Collective electronic transport close to metal-insulator or superconductor-insulator transitions

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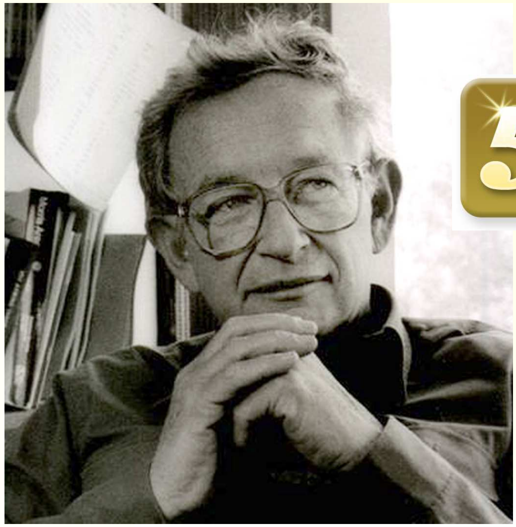
WHERE DISCOVERIES BEGIN

Newton Institute, Cambridge, 17th December, 2008

Outline

- Review of single-particle and many-body localization.
- Experiments suggesting purely electronic conduction in insulators
  (i.e. “many-body delocalization”).
- Theory of electron-assisted transport
  Major ingredient: strongly correlated, quantum glassy state of electrons close to the metal-insulator transition.
- Remnants of many-body localization close to the superconductor-to-insulator transition?

Review of localization and insulators



50 Years of Anderson Localization

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey

(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

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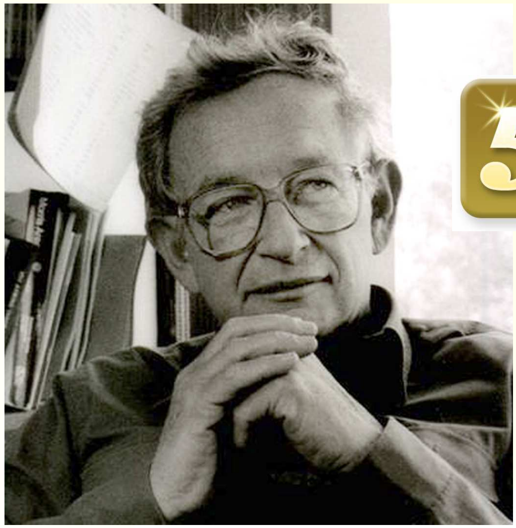

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Still little understanding beyond the simple model!!

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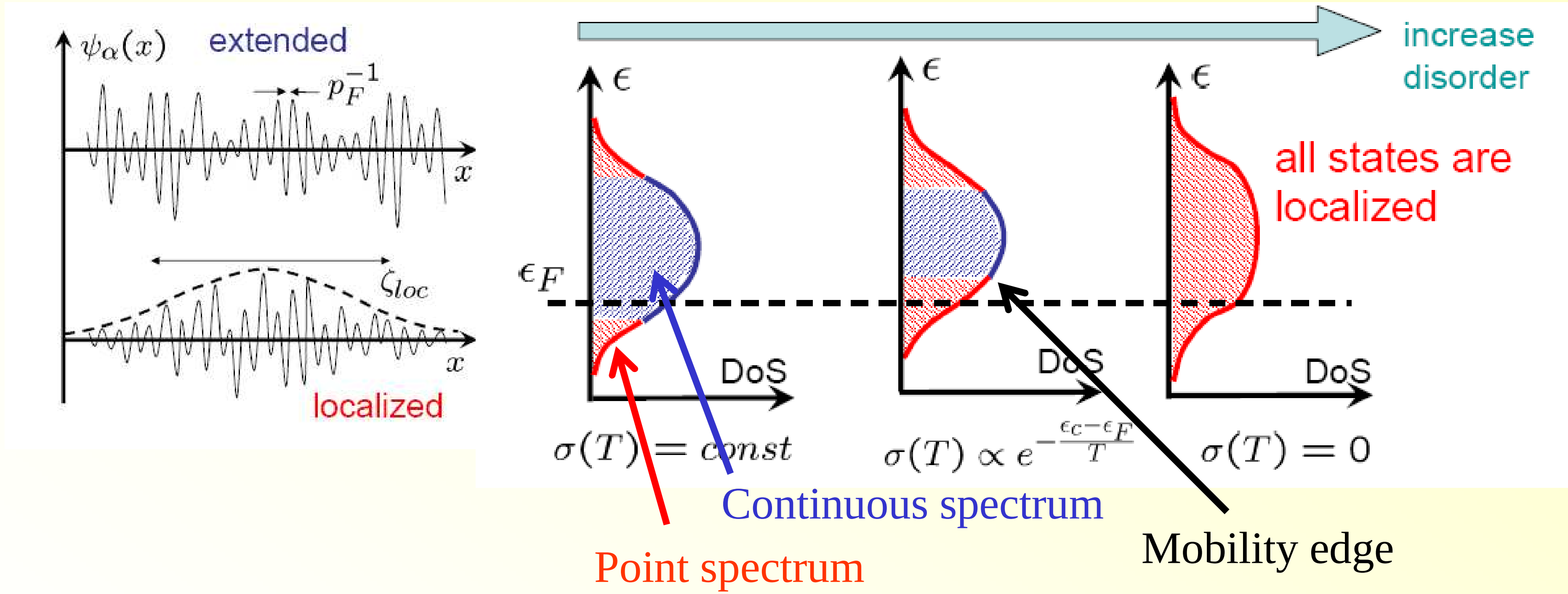

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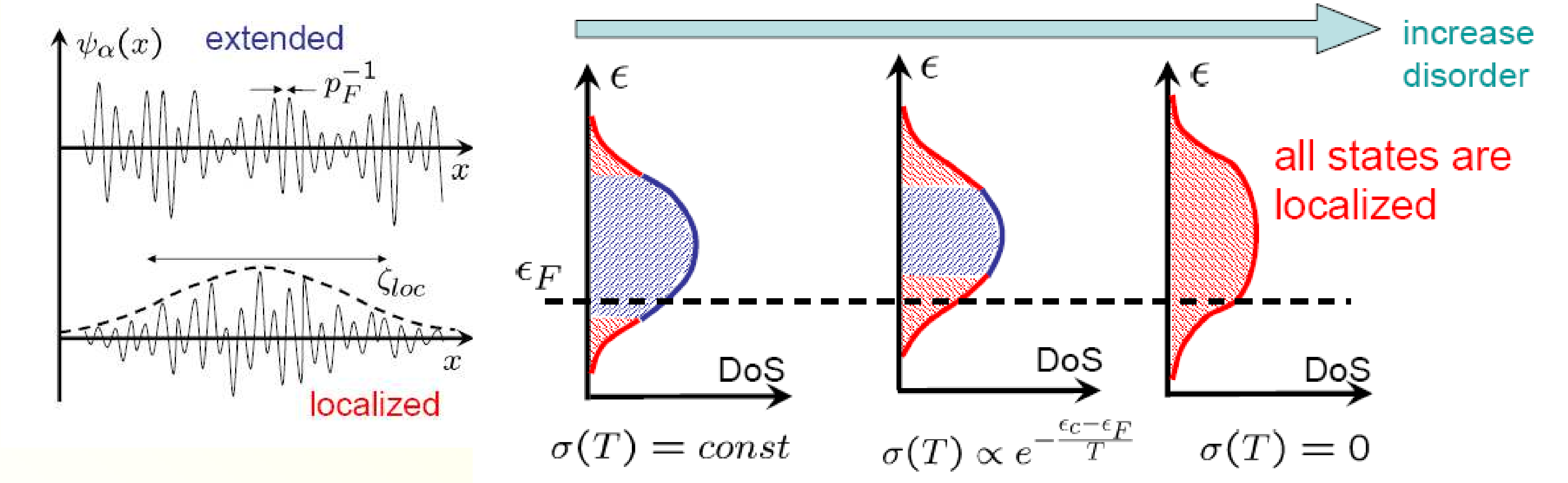
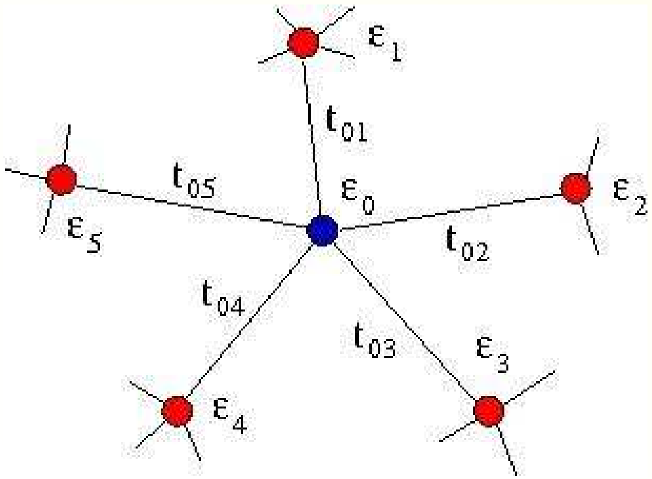


On the Bethe lattice: *(Abou-Chacra, Thouless, Anderson (1973))*



$$P(\varepsilon) = \frac{1}{\delta}$$

$$t < \# \frac{\delta}{k} = \# \frac{\delta}{2d} \quad \to \quad \text{localized}$$

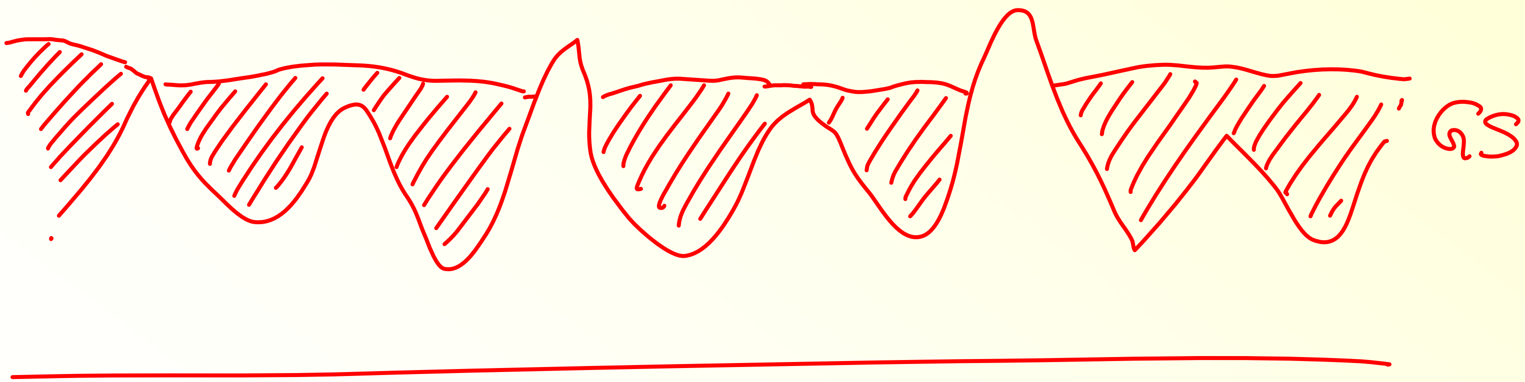
$$t > \# \frac{\delta}{k} = \# \frac{\delta}{2d} \quad \to \quad \text{extended}$$

Localization with interaction?

*L. Fleishman and P. W. Anderson, PRB, **21**, 2366 (1980).*

Q: Does localization persist in the presence of interactions?
In other words: Does conductivity vanish *exactly* without phonons?

$$H = -\psi^{+}(x)\Delta\psi(x) + \psi^{+}(x)V(x)\psi(x) + \psi^{+}(x)\psi^{+}(x')V_{\text{int}}(x-x')\psi(x')\psi(x)$$



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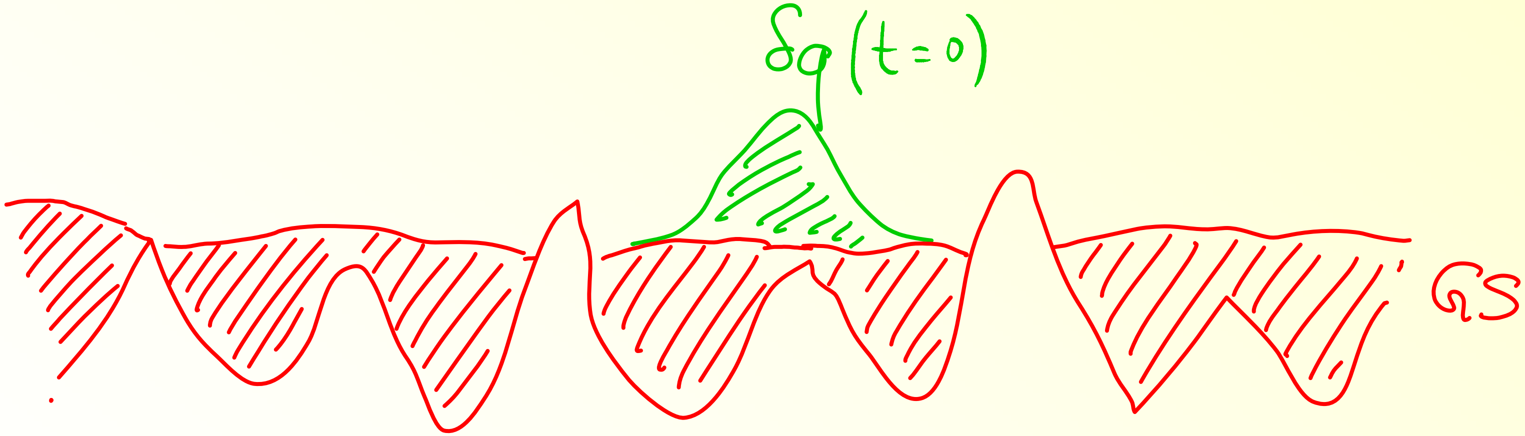

Create extra charge bump at origin:

$$|\Psi\rangle = \int dx \frac{e^{-x^2/2a^2}}{\sqrt{2\pi a}}\psi^{+}(x)|\Psi_{GS}\rangle$$

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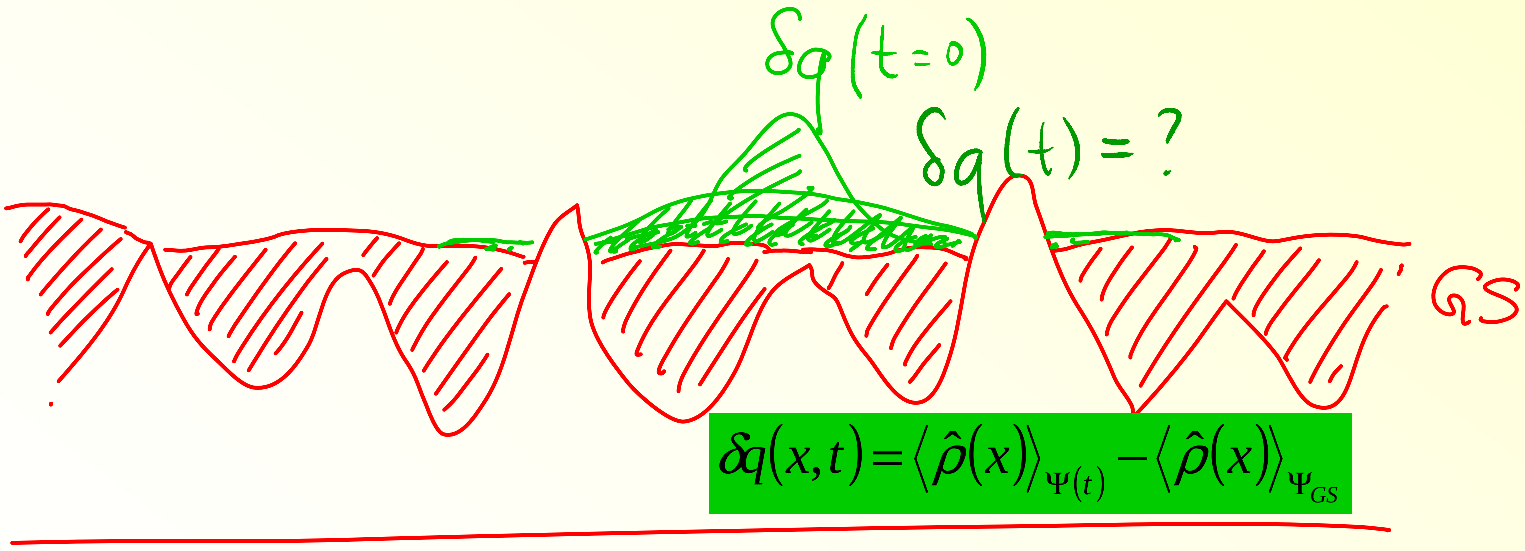

$$\delta q(x,t) = \langle \hat{\rho}(x) \rangle_{\Psi(t)} - \langle \hat{\rho}(x) \rangle_{\Psi_{GS}}$$

Time evolution?
Dynamic
localization?

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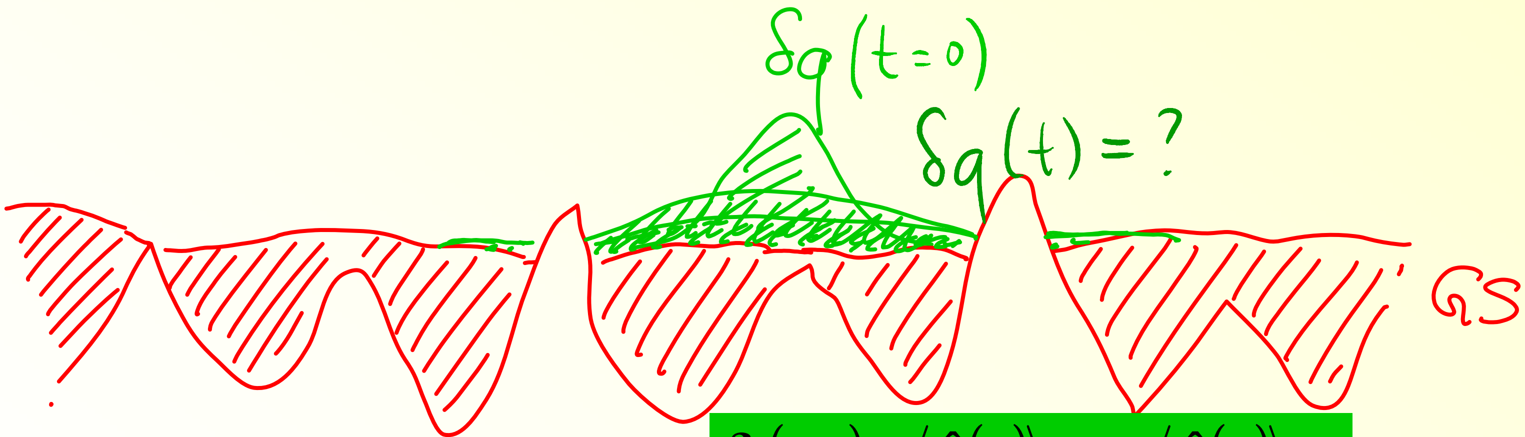

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Time evolution?
Dynamic localization?

$$\langle x^2\rangle_t = \frac{\int \delta q(x,t)x^2 dx}{\int \delta q(x,t)dx} = \begin{cases} Dt & \text{Delocalization} \\ \tilde{D}t^{\alpha},\ \alpha<1 & \text{Anomalous diffusion} \\ <\text{const.} & \text{Manybody localization} \end{cases}$$

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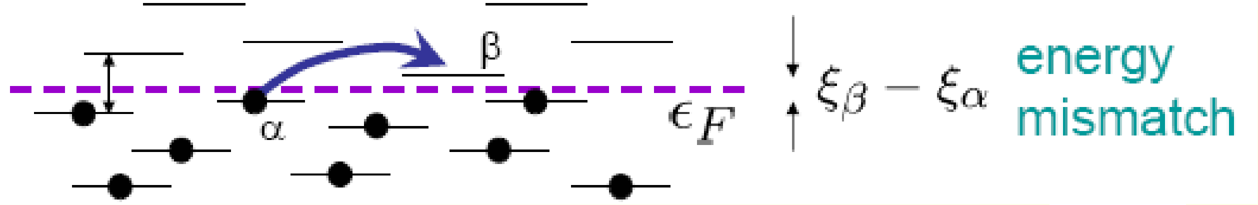

Single hop: Energy mismatch because of local point spectrum.
→ No charge transport at this level

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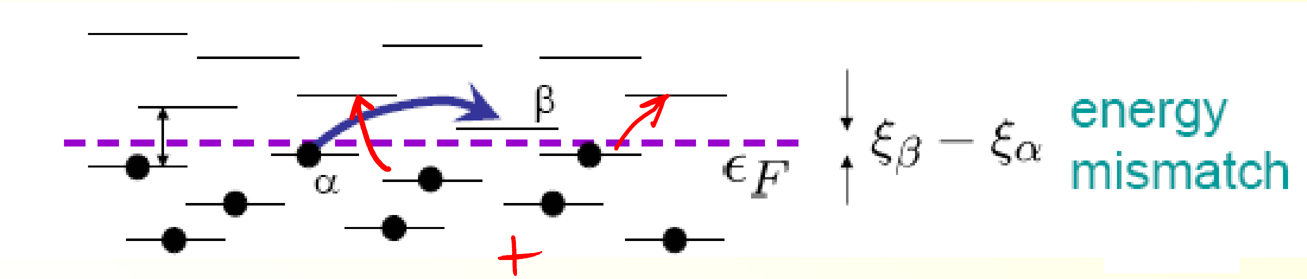


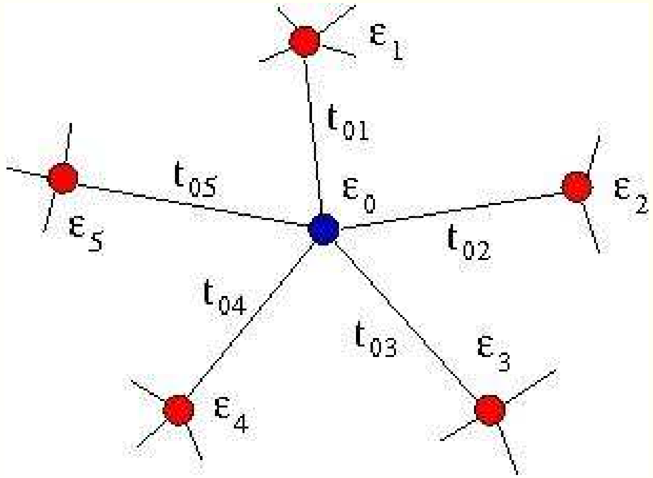
Multiparticle rearrangements:
Transition energies remain discrete for weak interactions and low T

Localization with interaction?

Investigation to all orders in perturbation theory:

*I. V. Gornyi, A. D. Mirlin, and D. G. Polyakov, PRL **95**, 206603 (2005).*
*D. M. Basko, I. L. Aleiner, and B. L. Altshuler, Ann. Phys. **321**, 1126 (2006).*

Assumption: Very weak interactions: $V_{int} \ll$ level spacing δ_ξ.
Conclusion: An energy crisis (i.e., a **metal-insulator transition** without phonons**)** occurs at high temperature due to "localization in Fockspace".



Argument:
Same as Anderson localization:
1) Sites $\rightarrow$ many body states

$$|\Psi_0\rangle = a_\alpha^+ |\Psi_{GS}\rangle$$

$$|\Psi_1\rangle = a_\gamma a_\beta^+ a_\alpha^+ |\Psi_{GS}\rangle \quad \text{etc.}$$

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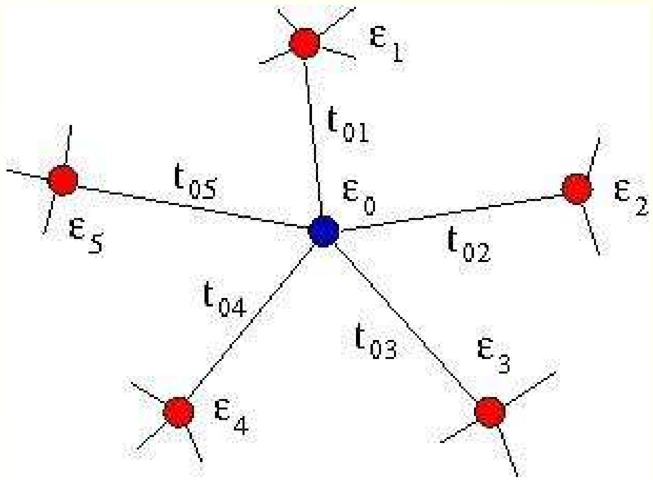

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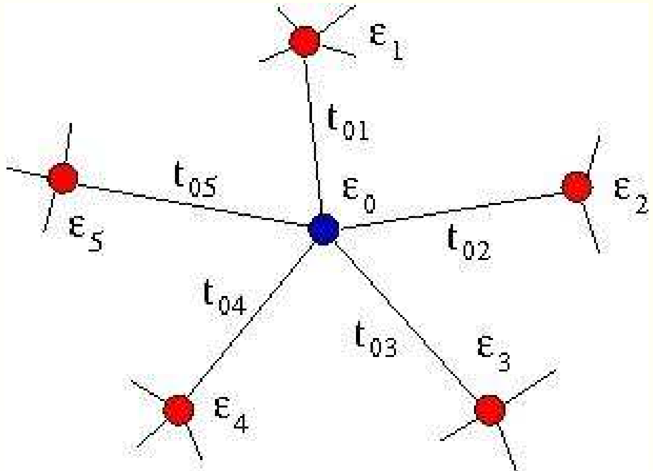

Could there be instantons??

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Implications of manybody localization

• A true quantum glass: non-ergodic systems, despite of interactions!

• Defeat of cardinal assumption of thermodynamics: that infinitesimal interactions will eventually lead to equilibration

• Perfect, collective insulators at finite T

• Quantum computing/information: Preserved quantum coherence due to limited entanglement of local degrees of freedom

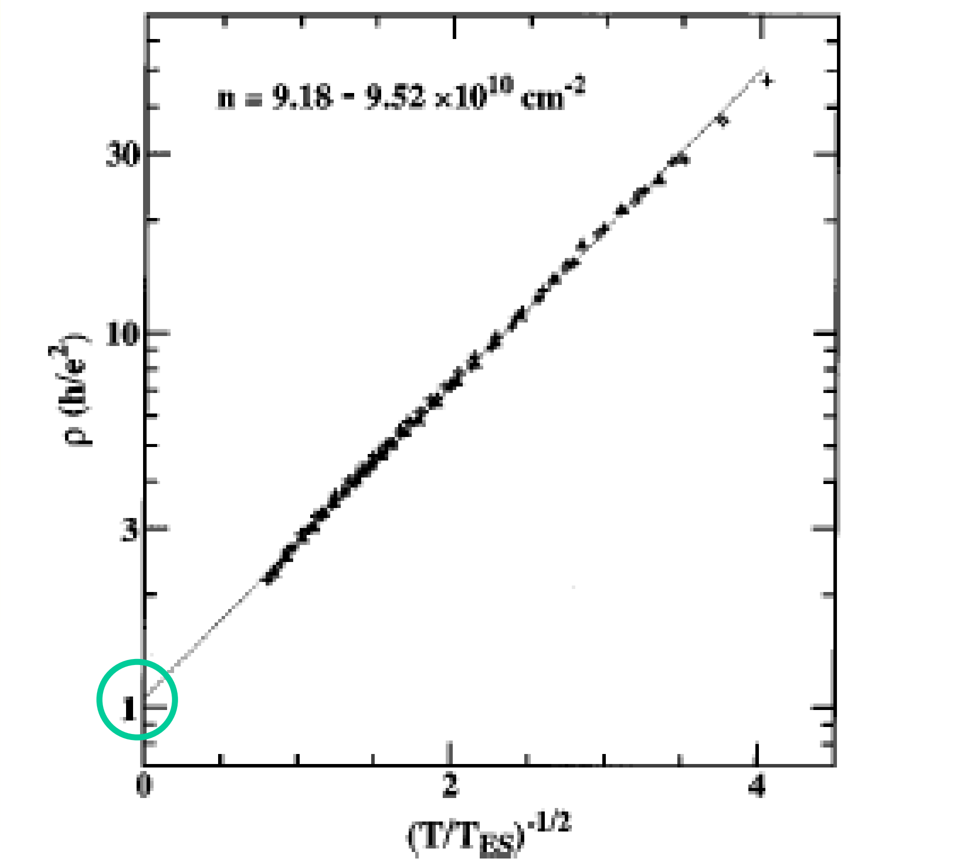
What about experiment?

- No metal-insulator transition observed at finite T
- Rather: Evidence for e-assisted hopping (many-body delocalization)

Why this difference from theoretical predictions!?

Electron assisted hopping

Doped GaAs/Al_xGa_{1-x}As heterostructure



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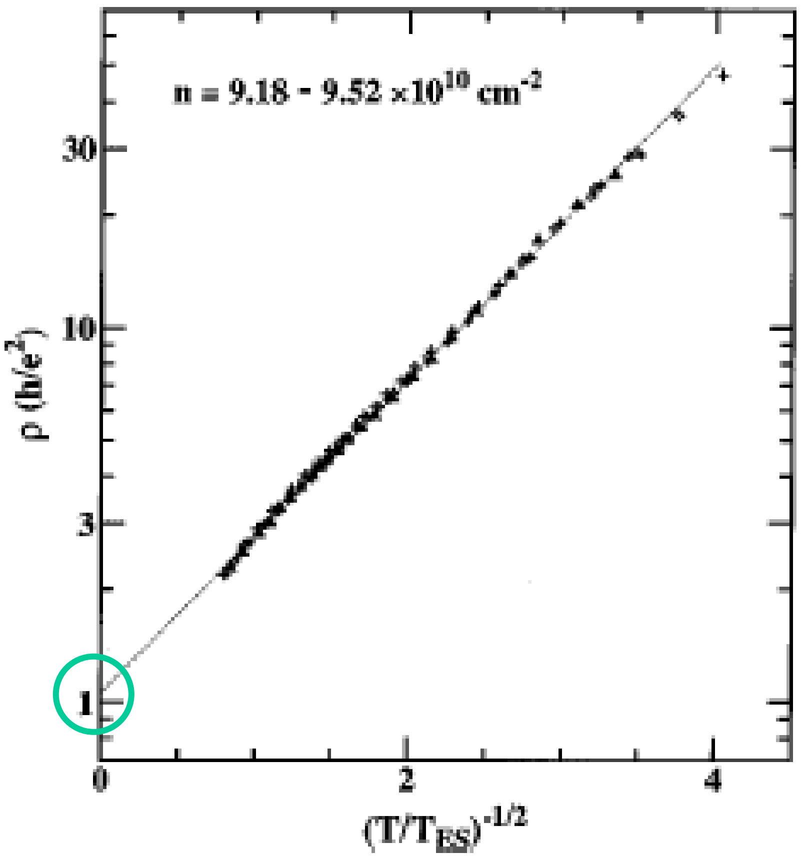

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Efros-Shklovskii variable range hopping:

$$\boxed{\rho_{2D} \approx \frac{h}{e^2} f\left(\frac{T}{T_0}\right) \exp\left[\left(\frac{T_0}{T}\right)^{1/2}\right]}$$

Nearly universal prefactor!

$f(1) = O(1)!$

In stark contrast with standard phonon-assisted hopping!

$$\left[\leftrightarrow\ f_{e-ph}(1) = O(10^4)\right]$$

Mott and Davies (1979), Aleiner et al. (1994)

Open Questions

Theory for electron-assisted transport in insulators ?

?

- Experimental evidence for e-assisted hopping
  $\longrightarrow$ Caveat in theories of manybody localization?
- Can one have an insulator *and* electron-electron interaction-induced conductivity at finite T?
- How to explain the nearly universal electronic prefactor h/e^2 ?

?

Model system

Electrons with disorder + Coulomb interactions in 3d or quasi 2d

$$\boxed{H = \underbrace{H_{kin} + V_{dis}}_{} + V_{Cb}}$$

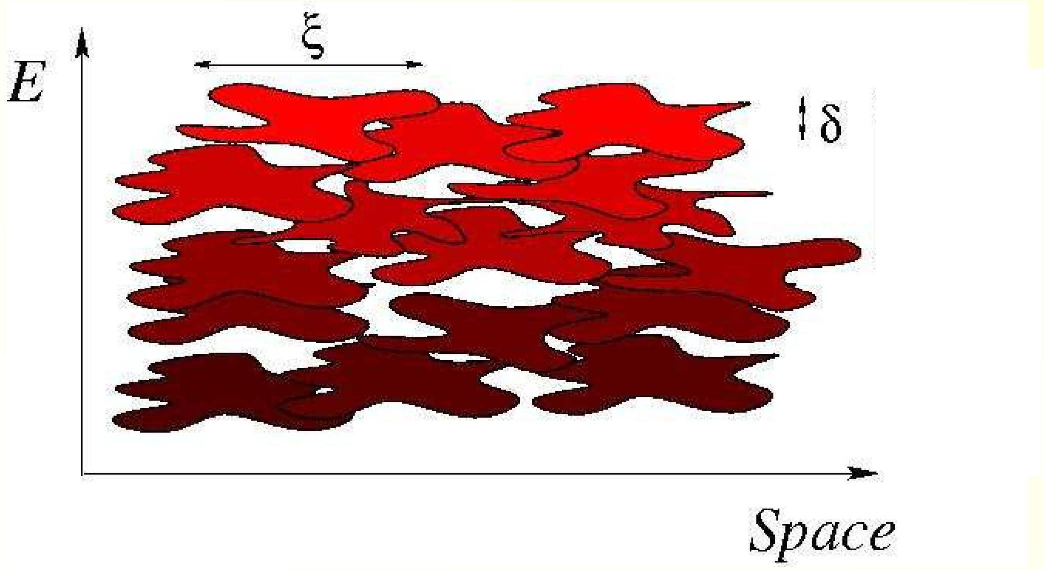
Single particle Anderson problem $\longrightarrow$ Diagonalize!

Assumption about disorder

Single particle problem close to the Anderson transition

$\rightarrow$ Large localization length $\xi \gg n^{-1/3}$,

$\rightarrow$ Small level spacing $\delta_{\xi} = \left(\nu \xi^{d}\right)^{-1}$



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Single particle energies

$$P(\varepsilon) = \frac{1}{\delta} = \nu \xi^3$$

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Single particle energies   Coulomb interaction (partial screening from high energy states)

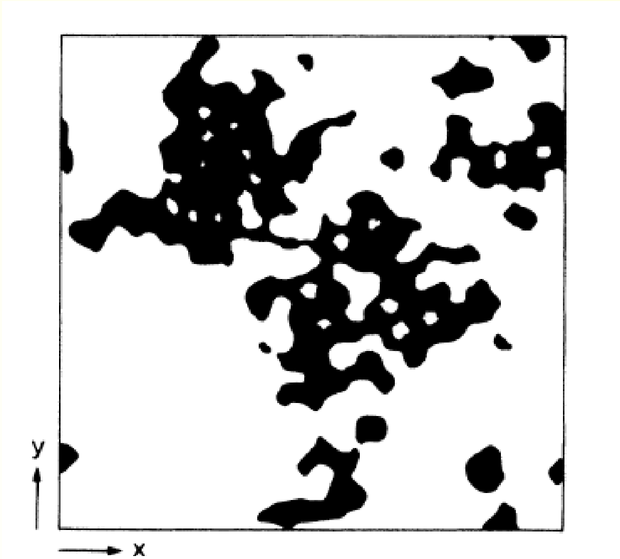
$$P(\varepsilon) = \frac{1}{\delta} = \nu \xi^3$$

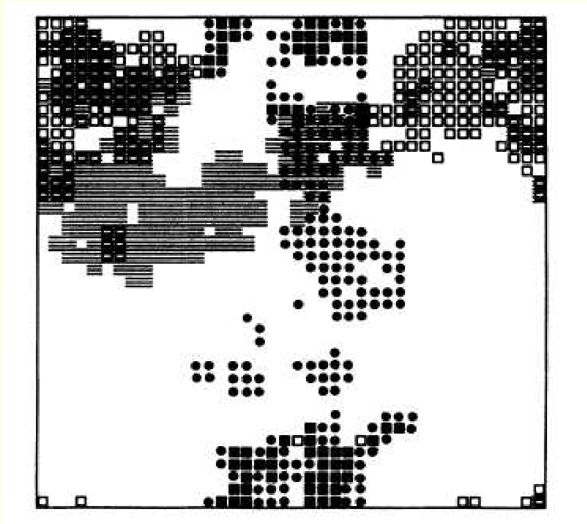
$$J_{ij} = \int dr dr' \frac{\left[\varphi_i^2(r) - \rho\right]\left[\varphi_j^2(r') - \rho\right]}{\kappa |r - r'|}$$

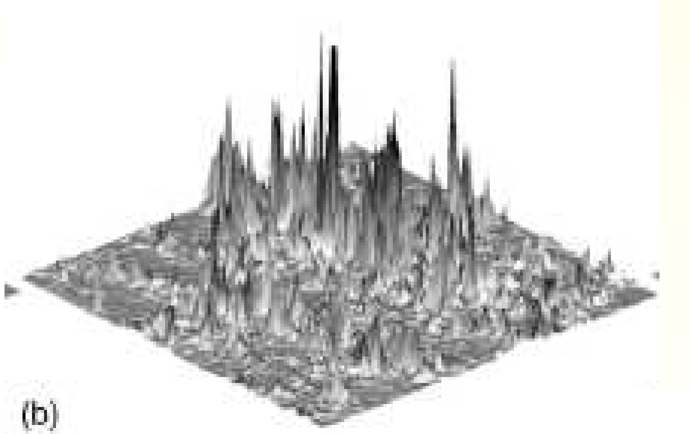
$$u_{ijkl} = \int dr dr' \frac{\varphi_i^*(r)\varphi_j(r)\varphi_k^*(r')\varphi_l(r')}{\kappa |r - r'|}$$

Wavefunctions at the mobility edge

Eigenstates of the non-interacting Anderson problem:
Spatially overlapping fractal wavefunctions







*H. Aoki, PRB, **33**, 7310 (1986).*

Theory: Mirlin et al.; Kravtsov et al.;

Coulomb interactions are strong at the Metal-insulator transition!

Scale of Coulomb interactions:

$$\boxed{J \sim \frac{e^2}{\kappa \xi}}$$

Level spacing:

$$\boxed{\delta \approx \frac{1}{\nu \xi^3}}$$

Scaling arguments + numerical and experimental indications:

$$\boxed{\frac{J}{\delta} \sim \frac{e^2 \nu \xi^3}{\kappa \xi} \sim \xi^{\alpha} \xrightarrow{\xi \to \infty} \infty; \quad \text{with } \alpha > 0.}$$

Conclusion: Coulomb interactions are strong and non-perturbative in the insulator!

Quantum electron glass

Theoretical model: Mean field-like quantum electron glass

$$H = \sum_i \varepsilon_i n_i + \sum_{i \neq j} n_i J_{ij} n_j + \sum_{i \neq j \neq k} t_{ijk} c_i^t c_j n_k + \sum_{i \neq j \neq k \neq l} u_{ijkl} c_i^t c_j c_k^t c_l$$

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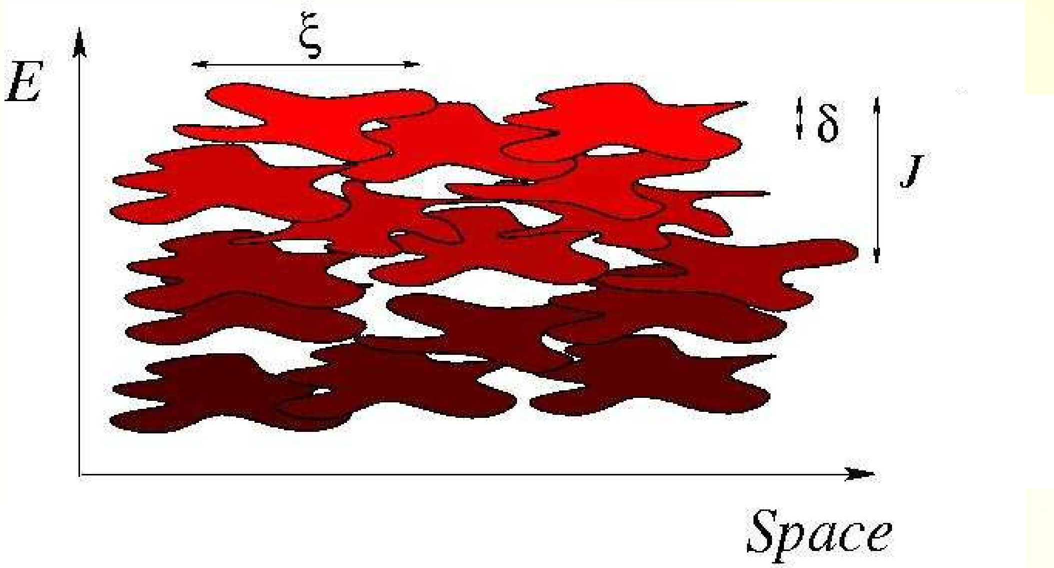

As $\xi \to \infty$,

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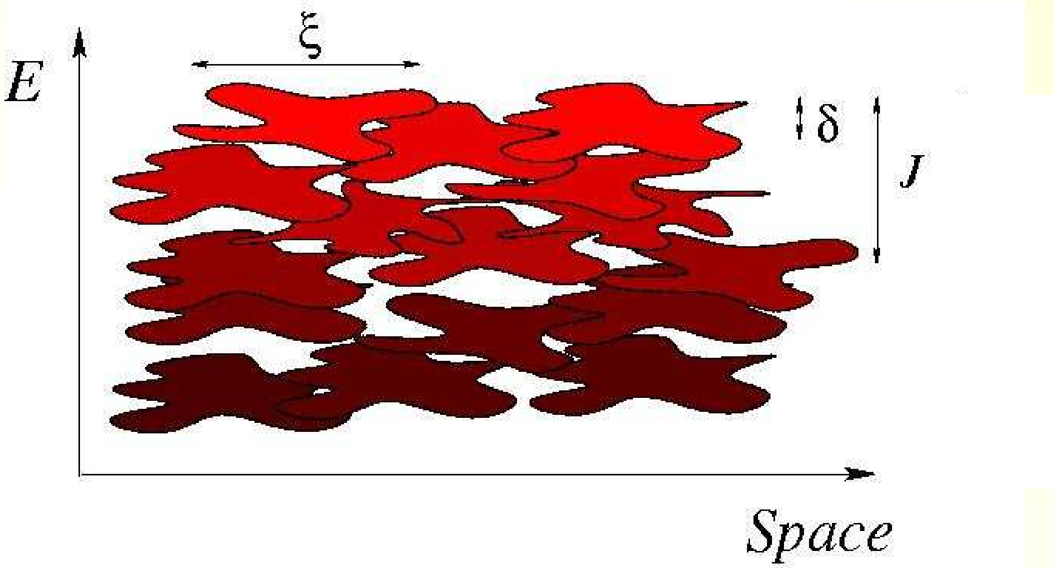

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Random signs → Frustration

→ Expect quantum glass state:
Many local minima with many soft collective excitations!

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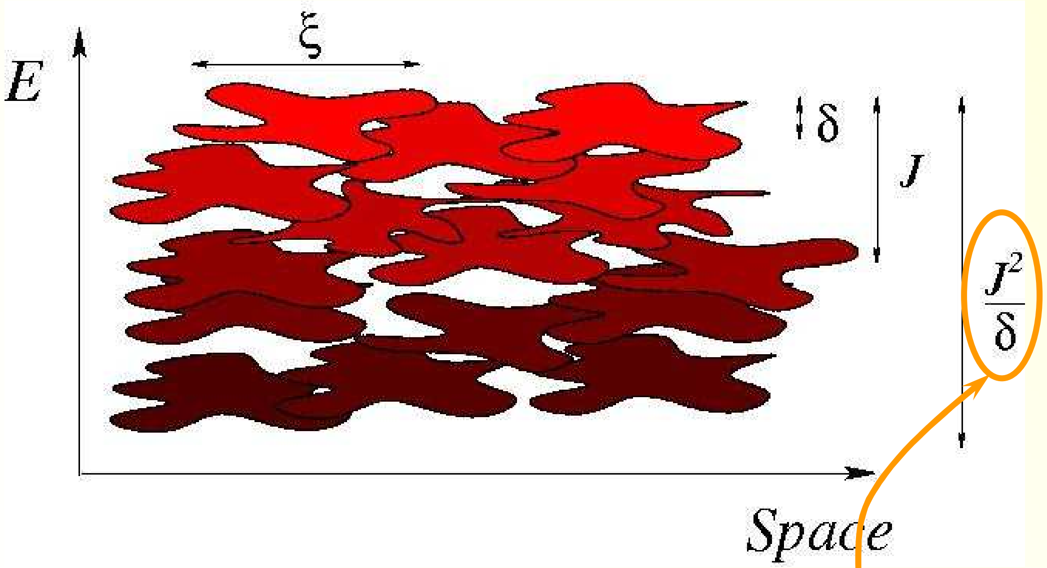

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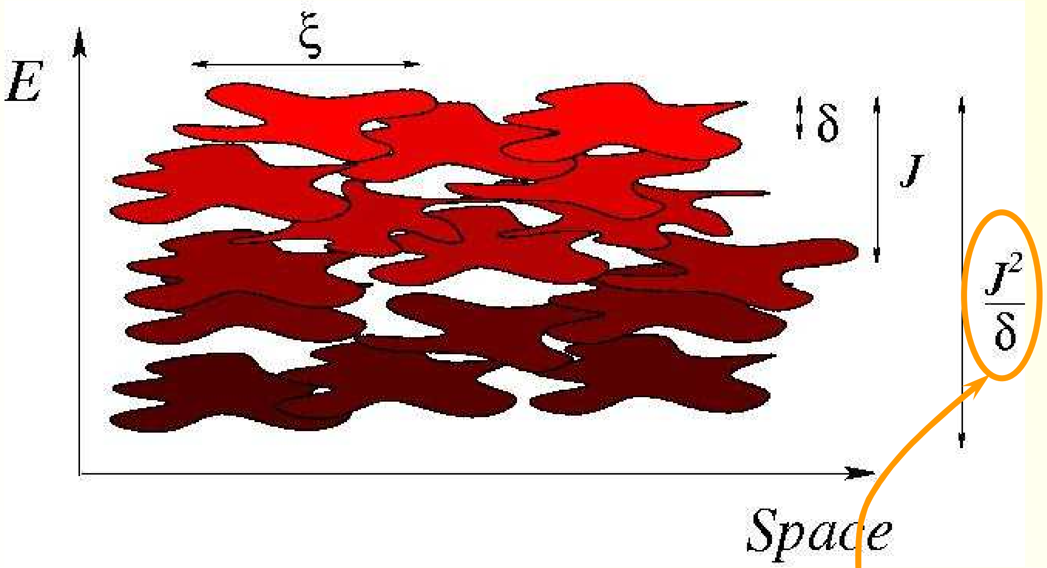

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Number of “active” neighbors of given electron:

$$N_{act} = z^2 >> 1$$

→ Large control parameter!

Quantum electron glass

Program:

- Understand the collective modes (plasmons) of the quantum electron glass within mean field theory.
- Infer the existence of a gapless phonon-like bath which can resolve the energy conservation problem in hopping conductivity.

Reduction to a quantum spin glass

Idea:

- Classical frustrated glass + quantum fluctuations

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- Dynamical mean field description (good for $z^2 >> 1$)

$$
\begin{aligned}
S_{\text{eff}} \; &= \int_0^\beta d\tau \left[\frac{1}{2} \sum_{i,j} \sigma_i^z(\tau) J_{ij} \sigma_j^z(\tau) + \sum_i (\epsilon_i - \mu) \sigma_i^z(\tau) \right] \\
&+ \sum_i \int_0^\beta d\tau' \int_0^\beta d\tau \, \sigma_i^+(\tau') G_i(\tau' - \tau) \sigma_i^-(\tau)
\end{aligned}
$$

Inertial, non-dissipative dynamics
↔ virtual exchange processes of electrons with
the “bath” of neighboring sites, no decay

Reduction to a quantum spin glass

Idea:

- Classical frustrated glass + quantum fluctuations
- Spin representation for level occupation: $\boxed{\sigma_i^z = \pm 1 \quad \Leftrightarrow \quad n_i = 1,0}$

- For the purpose of collective dynamics:

$\rightarrow$ Describe quantum fluctuations by a self-consistent effective transverse field t_{eff} with $\boxed{\frac{J^2}{\delta} >> t_{eff} >> J}$

$$\longrightarrow \boxed{H_{eff} = \sum_i \left(\varepsilon_i \sigma_i^z + t_{eff} \sigma_i^x \right) + \frac{1}{2} \sum_{i,j} \sigma_i^z J_{ij} \sigma_j^z}$$

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Aim:

- Obtain collective delocalized modes $\rightarrow$ continuous bath.
- Show that the system remains an insulator (single particle excitations remain sharp close to the Fermi level)
- Construct the theory of electron-assisted hopping.

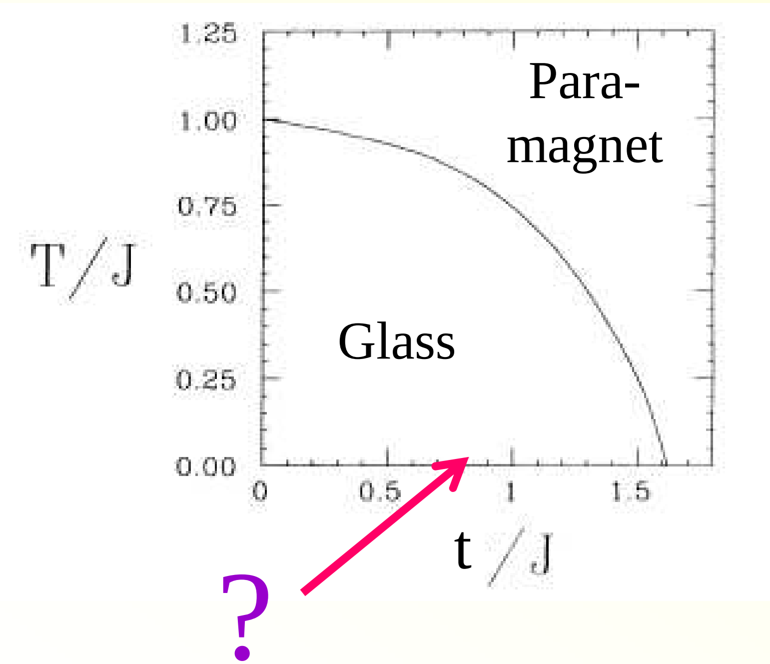
Quantum TAP equations

*(**T**houless, **A**nderson, **P**almer 1977: Classical SK model)*

$$H_{eff} = \sum_i \left(\varepsilon_i \sigma_i^z + t_{eff} \sigma_i^x \right) + \frac{1}{2} \sum_{i,j} \sigma_i^z J_{ij} \sigma_j^z$$

Transverse field Ising spin glass

(quantum Sherrington Kirkpatrick-model at z = ∞)



(Goldschmidt and Lai, PRL 1990)

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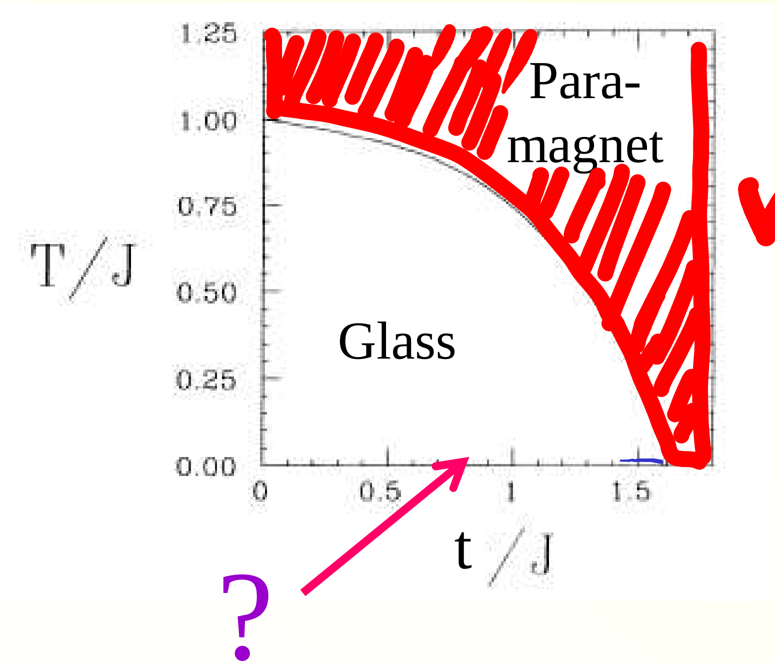

For infinite coordination z = ∞:
Phase transition into a glass state:
- Broken ergodicity
- Many long-lived metastable states

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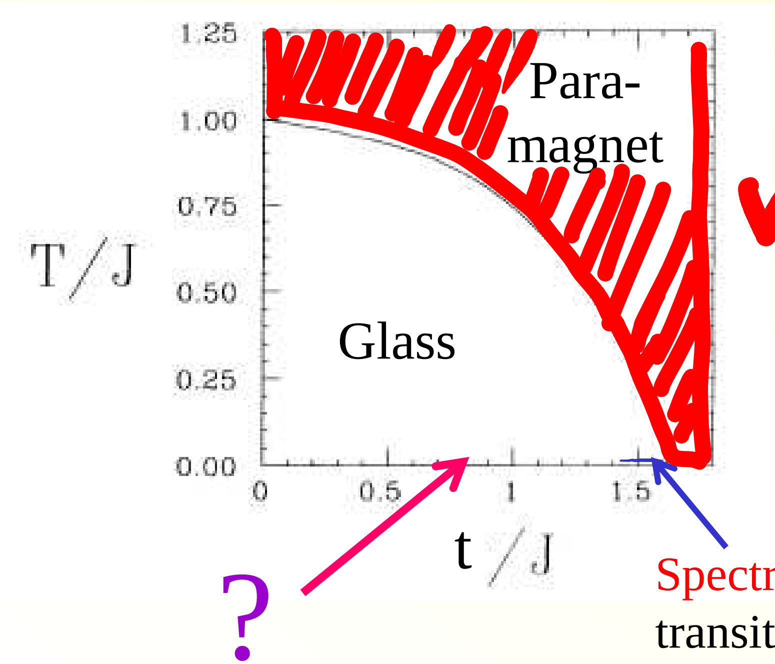

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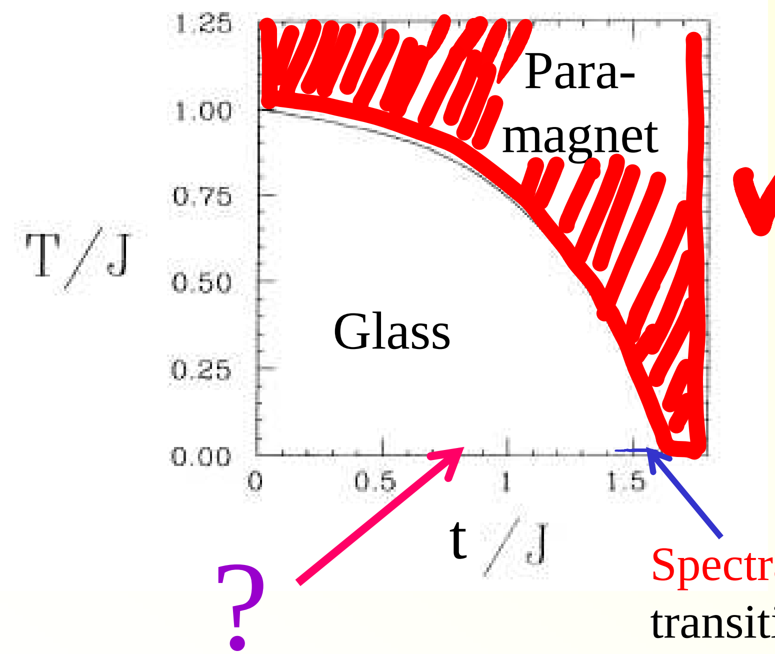

For infinite coordination z = ∞:
Phase transition into a glass state:
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- Many long-lived metastable states
- Self-organized criticality (marginal stability) of the states within the glass phase

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Constrained free energy as a function of magnetizations imposed by external auxiliary fields h_i^{ex} (total local field: $h_i = h_i^{ex} + \varepsilon_i$) at large z

$$G\left(\left\{\left\langle \sigma_i^z \right\rangle = m_i\right\}\right) = \sum_i \left(E_i(m_i) + h_i^{ex} m_i\right) - \frac{1}{2} \sum_{i \neq j} m_i J_{ij} m_j - \frac{1}{2} \sum_{i \neq j} J_{ij}^2 \int_0^\infty d\tau \chi_i(\tau) \chi_j(\tau)$$

$$\left.\begin{array}{l} E_i = -h_i/2 - \sqrt{(h_i/2)^2 + t_{eff}^2} \\ m_i = dE_i/dh_i \end{array}\right\} \Rightarrow E_i(m_i)$$

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Local minima $(\partial G/\partial m_i = 0)$ (in static approximation)

$$\rightarrow \quad \boxed{h_i = \varepsilon_i + \sum_{j\neq i} J_{ij} m_j - m_i \sum_{j\neq i} J_{ij}^2 \chi_j(m_j)\ ;\ m_i = m(h_i)}$$

N coupled random equations for $\{m_i\}$ with **exponentially many solutions!**

Quantum TAP equations

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Environment of a local minimum (potential landscape):

Hessian: $\mathrm{H}_{ij} = \partial^2 G \big/ \partial m_i \partial m_j = J_{ij} + \text{diagonal terms}$

$$Spec\left[H_{ij}\right] \equiv \rho_H\left(\lambda\right) = C\,\frac{\sqrt{\lambda \cdot t_{eff}}}{J^2}$$

(at small λ)

Gapless spectrum
(assured by marginal stability)
in the **whole** glass phase!

Soft collective modes

Spectrum of the Hessian $\leftrightarrow$
Distribution of “restoring forces”

$$Spec\left[H_{ij}\right] \equiv \rho_H(\lambda) = C\,\frac{\sqrt{\lambda \cdot t_{eff}}}{J^2}$$

Soft collective modes

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Semiclassics:

$\rightarrow$ N collective oscillators with mass $M \sim 1/t_{eff}$ and frequency $\omega = \sqrt{\lambda / M}$

$\longrightarrow$ Mode density $$\boxed{\rho(\omega) = C\,\frac{\omega^2}{t_{eff} J^2}}$$

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$\rightarrow$ Mode density

$$\rho(\omega) = C \frac{\omega^2}{t_{eff} J^2}$$

$\rightarrow$ **Continuous** bath with spectral function
(in the regime of **delocalized** modes!)

$$\chi''(\omega) = \frac{1}{M\omega} \rho(\omega) \sim \frac{\omega}{J^2}$$

Independent of t_{eff}!

Generalization of known spectral function at the quantum glass transition.
[Miller, Huse (SK model); Read, Ye, Sachdev (rotor models)]

Localization of collective modes ?

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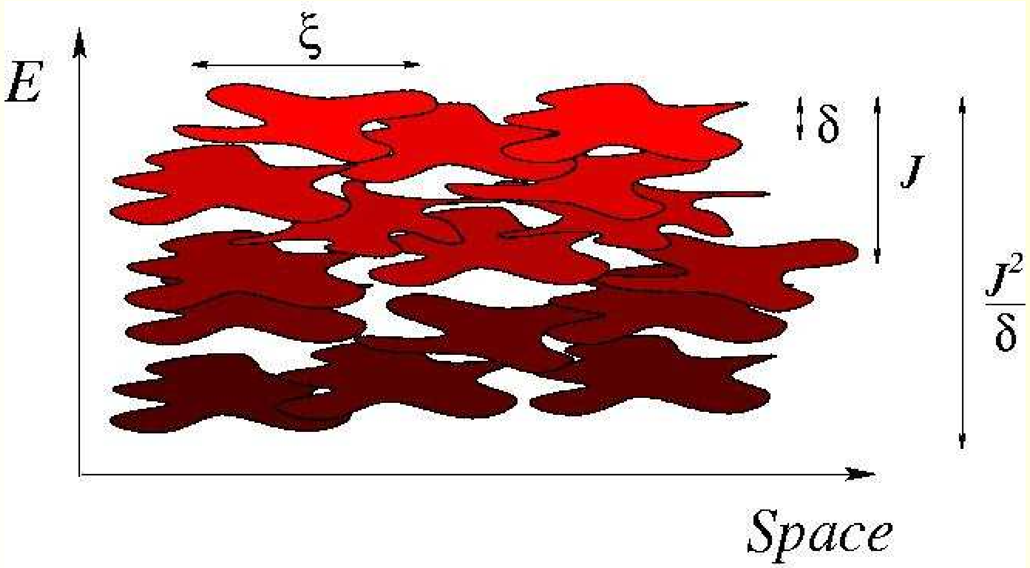
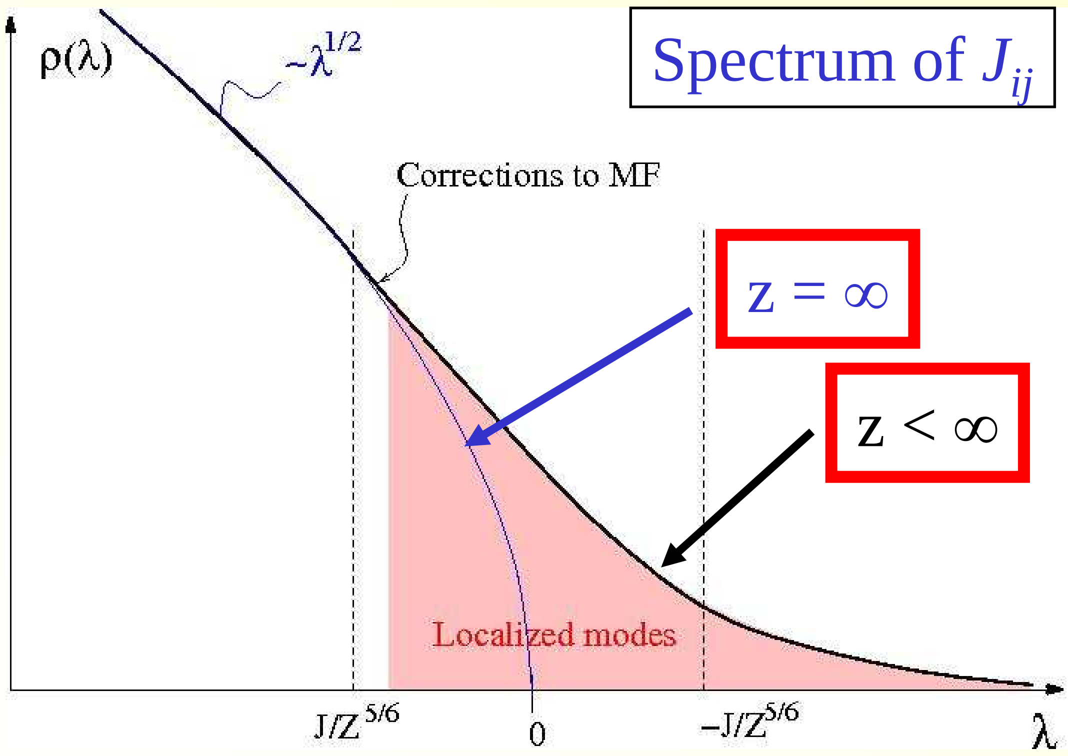
In 3D: Random matrix J_{ij} couples every localized level i to $z >> 1$ close spatial neighbors.



$\frac{J^2}{\delta} = z$

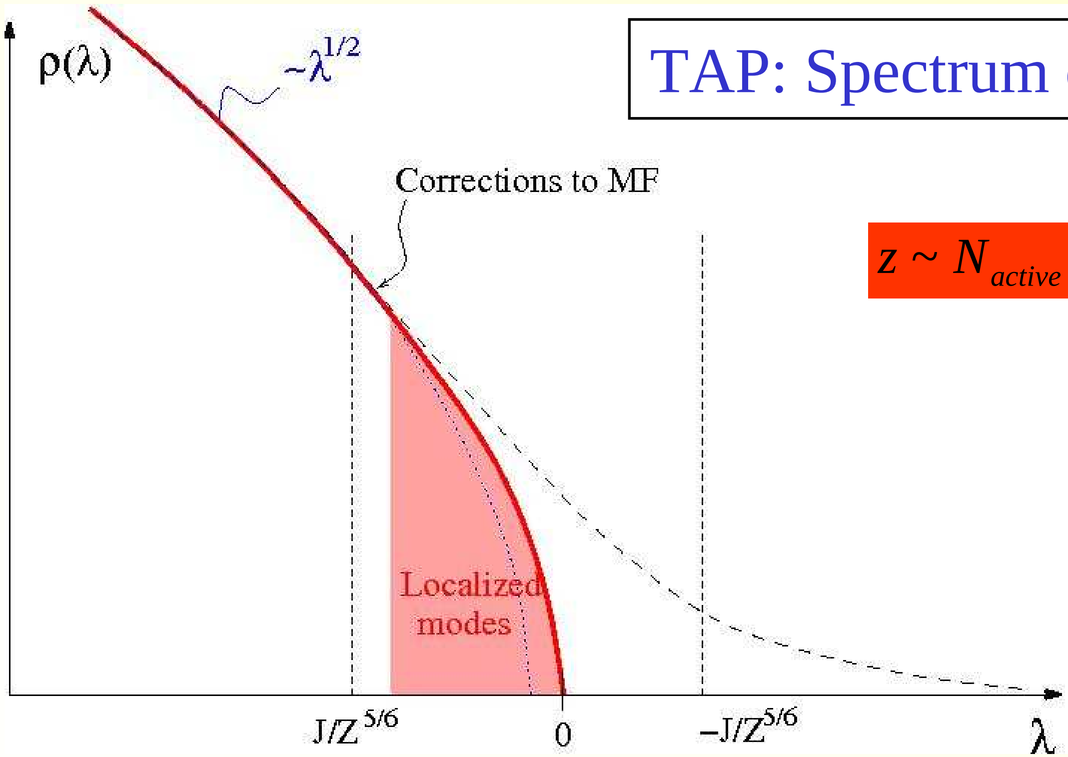
Localization of collective modes ?

Eigenvalue and eigenvector spectrum of a random matrix J_{ij} (3D)



Localization of collective modes ?

Eigenvalue and -vector spectrum of TAP Hessian H_{ij} (3d)

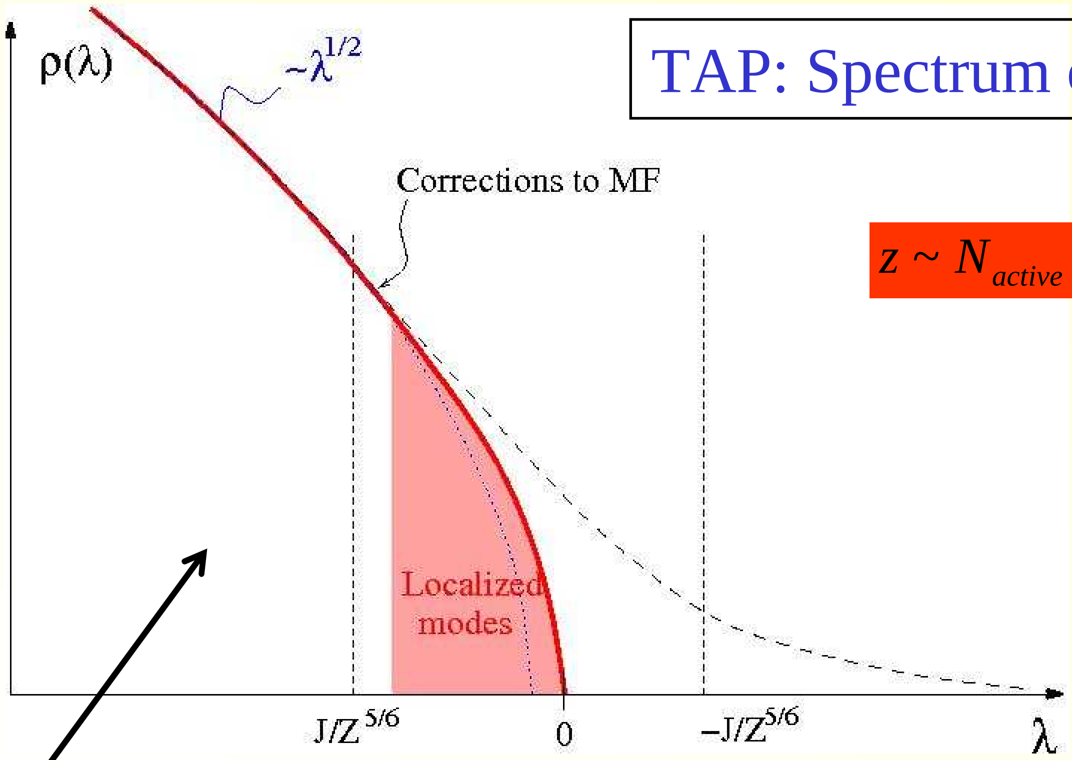


TAP: Spectrum of Hessian

$z \sim N_{active} >> 1$

Localization of collective modes ?

Eigenvalue and -vector spectrum of TAP Hessian H_{ij} (3d)



TAP: Spectrum of Hessian

$z \sim N_{active} >> 1$

Delocalized low-energy plasmons down to $E_{min} \sim Jz^{-1/6} << J$

Summary of results

- The quantum electron glass possesses a continuous bath of collective uncharged excitations, (which are beyond perturbation theory)

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Further, we have checked that:

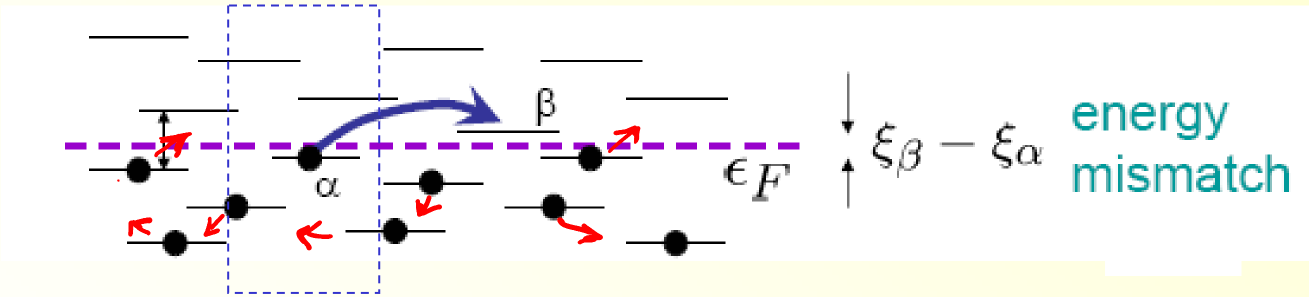
- Single particle excitations remain very sharp at the Fermi level:
  Level broadening from decay processes ($1/T_1$) and pure dephasing ($1/T_2$) is smaller than level spacing δ.

$\rightarrow$ The system remains an insulator: $\rho(T \rightarrow 0) \rightarrow \infty$

At finite temperature: conduction by hopping, stimulated by collective electron modes.

Bottom line: Variable range hopping

Electron hopping out
of localization volume



A collective mode (plasmon) can provide the exact energy difference in a single electron hop because of the continuous spectrum of the bath.

All electron levels acquire a finite if small width due to their coupling to plasmons. Hence, there is no manybody localization.

Bottom line: Variable range hopping

Variable range hopping

$$\boxed{\sigma(T) \approx \frac{\sigma_0}{\xi^{d-2}} \exp\left[-\left(\frac{T_0}{T}\right)^{1/2}\right]}$$

As in phonon-assisted hopping but with
different prefactor reflecting the plasmon bath!

• Stretched exponential in T:
Single electrons optimize activation energy vs transition probability (length of hops)
→ elementary resistors (Miller-Abrahams)
• Percolation problem for the network of resistors (Ambegaokar et al., Pollak, Shklovskii)

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Only two energy scales: T and $T_0 \approx J = \dfrac{e^2}{\kappa \xi}$

… (quasi 2d) $\longrightarrow$

$$\boxed{\sigma_0 \approx \frac{e^2}{h}\left(\frac{T}{T_0}\right)^{-\alpha} \qquad \alpha \approx 0.3}$$

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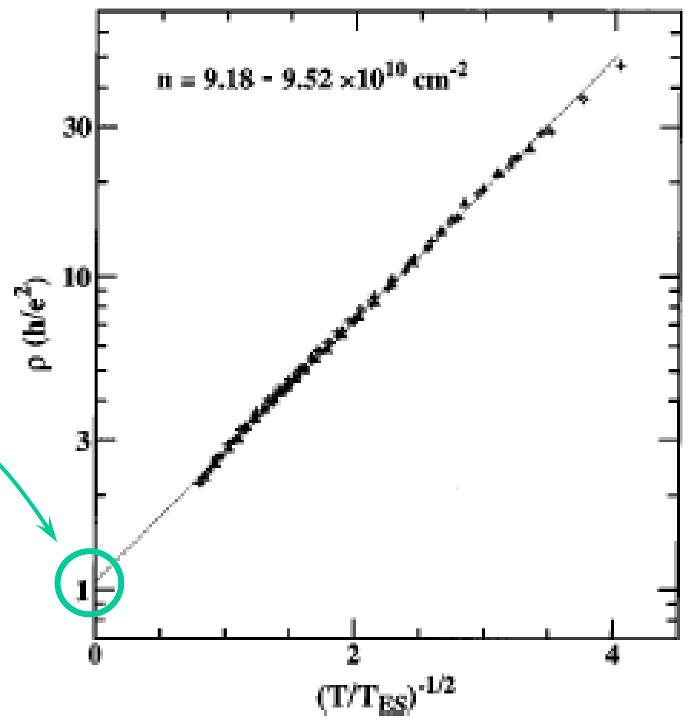
Doped GaAs/Al$_x$Ga$_{1-x}$As heterostructure

*S. I. Khondaker et al., PRB **59**, 4580 (1999)*



Many body localization: where to find it best?

Two problems:

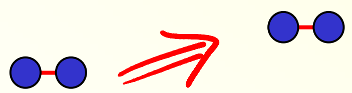
• Four-fermion scattering introduces strong quantum fluctuations

• Long range Coulomb interactions spoil localization, even at low density

Possible way out: insulators with strong superconducting correlations (fermions bound into preformed pairs), with suppressed/screened Coulomb interactions

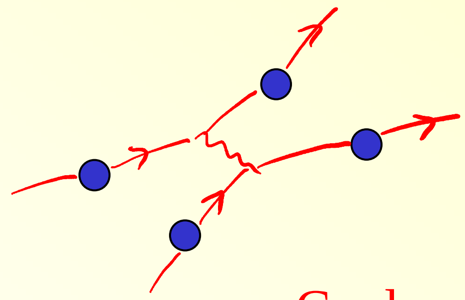
Why to expect many body localization at the SIT?

- Electrons are bound in localized pairs (Anderson pseudospins)
- Phase volume for inelastic processes is strongly reduced as compared to the single electron problem MIT



Cooper + hard core repulsion

$\leftrightarrow$



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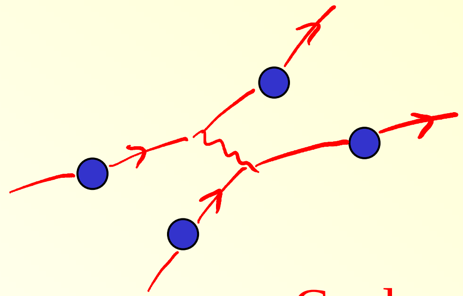

Cooper + hard core repulsion

Coulomb

Pairs: doubly occupied localized wavefunctions (hard core bosons)

$$H_{pair} = \sum_i \varepsilon_i \sigma_i^z + \sum_{ij} t_{ij} \sigma_i^+ \sigma_j^- \left(+ \sum_{ij} J_{ij} \sigma_i^z \sigma_j^z \right)$$

(Anderson, Ma+Lee, Feigelmann+Ioffe)

Disorder (→insulator)   Kinetic energy of pairs (→superconductivity)

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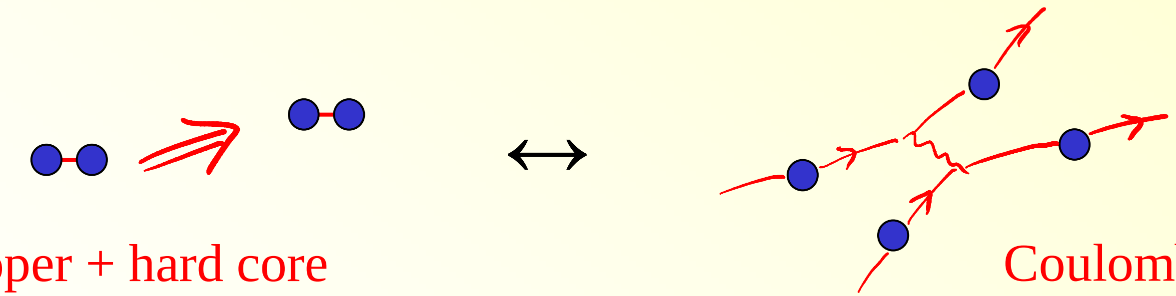

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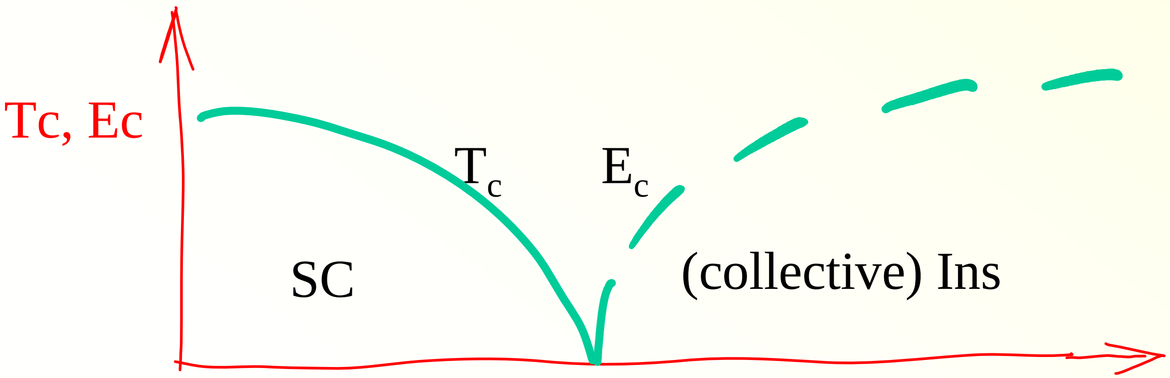

Conjecture (for insulator):
At T=0 all excitations with $E < E_c$ are localized
Experimental indications for such a secnario!

Conclusions

- Model for purely electron-assisted hopping in insulators.
- Collective soft modes provide a bath with continuous spectrum and ensure energy conservation during a hopping event. → No manybody localization expected close to the Metal-insulator transition
- Possibly different, and conceptually very interesting situation close to dirty superconductor-insulator transitions

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