Quantum glasses – Frustration and collective behavior at $T = 0$

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Arrow of time and ergodicity

The arrow of time

How to know the direction of increasing time?
- Entropy always increases
- Example: diffusion (continues forever in infinite systems)
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Interesting exception: quantum localized systems $\rightarrow$ time reversal
symmetry even in infinite systems!
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Ergodicity
Arrow of time and ergodicity

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Ergodicity

Fundamental postulate of thermodynamics:
State of maximal entropy (=equilibrium) is reached in finite time.

But: NO full equilibration when ergodicity is broken

Occurs in particular in a large class of disordered systems: Glasses $\rightarrow$ configurational entropy, memory, history dependence, etc.
Interrelation between arrow of time and ergodicity?

(both notions associated with time evolution and dynamics)

Look at two types of ergodicity breakers = « glasses »!
Glasses: defying equilibration

Two types of glasses
(i) Quantum localized systems (Anderson glasses)
(ii) Frustrated, disordered or amorphous systems
Glasses: defying equilibration

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Cold atoms

Single particle QM (Anderson)

\[ H = \sum_i \varepsilon_i n_i - t \sum_{\langle i,j \rangle} \left( c_i^+ c_j + \text{h.c.} \right) \]

No diffusion at large disorder!
No arrow of time!
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Cold atoms

Single particle QM (Anderson)  \[ H = \sum_i \varepsilon_i n_i - t \sum_{\langle i,j \rangle} (c_i^+ c_j + \text{h.c.}) \]

Many particles (Anderson, Fleishman, Altshuler et al., Mirlin et al., etc.)  \[ H = \sum_\alpha \varepsilon_\alpha n_\alpha - \sum_{\alpha,\beta,\gamma,\delta} V_{\alpha\beta\gamma\delta} (c_\alpha^+ c_\beta^+ c_\gamma c_\delta + \text{h.c.}) \]

No diffusion at large disorder!
No arrow of time!

No diffusion at large disorder!  
Transition to ‘super-insulator’ at finite T?!
Glasses: defying equilibration

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Examples:
• spin glasses

\[ H = \sum_{\langle i,j \rangle} J_{ij} s_i^z s_j^z \]
Glasses: defying equilibration

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Examples:
• spin glasses
• electron glasses
• dirty superconductors, underdoped high Tc’s
• defectful supersolids (He)

\[ H = \sum_{\langle i,j \rangle} J_{ij} s_i^z s_j^z + e^2 \sum_{\langle i,j \rangle} r_{ij} n_i n_j + \sum_i \epsilon_i n_i \]
Glasses: defying equilibration

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• many complex systems beyond physics (biology, economy, society)

High barriers in complex energy landscape

\[ H = \sum_{\langle i,j \rangle} J_{ij} s_i^z s_j^z \]
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Ergodicity breaking!
Transport (diffusion, arrow of time) ?!
Two types of glasses
(i) Quantum localized systems (Anderson glasses)
(ii) Frustrated, disordered or amorphous systems

Examples:
• spin glasses
\[ H = \sum_{\langle i,j \rangle} J_{ij} z_i z_j + \Gamma \sum_i s_i^z \]  
\text{[LiYHF]}  
Ergodicity breaking!

• electron glasses
\[ H = \sum_{\langle i,j \rangle} \frac{e^2}{r_{ij}} n_i n_j + \sum_i \epsilon_i n_i \]  
Transport (diffusion, arrow of time) ?!

• dirty superconductors, underdoped high Tc’s
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**Important to avoid in Q-computing!**
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Non-ergodicity AND no diffusion (no clear arrow of time)
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- **Non-ergodicity, BUT diffusion usually possible (arrow of time there)**
- **Non-ergodicity AND no diffusion (no clear arrow of time)**
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Joining two ingredients of ergodicity breaking:
(i) Mutual enhancement or competition?
(ii) Is there many particle-localization in quantum glasses?
Intriguing aspects of ‘spin glasses’

Classical glass: SK model

\[ H = - \sum_{i<j=1}^{N} \sigma_i^z J_{ij} \sigma_j^z \]

\[ \overline{J_{ij}} = 0 \]

\[ \overline{J_{ij}^2} = \frac{J^2}{N} \]

- **Thermodynamic** transition at \( T_c \) to a glass phase
- **Unusual** order parameter: \( Q_{EA} = \frac{1}{N} \sum_i \langle s_i \rangle^2 \)
- **Many** metastable states
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Glass phase is always (self-organized) critical! \((SK: Kondor-DeDominicis)\)
Power law correlations in whole glass phase! \((Droplets: Fisher-Huse)\)
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Physical consequences of this criticality:

- Pseudogap in the distribution of local fields! \((Palmer, Sommers)\)
  (electron glasses: Coulomb gap!) \((MM, Ioffe '04, MM, Pankov '07)\)
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\[ H = - \sum_{i<j=1}^{N} \sigma_i^z J_{ij} \sigma_j^z \]

\[ J_{ij} = 0 \]

\[ J_{ij}^2 = J^2 \]

\[ \bar{J}_{ij} = \frac{J^2}{N} \]

- **Thermodynamic** transition at \( T_c \) to a **glass phase**
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Glass phase is always (self-organized) critical! *(SK: Kondor-DeDominicis)*

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Physical consequences of this criticality:

- Pseudogap in the distribution of local fields! *(Palmer, Sommers)*
  (electron glasses: Coulomb gap!) *(MM, Ioffe '04, MM, Pankov '07)*

- Power law distributed avalanches + Barkhausen noise!
Intriguing aspects of ‘spin glasses’

What are the consequences of this criticality in the quantum versions?
Problem: Very little is known about quantum glasses!

= Strongly correlated, disordered quantum systems!

Our strategy:

1. Solve mean field models (infinite connectivity) - highly non-trivial!
2. Obtain physical understanding
3. Extend to finite dimensions (large but finite connectivity)
Quantum glass models

Disorder: frustration vs. localization?

• Collective excitations in quantum glasses?

Transverse field Ising spin glass (Sherrington-Kirkpatrick SK)

\[ H = -\sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z - \Gamma \sum_i \sigma_i^x \]

\[ \bar{J}_{ij} = 0 \]

\[ \bar{f}_{ij}^2 = \frac{f^2}{N} \]

• Glassiness and superfluidity - (spin ½ = hard core bosons)

\[ \sigma_i^z \leftrightarrow 2n_i - 1 \]

“Superglass” = glassy supersolid

\[ H = -\sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z - \frac{t}{N} \sum_{i<j} \sigma_i^x \sigma_j^x \]
Quantum SK: Known properties

Transverse field SK model (fully connected, random Ising)

\[ H = -\sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z - \Gamma \sum_i \sigma_i^x \]

Goldschmidt, Lai, PRL (’90):
Static approximation

There is a quantum glass transition also at \( \Gamma > 0 \)
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Spectral gap closes (Miller, Huse ‘93)
Seems to remains closed in the glass!!
Why?? (Read, Sachdev, Ye ‘93)
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(Read, Sachdev, Ye ‘93)
Understanding the quantum glass

MM, Ioffe ’07

1. Physical approach

→ Nature of excitations
→ Approach generalizable to finite dimensions
Effective potential

Construct free energy functional $G(m)$ imposing magnetization $m$
Effective potential

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- Two states!
- Barrier becomes very large at low T
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Construct free energy functional $G(m)$ imposing magnetization $m$

- Two states!
- Barrier becomes very large at low T

- Many states!
- Hierarchical valleys
- Barriers never very large: As if always at criticality!

![Diagram showing ferromagnet and spin glass phases with temperature dependence](image)
Low energy excitations

**XY or Heisenberg ferromagnet:**

Goldstone modes:

Soft collective excitations along flat directions of the potential

What about Ising glasses?
Low energy excitations

Two possibilities:
Isolated stable minimum in the potential landscape

Many valleys with rather flat interconnections!
(in configuration space)
Low energy excitations

Two possibilities:

Isolated stable minimum in the potential landscape

Many valleys with rather flat interconnections! (in configuration space)
Effective potential

(Thouless, Anderson, Palmer ‘77: Classical SK model; Biroli, Cugliandolo ‘01, MM, Ioffe ‘07)

\[ H = - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z - \Gamma \sum_i \sigma_i^x \]

Effective potential (exact at \( N = \infty \))

\[ G(\{\langle \sigma_i^z \rangle = m_i \}) = -\Gamma \sum_i \sqrt{1 - m_i^2} - \frac{1}{2} \sum_{i \neq j} m_i J_{ij} m_j - \frac{1}{2} \sum_{i \neq j} J_{ij}^2 \int_0^\infty d\tau \chi_i(\tau) \chi_j(\tau) + O(1/N) \]

Static approximation for susceptibility: \( \chi_i(\omega \to 0) = dm_i/dh_i \approx \chi_i(m_i) \)
Effective potential

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Static approximation for susceptibility: \( \chi_i(\omega \to 0) = dm_i/dh_i \approx \chi_i(m_i) \)

Local minima \( (\partial G/\partial m_i = 0) \) (in static approximation)

\[ \frac{\Gamma m_i}{\sqrt{1 - m_i^2}} = \sum_{j \neq i} J_{ij} m_j - m_i \sum_{j \neq i} J_{ij}^2 \chi_j(m_j) \]

Effective field on \( \sigma_i \) in \( z \)-direction

N coupled random equations for \( \{m_i\}\).

With \( \sim \) \( \text{Exp}[\alpha N] \) solutions!
Quantum TAP equations

\textbf{Local minima} \quad (\partial G/\partial m_i = 0)

\[
\frac{\Gamma m_i}{\sqrt{1 - m_i^2}} = \sum_{j \neq i} J_{ij} m_j - m_i \sum_{j \neq i} J_{ij}^2 \chi_j (m_j)
\]

\textbf{Environment of a local minimum}

\textbf{Curvatures (Hessian):} \quad H_{ij} = \partial^2 G/\partial m_i \partial m_j = J_{ij} + \text{diagonal terms}
Quantum TAP equations

Thouless, Anderson, Palmer ’77 (Classical); Biroli, Cugliandolo ’01; MM, Ioffe ’07 (quantum)

Local minima \( (\partial G/\partial m_i = 0) \)

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Environmental of a local minimum

Curvatures (Hessian): \( H_{ij} = \partial^2 G/\partial m_i \partial m_j = J_{ij} + \text{diagonal terms} \)

Standard resolvent technique

Spectrum of “spring constants” in a minimum

\[
\text{Spec} \left[ H_{ij} \right] \equiv \rho_H(\lambda) = \text{const} \times \frac{\sqrt{\lambda \Gamma}}{J^2}
\]

Gapless spectrum in the whole glass phase, ensured by marginality of minima!

(at small \( \lambda \): No Gap!)
Soft collective modes

Spectrum of “spring constants”

\[
\text{Spec}\left[H_{ij}\right] \equiv \rho_H(\lambda) = \text{const} \times \frac{\sqrt{\lambda \Gamma}}{J^2}
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Soft collective modes

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Semiclassical picture:

→ \( N \) collective oscillators with mass \( M \sim 1/\Gamma \):

Frequency \( \omega = \sqrt{\frac{\lambda \Gamma}{M}} \)
Soft collective modes

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\[ \omega = \sqrt{\frac{\lambda}{M}} \]

→ Mode density

\[ \rho (\omega) = \text{const} \times \frac{\omega^2}{\Gamma J^2} \]
Soft collective modes

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Semiclassical picture:

\( \rightarrow N \) collective oscillators with mass \( M \sim 1/\Gamma \):

- Frequency \( \omega = \sqrt{\frac{\lambda}{M}} \)
- Mean square displacement \( \langle x^2 \rangle_\omega = \frac{1}{M \omega} \)
- Mode density \( \rho(\omega) = \text{const} \times \frac{\omega^2}{\Gamma J^2} \)

Susceptibility:

Collective modes form a bath with Ohmic spectral function

\[ \chi''(\omega) = \frac{1}{M \omega} \rho(\omega) \sim \frac{\omega}{J^2} \]
Soft collective modes

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Physical picture + generalization of a known result at the glass transition!

[Miller, Huse (SK model); Read, Ye, Sachdev (rotor models)]
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Prediction: Independent of \( \Gamma \) for \( \omega \ll \Gamma \)

Physical picture + generalization of a known result at the glass transition!

[Miller, Huse (SK model); Read, Ye, Sachdev (rotor models)]
Rigorous confirmation

A. Andreanov, MM, ‘10

$$H = - \sum_{i < j} \sigma_i^z J_{ij} \sigma_j^z - \Gamma \sum_i \sigma_i^x$$
Rigorous confirmation

Mean field equations (exact for $N = \infty$)  

$$
\exp[-\beta F_{\text{eff}}] = \text{Tr } T \exp S_{\text{eff}}
$$

$S_{\text{eff}} = J^2 \int_0^\beta \int d\tau d\tau' \left[ \sum_{a<b} Q_{ab} \sigma_a^z(\tau) \sigma_b^z(\tau') + \frac{1}{2} \sum_a Q_{aa}(\tau - \tau') \sigma_a^z(\tau) \sigma_a^z(\tau') \right] + \Gamma \sum_a \int_0^\beta d\tau \sigma_a^x(\tau)
$

Selfconsistency:

$$
Q_{aa}(\tau - \tau') = \langle \sigma_a^z(\tau) \sigma_a^z(\tau') \rangle_{S_{\text{eff}}}
$$

$$
Q_{ab} = \langle \sigma_a^z(\tau) \sigma_b^z(\tau') \rangle_{S_{\text{eff}}} \to q(x)
$$

Quantum impurity problem!

Full replica symmetry broken solution!

A. Andreanov, MM, '10
Rigorous confirmation

Mean field equations (exact for $N = \infty$)

(Replica trick: $n \to 0$ system copies)

$$\exp[-\beta F_{\text{eff}}] = \text{Tr} \mathbf{T} \exp \mathcal{S}_{\text{eff}}$$

$$\mathcal{S}_{\text{eff}} = J^2 \int_0^\beta d\tau d\tau' \left\{ \sum_{a<b} Q_{ab} \sigma_a^z(\tau) \sigma_b^z(\tau') + \frac{1}{2} \sum_a Q_{aa} (\tau - \tau') \sigma_a^z(\tau) \sigma_a^z(\tau') \right\} + \Gamma \sum_a \int_0^\beta d\tau \sigma_a^x(\tau)$$

Selfconsistency:

$$Q_{aa} (\tau - \tau') = \langle \sigma_a^z(\tau) \sigma_a^z(\tau') \rangle_{\text{eff}}$$

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\]

Quantum impurity problem!

Full replica symmetry broken solution!

To confirm:

\[
\Gamma \ll \Gamma_c \rightarrow \langle \sigma^z_a(\tau) \sigma^z_a(\tau') \rangle = \text{const} \times \frac{|\omega|}{J^2}
\]

i) Glass always gapless; Ohmic spectral function

ii) Const is independent of \(\Gamma\)
Rigorous confirmation

Mean field equations (exact for $N = \infty$)

$$\exp[-\beta F_{\text{eff}}] = \text{Tr} \mathbf{T} \exp \mathcal{S}_{\text{eff}}$$

$$\mathcal{S}_{\text{eff}} = J^2 \int_0^\beta \int d\tau d\tau' \left[ \sum_{a < b} Q_{ab} \sigma_a^z(\tau) \sigma_b^z(\tau') + \frac{1}{2} \sum_a Q_{aa}(\tau - \tau') \sigma_a^z(\tau) \sigma_a^z(\tau') \right] + \Gamma \sum_a \int_0^\beta d\tau' \sigma_a^x(\tau)$$

Selfconsistency:

$$Q_{aa}(\tau - \tau') = \langle \sigma_a^z(\tau) \sigma_a^z(\tau') \rangle_{\text{eff}}$$

$$Q_{ab} = \langle \sigma_a^z(\tau) \sigma_b^z(\tau') \rangle_{\text{eff}} \rightarrow q(x)$$

To confirm:

$$\Gamma \ll \Gamma_c \rightarrow \langle \sigma_a^z(\tau) \sigma_a^z(\tau') \rangle = \text{const} \times \frac{|\omega|}{J^2}$$

i) Glass always gapless; Ohmic spectral function

ii) Const is independent of $\Gamma$

Quantum impurity problem!

Full replica symmetry broken solution!

Ensures marginality, i.e. gaplessness!
Rigorous confirmation

Mean field equations (exact for \( N = \infty \))

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\exp[-\beta F_{\text{eff}}] = \text{Tr} \mathcal{T} \exp S_{\text{eff}}
\]

\[
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Q_{aa}(\tau - \tau') = \langle \sigma_a^z(\tau) \sigma_a^z(\tau') \rangle_{S_{\text{eff}}}
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- i) Glass always gapless; Ohmic spectral function
- ii) Const is independent of \( \Gamma \)

\[ H = -\sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z - \Gamma \sum_i \sigma_i^x \]

Quantum impurity problem!

Full replica symmetry broken solution!

- Physical insight → Find selfconsistent solution for a rescaled problem that is independent of \( \Gamma \)!
Rigorous confirmation

Mean field equations (exact for $N = \infty$)

(Replica trick: $n \to 0$ system copies)

$$
\exp[-\beta F_{\text{eff}}] = \text{Tr} \mathbf{T} \exp S_{\text{eff}}
$$

$$
S_{\text{eff}} = J^2 \int_0^\beta \int d\tau d\tau' \left[ \sum_{a < b} Q_{ab} \sigma_a^z(\tau) \sigma_b^z(\tau') + \frac{1}{2} \sum_a Q_{aa} (\tau - \tau') \sigma_a^z(\tau) \sigma_a^z(\tau') \right] + \Gamma \sum_a \int_0^\beta d\tau \sigma_a^x(\tau)
$$

Selfconsistency:

$$
Q_{aa}(\tau - \tau') = \langle \sigma_a^z(\tau) \sigma_a^z(\tau') \rangle_{S_{\text{eff}}}
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To confirm:

$$
\Gamma \ll \Gamma_c \to \langle \sigma_a^z(\tau) \sigma_a^z(\tau') \rangle = \text{const} \times \frac{|\omega|}{J^2}
$$

i) Glass always gapless; Ohmic spectral function

ii) Const $\approx 0.47$ independent of $\Gamma$ ← Physical insight → Find selfconsistent solution for a rescaled problem that is independent of $\Gamma$!
Rigorous confirmation

A. Andreanov, MM, ‘10

Take home message for mean field (SK):

• Effective potential approach $\rightarrow$ consistent physical picture of the low frequency dynamics

• Quantum glass is “self-organized critical” (gapless)

• The dynamics of active spins (for $\omega < \Gamma$) remains almost independent of $\Gamma$, despite mass and spring constants renormalizing!
What survives beyond mean field?

SK ↔
What survives beyond mean field?

- Gapless collective modes?
- Spatial properties (localization?)

Expected:
Large connectivity → Mean field describes well the spectrum, except at the lowest energies
Beyond mean field

Quantum ‘spin glass’ with

• Exchange matrix $J_{ij}$ random, $|J_{ij}| \sim J$
• Large connectivity $z$

Argued to be a relevant model for electron glasses close to Mott-Anderson (M-I) transition  

$MM, Ioffe \ '07$
Beyond mean field

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$MM, Ioffe ‘07$

Repeat effective potential + semi-classics analysis!
Beyond mean field

Quantum ‘spin glass’ with

• Exchange matrix $J_{ij}$ random, $|J_{ij}| \sim J$
• Large connectivity $z$

Spectrum of spring constants (Hessian $H_{ij}$) ($d>3$)

Semicircle + Corrections in regime $\sim 1/z^{5/6}$:
• localization
• spectral deviations

Delocalized spectrum!
Beyond mean field

Quantum ‘spin glass’ with

- Exchange matrix $J_{ij}$ random, $|J_{ij}| \sim J$
- Large connectivity $z$

Delocalized low-energy excitations, at least down to $\omega_{\text{min}} \sim J z^{-1/6} \ll J \approx h_{\text{loc}}^{\text{typ}}$

$\infty > z \gg 1$

Semicircle
+ Corrections in regime $\sim 1/z^{5/6}$:
  - localization
  - spectral deviations
+ semiclassical treatment

Spectrum of spring constants (Hessian $H_{ij}$) ($d>3$)
Beyond mean field?

Conclusions from our reasoning:

Criticality of quantum glass
→ large density of soft collective modes
delocalized to very low energies

→ Spin glass-type ergodicity breaking counteracts quantum localization!

→ Instead it enhances transport of energy and charge
(and decoherence in qubits)!
Beyond mean field?

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→ NO many body localization! (only with weak interactions!)
  Especially, not to be expected close to the MIT!
Beyond mean field?

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→ NO many body localization! (only with weak interactions)
  Especially, not to be expected close to the MIT!

Interesting open questions:

• Mobility edge for many body excitations?
• What happens at the quantum phase transition? (cf. MM ’09 regarding Bose glass)
S=1/2 ↔ hard core bosons

Bose condensation in disorder?
$S = \frac{1}{2} \leftrightarrow$ hard core bosons

How do glassy order and superfluidity compete?

Possibility of a “superglass”? 

= amorphous+glassy supersolid

(e.g. dirty bosons [preformed pairs] with Coulomb frustration)
Motivation

Supersolidity observed in **defectful** (glassy) quantum solids

*(Kim&Chan, Reppy, Dalibard)*

Torsional oscillator filled with Helium:

Superfluid fraction observed in the solid phase via **non-classical rotational inertia**, + anomalous shear properties etc.
Superglasses ?!

Xiaoquan Yu, MM '10

\[ H = -\Gamma \sum_{i} \sigma_{i}^{x} - \sum_{i<j} \sigma_{i}^{z} J_{ij} \sigma_{j}^{z} \]

\[ H = -\frac{t}{N} \sum_{i<j} \sigma_{i}^{x} \sigma_{j}^{x} - \sum_{i<j} \sigma_{i}^{z} J_{ij} \sigma_{j}^{z} \]

“Self-generated transverse field”
Superglasses?!  

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“Self-generated transverse field”

Xiaoquan Yu, MM ‘10

New aspect:

1\textsuperscript{st} order transition?  
Or superglass?
Superglasses ?!

Xiaoquan Yu, MM ‘10

\[ H = -\Gamma \sum_i \sigma_i^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z \]

\[ H = -\frac{t}{N} \sum_{i<j} \sigma_i^x \sigma_j^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z \]

Competing order parameters:

\[ M = \frac{1}{N} \langle \sigma_i^x \rangle \]

\[ q_{EA} = \frac{1}{N} \langle \sigma_i^z \rangle^2 \]

M signals superfluidity of hard core bosons \( \sigma_i^z \leftrightarrow 2n_i - 1 \)
Superglasses ?!

Xiaoquan Yu, MM '10

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Competing order parameters:

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M = \frac{1}{N} \langle \sigma_i^x \rangle \\
q_{\text{EA}} = \frac{1}{N} \langle \sigma_i^z \rangle^2
\]

\( M \) signals superfluidity of hard core bosons \( \sigma_i^z \leftrightarrow 2n_i - 1 \)

If \( M \) and \( q_{\text{EA}} \) exist at same time \( \rightarrow \) Meanfield model of a superglass!
Superglass

Tam, Geraedts, Inglis, Gingras, Melko, PRL (10)
Carleo, Tarzia, Zamponi PRL (09)

QMC (3d)  “Mean field”

\[
H = - \sum_{\langle ij \rangle} V_{ij} (n_i - 1/2)(n_j - 1/2) - t \sum_{\langle ij \rangle} (b_i^+ b_j + b_i b_j^+),
\]

Quenched randomness

QMC and cavity

\[
\hat{H} = -t \sum_{\langle i,j \rangle} [b_i^+ b_j + b_i b_j^+] + V \sum_{\langle i,j \rangle} n_i n_j,
\]

Random, frustrated “Bethe” lattice
<< Superglass >>

Tam, Geraedts, Inglis, Gingras, Melko, PRL (10)
Carleo, Tarzia, Zamponi PRL (09)

QMC (3d)  "Mean field"

Quenched randomness

\[ H = - \sum_{\langle ij \rangle} V_{ij} (n_i - 1/2)(n_j - 1/2) - t \sum_{\langle ij \rangle} \left( b_i^\dagger b_j + b_i b_j^\dagger \right), \]

Random, frustrated "Bethe" lattice

QMC and cavity

\[ \hat{H} = -t \sum_{\langle i,j \rangle} \left[ b_i^\dagger b_j + b_i b_j^\dagger \right] + V \sum_{\langle i,j \rangle} n_i n_j. \]
Superglass

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QMC (3d)  "Mean field"

\[ H = - \sum_{\langle ij \rangle} V_{ij}(n_i - 1/2)(n_j - 1/2) - t \sum_{\langle ij \rangle} \left( b_i^\dagger b_j + b_i b_j^\dagger \right), \]

Quenched randomness

?? Low T behavior ? – QPT ? - Local structure of the superglass ??
Mean field superglass

\( H = -\frac{t}{N} \sum_{i<j} \sigma_i^x \sigma_j^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z \)

\( M = \frac{1}{N} \langle \sigma_i^x \rangle \)

\( q_{EA} = \frac{1}{N} \langle \sigma_i^z \rangle^2 \)
Mean field superglass

\[ H = -\frac{t}{N} \sum_{i<j} \sigma_i^x \sigma_j^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z \]

\[ M = \frac{1}{N} \langle \sigma_i^x \rangle \]

\[ q_{EA} = \frac{1}{N} \langle \sigma_i^z \rangle^2 \]

- Obtain T = 0 phase transition glass-to-superglass exactly! (BCS instability of glass!)

- For superfluid-to-superglass transition: Use static approximation (but exact upper bound)

- Analytical proof of existence of superglass phase!
Phase diagram

Classical SK transition
Phase diagram

BCS-like condition: \( t \int dy P(y) \chi_{y}^{xx} = 1 \).

Suppression of superfluidity by the pseudogap!
Transition at finite \( J/t \approx 1.00 \)
Phase diagram

BCS-like condition: \( t \int dy P(y) \chi_{xy} = 1 \).

Suppression of superfluidity by the pseudogap!
Transition at finite \( J/t \approx 1.00 \).

Analogue for \( 1/r \) (Coulomb gap) in \( d<\infty \)? Expect different fractality of the condensate!
Local structure of the superglass

- Superfluid and glass try to avoid each other:
  \[ \langle s^x_i \rangle \text{ and } \langle s^z_i \rangle^2 \text{ are anticorrelated} \]
Local structure of the superglass

- Superfluid and glass try to avoid each other:
  \[ \langle s_i^x \rangle \text{ and } \langle s_i^z \rangle^2 \text{ are anticorrelated} \]

- Superfluid order non-monotonous with T!

Superfluid enhanced due to weakened glassy disorder
Conclusions

• Understanding of collective low energy excitations in quantum glasses

• Glassy order counteracts many particle quantum localization
  Ergodicity is broken – but arrow of time is intact!

• Frustrated bosons: superfluid and glassy order can coexist
Conclusions

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• Glassy order counteracts many particle quantum localization
  Ergodicity is broken – but arrow of time is intact!

• Frustrated bosons: superfluid and glassy order can coexist

Perspectives

Quantum phase transitions in disordered systems
(MIT, SIT, Bose glass, quantum glass transition):

• Nature of excitations and many particle localization?
  Where/when does it occur? Implications on quantum information?
• New effects due to competition of different orders
• Quantum annealing, adiabatic quantum computation?