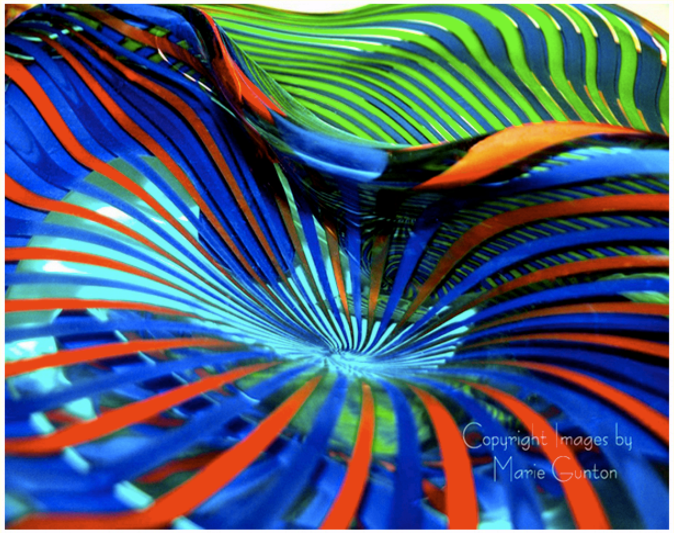
Quantum glasses – Frustration and collective behavior at T = 0

Markus Müller (ICTP Trieste)

Collaborators:
Alexei Andreanov (ICTP)
Xiaoquan Yu (SISSA)
Lev Ioffe (Rutgers)



Universität zu Köln, 17. November 2010

Arrow of time and ergodicity

The arrow of time

How to know the direction of increasing time?

- Entropy always increases
- Example: diffusion (continues forever in infinite systems)

$$\longrightarrow t \quad \neq \quad \longleftarrow t$$

Arrow of time and ergodicity

The arrow of time

How to know the direction of increasing time?

- Entropy always increases
- Example: diffusion (continues forever in infinite systems)

$$\xrightarrow{\qquad\qquad} t \qquad \neq \qquad \xleftarrow{\qquad\qquad} t$$

Interesting exception: quantum localized systems → time reversal symmetry even in infinite systems!

Arrow of time and ergodicity

The arrow of time

How to know the direction of increasing time?

- Entropy always increases
- Example: diffusion (continues forever in infinite systems)

$\longrightarrow t \neq \longleftarrow t$

Interesting exception: quantum localized systems → time reversal symmetry even in infinite systems!

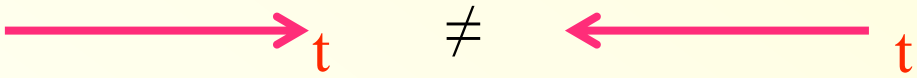
Ergodicity

Arrow of time and ergodicity

The arrow of time

How to know the direction of increasing time?

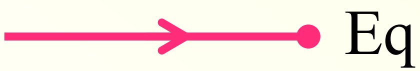
- Entropy always increases
- Example: diffusion (continues forever in infinite systems)



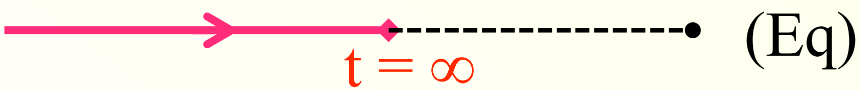
Interesting exception: quantum localized systems → time reversal symmetry even in infinite systems!

Ergodicity

Fundamental postulate of thermodynamics:
State of maximal entropy (=equilibrium) is reached in finite time.



But: NO full equilibration when ergodicity is broken



Occurs in particular in a large class of disordered systems: Glasses
→ configurational entropy, memory, history dependence, etc.

Interrelation between arrow of time and ergodicity?

(both notions associated with time evolution and dynamics)

Look at two types of ergodicity breakers = « glasses »!

Glasses: defying equilibration

Two types of glasses

(i) Quantum localized systems (Anderson glasses)

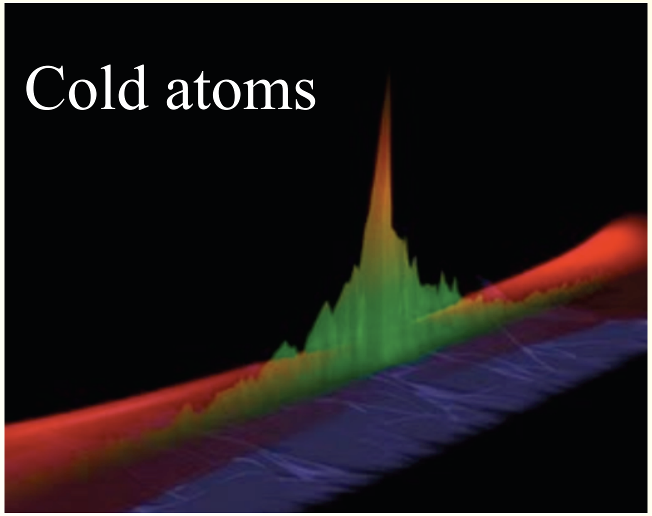
(ii) Frustrated, disordered or amorphous systems

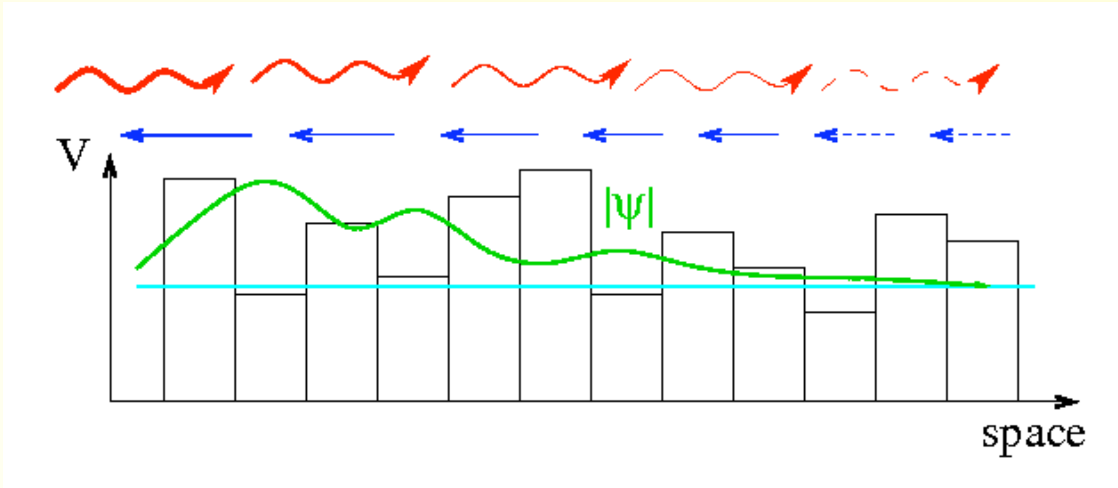
Glasses: defying equilibration

Two types of glasses

→ (i) Quantum localized systems (Anderson glasses)

(ii) Frustrated, disordered or amorphous systems





Single particle QM (Anderson)

$$H = \sum_i \varepsilon_i n_i - t \sum_{\langle i,j \rangle} \left(c_i^+ c_j + \text{h.c.} \right)$$

No diffusion at large disorder!

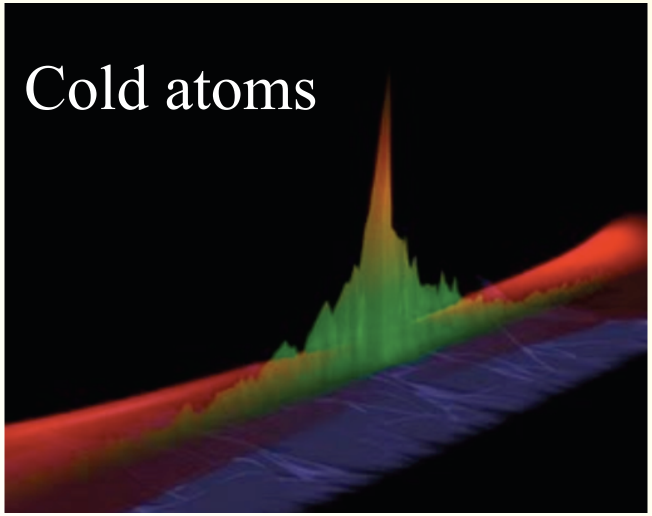
No arrow of time!

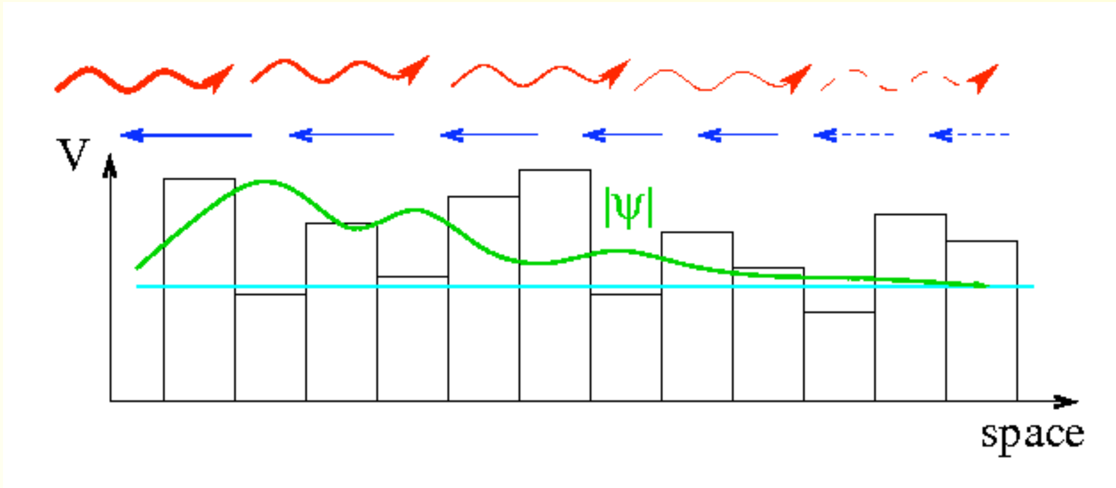
Glasses: defying equilibration

Two types of glasses

→ (i) Quantum localized systems (Anderson glasses)

(ii) Frustrated, disordered or amorphous systems





Single particle QM (Anderson) $$H=\sum_i \varepsilon_i n_i - t\sum_{\langle i,j\rangle}\left(c_i^+ c_j + \text{h.c.}\right)$$

No diffusion at large disorder!

No arrow of time!

Many particles (Anderson, Fleishman, Altshuler et al., Mirlin et al., etc.)

$$H=\sum_\alpha \varepsilon_\alpha n_\alpha - \sum_{\alpha,\beta,\gamma,\delta} V_{\alpha\beta\gamma\delta}\left(c_\alpha^+ c_\beta^+ c_\gamma c_\delta + \text{h.c.}\right)$$

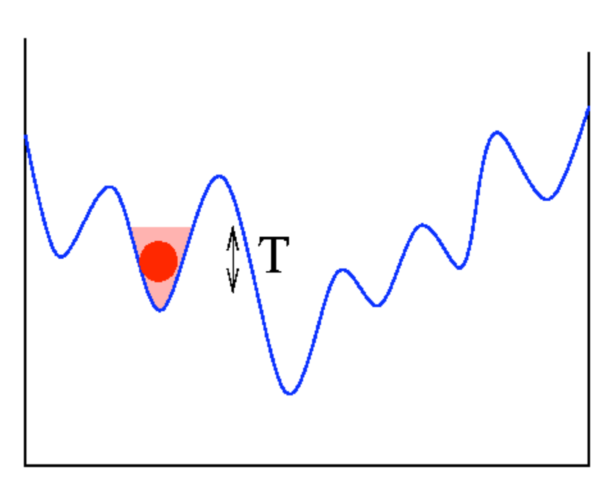
No diffusion at large disorder! && Transition to 'super-insulator' at finite T?!

Glasses: defying equilibration

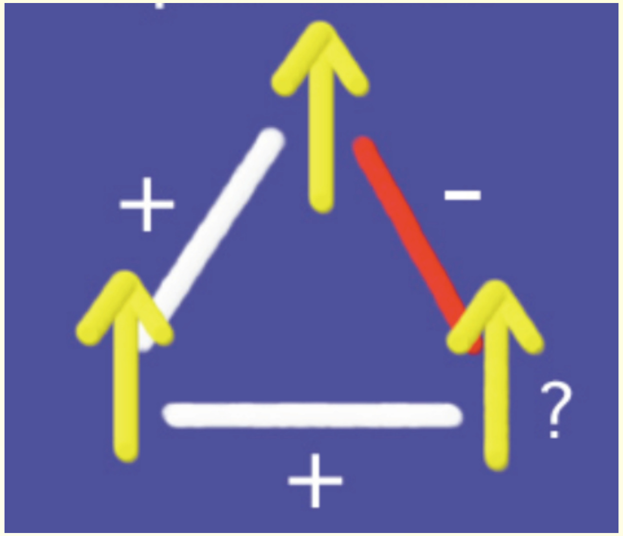
Two types of glasses

(i) Quantum localized systems (Anderson glasses)

→ (ii) Frustrated, disordered or amorphous systems



High barriers in complex energy landscape



Examples:

- spin glasses

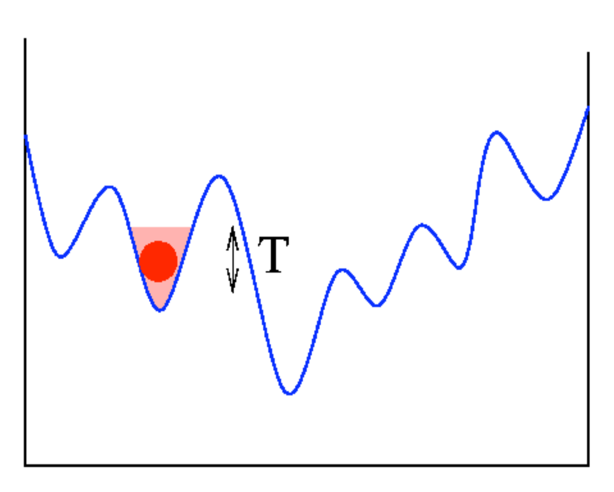
$$H = \sum_{\langle i,j \rangle} J_{ij} s_i^z s_j^z$$

Glasses: defying equilibration

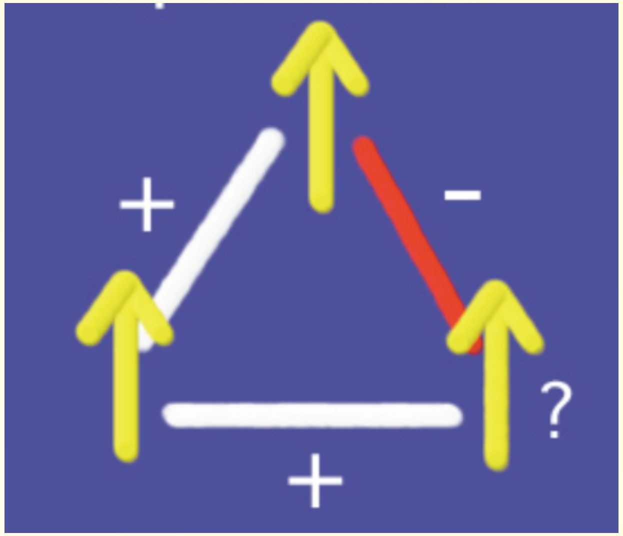
Two types of glasses

(i) Quantum localized systems (Anderson glasses)

→ (ii) Frustrated, disordered or amorphous systems



High barriers in complex energy landscape



Examples:

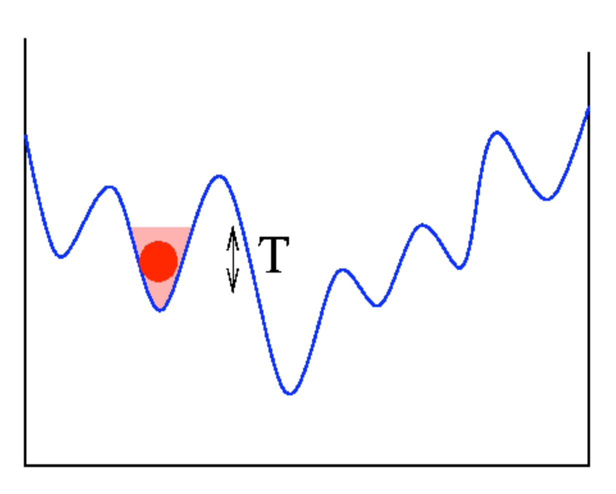
- spin glasses $$H = \sum_{\langle i,j \rangle} J_{ij} s_i^z s_j^z$$
- electron glasses $$H = \sum_{\langle i,j \rangle} \frac{e^2}{r_{ij}} n_i n_j + \sum_i \varepsilon_i n_i$$
- dirty superconductors, underdoped high Tc's
- defectful supersolids (He)

Glasses: defying equilibration

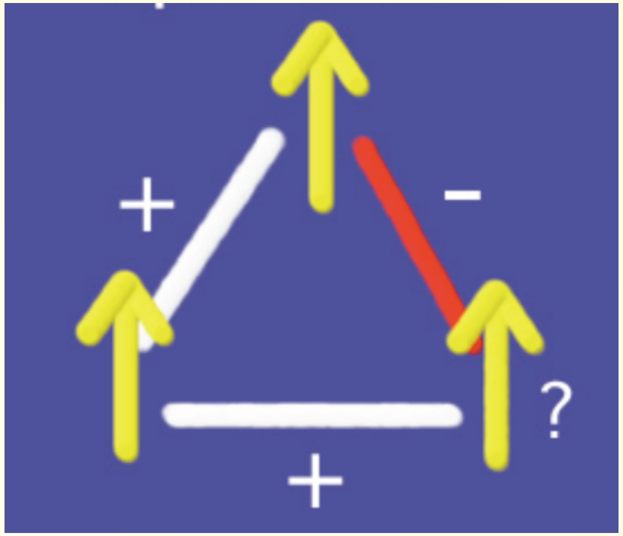
Two types of glasses

(i) Quantum localized systems (Anderson glasses)

→ (ii) Frustrated, disordered or amorphous systems



High barriers in complex energy landscape



Examples:

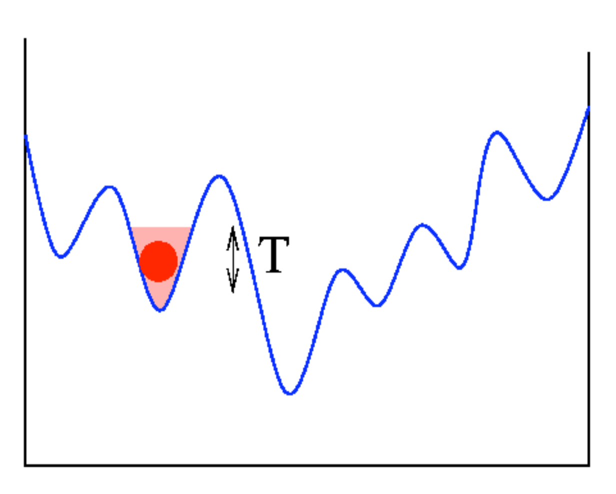
- spin glasses $$H = \sum_{\langle i,j \rangle} J_{ij} s_i^z s_j^z$$
- electron glasses $$H = \sum_{\langle i,j \rangle} \frac{e^2}{r_{ij}} n_i n_j + \sum_i \varepsilon_i n_i$$
- dirty superconductors, underdoped high Tc's
- defectful supersolids (He)
- many complex systems beyond physics (biology, economy, society)

Glasses: defying equilibration

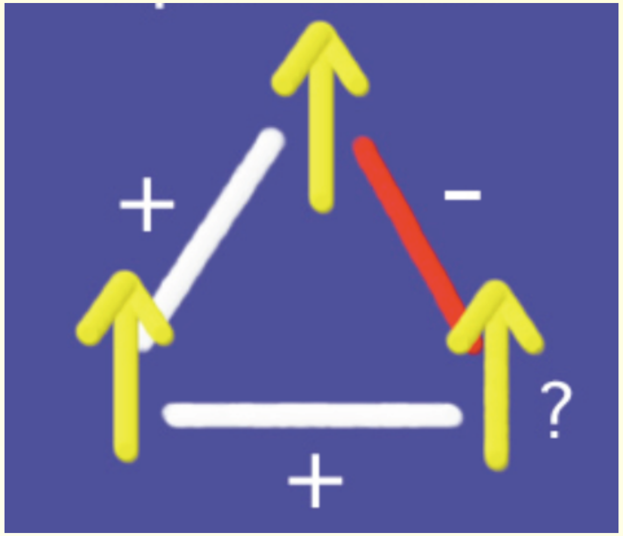
Two types of glasses

(i) Quantum localized systems (Anderson glasses)

→ (ii) Frustrated, disordered or amorphous systems



High barriers in complex energy landscape



Examples:

- spin glasses $$H = \sum_{\langle i,j \rangle} J_{ij} s_i^z s_j^z$$
- electron glasses $$H = \sum_{\langle i,j \rangle} \frac{e^2}{r_{ij}} n_i n_j + \sum_i \varepsilon_i n_i$$
- dirty superconductors, underdoped high Tc's
- defectful supersolids (He)
- many complex systems beyond physics (biology, economy, society)

Ergodicity breaking!

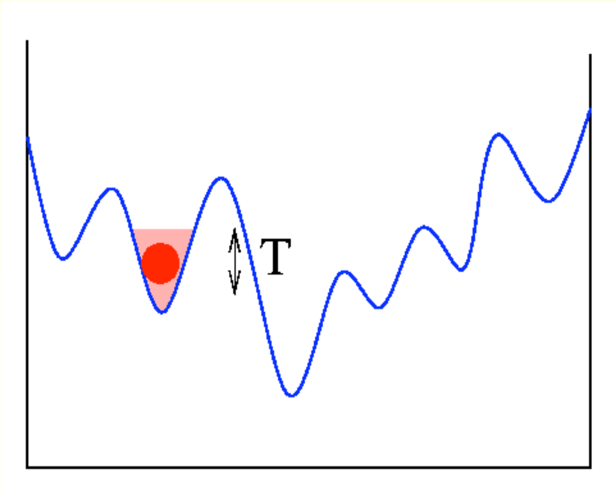
Transport (diffusion, arrow of time) ?!

Glasses: defying equilibration

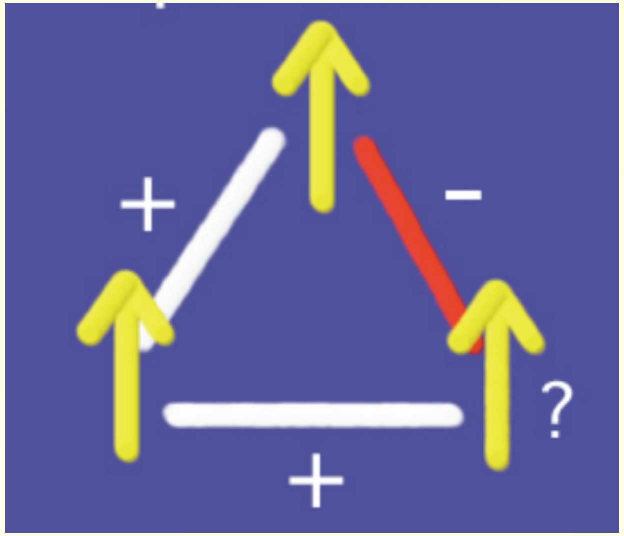
Two types of glasses

(i) Quantum localized systems (Anderson glasses)

→ (ii) Frustrated, disordered or amorphous systems



High barriers in complex energy landscape



Examples:

- spin glasses $H=\sum_{\langle i,j\rangle} J_{ij}s_i^z s_j^z + \Gamma\sum_i s_i^x$ [LiYHF] Ergodicity breaking!
- electron glasses $H=\sum_{\langle i,j\rangle}\frac{e^2}{r_{ij}}n_i n_j + \sum_i \varepsilon_i n_i + \sum_{i,j} t_{ij}c_i^+ c_j$ Transport (diffusion, arrow of time) ?!
- dirty superconductors, underdoped high Tc's **Quantum?** $\left[H_{cl}, H_q\right]\neq 0$
- defectful supersolids (He)
- many complex systems beyond physics (biology, economy, society)

Two types of « glasses »

Frustrated disordered systems	Quantum localized systems
e.g. Spin glass	e.g. Anderson insulator (Fermi glass)

Two types of « glasses »

Frustrated disordered systems	Quantum localized systems
e.g. Spin glass	e.g. Anderson insulator (Fermi glass)
Arbitrarily large energy barriers ΔE between metastable states	Too small matrix elements between distant states in Hilbert space

Two types of « glasses »

Frustrated disordered systems	**Quantum localized systems**
e.g. Spin glass	e.g. Anderson insulator (Fermi glass)
Arbitrarily large energy barriers ΔE between metastable states	Too small matrix elements between distant states in Hilbert space
Destroyed by large T	Robust to T (if fully localized)
Robust to quantum dephasing/ coupling to environment	Destroyed by a dephasing bath Important to avoid in Q-computing!

Two types of « glasses »

Frustrated disordered systems	**Quantum localized systems**
e.g. Spin glass	e.g. Anderson insulator (Fermi glass)
Arbitrarily large energy barriers ΔE between metastable states	Too small matrix elements between distant states in Hilbert space
Destroyed by large T	Robust to T (if fully localized)
Robust to quantum dephasing/ coupling to environment	Destroyed by a dephasing bath Important to avoid in Q-computing!

↓

Non-ergodicity AND no diffusion
(no clear arrow of time)

Two types of « glasses »

Frustrated disordered systems	**Quantum localized systems**
e.g. Spin glass	e.g. Anderson insulator (Fermi glass)
Arbitrarily large energy barriers ΔE between metastable states	Too small matrix elements between distant states in Hilbert space
Destroyed by large T	Robust to T (if fully localized)
Robust to quantum dephasing/ coupling to environment	Destroyed by a dephasing bath Important to avoid in Q-computing!

↓ Non-ergodicity, BUT diffusion usually possible (arrow of time there)

↓ Non-ergodicity AND no diffusion (no clear arrow of time)

Two types of « glasses »

Frustrated disordered systems	**Quantum localized systems**
e.g. Spin glass	e.g. Anderson insulator (Fermi glass)
Arbitrarily large energy barriers ΔE between metastable states	Too small matrix elements between distant states in Hilbert space
Destroyed by large T	Robust to T (if fully localized)
Robust to quantum dephasing/ coupling to environment	Destroyed by a dephasing bath Important to avoid in Q-computing!

↓ Non-ergodicity, BUT diffusion usually possible (arrow of time there)

↓ Non-ergodicity AND no diffusion (no clear arrow of time)

Joining two ingredients of ergodicity breaking:

(i) Mutual enhancement or competition?

(ii) Is there many particle-localization in quantum glasses?

Intriguing aspects of ‘spin glasses’

Classical glass: SK model

$$H = -\sum_{i<j=1}^{N} \sigma_i^z J_{ij} \sigma_j^z$$

$$\overline{J_{ij}} = 0$$

$$\overline{J_{ij}^2} = \frac{J^2}{N}$$

- Thermodynamic transition at T_c to a glass phase
- Unusual order parameter: $Q_{EA} = \frac{1}{N}\sum_i \langle s_i \rangle^2$
- Many metastable states

Intriguing aspects of 'spin glasses'

Classical glass: SK model

$$H = -\sum_{i<j=1}^{N} \sigma_i^z J_{ij} \sigma_j^z$$

$$\overline{J_{ij}} = 0$$

$$\overline{J_{ij}^2} = \frac{J^2}{N}$$

- Thermodynamic transition at T_c to a glass phase
- Unusual order parameter: $Q_{EA} = \frac{1}{N}\sum_i \langle s_i \rangle^2$
- Many metastable states

Glass phase is always (self-organized) critical! *(SK: Kondor-DeDominicis)*

Power law correlations in whole glass phase! *(Droplets: Fisher-Huse)*

Intriguing aspects of 'spin glasses'

Classical glass: SK model

$$H = -\sum_{i<j=1}^{N} \sigma_i^z J_{ij} \sigma_j^z$$

$$\overline{J_{ij}} = 0$$

$$\overline{J_{ij}^2} = \frac{J^2}{N}$$

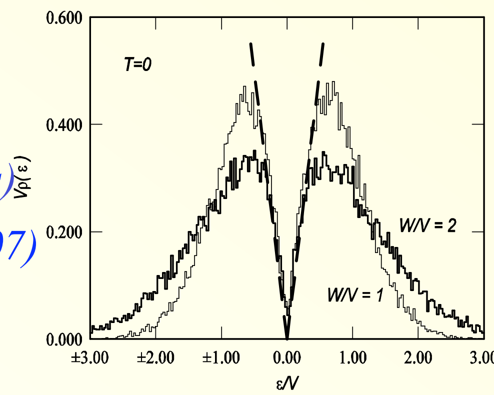
- Thermodynamic transition at T_c to a glass phase
- Unusual order parameter: $Q_{EA} = \frac{1}{N}\sum_i \langle s_i \rangle^2$
- Many metastable states

Glass phase is always (self-organized) critical! *(SK: Kondor-DeDominicis)*

Power law correlations in whole glass phase! *(Droplets: Fisher-Huse)*

Physical consequences of this criticality:

- Pseudgap in the distribution of local fields! *(Palmer, Sommers)*
  (electron glasses: Coulomb gap!) *(MM, Ioffe '04, MM, Pankov '07)*



Intriguing aspects of ‘spin glasses’

Classical glass: SK model

$$H = -\sum_{i<j=1}^{N} \sigma_i^z J_{ij} \sigma_j^z$$

$$\overline{J_{ij}} = 0$$

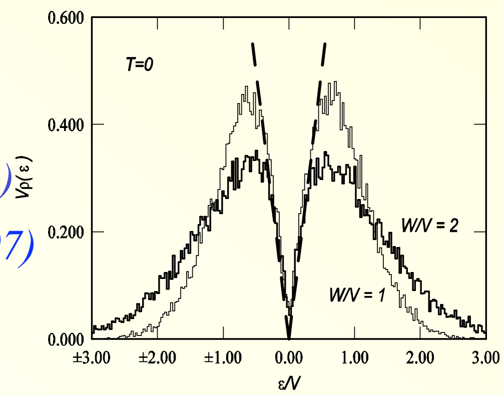
$$\overline{J_{ij}^2} = \frac{J^2}{N}$$

- Thermodynamic transition at T_c to a glass phase
- Unusual order parameter: $Q_{EA} = \frac{1}{N}\sum_i \langle s_i \rangle^2$
- Many metastable states

Glass phase is always (self-organized) critical! *(SK: Kondor-DeDominicis)*
Power law correlations in whole glass phase! *(Droplets: Fisher-Huse)*

Physical consequences of this criticality:

- Pseudgap in the distribution of local fields! *(Palmer, Sommers)*
  (electron glasses: Coulomb gap!) *(MM, Ioffe ’04, MM, Pankov ’07)*



- Power law distributed avalanches + Barkhausen noise!
 *(**Numeric**: Pazmandi et al ’99, **Analytic**: Le Doussal, MM, Wiese ‘10)*

Intriguing aspects of 'spin glasses'

? What are the consequences of this criticality in the quantum versions? ?

Problem: Very little is known about quantum glasses!

= Strongly correlated, disordered quantum systems!

Our strategy:

1. Solve mean field models (infinite connectivity) - highly non-trivial!
2. Obtain physical understanding
3. Extend to finite dimensions (large but finite connectivity)

Quantum glass models

Disorder: frustration vs. localization?

- Collective excitations in quantum glasses ?

Transverse field Ising spin glass
(Sherrington-Kirkpatrick SK)

$$H = -\sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z - \Gamma \sum_i \sigma_i^x$$

$$\overline{J_{ij}} = 0$$

$$\overline{J_{ij}^2} = \frac{J^2}{N}$$

- Glassiness and superfluidity - (spin ½ = hard core bosons)

$$\sigma_i^z \leftrightarrow 2n_i - 1$$

"Superglass" = glassy supersolid

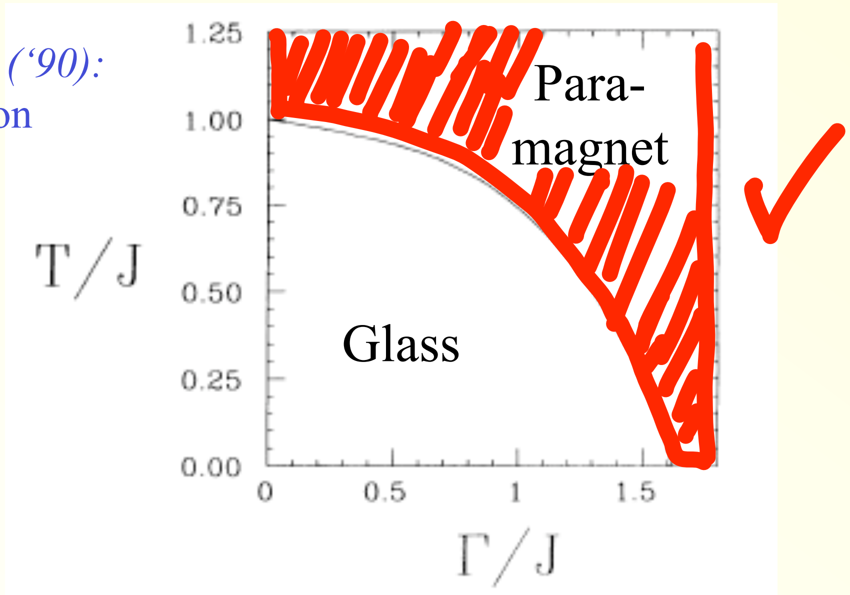
$$H = -\sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z - \frac{t}{N} \sum_{i<j} \sigma_i^x \sigma_j^x$$

Quantum SK: Known properties

Transverse field SK model (fully connected, random Ising)

$$H = -\sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z - \Gamma \sum_i \sigma_i^x$$

Goldschmidt, Lai, PRL ('90):
Static approximation



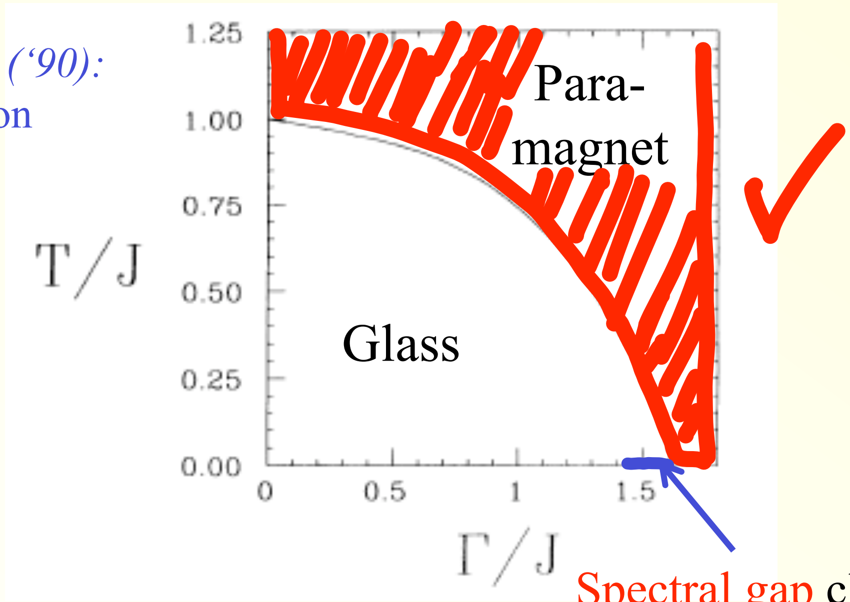
There *is* a quantum glass transition also at $\Gamma > 0$

Quantum SK: Known properties

Transverse field SK model (fully connected, random Ising)

$$H = -\sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z - \Gamma \sum_i \sigma_i^x$$

Goldschmidt, Lai, PRL ('90):
Static approximation



There *is* a quantum glass transition also at $\Gamma > 0$

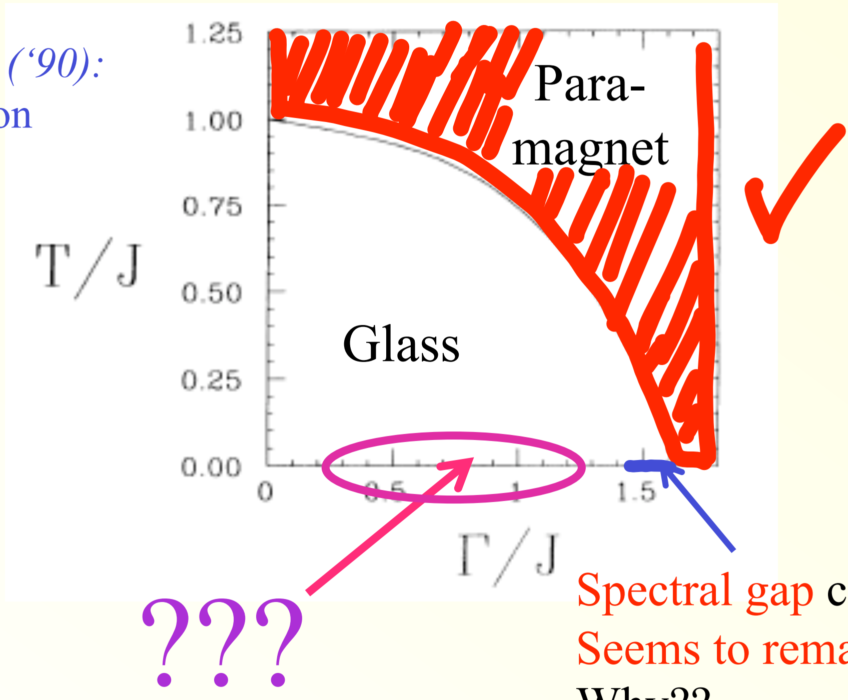
Spectral gap closes *(Miller, Huse '93)*
Seems to remains closed in the glass!!
Why?? *(Read, Sachdev, Ye '93)*

Quantum SK: Known properties

Transverse field SK model (fully connected, random Ising)

$$H = -\sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z - \Gamma \sum_i \sigma_i^x$$

Goldschmidt, Lai, PRL ('90):
Static approximation



There *is* a quantum glass transition also at $\Gamma > 0$

Spectral gap closes *(Miller, Huse '93)*
Seems to remains closed in the glass!!
Why?? *(Read, Sachdev, Ye '93)*

Understanding the quantum glass

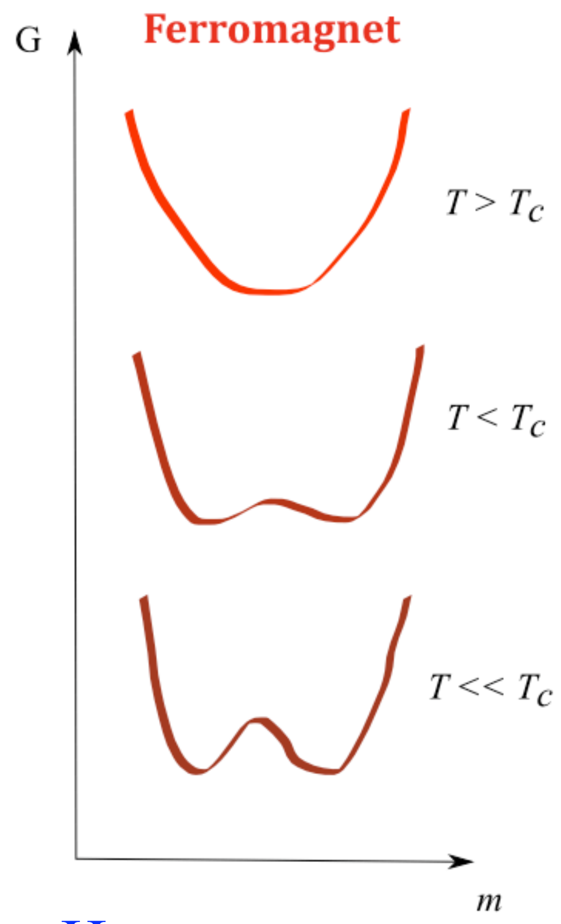
MM, Ioffe '07

1. Physical approach

→ Nature of excitations
→ Approach generalizable
to finite dimensions

Effective potential

Construct free energy functional *G(m)* imposing magnetization *m*

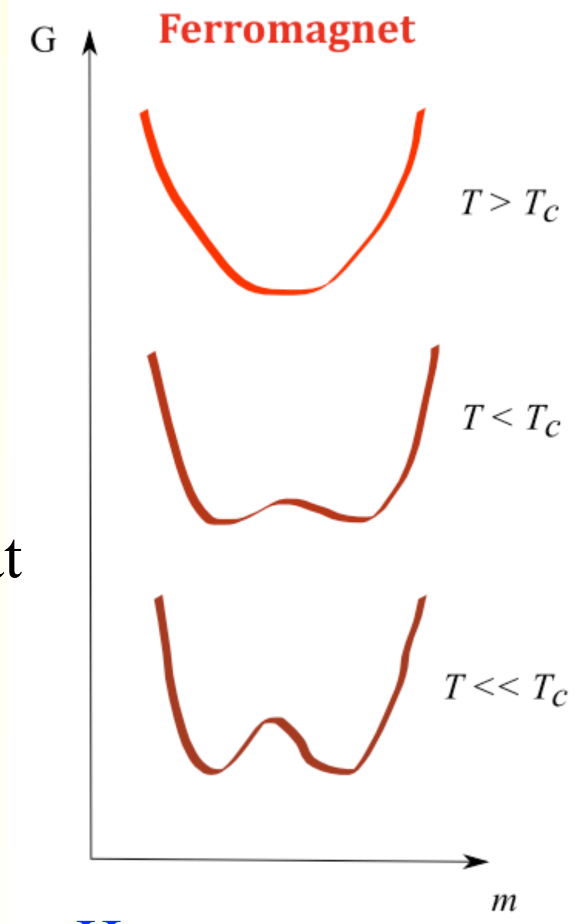


Homogeneous *m*

Effective potential

Construct free energy functional *G(m)* imposing magnetization *m*

- Two states!
- Barrier becomes very large at low T

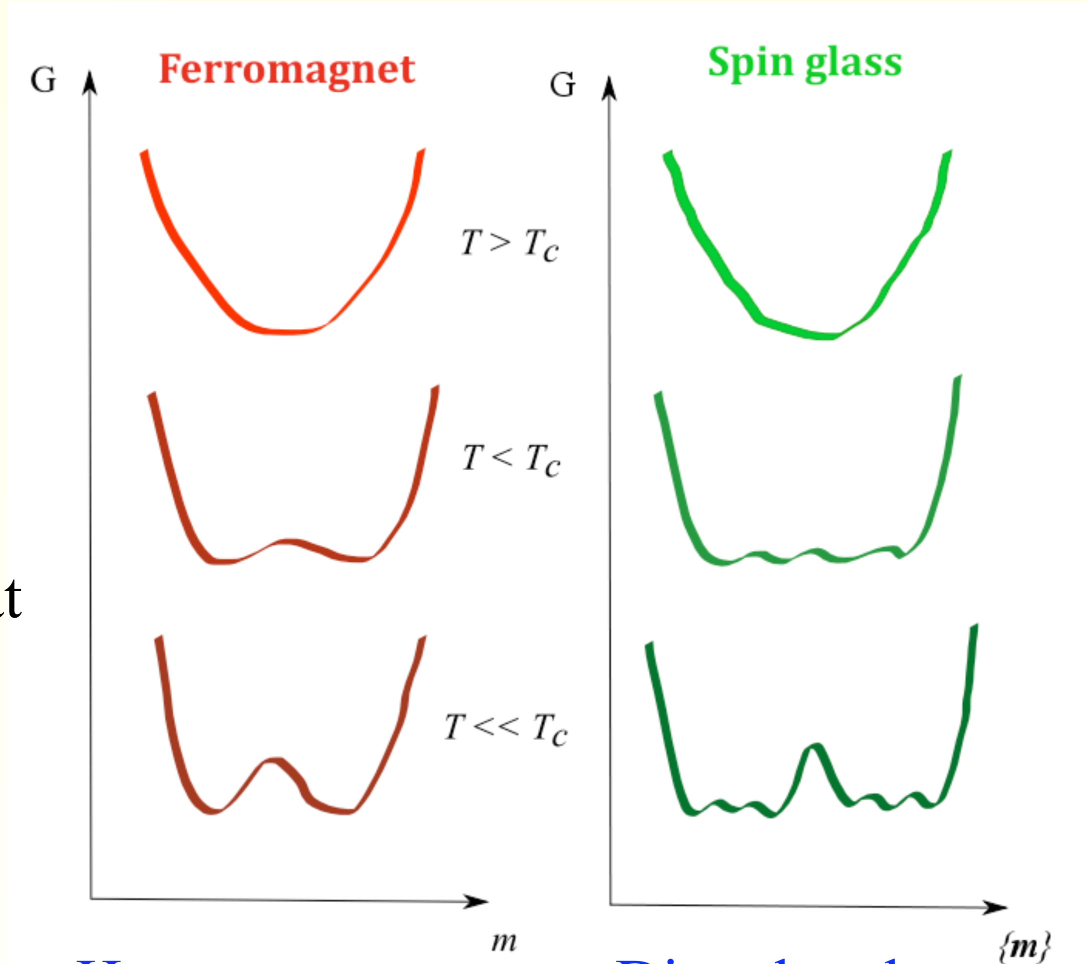


Homogeneous *m*

Effective potential

Construct free energy functional *G(m)* imposing magnetization *m*

- Two states!
- Barrier becomes very large at low T

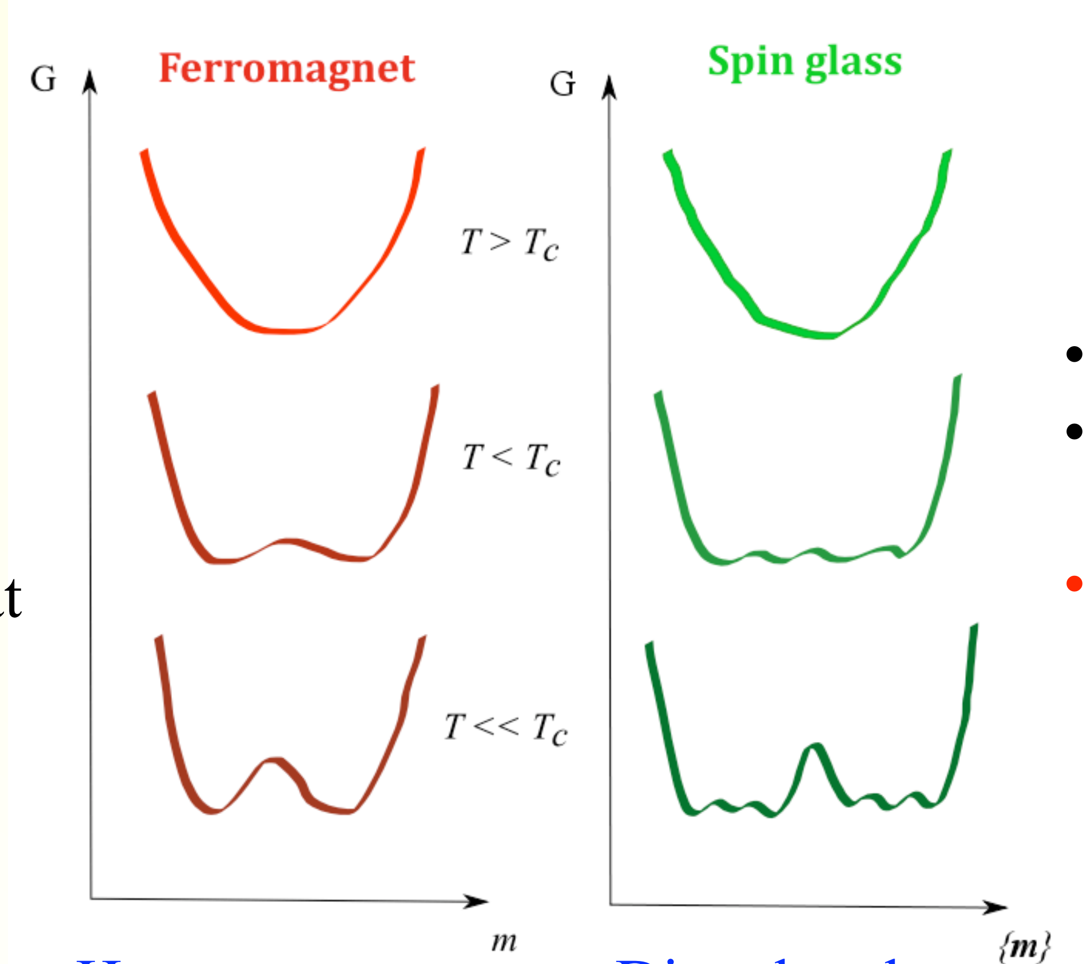


Homogeneous m Disordered pattern of m_i

Effective potential

Construct free energy functional *G(m)* imposing magnetization *m*

- Two states!
- Barrier becomes very large at low T



- Many states!
- Hierarchical valleys
- Barriers never very large: As if always at criticality!

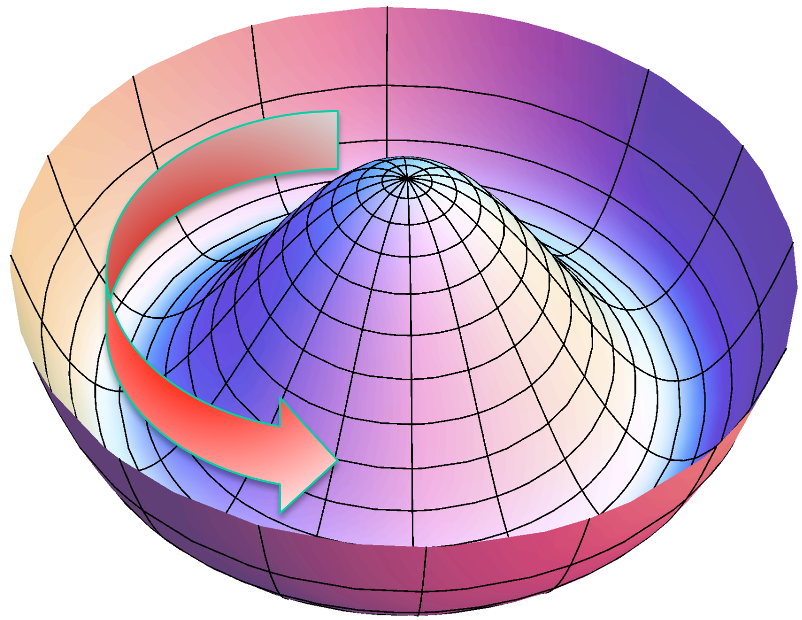
Homogeneous m Disordered pattern of m_i

Low energy excitations

XY or Heisenberg ferromagnet:

Goldstone modes:

Soft collective excitations along
flat directions of the potential



What about Ising glasses?

Low energy excitations

Two possibilities:

Isolated stable minimum in the potential landscape

Many valleys with rather flat interconnections!

(in configuration space)

Low energy excitations

Two possibilities:

Isolated stable minimum in the potential landscape

Many valleys with rather flat interconnections!
(in configuration space)

Effective potential

(Thouless, Anderson, Palmer '77: Classical SK model;   Biroli, Cugliandolo '01, MM, Ioffe '07)

$$H = -\sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z - \Gamma \sum_i \sigma_i^x$$

Effective potential (exact at N = ∞)

$$G\left(\left\{\left\langle \sigma_i^z \right\rangle = m_i\right\}\right) = -\Gamma \sum_i \sqrt{1-m_i^2} - \frac{1}{2}\sum_{i\neq j} m_i J_{ij} m_j - \frac{1}{2}\sum_{i\neq j} J_{ij}^2 \int_0^\infty d\tau \chi_i(\tau)\chi_j(\tau) + O\left(\sqrt{1/N}\right)$$

Static approximation for susceptibility: $\chi_i(\omega \to 0) = dm_i/dh_i \approx \chi_i(m_i)$

Effective potential

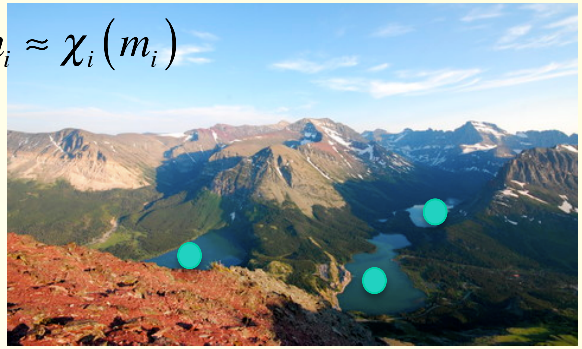
(Thouless, Anderson, Palmer '77: Classical SK model;   Biroli, Cugliandolo '01, MM, Ioffe '07)

$$H = -\sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z - \Gamma \sum_i \sigma_i^x$$

Effective potential (exact at N = ∞)

$$G\left(\left\{\left\langle \sigma_i^z \right\rangle = m_i\right\}\right) = -\Gamma \sum_i \sqrt{1-m_i^2} - \frac{1}{2}\sum_{i\neq j} m_i J_{ij} m_j - \frac{1}{2}\sum_{i\neq j} J_{ij}^2 \int_0^\infty d\tau \chi_i(\tau)\chi_j(\tau) + O\left(\sqrt{1/N}\right)$$

Static approximation for susceptibility: $\chi_i(\omega \to 0) = dm_i/dh_i \approx \chi_i(m_i)$



Local minima $\left(\partial G/\partial m_i = 0\right)$ (in static approximation)

$$\rightarrow \quad \frac{\Gamma m_i}{\sqrt{1-m_i^2}} = \sum_{j\neq i} J_{ij} m_j - m_i \sum_{j\neq i} J_{ij}^2 \chi_j\left(m_j\right)$$

Effective field on σ_i in z-direction

N coupled random equations for $\{m_i\}$.
With ~ Exp[αN] solutions!

Quantum TAP equations

***T**houless, **A**nderson, **P**almer '77 (Classical); Biroli, Cugliandolo '01; MM, Ioffe '07 (quantum)*

Local minima $\left(\partial G/\partial m_i = 0\right)$

$$\frac{\Gamma m_i}{\sqrt{1-m_i^2}} = \sum_{j\neq i} J_{ij} m_j - m_i \sum_{j\neq i} J_{ij}^2 \chi_j\left(m_j\right)$$

Environment of a local minimum

Curvatures (Hessian): $\mathrm{H}_{ij} = \partial^2 G \big/ \partial m_i \partial m_j = J_{ij} + \text{diagonal terms}$

Quantum TAP equations

Thouless, Anderson, Palmer '77 (Classical); Biroli, Cugliandolo '01; MM, Ioffe '07 (quantum)

Local minima $\left(\partial G/\partial m_i = 0\right)$

$$\frac{\Gamma m_i}{\sqrt{1-m_i^2}} = \sum_{j\neq i} J_{ij} m_j - m_i \sum_{j\neq i} J_{ij}^2 \chi_j\left(m_j\right)$$

$$Spec\left[H_{ij}\right] \equiv \rho_H(\lambda) = const \times \frac{\sqrt{\lambda\Gamma}}{J^2}$$

Environment of a local minimum

Curvatures (Hessian): $\mathrm{H}_{ij} = \partial^2 G \big/ \partial m_i \partial m_j = J_{ij} + \text{diagonal terms}$

Standard resolvent technique

Spectrum of "spring constants" in a minimum

$$Spec\left[H_{ij}\right] \equiv \rho_H(\lambda) = const \times \frac{\sqrt{\lambda\Gamma}}{J^2}$$

(at small λ: No Gap!)

Gapless spectrum
in the **whole** glass phase,
ensured by **marginality of minima**!

Soft collective modes

Spectrum of "spring constants"

$$Spec\left[H_{ij} \right] \equiv \rho_H(\lambda) = const \times \frac{\sqrt{\lambda \Gamma}}{J^2}$$

Soft collective modes

Spectrum of "spring constants"

$$Spec\left[H_{ij}\right] \equiv \rho_H(\lambda) = const \times \frac{\sqrt{\lambda \Gamma}}{J^2}$$

Semiclassical picture:

→ N collective oscillators with mass M ~ 1/Γ: Frequency $\omega = \sqrt{\lambda / M}$

Soft collective modes

Spectrum of "spring constants" $$Spec\left[H_{ij}\right] \equiv \rho_H(\lambda) = const \times \frac{\sqrt{\lambda\Gamma}}{J^2}$$

Semiclassical picture:

→ N collective oscillators with mass M ~ 1/Γ: Frequency $\omega = \sqrt{\lambda/M}$

⟶ Mode density $$\rho(\omega) = \text{const} \times \frac{\omega^2}{\Gamma J^2}$$

Soft collective modes

Spectrum of "spring constants" $$Spec\left[H_{ij}\right] \equiv \rho_H(\lambda) = const \times \frac{\sqrt{\lambda\Gamma}}{J^2}$$

Semiclassical picture:

$\rightarrow$ N collective oscillators with mass M ~ 1/Γ: Frequency $\omega = \sqrt{\lambda / M}$

$\longrightarrow$ Mode density $$\rho(\omega) = \text{const} \times \frac{\omega^2}{\Gamma J^2}$$

Mean square displacement

$$\langle x^2 \rangle_\omega = \frac{1}{M\omega}$$

Susceptibility:

$\longrightarrow$ Collective modes form a bath with Ohmic spectral function

$$\chi''(\omega) = \frac{1}{M\omega}\rho(\omega) \sim \frac{\omega}{J^2}$$

Soft collective modes

Spectrum of "spring constants"

$$Spec\left[H_{ij}\right] \equiv \rho_H(\lambda) = const \times \frac{\sqrt{\lambda\Gamma}}{J^2}$$

Semiclassical picture:

→ N collective oscillators with mass M ~ 1/Γ:

Frequency $\omega = \sqrt{\lambda / M}$

→ Mode density

$$\rho(\omega) = \text{const} \times \frac{\omega^2}{\Gamma J^2}$$

Mean square displacement

$$\left\langle x^2 \right\rangle_\omega = \frac{1}{M\omega}$$

→ **Susceptibility:**
Collective modes form a bath with Ohmic spectral function

$$\chi''(\omega) = \frac{1}{M\omega}\rho(\omega) \sim \frac{\omega}{J^2}$$

Physical picture + generalization of a known result at the glass transition!
[Miller, Huse (SK model); Read, Ye, Sachdev (rotor models)]

Soft collective modes

Spectrum of "spring constants"

$$Spec\left[H_{ij}\right] \equiv \rho_H(\lambda) = const \times \frac{\sqrt{\lambda\Gamma}}{J^2}$$

Semiclassical picture:

$\rightarrow$ *N* collective oscillators with mass M ~ 1/Γ:

Frequency $\omega = \sqrt{\lambda / M}$

→ Mode density

$$\rho(\omega) = \text{const} \times \frac{\omega^2}{\Gamma J^2}$$

Mean square displacement

$$\left\langle x^2 \right\rangle_\omega = \frac{1}{M\omega}$$

→ **Susceptibility:**
Collective modes form a bath
with Ohmic spectral function

$$\chi''(\omega) = \frac{1}{M\omega}\rho(\omega) \sim \frac{\omega}{J^2}$$

Prediction:
Independent of
Γ for ω << Γ !!

Physical picture + generalization of a known result at the glass transition!
[Miller, Huse (SK model); Read, Ye, Sachdev (rotor models)]

Rigorous confirmation

A. Andreanov, MM, '10

$$H = -\sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z - \Gamma \sum_i \sigma_i^x$$

Rigorous confirmation

A. Andreanov, MM, '10

Mean field equations (exact for N = ∞)

$$H = -\sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z - \Gamma \sum_i \sigma_i^x$$

(Replica trick:
n→0 system copies)

$$\exp[-\beta F_{\mathrm{eff}}] = \mathrm{Tr}\,\mathbf{T} \exp \mathcal{S}_{\mathrm{eff}}$$

$$\mathcal{S}_{\mathrm{eff}} = J^2 \iint_0^\beta d\tau d\tau' \left[\sum_{a<b} Q_{ab} \sigma_a^z(\tau) \sigma_b^z(\tau') + \frac{1}{2} \sum_a Q_{aa}(\tau - \tau') \sigma_a^z(\tau) \sigma_a^z(\tau') \right] + \Gamma \sum_a \int_0^\beta d\tau\, \sigma_a^x(\tau)$$

Selfconsistency:

$$Q_{aa}(\tau - \tau') = \langle \sigma_a^z(\tau) \sigma_a^z(\tau') \rangle_{\mathcal{S}_{\mathrm{eff}}}$$

Quantum impurity problem!

$$Q_{ab} = \langle \sigma_a^z(\tau) \sigma_b^z(\tau') \rangle_{\mathcal{S}_{\mathrm{eff}}} \to q(x)$$

Full replica symmetry
broken solution!

Rigorous confirmation

A. Andreanov, MM, '10

Mean field equations (exact for N = ∞)

(Replica trick:
n→0 system copies)

$$\exp[-\beta F_{\text{eff}}] = \text{Tr}\,\mathbf{T}\exp\mathcal{S}_{\text{eff}}$$

$$H = -\sum_{i<j}\sigma_i^z J_{ij}\sigma_j^z - \Gamma\sum_i \sigma_i^x$$

$$\mathcal{S}_{\text{eff}} = J^2 \iint_0^\beta d\tau d\tau' \left[\sum_{a<b} Q_{ab}\sigma_a^z(\tau)\sigma_b^z(\tau') + \frac{1}{2}\sum_a Q_{aa}(\tau-\tau')\sigma_a^z(\tau)\sigma_a^z(\tau')\right] + \Gamma\sum_a \int_0^\beta d\tau\, \sigma_a^x(\tau)$$

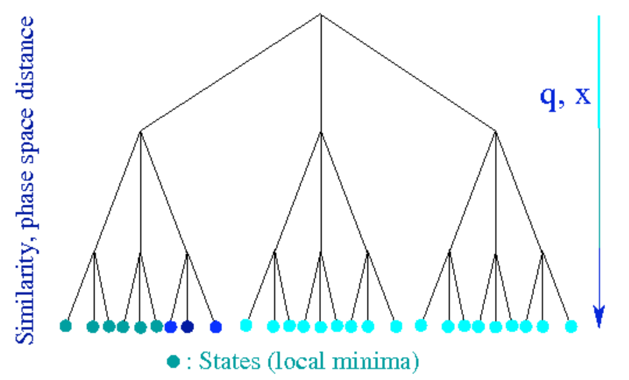
Selfconsistency:

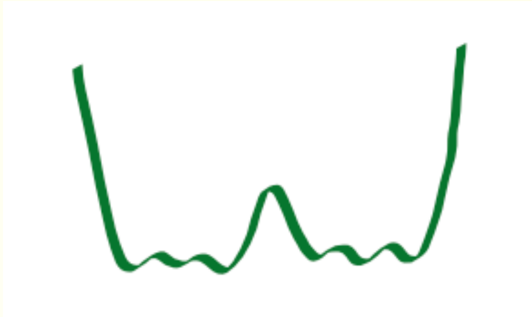
$$Q_{aa}(\tau-\tau') = \langle\sigma_a^z(\tau)\sigma_a^z(\tau')\rangle_{\mathcal{S}_{\text{eff}}}$$

$$Q_{ab} = \langle\sigma_a^z(\tau)\sigma_b^z(\tau')\rangle_{\mathcal{S}_{\text{eff}}} \to q(x)$$

Quantum impurity problem!

Full replica symmetry broken solution!





Rigorous confirmation

A. Andreanov, MM, '10

Mean field equations (exact for N = ∞)

(Replica trick:
n→0 system copies)

$$\exp[-\beta F_{\rm eff}] = \mathrm{Tr}\,\mathbf{T}\exp\mathcal{S}_{\rm eff}$$

$$H = -\sum_{i<j}\sigma_i^z J_{ij}\sigma_j^z - \Gamma\sum_i \sigma_i^x$$

$$\mathcal{S}_{\rm eff} = J^2 \iint_0^\beta d\tau d\tau' \left[\sum_{a<b} Q_{ab}\sigma_a^z(\tau)\sigma_b^z(\tau') + \frac{1}{2}\sum_a Q_{aa}(\tau-\tau')\sigma_a^z(\tau)\sigma_a^z(\tau')\right] + \Gamma\sum_a \int_0^\beta d\tau\, \sigma_a^x(\tau)$$

Selfconsistency:

$$Q_{aa}(\tau-\tau') = \langle\sigma_a^z(\tau)\sigma_a^z(\tau')\rangle_{\mathcal{S}_{\rm eff}}$$

Quantum impurity problem!

$$Q_{ab} = \langle\sigma_a^z(\tau)\sigma_b^z(\tau')\rangle_{\mathcal{S}_{\rm eff}} \to q(x)$$

Full replica symmetry
broken solution!

To confirm:

$$\Gamma \ll \Gamma_c \to \langle\sigma_a^z(\tau)\sigma_a^z(\tau')\rangle = \mathrm{const}\times\frac{|\omega|}{J^2}$$

i) Glass always gapless; Ohmic spectral function
ii) Const is independent of Γ

Rigorous confirmation

A. Andreanov, MM, '10

Mean field equations (exact for N = ∞)

(Replica trick:
n→0 system copies)

$$\exp[-\beta F_{\text{eff}}] = \operatorname{Tr} \mathbf{T} \exp \mathcal{S}_{\text{eff}}$$

$$H = -\sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z - \Gamma \sum_i \sigma_i^x$$

$$\mathcal{S}_{\text{eff}} = J^2 \int_0^\beta \!\! \int d\tau d\tau' \left[\sum_{a<b} Q_{ab} \sigma_a^z(\tau) \sigma_b^z(\tau') + \frac{1}{2} \sum_a Q_{aa}(\tau - \tau') \sigma_a^z(\tau) \sigma_a^z(\tau') \right] + \Gamma \sum_a \int_0^\beta d\tau \, \sigma_a^x(\tau)$$

Selfconsistency:

$$Q_{aa}(\tau - \tau') = \langle \sigma_a^z(\tau) \sigma_a^z(\tau') \rangle_{\mathcal{S}_{\text{eff}}}$$

Quantum impurity problem!

$$Q_{ab} = \langle \sigma_a^z(\tau) \sigma_b^z(\tau') \rangle_{\mathcal{S}_{\text{eff}}} \to q(x)$$

Full replica symmetry broken solution!

To confirm:

$$\Gamma \ll \Gamma_c \to \langle \sigma_a^z(\tau) \sigma_a^z(\tau') \rangle = \text{const} \times \frac{|\omega|}{J^2}$$

Ensures marginality, i.e. gaplessness!

i) Glass always gapless; Ohmic spectral function
ii) Const is independent of Γ

Rigorous confirmation

A. Andreanov, MM, '10

Mean field equations (exact for N = ∞)

(Replica trick: n→0 system copies)

$$\exp[-\beta F_{\text{eff}}] = \text{Tr}\,\mathbf{T}\exp \mathcal{S}_{\text{eff}}$$

$$H = -\sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z - \Gamma \sum_i \sigma_i^x$$

$$\mathcal{S}_{\text{eff}} = J^2 \iint_0^\beta d\tau d\tau' \left[\sum_{a<b} Q_{ab} \sigma_a^z(\tau) \sigma_b^z(\tau') + \frac{1}{2} \sum_a Q_{aa}(\tau - \tau') \sigma_a^z(\tau) \sigma_a^z(\tau') \right] + \Gamma \sum_a \int_0^\beta d\tau\, \sigma_a^x(\tau)$$

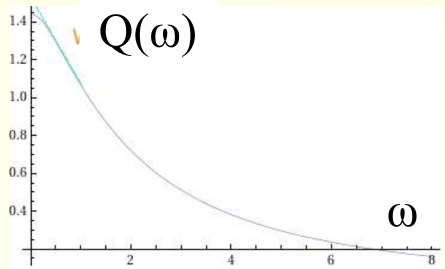
Selfconsistency:

$$Q_{aa}(\tau - \tau') = \langle \sigma_a^z(\tau) \sigma_a^z(\tau') \rangle_{\mathcal{S}_{\text{eff}}}$$

Quantum impurity problem!

$$Q_{ab} = \langle \sigma_a^z(\tau) \sigma_b^z(\tau') \rangle_{\mathcal{S}_{\text{eff}}} \rightarrow q(x)$$

Full replica symmetry broken solution!



To confirm:

$$\Gamma \ll \Gamma_c \rightarrow \langle \sigma_a^z(\tau) \sigma_a^z(\tau') \rangle = \text{const} \times \frac{|\omega|}{J^2}$$

i) Glass always gapless; Ohmic spectral function

ii) Const is independent of Γ ← Physical insight → Find selfconsistent solution for a rescaled problem that is independent of Γ!

Rigorous confirmation

A. Andreanov, MM, '10

Mean field equations (exact for N = ∞)

(Replica trick:
n→0 system copies)

$$\exp[-\beta F_{\rm eff}] = \, {\rm Tr}\, \mathbf{T} \exp \mathcal{S}_{\rm eff}$$

$$H = -\sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z - \Gamma \sum_i \sigma_i^x$$

$$\mathcal{S}_{\rm eff} = J^2 \iint_0^\beta d\tau d\tau' \left[\sum_{a<b} Q_{ab} \sigma_a^z(\tau) \sigma_b^z(\tau') + \frac{1}{2} \sum_a Q_{aa}(\tau - \tau') \sigma_a^z(\tau) \sigma_a^z(\tau') \right] + \Gamma \sum_a \int_0^\beta d\tau \, \sigma_a^x(\tau)$$

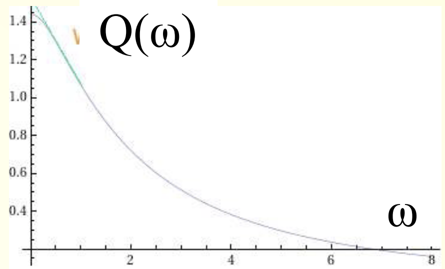
Selfconsistency:

$$Q_{aa}(\tau - \tau') = \langle \sigma_a^z(\tau) \sigma_a^z(\tau') \rangle_{\mathcal{S}_{\rm eff}}$$

Quantum impurity problem!

$$Q_{ab} = \langle \sigma_a^z(\tau) \sigma_b^z(\tau') \rangle_{\mathcal{S}_{\rm eff}} \to q(x)$$

Full replica symmetry
broken solution!



To confirm:

$$\Gamma \ll \Gamma_c \to \langle \sigma_a^z(\tau) \sigma_a^z(\tau') \rangle = {\rm const} \times \frac{|\omega|}{J^2}$$

i) Glass always gapless; Ohmic spectral function
ii) Const **≅0.47** independent of Γ ← Physical insight → Find selfconsistent solution for a rescaled problem that is independent of Γ!

Rigorous confirmation

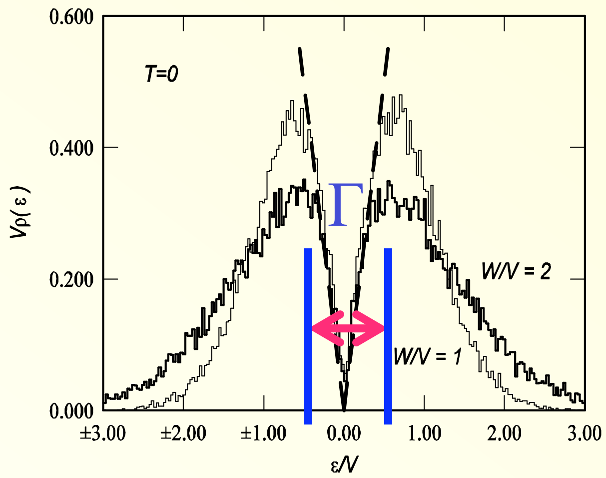
A. Andreanov, MM, '10

Take home message for mean field (SK):

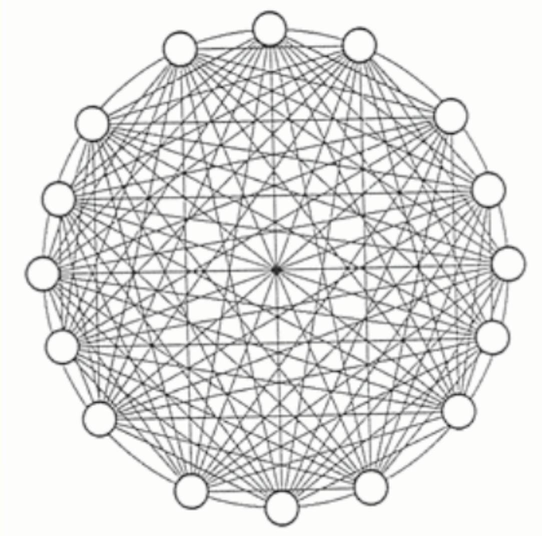
- Effective potential approach → consistent physical picture of the low frequency dynamics

- Quantum glass is "self-organized critical" (gapless)

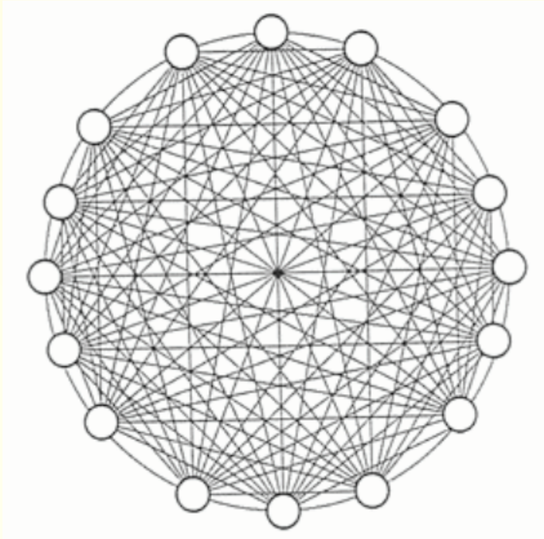
- The dynamics of active spins (for $\omega < \Gamma$) remains almost independent of Γ, despite mass and spring constants renormalizing!



What survives beyond mean field?

SK $\leftrightarrow$ 

What survives beyond mean field?

SK $\leftrightarrow$ 

- Gapless collective modes?
- **Spatial properties (localization?)**

Expected:

Large connectivity $\rightarrow$ Mean field describes well the spectrum, except at the lowest energies

Beyond mean field

Quantum 'spin glass' with

- Exchange matrix J_{ij} random, $|J_{ij}| \sim J$
- Large connectivity z

Argued to be a relevant model for electron glasses close to Mott-Anderson (M-I) transition *MM, Ioffe '07*

Beyond mean field

Quantum 'spin glass' with

- Exchange matrix J_{ij} random, $|J_{ij}| \sim J$
- Large connectivity z

Argued to be a relevant model for electron glasses
close to Mott-Anderson (M-I) transition *MM, Ioffe '07*

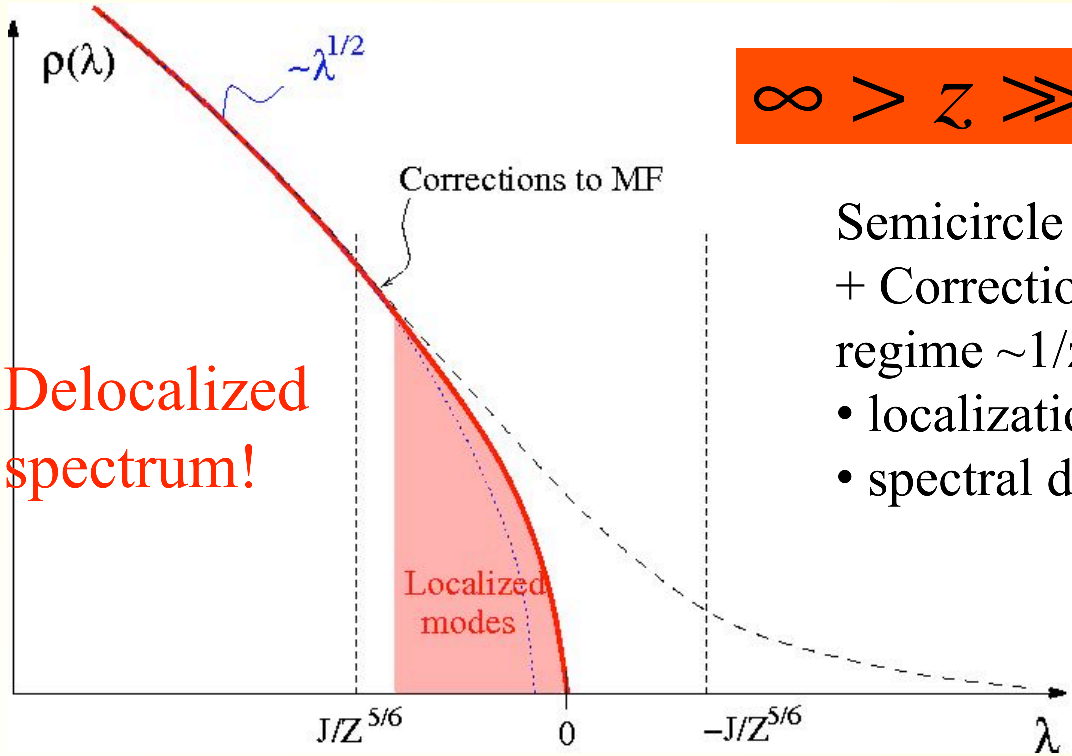
Repeat effective potential + semi-classics analysis!

Beyond mean field

Quantum 'spin glass' with

- Exchange matrix J_{ij} random, $|J_{ij}| \sim J$
- Large connectivity z

Spectrum of spring constants (Hessian H_{ij}) (d>3)



$\infty > z \gg 1$

Delocalized spectrum!

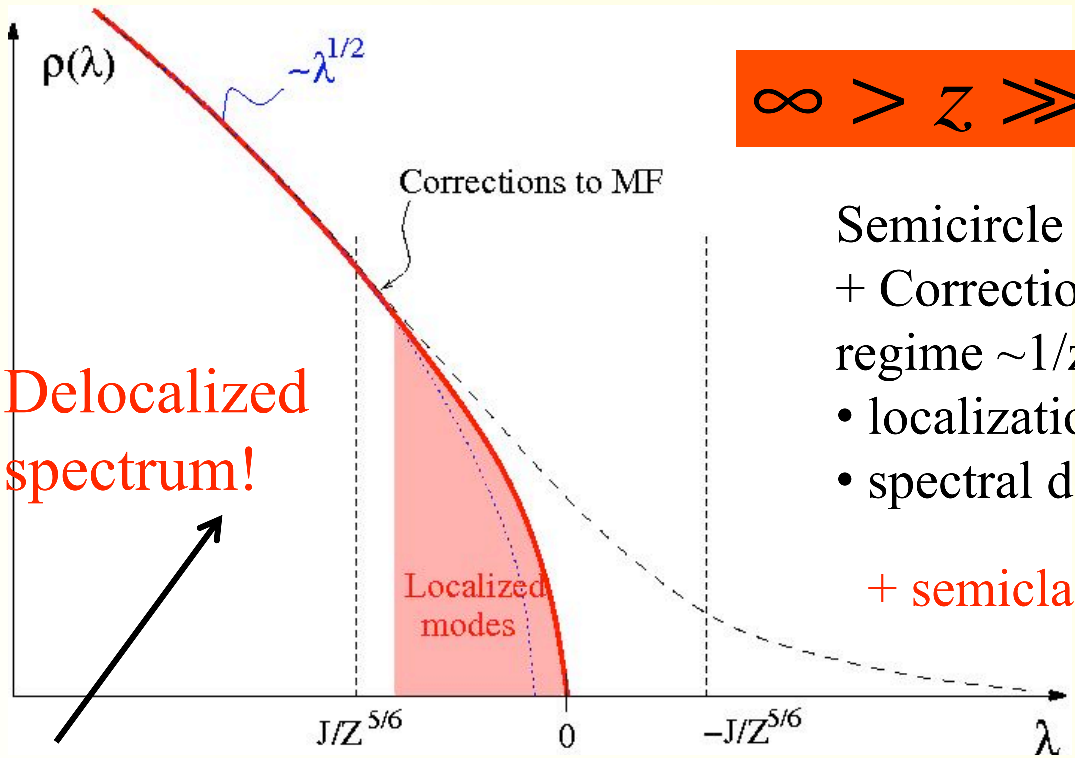
Semicircle
+ Corrections in regime $\sim 1/z^{5/6}$:
- localization
- spectral deviations

Beyond mean field

Quantum 'spin glass' with

- Exchange matrix J_{ij} random, $|J_{ij}| \sim J$
- Large connectivity z

Spectrum of spring constants (Hessian H_{ij}) (d>3)



$\infty > z \gg 1$

Delocalized spectrum!

Semicircle
+ Corrections in regime $\sim 1/z^{5/6}$:

- localization
- spectral deviations

+ semiclassical treatment

Delocalized low-energy excitations, *at least* down to $\omega_{\min} \sim Jz^{-1/6} \ll J \simeq h_{loc}^{typ}$

Beyond mean field?

Conclusions from our reasoning:

Criticality of quantum glass
→ large density of soft collective modes
delocalized to very low energies

→ Spin glass-type ergodicity breaking counteracts quantum localization!

→ Instead it enhances transport of energy and charge
(and decoherence in qubits)!

Beyond mean field?

Conclusions from our reasoning:

Criticality of quantum glass
→ large density of soft collective modes
delocalized to very low energies

→ Spin glass-type ergodicity breaking counteracts quantum localization!

→ Instead it enhances transport of energy and charge
(and decoherence in qubits)!

→ **NO many body localization! (only with weak interactions!)**
Especially, not to be expected close to the MIT!

Beyond mean field?

Conclusions from our reasoning:

Criticality of quantum glass
→ large density of soft collective modes
delocalized to very low energies

→ Spin glass-type ergodicity breaking counteracts quantum localization!

→ Instead it enhances transport of energy and charge
(and decoherence in qubits)!

→ **NO many body localization! (only with weak interactions)**
Especially, not to be expected close to the MIT!

Interesting open questions:

- Mobility edge for many body excitations?
- What happens at the quantum phase transition? *(cf. MM '09 regarding Bose glass)*

S=1/2 ↔ hard core bosons

Bose condensation in disorder?

S=1/2 $\leftrightarrow$ hard core bosons

How do glassy order and superfluidity compete?

Possibility of a "superglass" ?

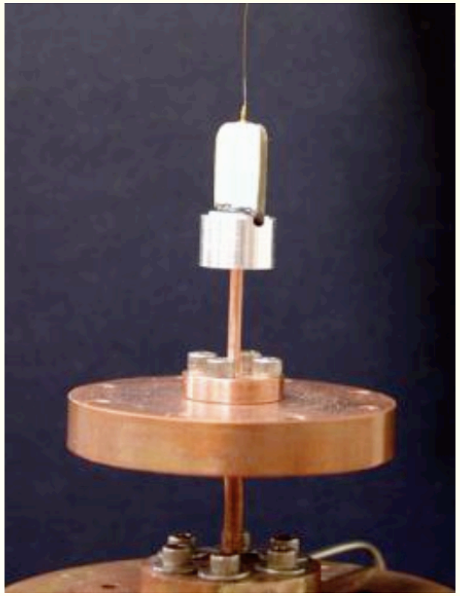
= amorphous+glassy supersolid

(e.g. dirty bosons [preformed pairs] with Coulomb frustration)

Motivation

Supersolidity observed in **defectful (glassy) quantum solids**

(Kim&Chan, Reppy, Dalibard)



Torsional oscillator filled with Helium:

Superfluid fraction observed in the solid phase **via non-classical rotational inertia,** + anomalous shear properties etc.

Superglasses ?!

Xiaoquan Yu, MM '10

$$H = -\Gamma \sum_i \sigma_i^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z$$

$$H = -\frac{t}{N} \sum_{i<j} \sigma_i^x \sigma_j^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z$$

"Self-generated transverse field"

Superglasses ?!

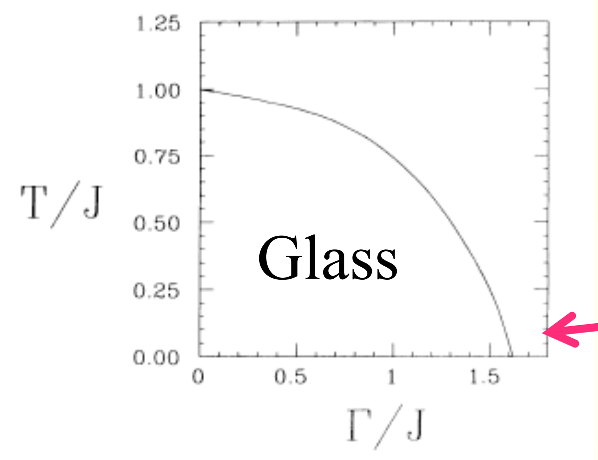
Xiaoquan Yu, MM '10

$$H = -\Gamma \sum_i \sigma_i^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z$$

$$H = -\frac{t}{N} \sum_{i<j} \sigma_i^x \sigma_j^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z$$

"Self-generated transverse field"

New aspect:



1[st] order transition?
Or superglass?

Superglasses ?!

Xiaoquan Yu, MM '10

$$H = -\Gamma \sum_i \sigma_i^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z$$

$\updownarrow$

$$H = -\frac{t}{N} \sum_{i<j} \sigma_i^x \sigma_j^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z$$

Competing order parameters: $M = \frac{1}{N} \langle \sigma_i^x \rangle$ $\quad q_{\mathrm{EA}} = \frac{1}{N} \langle \sigma_i^z \rangle^2$

M signals superfluidity of hard core bosons $\quad \sigma_i^z \leftrightarrow 2n_i - 1$

Superglasses ?!

Xiaoquan Yu, MM '10

$$H = -\Gamma \sum_i \sigma_i^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z$$

$$\updownarrow$$

$$H = -\frac{t}{N} \sum_{i<j} \sigma_i^x \sigma_j^x - \sum_{i<j} \sigma_i^z J_{ij} \sigma_j^z$$

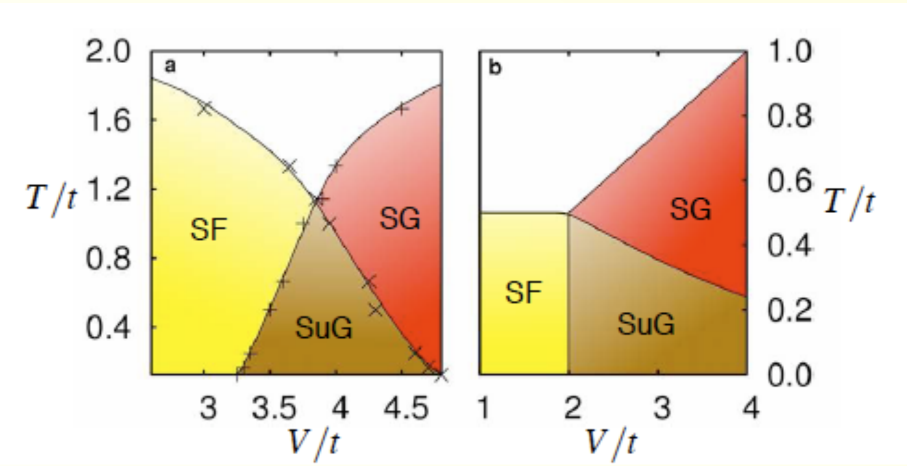
Competing order parameters: $M = \frac{1}{N} \langle \sigma_i^x \rangle$ $\quad q_{\mathrm{EA}} = \frac{1}{N} \langle \sigma_i^z \rangle^2$

M signals superfluidity of hard core bosons $\quad \sigma_i^z \leftrightarrow 2n_i - 1$

If M and q_{EA} exist at same time → Meanfield model of a superglass!

« Superglass »

Tam, Geraedts, Inglis, Gingras, Melko, PRL (10)

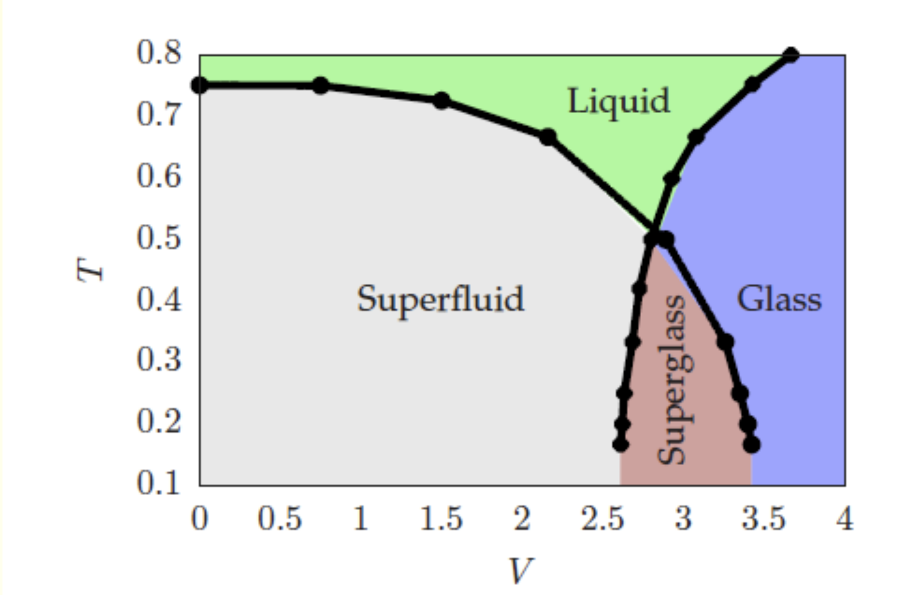


QMC (3d) “Mean field”

$$H = -\sum_{\langle ij \rangle} V_{ij}(n_i - 1/2)(n_j - 1/2) - t \sum_{\langle ij \rangle} (b_i^\dagger b_j + b_i b_j^\dagger),$$

Quenched randomness

Carleo, Tarzia, Zamponi PRL (09)



$$\widehat{\mathcal{H}} = -t \sum_{\langle i,j \rangle} \left[b_i^\dagger b_j + b_i b_j^\dagger \right] + V \sum_{\langle i,j \rangle} n_i n_j ;$$

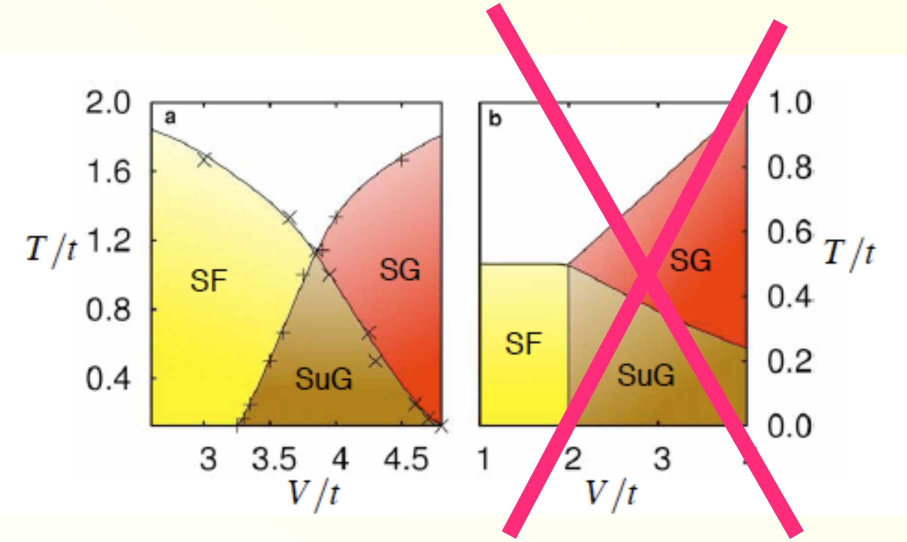
QMC and cavity

Random, frustrated “Bethe” lattice

« Superglass »

Tam, Geraedts, Inglis, Gingras, Melko, PRL (10)

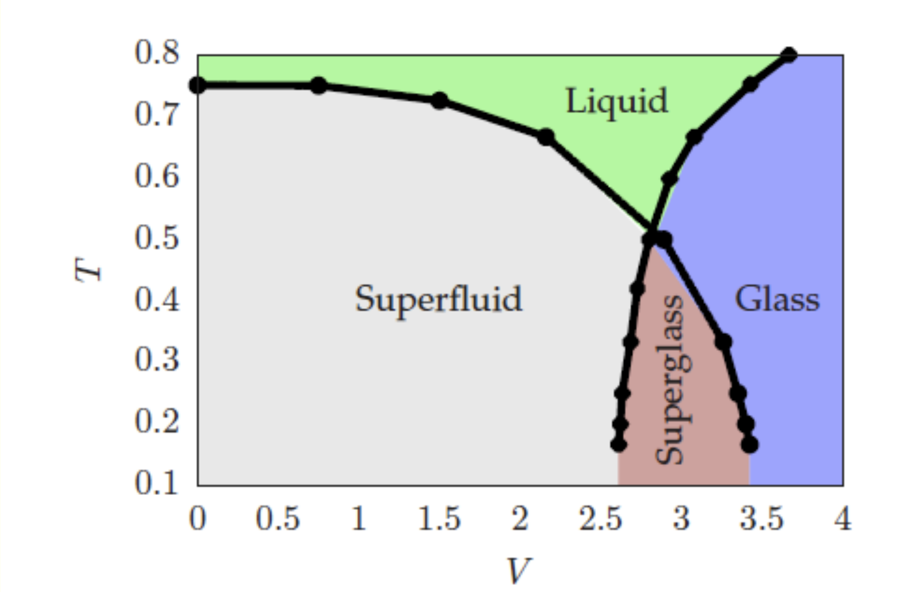
Carleo, Tarzia, Zamponi PRL (09)



QMC (3d) “Mean field”

$$H = -\sum_{\langle ij \rangle} V_{ij}(n_i - 1/2)(n_j - 1/2) - t \sum_{\langle ij \rangle} (b_i^\dagger b_j + b_i b_j^\dagger),$$

Quenched randomness



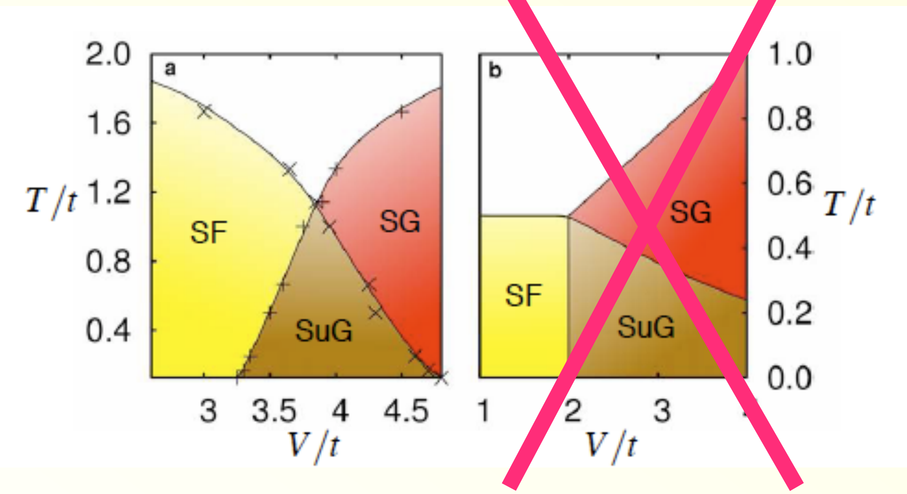
$$\hat{\mathcal{H}} = -t \sum_{\langle i,j \rangle} \left[b_i^\dagger b_j + b_i b_j^\dagger \right] + V \sum_{\langle i,j \rangle} n_i n_j ;$$

QMC and cavity

Random, frustrated “Bethe” lattice

« Superglass »

Tam, Geraedts, Inglis, Gingras, Melko, PRL (10)

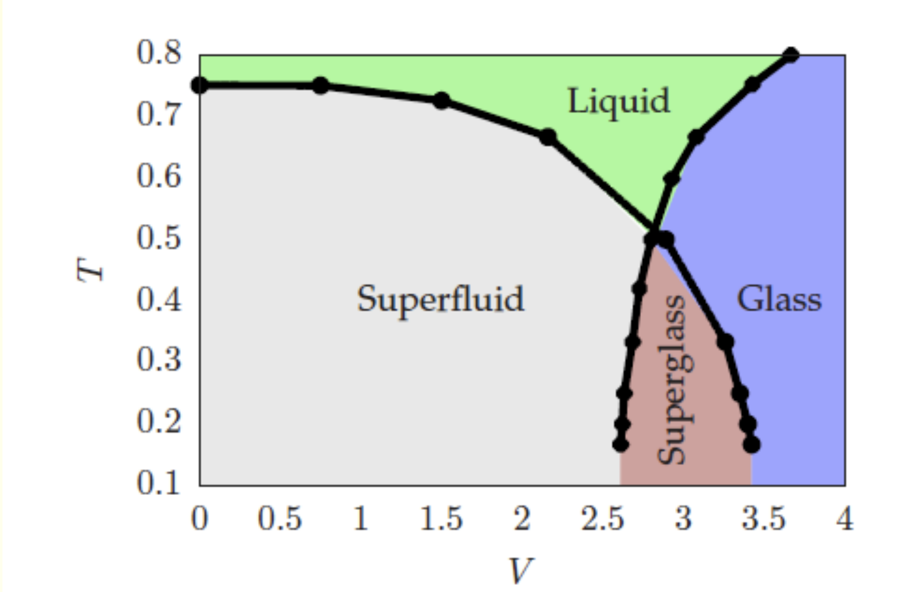


QMC (3d) “Mean field”

$$H = -\sum_{\langle ij \rangle} V_{ij}(n_i - 1/2)(n_j - 1/2) - t \sum_{\langle ij \rangle} (b_i^\dagger b_j + b_i b_j^\dagger),$$

Quenched randomness

Carleo, Tarzia, Zamponi PRL (09)



$$\widehat{\mathcal{H}} = -t \sum_{\langle i,j \rangle} \left[b_i^\dagger b_j + b_i b_j^\dagger \right] + V \sum_{\langle i,j \rangle} n_i n_j ,$$

QMC and cavity

Random, frustrated “Bethe” lattice

?? Low T behavior ? – QPT ? - Local structure of the superglass ??

Mean field superglass

X. Yu, MM '10

$$M = \frac{1}{N}\langle \sigma_i^x \rangle$$

$$H = -\frac{t}{N}\sum_{i<j}\sigma_i^x\sigma_j^x - \sum_{i<j}\sigma_i^z J_{ij}\sigma_j^z$$

$$q_{\mathrm{EA}} = \frac{1}{N}\langle \sigma_i^z \rangle^2$$

Mean field superglass

X. Yu, MM '10

$$H = -\frac{t}{N}\sum_{i<j}\sigma_i^x\sigma_j^x - \sum_{i<j}\sigma_i^z J_{ij}\sigma_j^z$$

$$M = \frac{1}{N}\langle\sigma_i^x\rangle$$

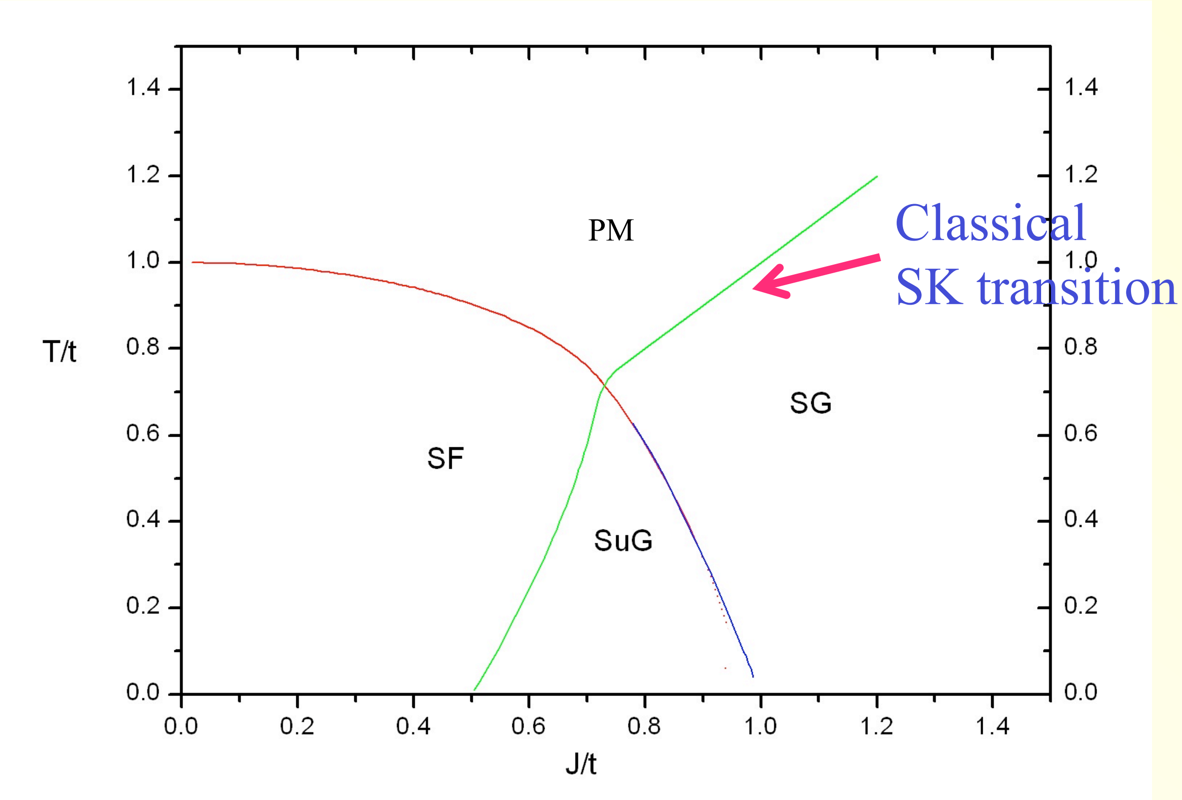
$$q_{\mathrm{EA}} = \frac{1}{N}\langle\sigma_i^z\rangle^2$$

- Obtain T = 0 phase transition glass-to-superglass exactly! (BCS instability of glass!)

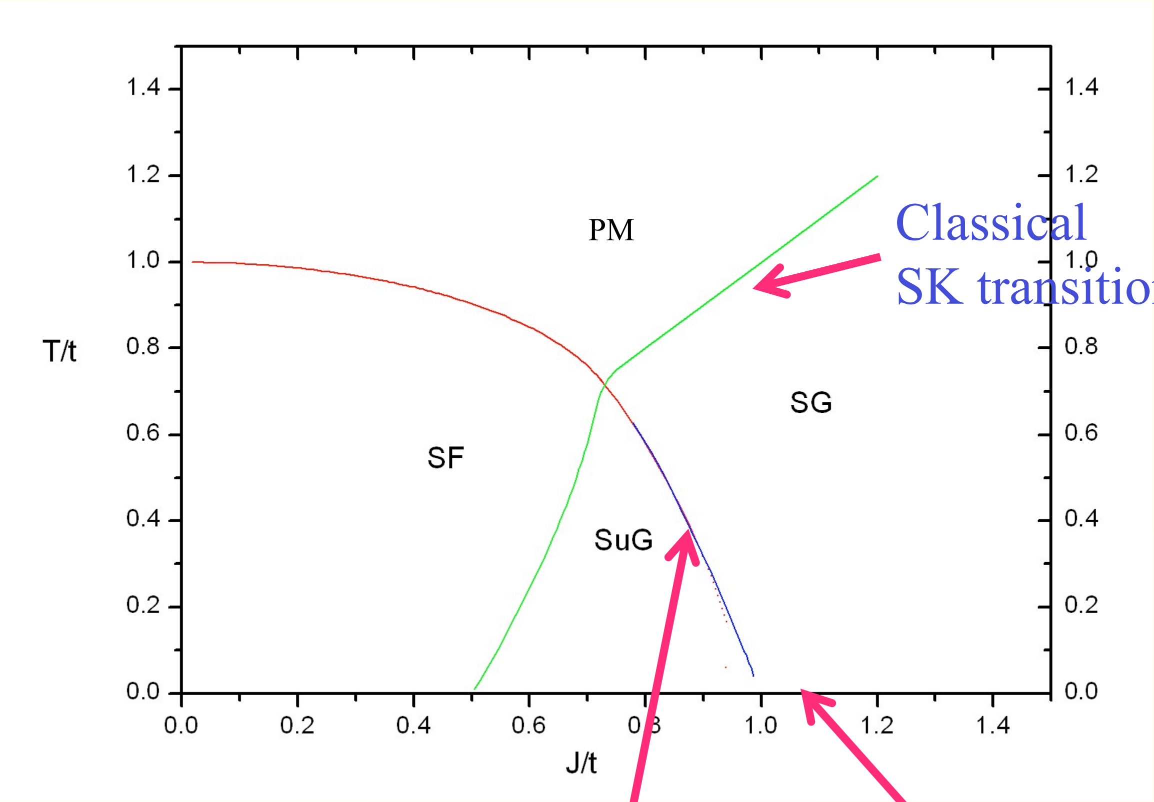
- For superfluid-to-superglass transition: Use static approximation (but exact upper bound)

- Analytical proof of existence of superglass phase!

Phase diagram



Phase diagram



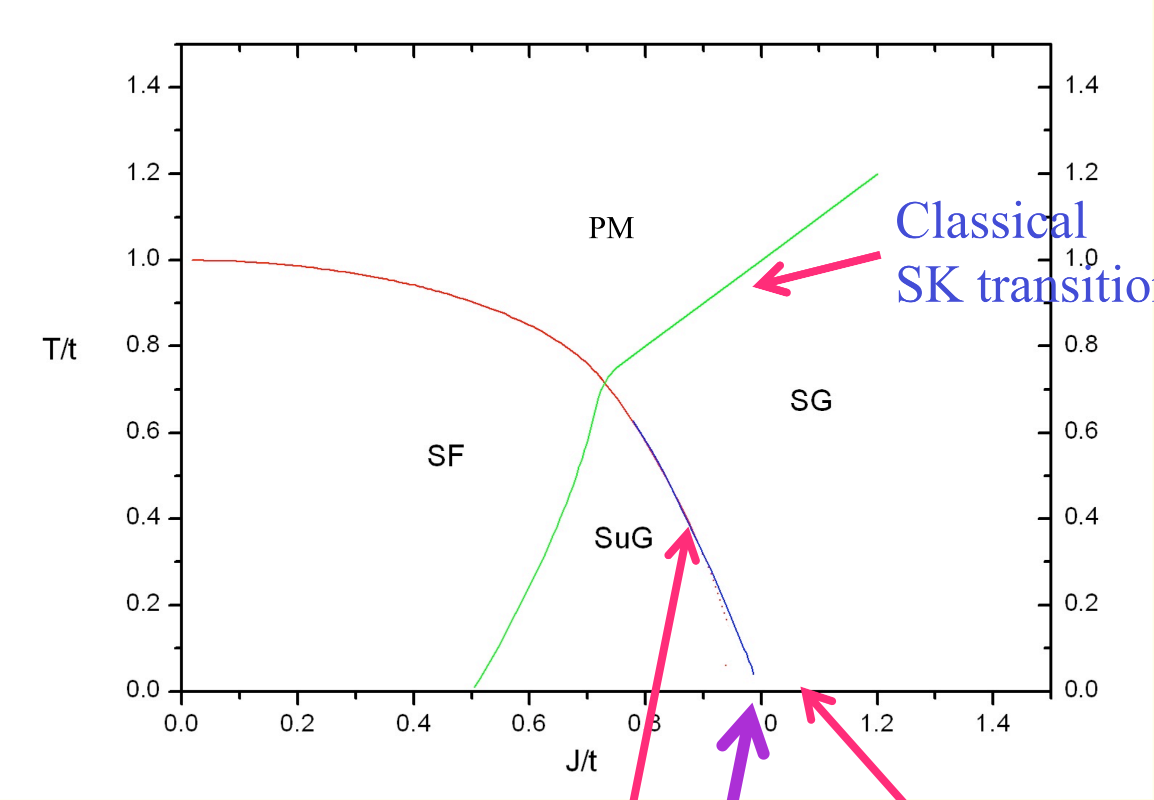
Classical SK transition

BCS-like condition: $t\int dy P(y)\chi_y^{xx} = 1.$

Suppression of superfluidity by the pseudogap!

Transition at finite J/t ≅ 1.00

Phase diagram



Classical SK transition

BCS-like condition: $$t \int dy P(y) \chi_y^{xx} = 1.$$

Suppression of superfluidity by the pseudogap!

Transition at finite J/t ≅ 1.00

Analogue for 1/r (Coulomb gap) in d<∞? Expect different fractality of the condensate!

Local structure of the superglass

- Superfluid and glass try to avoid each other:

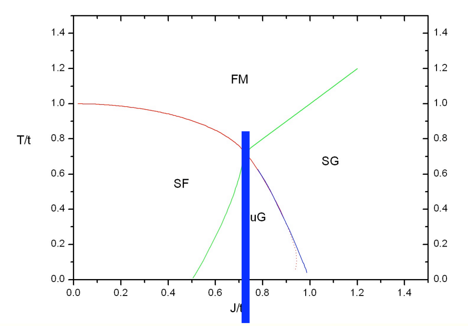
$$\langle s_i^x \rangle \text{ and } \langle s_i^z \rangle^2 \text{ are anticorrelated}$$

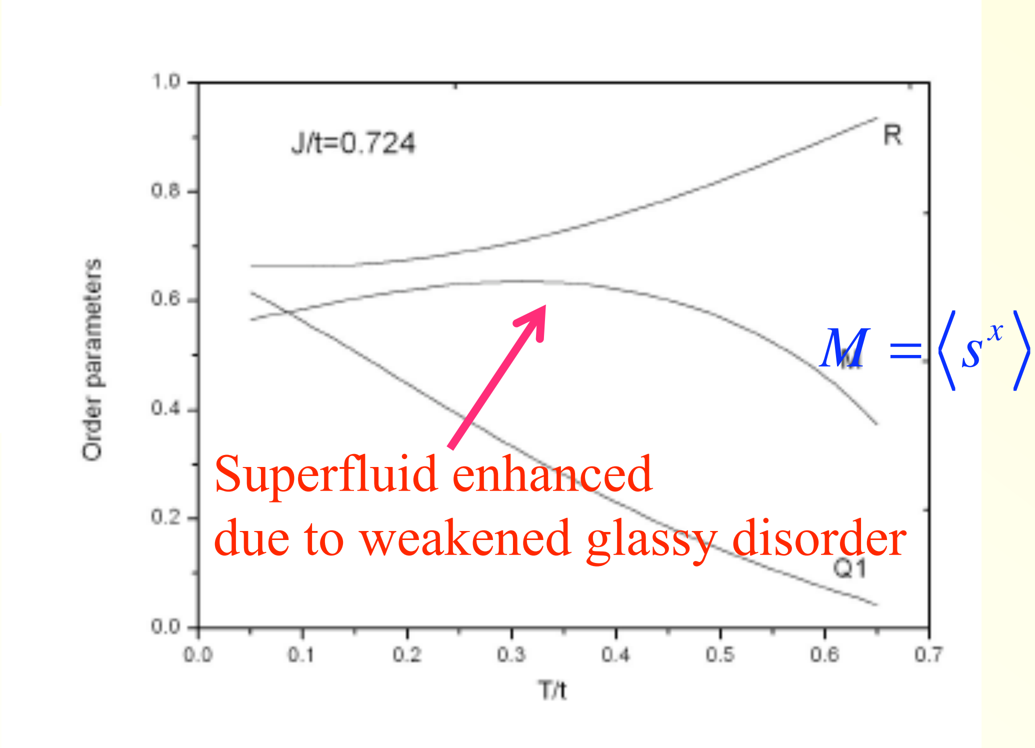
Local structure of the superglass

- Superfluid and glass try to avoid each other:

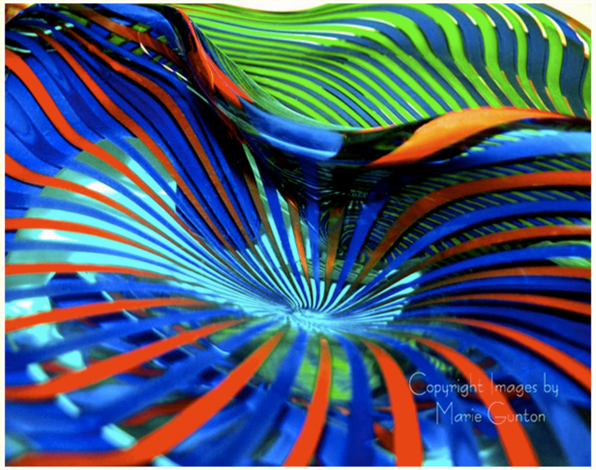
  $\langle s_i^x \rangle$ and $\langle s_i^z \rangle^2$ are anticorrelated

- Superfluid order non-monotonous with T!





Conclusions

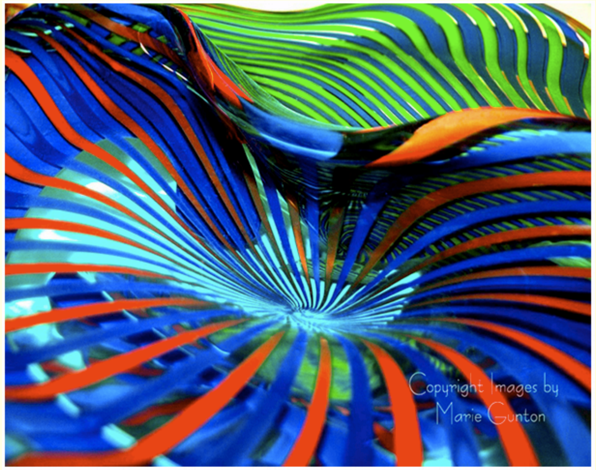


- Understanding of collective low energy excitations in quantum glasses

- Glassy order counteracts many particle quantum localization
  Ergodicity is broken – but arrow of time is intact!

- Frustrated bosons: superfluid and glassy order can coexist

Conclusions



- Understanding of collective low energy excitations in quantum glasses

- Glassy order counteracts many particle quantum localization
  Ergodicity is broken – but arrow of time is intact!

- Frustrated bosons: superfluid and glassy order can coexist

Perspectives

Quantum phase transitions in disordered systems
(MIT, SIT, Bose glass, quantum glass transition):

- Nature of excitations and many particle localization?
  Where/when does it occur? Implications on quantum information?
- New effects due to competition of different orders
- Quantum annealing, adiabatic quantum computation?