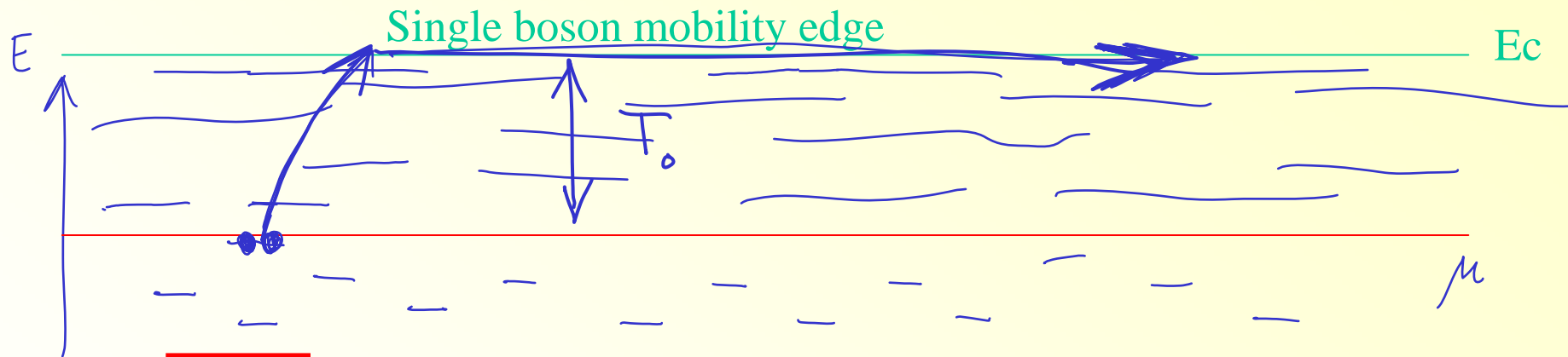
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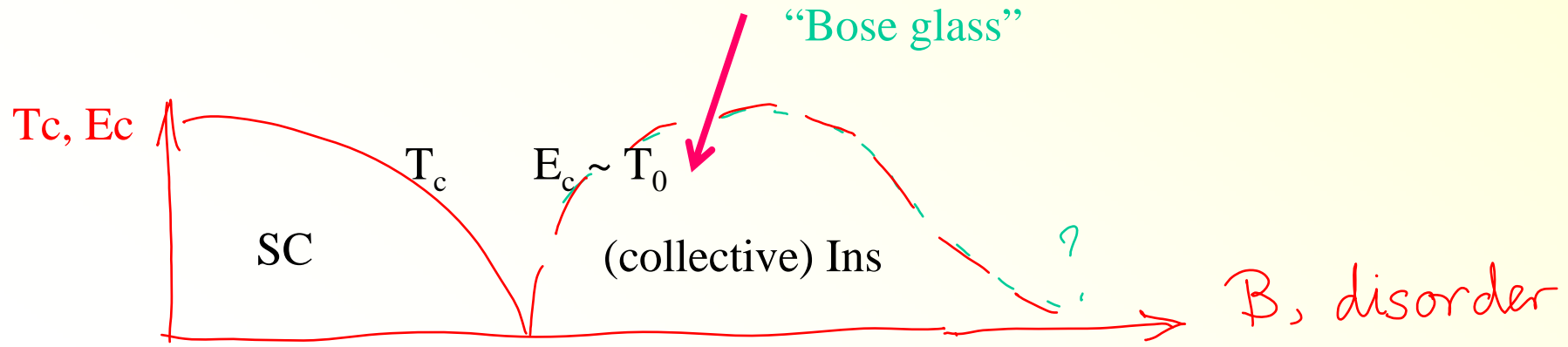


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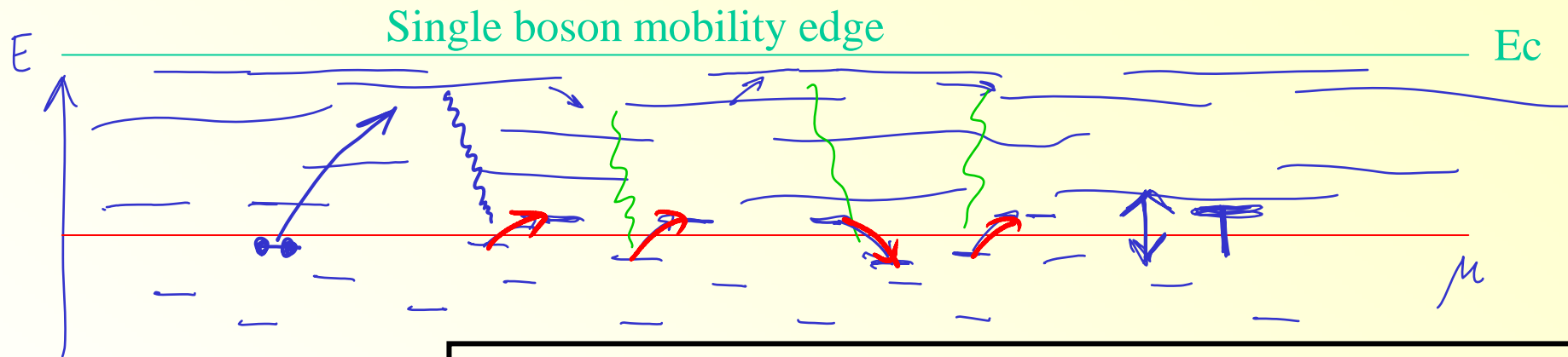
$T \sim 0$

Simple activation!
 $T_0 \sim$ typical hopping amplitude of preformed pairs
 $\sim T_c$ close to SIT (Ma + Lee!)



B , disorder

Transport in the collective insulator



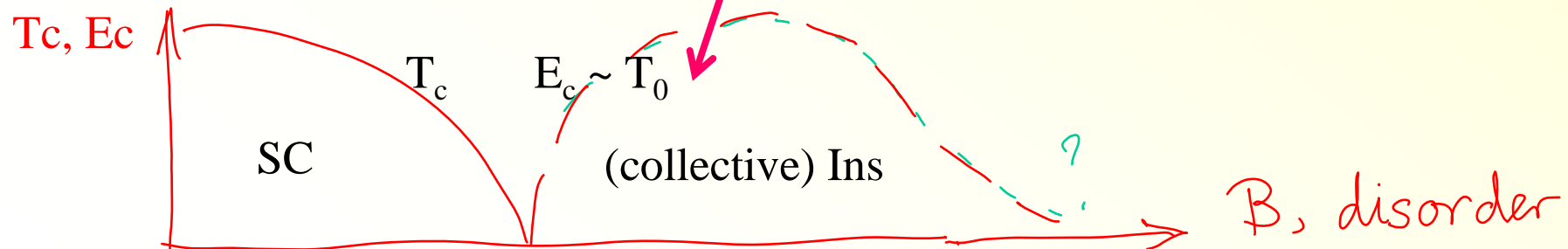
$$E_c > T > 0$$

Diffusion already at energy where

$$L_{inel}(T; E^*(T)) < \xi(E^*(T))$$

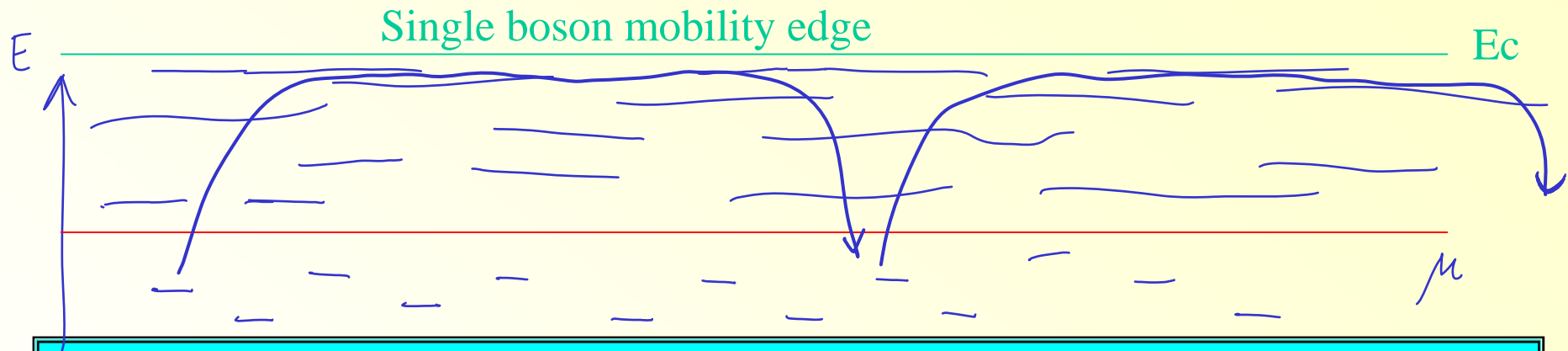
$$E^*(T) = E_c - AT^\gamma - \dots \quad (\gamma = 1/vd)$$

Overactivation! (cf.: *Semiconductors: Mott, Thomas, Overhof, 1988*)

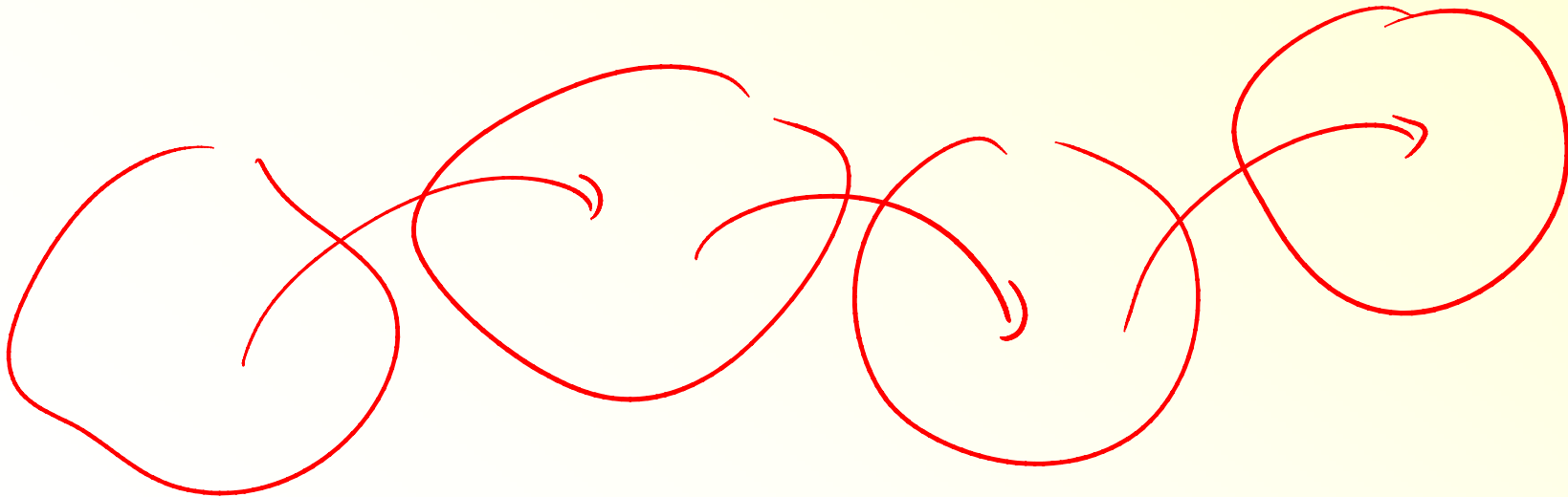


Transport in the collective insulator

Transport on large scales:



- Essential ingredient: **Elementary step** of transport is **simply activated** (no VRH)!
- Eventual d.c. transport is **percolative in nature** as in ANY disordered insulator



Experimental recall: Summary II

- Transport is **simply activated** at **low T** over several orders of magnitude
- There is a tendency to
 - **overactivation** close to the SIT
(saturating to simple activation at low T)
Highly unusual in a disordered system!
 - **subactivation** beyond the MR peak (at lowest T)

Experimental recall: Summary II

- Transport is **simply activated** at **low T** over several orders of magnitude

Activation to mobility edge of pairs!

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Highly unusual in a disordered system!

T-induced lowering of diffusion edge

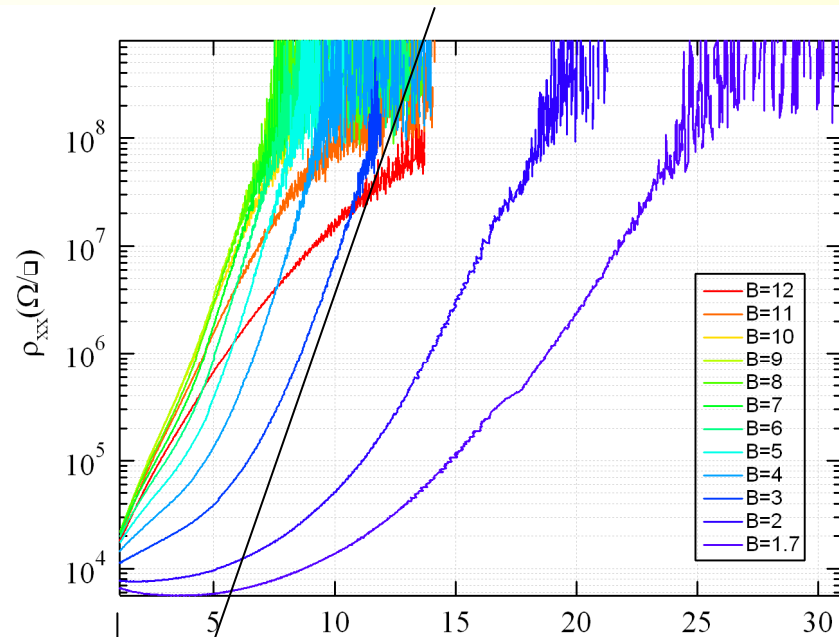
- **subactivation** beyond the MR peak (at lowest T)

VRH of depaired electrons,

Destruction of manybody localization due to single electrons and their stronger tendency to delocalize.

Overactivation near the SIT

B. Sacépé et al. (unpublished - 2008) – InO_x



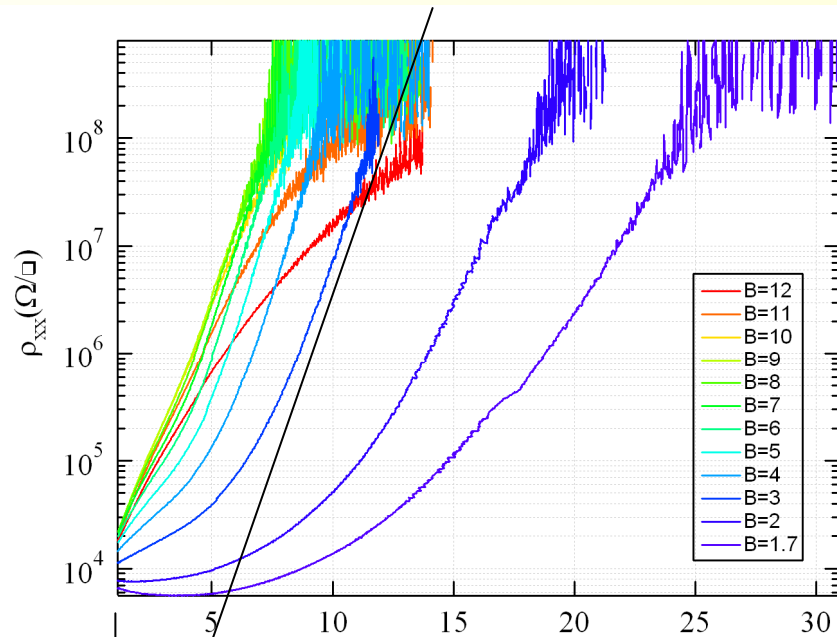
$$R = R_0 \exp\left[\frac{E_0}{T}\right]$$

$R_0 \ll h/4e^2$!?

Overactivation near the SIT

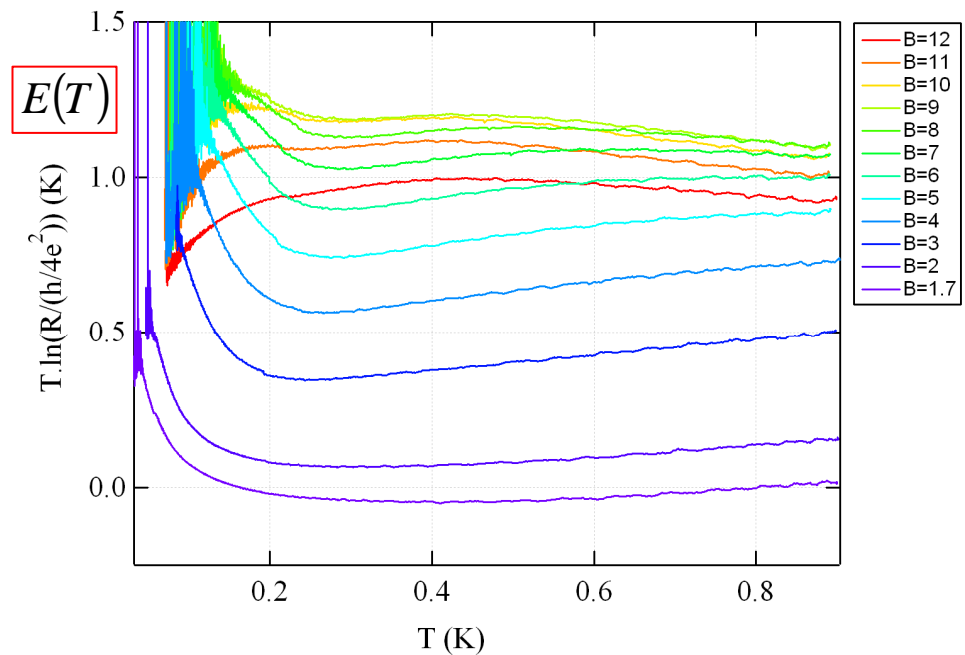
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Activation energy plot!



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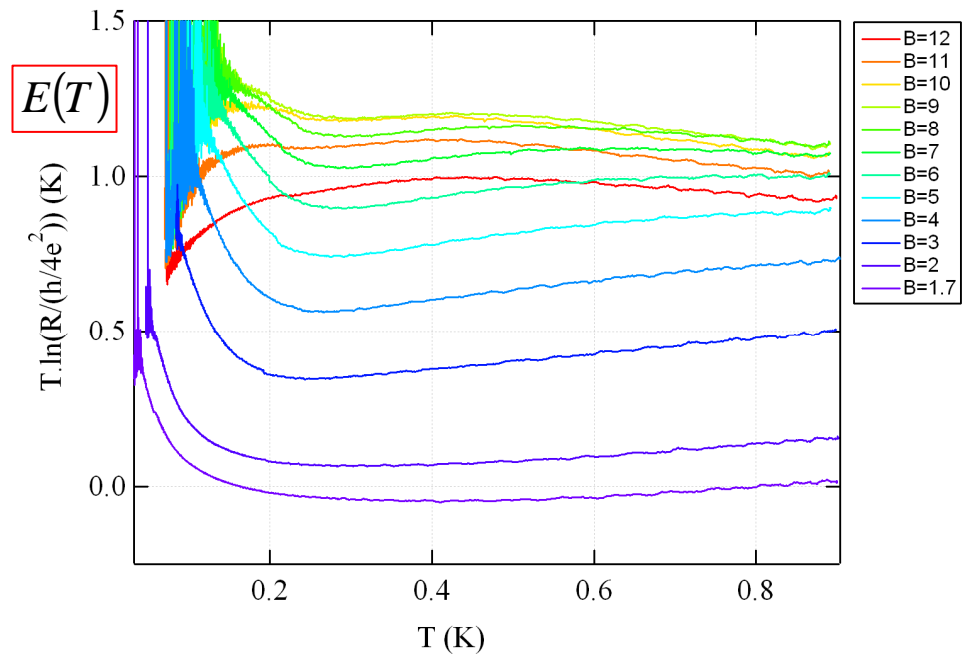
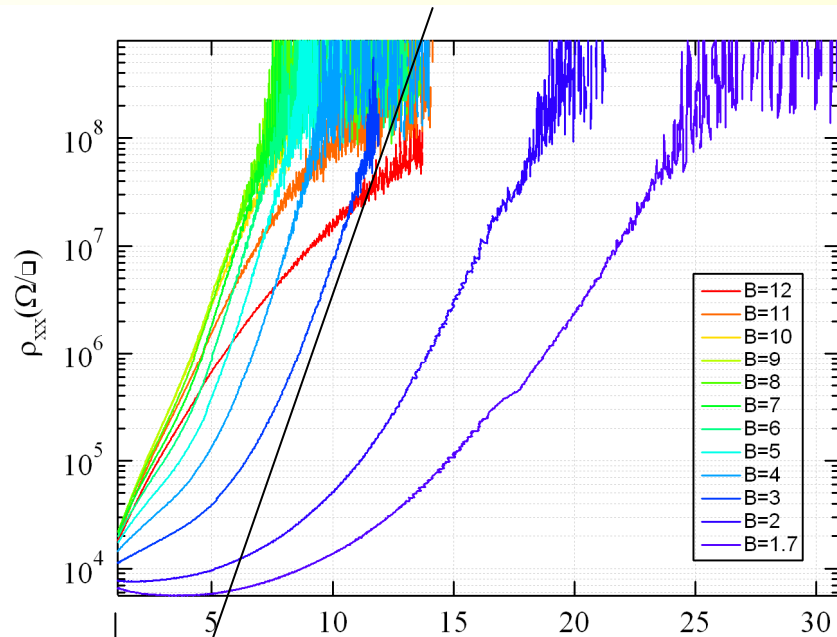
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Different interpretation:
T-dependent mobility edge:

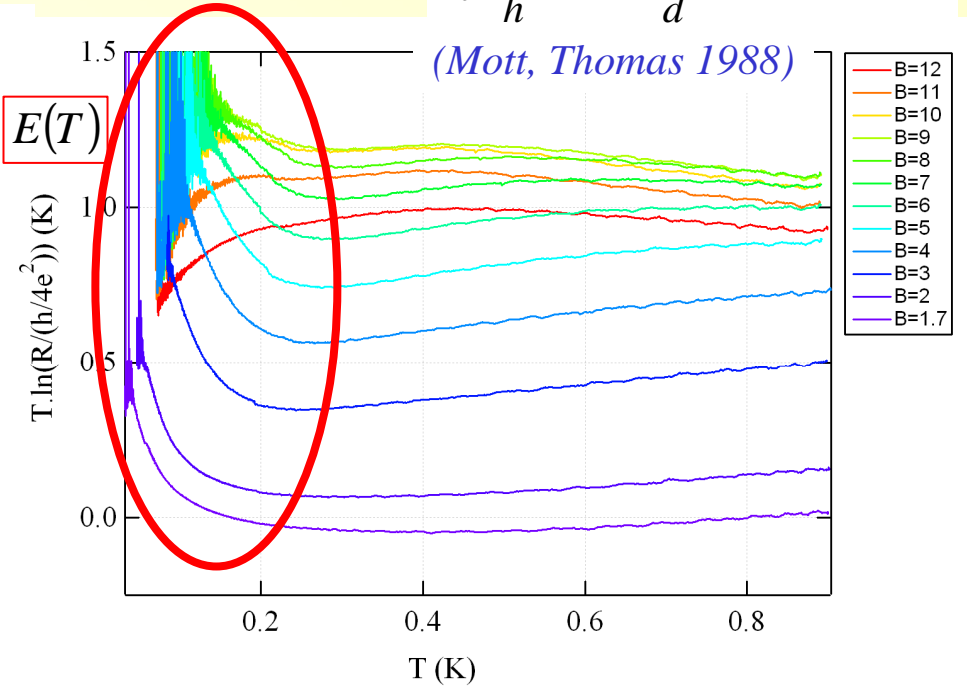
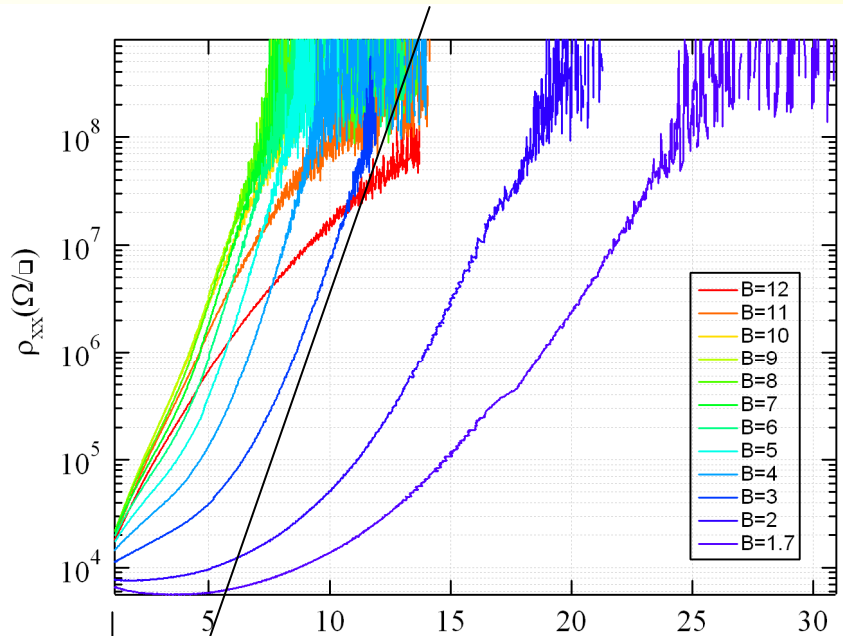
$$R = C \frac{L_\varphi(T)}{d} \frac{h}{4e^2} \exp \left[\frac{E^*(T)}{T} \right]$$

Overactivation near the SIT

B. Sacépé et al. (unpublished - 2008) – InO_x

Approximate: $E^*(T) = E_0 - BT - \dots$

→ Prefactor: $R_0 \frac{4e^2}{h} = C \frac{L_\phi(T)}{d} \exp[-B] \ll 1$



$$R = R_0 \exp\left[\frac{E_0}{T}\right]$$

$$R_0 \ll h/4e^2 \text{ !?}$$

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Summary

- Global charge gap for pairs unlikely due to disorder (except for distorted Wigner crystal of pairs or granular superconductors):
 - Remaining consistent model for simple activation:
Conductivity of pairs at their mobility edge.
- **Variable range hopping excluded** by remnant of **many body localization** in the low energy sector.
- Dephasing of nearly delocalized states
 - diffusion below the mobility edge
 - might explain observed **overactivation** and an apparently very small pre-exponential factor R_0 .
- Destruction of many body localization by depairing (high B) reestablishes VRH of single e's → **subactivation**.