Solution of the quantum Ising spin glass and the superglasses

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The arrow of time, IHP Paris, 12th October, 2010

Arrow of time and ergodicity

The arrow of time

How to know the direction of increasing time?

- Entropy always increases
- Example: diffusion (continues forever in infinite systems)

$$\xrightarrow{\qquad\qquad} t \quad \neq \quad \xleftarrow{\qquad\qquad} t$$

Interesting exception: quantum localized systems → time reversal symmetry even in infinite systems

Ergodicity

Fundamental postulate of thermodynamics:
State of maximal entropy (=equilibrium) is reached in finite time.

$$\xrightarrow{\qquad\qquad} \bullet \ \mathrm{Eq}$$

But: NO full equilibration when ergodicity is broken

$$\underset{t\,=\,\infty}{\xrightarrow{\qquad\qquad}} \text{- - - - - - - -} \bullet \quad (\mathrm{Eq})$$

Occurs in particular in certain disordered systems: Glasses

Interrelation between arrow of time and ergodicity?

Two notions associated with time evolution and dynamics

Consider two types of ergodicity breakers or « glasses »!

Two types of « glasses »

Frustrated disordered systems	Quantum localized systems
e.g. Spin glass	e.g. Anderson insulator (Fermi glass)
Arbitrarily large energy barriers ΔE between metastable states ΔE >> temperature T (classical) ΔE >> tunneling Γ (quantum)	Vanishingly small matrix elements between distant states in Hilbert space (no energy barriers necessary)
Pure states differ in **global** density profiles	Non-ergodic sectors differ by **local** density matrices
Number of pure states: Mean field: exponential in N Finite dimensions ?	Exponential number of non-ergodic sectors $N \sim \exp[V / L_{loc}^d]$
Destroyed by large T	Robust to T (if fully localized)
Robust to dephasing/coupling to environment	Destroyed by a dephasing bath

Non-ergodicity BUT
diffusion in general still possible

Non-ergodicity AND no diffusion
(no clear arrow of time)

Two types of « glasses »

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Two mechanisms of ergodicity breaking:
Mutual enhancement or competition?

Mean field quantum glass models

- Nature and spectrum of collective excitations in quantum glasses ?

Transverse field Ising spin glass
(Sherrington-Kirkpatrick SK)

$$\boxed{H = -\Gamma \sum_i \sigma_i^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z}$$

$$\overline{J_{ij}} = 0$$

$$\overline{J_{ij}^2} = \frac{J^2}{N}$$

→ Deep quantum glass phase:
find Ohmic bath of collective modes; extrapolate to finite d

- Interplay of glassiness and superfluidity - delocalization of hard core bosons: $\sigma_i^z \leftrightarrow 2n_i - 1$

Meanfield model for "superglass"
= amorphous, glassy supersolid

$$\boxed{H = -\frac{t}{N} \sum_{i<j} \sigma_i^x \sigma_j^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z}$$

→ superglass, interesting features due to competition
superfluidity ↔ glass order

Transverse field SK model

Static approximation
A. Bray, M. Moore JPC ('88); Y.Y. Goldschmidt and P.Y. Lai, PRL ('90)

Critical behavior at the Q-Glass transition
J. Miller, D. A. Huse, Phys. Rev. Lett. ('93)

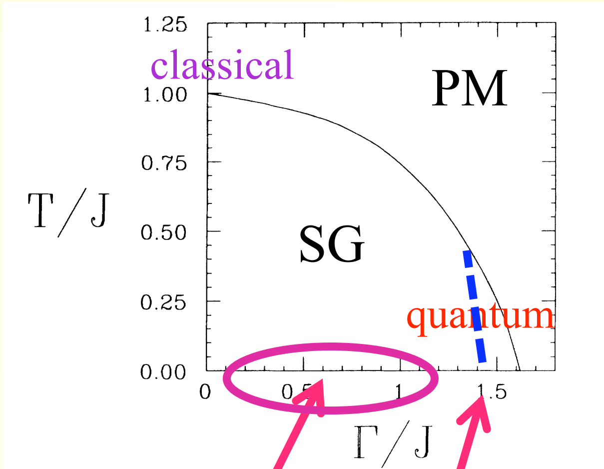
Effective field theory (Landau expansion)
S. Sachdev, N. Read, J. Ye ('93,'95)
K. Takahashi PRB ('07)

QMC in the paramagnet, critical line
M. J. Rozenberg, D. R. Grempel, PRL **('98)**;

Exact diagonalization in finite N~20 SK model
L. Arrachea, M. J. Rozenberg, PRL ('01)

Other mean field quantum glasses:
Spherical, O(N), p-spin type models – often QPT is first order, *unlike* SK!
T. Nieuwenhuizen; F. Ritort; G. Biroli, L. Cugliandolo et al.

Phase diagram



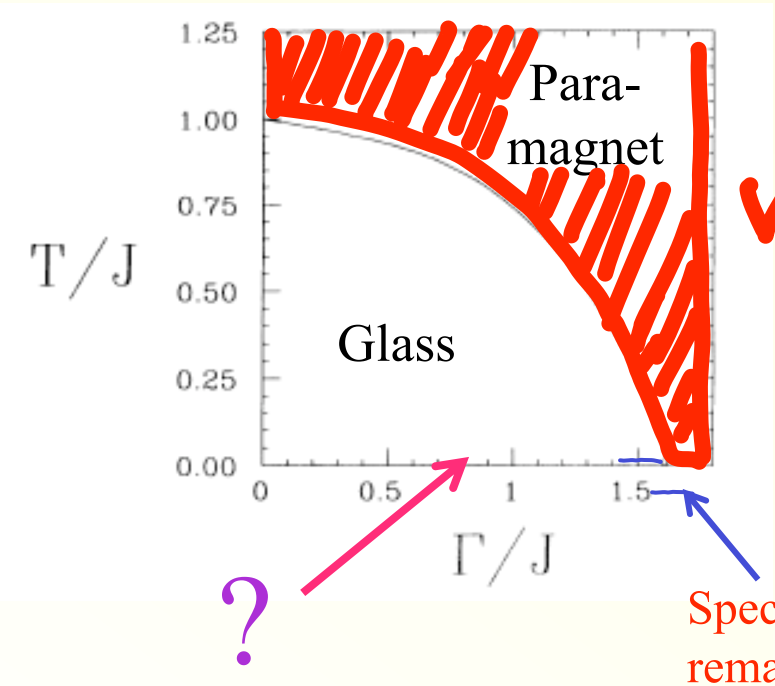
Goldschmidt, Lai, PRL ('90):
Static approx

But ??

SK: Known properties

Transverse field Ising spin glass

$$H = -\Gamma \sum_i \sigma_i^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z$$



SK: mean field limit,
Infinite coordination z = ∞:

Phase transition into a glass state:

- Many long-lived metastable states
- replica symmetry breaking

Spectral gap closes *(Miller, Huse PRL 1993)*
remains closed in the glass phase!
Read, Sachdev, Ye, PRL (1993)

Quantum TAP equations

*(**T**houless, **A**nderson, **P**almer '77: Classical SK model;   Biroli, Cugliandolo '01, MM, Ioffe '07)*

$$\boxed{H = -\Gamma \sum_i \sigma_i^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z}$$

How to obtain insight into excitations and dynamics in the deep quantum glass?

1. Physical, qualitative arguments
   à la T.A.P. (Thouless, Anderson, Palmer)
2. Rigorous solution of mean field equations

Quantum TAP equations

*(**T**houless, **A**nderson, **P**almer '77: Classical SK model; Biroli, Cugliandolo '01, MM, Ioffe '07)*

$$\boxed{H = -\Gamma \sum_i \sigma_i^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z}$$

Effective potential as a function of magnetizations imposed by external auxiliary fields h_i^{ex} :

$$G\left(\left\{\left\langle \sigma_i^z \right\rangle = m_i\right\}\right) = \sum_i G_i^0(m_i) - \frac{1}{2}\sum_{i \neq j} m_i J_{ij} m_j - \frac{1}{2}\sum_{i \neq j} J_{ij}^2 \int_0^\infty d\tau \chi_i(\tau) \chi_j(\tau) + O\left(\sqrt{1/N}\right)$$

$$G_i^0(m_i) = -\Gamma \sqrt{1 - m_i^2}$$

$$\chi_i(\omega \to 0) = dm_i / dh_i \approx \chi_i(m_i)$$

Local minima $\left(\partial G / \partial m_i = 0\right)$ (in static approximation)

$$\longrightarrow \quad \boxed{h_i \equiv \frac{\partial G_i^0}{\partial m_i} = \frac{\Gamma m_i}{\sqrt{1 - m_i^2}} = \sum_{j \neq i} J_{ij} m_j - m_i \sum_{j \neq i} J_{ij}^2 \chi_j(m_j)}$$

N coupled random equations for $\{m_i\}$ with ~ **exp[α N] solutions!**

Quantum TAP equations

*(**T**houless, **A**nderson, **P**almer '77: Classical SK model; Biroli, Cugliandolo '01, MM, Ioffe '07)*

$$H = -\Gamma \sum_i \sigma_i^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z$$

Local minima $\left(\partial G / \partial m_i = 0\right)$

$$h_i \equiv \frac{\partial G_i^0}{\partial m_i} = \frac{\Gamma m_i}{\sqrt{1-m_i^2}} = \sum_{j\neq i} J_{ij} m_j - m_i \sum_{j\neq i} J_{ij}^2 \chi_j\left(m_j\right)$$

Environment of a local minimum (potential landscape):

Hessian: $\mathrm{H}_{ij} = \partial^2 G / \partial m_i \partial m_j = J_{ij} + \text{diagonal terms}$

Spectrum of curvatures in a minimum

$$Spec\left[H_{ij}\right] \equiv \rho_H(\lambda) = const \times \frac{\sqrt{\lambda \Gamma}}{J^2}$$

(at small λ)

Gapless spectrum
in the **whole** glass phase!

Ensured by marginality, see below!

Soft collective modes

Spectrum of curvatures $\leftrightarrow$
Distribution of "restoring forces"

$$\boxed{Spec\left[H_{ij}\right] \equiv \rho_H(\lambda) = const \times \frac{\sqrt{\lambda\Gamma}}{J^2}}$$

Semiclassical picture:

$\rightarrow$ *N* collective oscillators with mass M ~ 1/Γ and frequency $\omega = \sqrt{\lambda/M}$

$\rightarrow$ Mode density $\boxed{\rho(\omega) = \mathrm{const} \times \frac{\omega^2}{\Gamma J^2}}$

$\rightarrow$ **Continuous** bath with
Ohmic spectral function

$$\boxed{\chi''(\omega) = \frac{1}{M\omega}\rho(\omega) \sim \frac{\omega}{J^2}}$$

Independent of Γ
for ω < Γ !!

Generalization of known spectral function at the quantum glass transition!
[Miller, Huse (SK model); Read, Ye, Sachdev (rotor models)]

Rigorous confirmation

A. Andreanov, MM, '10

Mean field equations

$$\exp[-\beta F_{\text{eff}}] = \operatorname{Tr} \mathbf{T} \exp \mathcal{S}_{\text{eff}}$$

$$\mathcal{S}_{\text{eff}} = J^2 \iint_0^\beta d\tau d\tau' \left[\sum_{a<b} Q_{ab} \sigma_a^z(\tau) \sigma_b^z(\tau') + \frac{1}{2} \sum_a Q_{aa}(\tau - \tau') \sigma_a^z(\tau) \sigma_a^z(\tau') \right] + \Gamma \sum_a \int_0^\beta d\tau \, \sigma_a^x(\tau)$$

Selfconsistency:

$$Q_{aa}(\tau - \tau') = \langle \sigma_a^z(\tau) \sigma_a^z(\tau') \rangle_{\mathcal{S}_{\text{eff}}}$$

$$Q_{ab} = \langle \sigma_a^z(\tau) \sigma_b^z(\tau') \rangle_{\mathcal{S}_{\text{eff}}} \to q(x)$$

To prove:

$$\Gamma \ll \Gamma_c \to \langle \sigma_a^z(\tau) \sigma_a^z(\tau') \rangle = \text{const} \times \frac{|\omega|}{J^2}$$

i) Permanent gaplessness, Ohmic spectral function
ii) Constant is independent of Γ

Rigorous confirmation

A. Andreanov, MM, '10

1) Gaplessness + Ohmic spectrum

$$\mathcal{S}_{\text{eff}} = J^2 \iint_0^\beta d\tau d\tau' \left[\sum_{a<b} Q_{ab}\sigma_a^z(\tau)\sigma_b^z(\tau') + \frac{1}{2}\sum_a Q_{aa}(\tau-\tau')\sigma_a^z(\tau)\sigma_a^z(\tau') \right] + \Gamma \sum_a \int_0^\beta d\tau\, \sigma_a^x(\tau)$$

- Full replica symmery breaking ansatz for Q_{ab}
- Hubbard Stratonovich off-diagonal part $\rightarrow$ distribution P(y) of frozen fields $y_i = \Sigma J_{ij} m_j$
- Obtain susceptibility in a given frozen field y

$$\chi_\omega(y) = \langle \sigma_{-\omega}^z \sigma_\omega^z \rangle_{y,c} = \frac{\widetilde{\Pi}_\omega(y)}{1 - J^2 R(\omega)\widetilde{\Pi}_\omega(y)}$$

Proper polarizability

$$\widetilde{\Pi}_{\omega\to 0}(y) = \widetilde{\Pi}_0 - A(y)\omega^2$$

$$R(\tau) = Q_{aa}(\tau) - Q_{aa}(\infty) = Q_{aa}(\tau) - q(x \to 1)$$

- Selfconsistency equation $\langle \sigma_a^z(\tau)\sigma_a^z(\tau')\rangle_{\mathcal{S}_{\text{eff}},c} \equiv R(\omega) = \int dy P(y)\chi_\omega(y)$
- Low frequency: $R(\omega) = R_0 + \delta R_\omega$

$$\boxed{\delta R_\omega = c|\omega| + O(\omega^2)}$$

- Marginal stability everywhere in the glass (full RSB) $\int dy P(y)\chi_0^2(y) = 1$

Rigorous confirmation

A. Andreanov, MM, '10

$$\boxed{\delta R_\omega = c|\omega| + O(\omega^2)}$$

2) Proof that coefficient c is independent of Γ:

- Assume scaling functions, such as:

$$P(y) \sim \frac{\Gamma^2}{J^2} \begin{cases} |y| & J \gg y \gg \Gamma \\ \Gamma & y \ll \Gamma \end{cases}$$

Linear pseudogap in
frozen field distribution!
(Palmer, Thouless; Pankov '06)

$$\chi_\omega(y) = \frac{1}{\Gamma} \hat{\chi}(y/\Gamma, \omega/\Gamma)$$

- Obtain a selfconsistency problem which is independent of Γ,
  its solution yields c = O(1) !

What survives of the phenomenology beyond mean field?

- Criticality ?

- Gapless collective modes ?

Expect:
Large connectivity → MF describes well the spectrum, except at the lowest energies

Beyond mean field

Quantum spin glass with

- Exchange matrix J_{ij} random, $|J_{ij}| \sim J$
- Large connectivity z

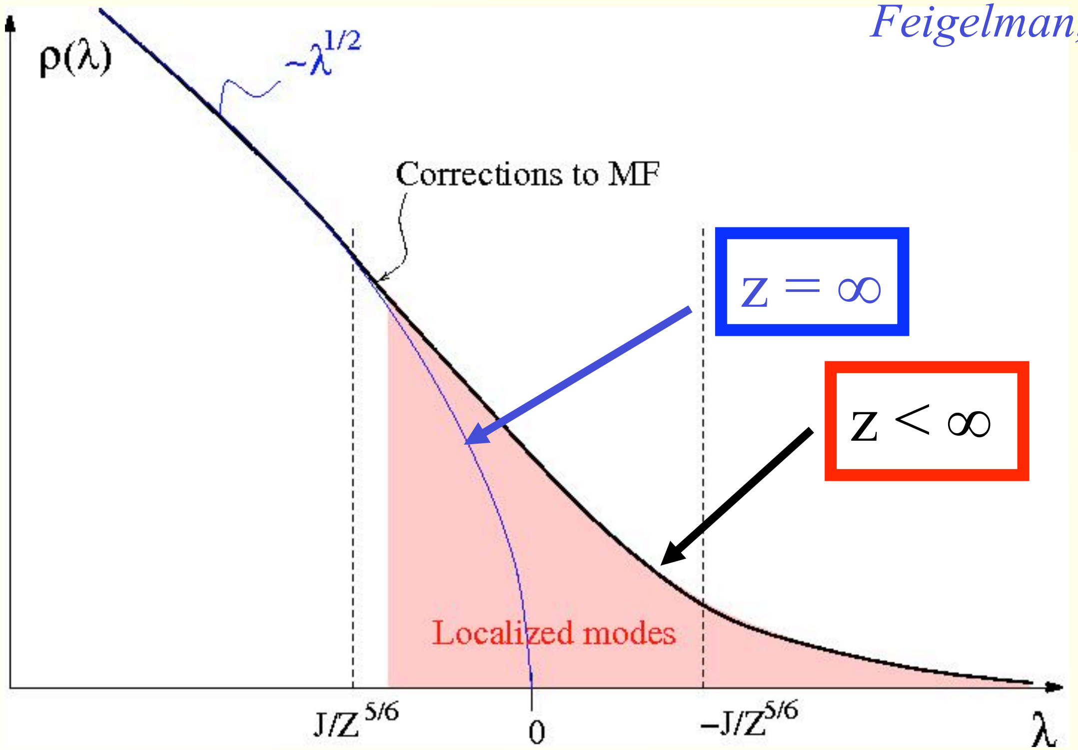
Repeat Quantum-TAP / semi-classical analysis!

Beyond mean field

Quantum spin glass with
- Exchange matrix J_{ij} random, $|J_{ij}| \sim J$
- Large connectivity z

Eigenvalue and eigenvector spectrum of random matrix J_{ij} (D>2)

Feigelman, Ioffe, Dotsenko (95)



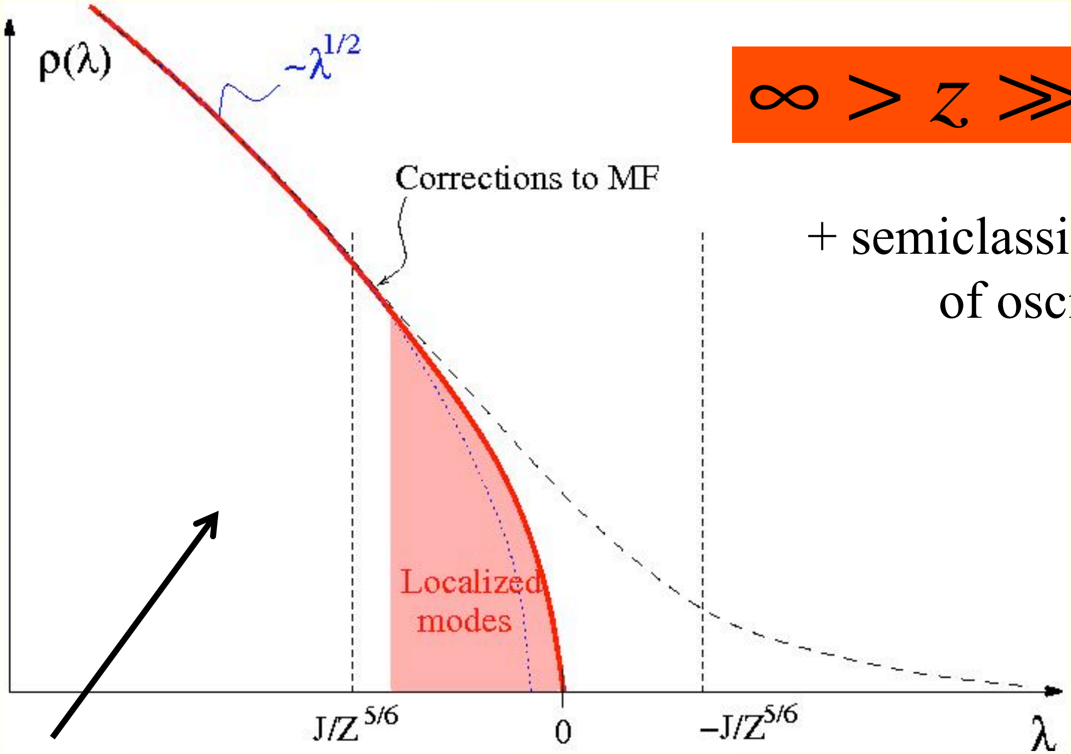
Semicircle
+ Corrections in regime $\sim 1/z^{5/6}$:
- localization
- spectral deviations

Beyond mean field

Quantum spin glass with

- Exchange matrix J_{ij} random, $|J_{ij}| \sim J$
- Large connectivity z

Eigenvalue spectrum of the TAP Hessian H_{ij} (d>3)



$\infty > z \gg 1$

+ semiclassical treatment of oscillators

Delocalized low-energy excitations, *at least* down to $\omega_{\min} \sim Jz^{-1/6} \ll J \simeq h_{loc}^{typ}$

Beyond mean field?

Preliminary conclusions

• Criticality of quantum glass
→ large density of **soft collective modes**
delocalized to very low energies
(provided the connectivity of the exchange matrix is large enough)
Similar to boson peak in structural gasses!

• Marginality reflects the competition of many metastable minima:

→ Spin glass-type ergodicity breaking counteracts quantum localization.

→ Instead it may enhance transport of energy and charge, as well as decoherence (as compared to a pure, ordered, but gapped phase).

Open questions: How does the low energy spectrum really look like?
Is there a mobility gap for many body excitations?

Ordering transitions in random systems

Hertz, Fleishman, Anderson, PRL (79)
Bray, Moore, J. Phys. C (80)

Scenario for the ordering transitions in

- Disordered magnets
- Spin glasses
- Dirty superfluids (SI transition)

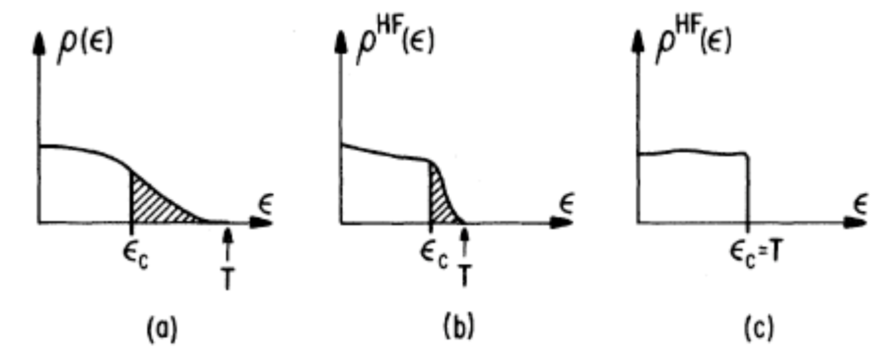


FIG. 1. Hartree-Fock density of states at three different temperatures (schematic): (a) For high T, $\rho^{\mathrm{HF}} \approx \rho$ = density of eigenvalues of $\underline{J}$; (b) for intermediate T, tail of localized states moves to keep to the left of T; (c) for T reaching the mobility edge, no localized states remain.

Hamiltonian

$$\frac{1}{2}\sum_{x_1x_2}(T\delta_{x_1x_2}-J_{x_1x_2})\vec{S}_{x_1}\cdot\vec{S}_{x_2}+\frac{1}{4}U\sum_{x_1}(\vec{S}_{x_1}{}^2)^2$$

Susceptibility matrix (≥ 0!)

$$\chi_{m_1m_2}{}^{\mu\mu}=T^{-1}\langle S_{m_1}{}^{\mu}S_{m_2}{}^{\mu}\rangle$$

Hartree-Fock Hamiltonian

$$\langle x\mu|H^{\mathrm{HF}}|x'\mu'\rangle =[J_{xx'}-u\textstyle\sum_{\lambda}G_{xx}{}^{\lambda\lambda}\delta_{xx'}]\delta_{\mu\mu'}-uG_{xx}{}^{\mu\mu'}\delta_{xx'}$$

→ Transition when extended Hartree Fock state condenses

→ HF modes at finite energy are delocalized

→ Does a mobility edge re-emerge after the SG transition??

Ordering transitions in random systems

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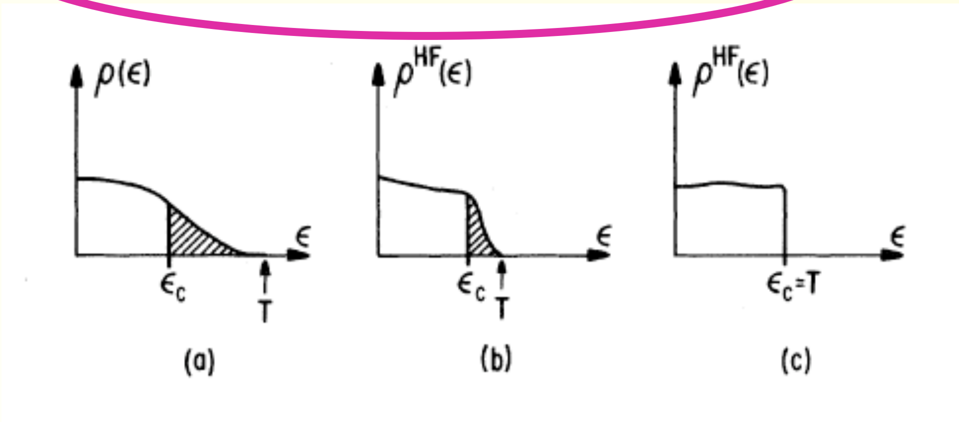

Hamiltonian

$$\frac{1}{2}\sum_{x_1x_2}(T\delta_{x_1x_2}-J_{x_1x_2})\vec{S}_{x_1}\cdot\vec{S}_{x_2}+\frac{1}{4}U\sum_{x_1}(\vec{S}_{x_1}{}^2)^2$$

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Remark: Dirty bosons: transition when single particle density matrix acquires delocalized zero-mode. – Expect a finite mobility edge of (many-body) excitations and Arrhenius transport before the SI transition! (d $\geq$ 3)
MM '09; Ioffe, Mézard '09, Feigelman, Ioffe, Mézard '10

→ Transition when extended Hartree Fock state condenses

→ HF modes at finite energy are delocalized

→ Does a mobility edge re-emerge after the SG transition??

What about glassiness AND superfluidity?

"superglasses" = amorphous, glassy supersolids

Motivation: supersolidity observed in **defectful (glassy?) quantum solids**

(Chan, Dalibard)

Superglasses ?!

Xiaoquan Yu, MM '10

$$\boxed{H = -\Gamma \sum_i \sigma_i^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z}$$

$$\Longleftrightarrow$$

$$\boxed{H = -\frac{t}{N} \sum_{i<j} \sigma_i^x \sigma_j^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z}$$

"Self-generated transverse field"

Superglasses ?!

Xiaoquan Yu, MM '10

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$$\leftrightarrow$$

$$H = -\frac{t}{N} \sum_{i<j} \sigma_i^x \sigma_j^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z$$

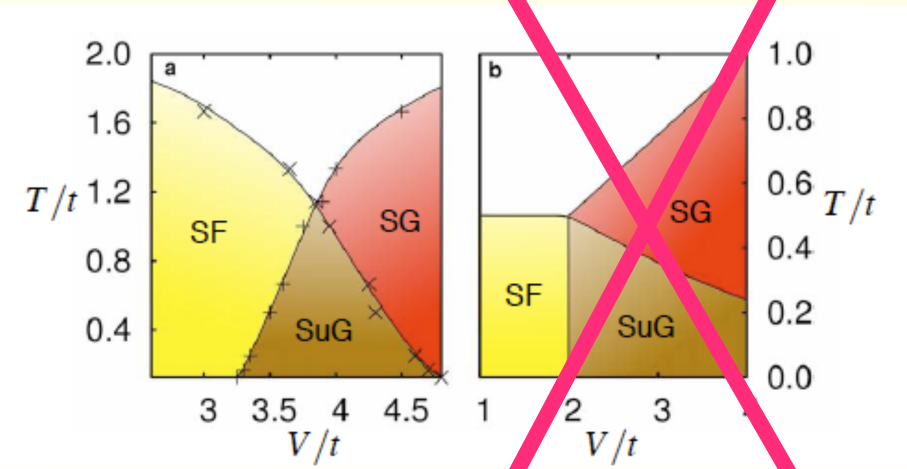
Competing order parameters: $M = \frac{1}{N} \langle \sigma_i^x \rangle$ $\quad q_{\mathrm{EA}} = \frac{1}{N} \langle \sigma_i^z \rangle^2$

M signals ferromagnetism or
superfluidity of hard core bosons $\quad \sigma_i^z \leftrightarrow 2n_i - 1$

If M and q_{EA} exist simultaneously → Meanfield model for superglasses

« Superglass »

Gingras, Melko et al PRL (10)



QMC (3d) "Mean field"

$$H = -\sum_{\langle ij \rangle} V_{ij}(n_i - 1/2)(n_j - 1/2) - t \sum_{\langle ij \rangle} (b_i^\dagger b_j + b_i b_j^\dagger),$$

Quenched randomness

? Low T behavior – QPT ?

? Local structure of the superglass ?

Carleo, Tarzia, Zamponi PRL (09)

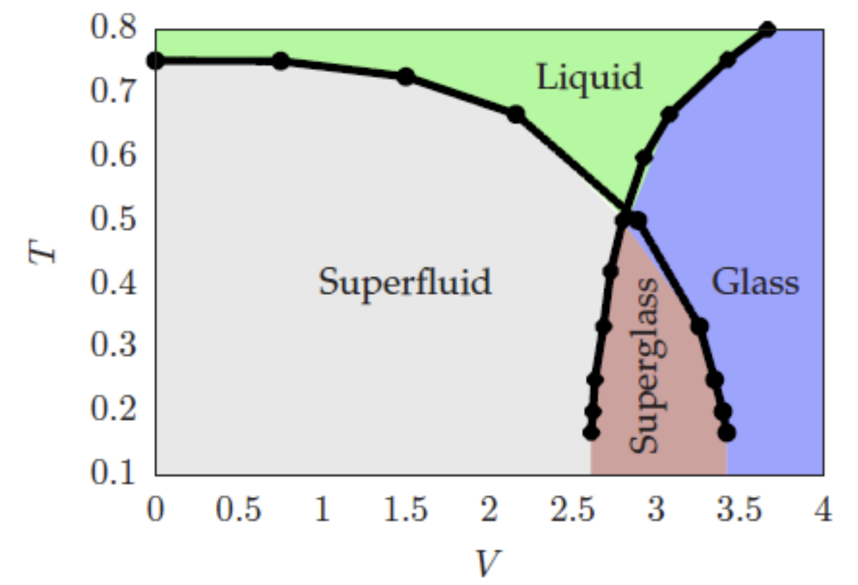


Figure 2: Finite temperature phase diagram at half-filling.

QMC and cavity

Random, frustrated lattice

$$\widehat{\mathcal{H}} = -t \sum_{\langle i,j \rangle} \left[b_i^\dagger b_j + b_i b_j^\dagger \right] + V \sum_{\langle i,j \rangle} n_i n_j,$$

Mean field superglass

Xiaoquan Yu, MM '10

$$M = \frac{1}{N}\langle \sigma_i^x \rangle$$

$$\boxed{H = -\frac{t}{N}\sum_{i<j}\sigma_i^x\sigma_j^x - \sum_{i<j}\sigma_i^z J_{ij}\sigma_j^z}$$

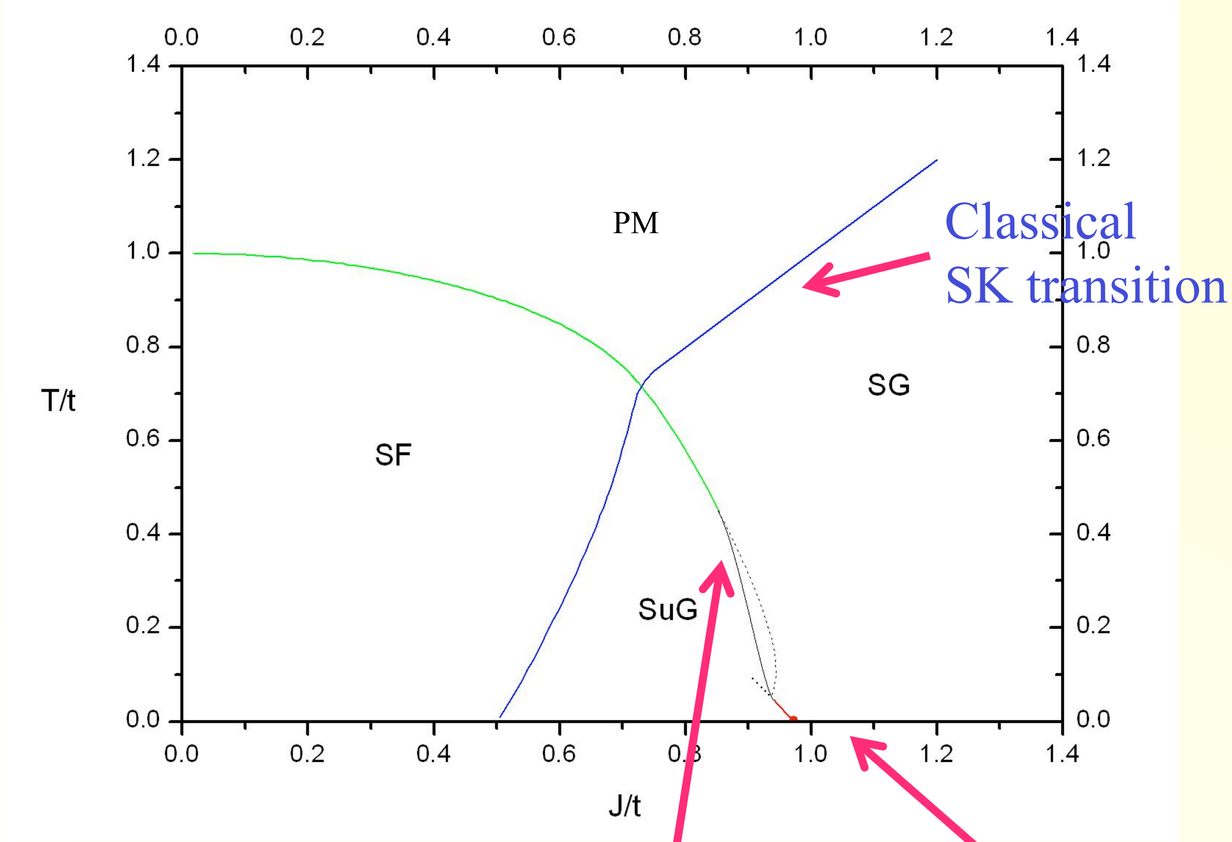
$$q_{\mathrm{EA}} = \frac{1}{N}\langle \sigma_i^z \rangle^2$$

- Static approximation is exact when M = 0!

→ obtain T = 0 phase transition glass-to-superglass exactly!

- For superfluid-to-superglass transition: Use static approximation

Phase diagram



$$t \int dy P(y) \chi_y^{xx} = 1.$$

$$\chi_y^{xx} = \frac{\partial \langle s^x \rangle_y}{\partial h_x} \stackrel{T \to 0}{\longrightarrow} \frac{1}{y}$$

Suppression of superfluidity by the linear pseudogap!

Transition at finite J/t ≅ 0.97

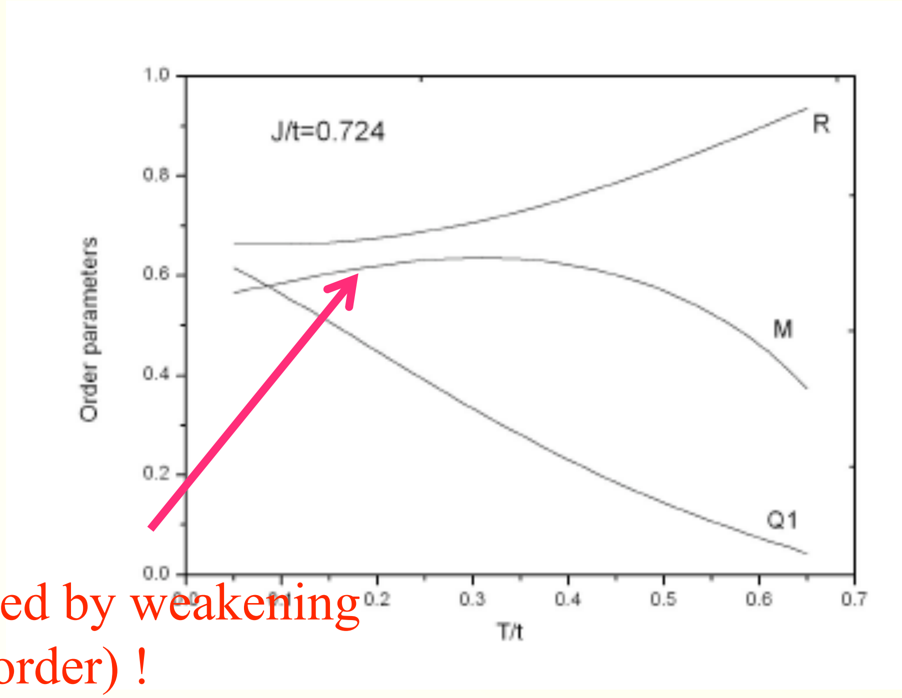
Analogue with LR interactions in finite d?

Structure of the superglass

- Superfluid and glass try to avoid each other:

$$\langle s_i^x \rangle \text{ and } \langle s_i^z \rangle^2 \text{ are anticorrelated}$$

- Superfluid order parameter M is non-monotonous with T



Superfluid enhanced by weakening
of the glass (= disorder) !

Conclusions

- Quantum spin glass:

  - Low energy excitations of the MF quantum glass: collective oscillators → Ohmic bath
  - Extrapolation to finite dimensions: spin glass order favors soft collective excitations, tendentially counteracting quantum localization

  Ergodicity broken – Arrow of time intact

- Superglass:

  - Coexistence of superfluid and glassy density order
  - Long range interactions suppress density of states and thus superfluidity down to $T = 0$
  - Non-monotonous superfluid order in the superglass