

Transport and many-body localization in bosonic insulators and disordered magnets

Markus Müller

Ann. Phys. (Berlin) 18, 849 (2009)

Discussions with

M. Feigel'man, M.P.A. Fisher, L.
Ioffe, V. Kravtsov

Experiments: B. Sacépé (Grenoble),
D. Shahar (Weizmann),
T. Baturina (Novosibirsk)



Abdus Salam
International
Center of
Theoretical
Physics

ICTP Workshop on emergent new states of matter, 5-9 July, 2010

Outline

- The disordered superconductor-insulator transition (SIT) – dirty bosons and random XY-magnets
- Review of various puzzling transport experiments in the Bose glass phase
- Resolution based on:
 - Characterization of the insulator by its spectral properties
 - Consequences for transport: $R(T)$
 - "Many-body localization" and its precursors

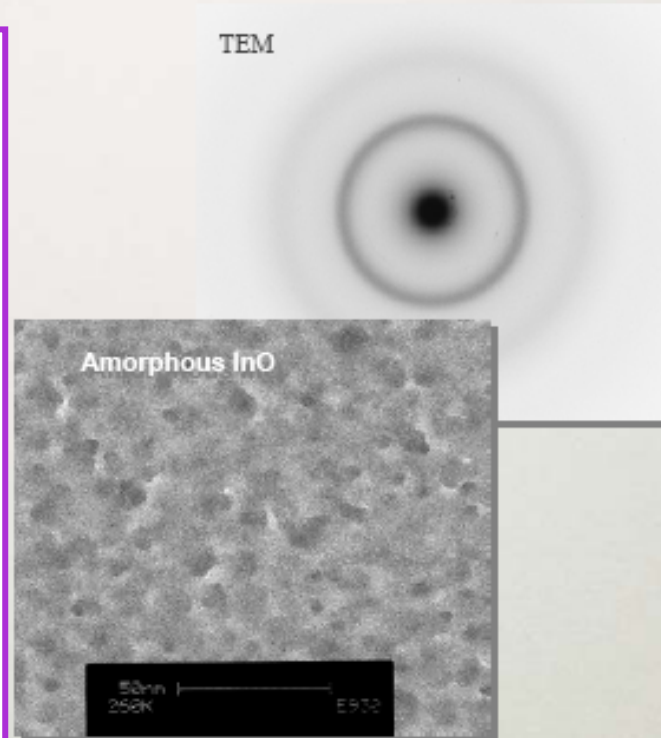
Indium-oxide (InO_x)

Indium-oxide: One of the materials used in the experiments discussed here

(Sambandamurthy, Shahar, Sacépé)

- Strong disorder
- Tunable disorder

Similar experiments in TiN films *(Baturina)*



SI transition in thin films

M. Strongin, et. al., Phys. Rev. B1, 1078 (1970).

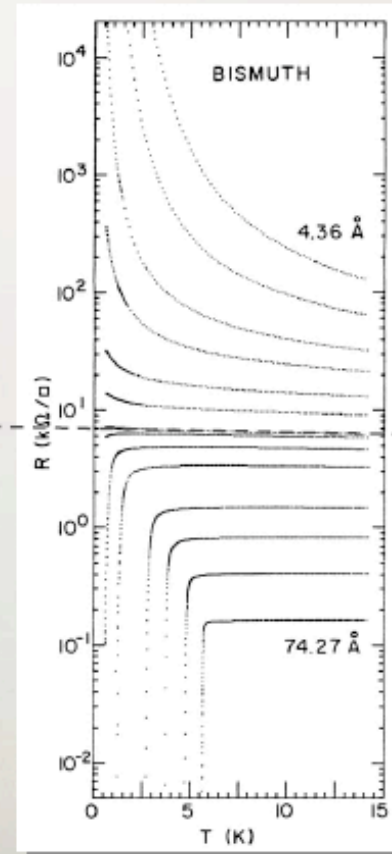
D. B. Haviland, Y. Liu, and A. M. Goldman, Phys. Rev. Lett. 62, 2180 (1989)...

Thickness tuned transition

T = 0 transition

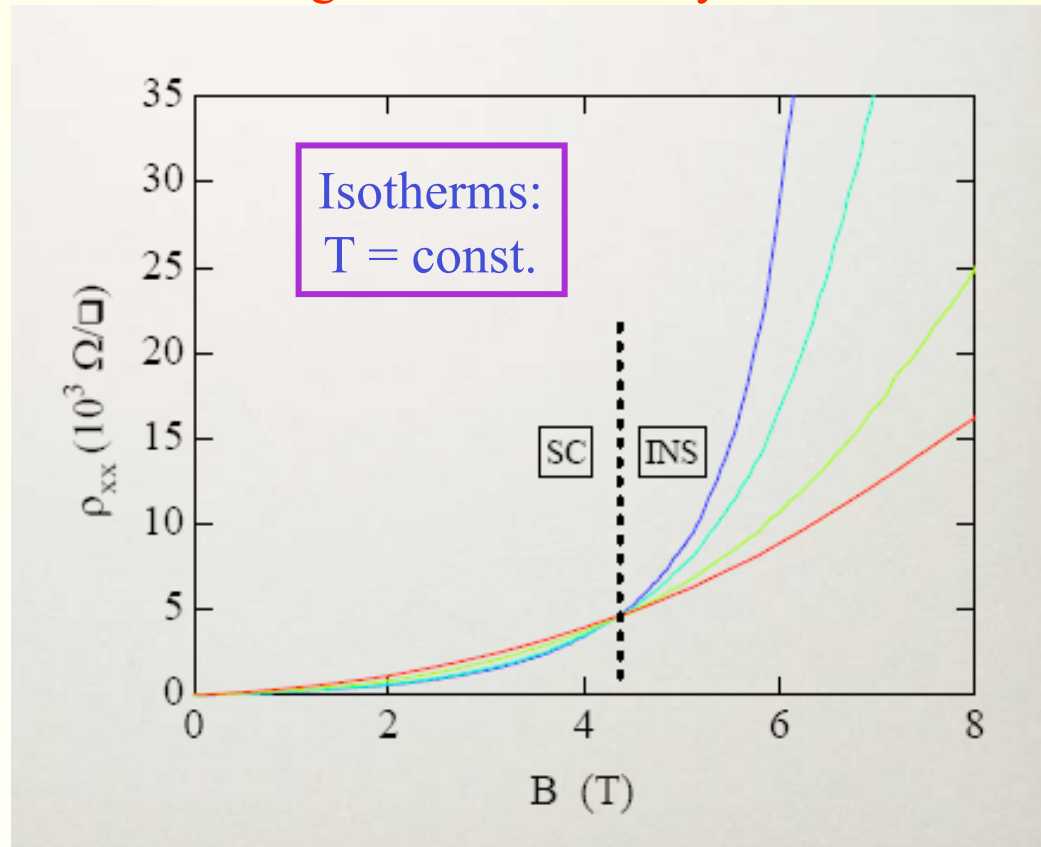
Review: Finkl'stein ('94),
Markovic and Goldman ('98).

2D



Field driven transition

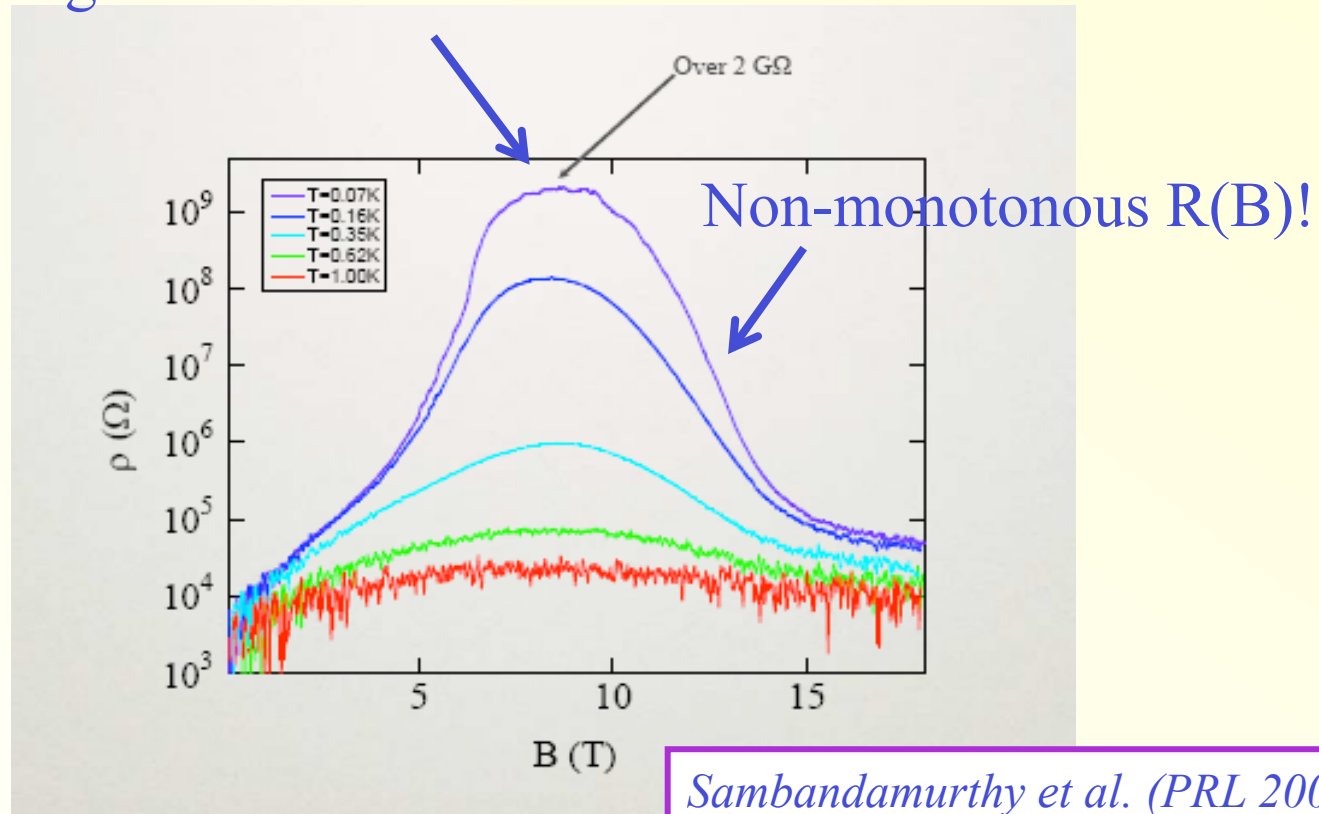
Magnetic field destroys SC!



Gantmakher, Shahar, Kapitulnik, Goldman, Baturina

Insulator: Giant magnetoresistance

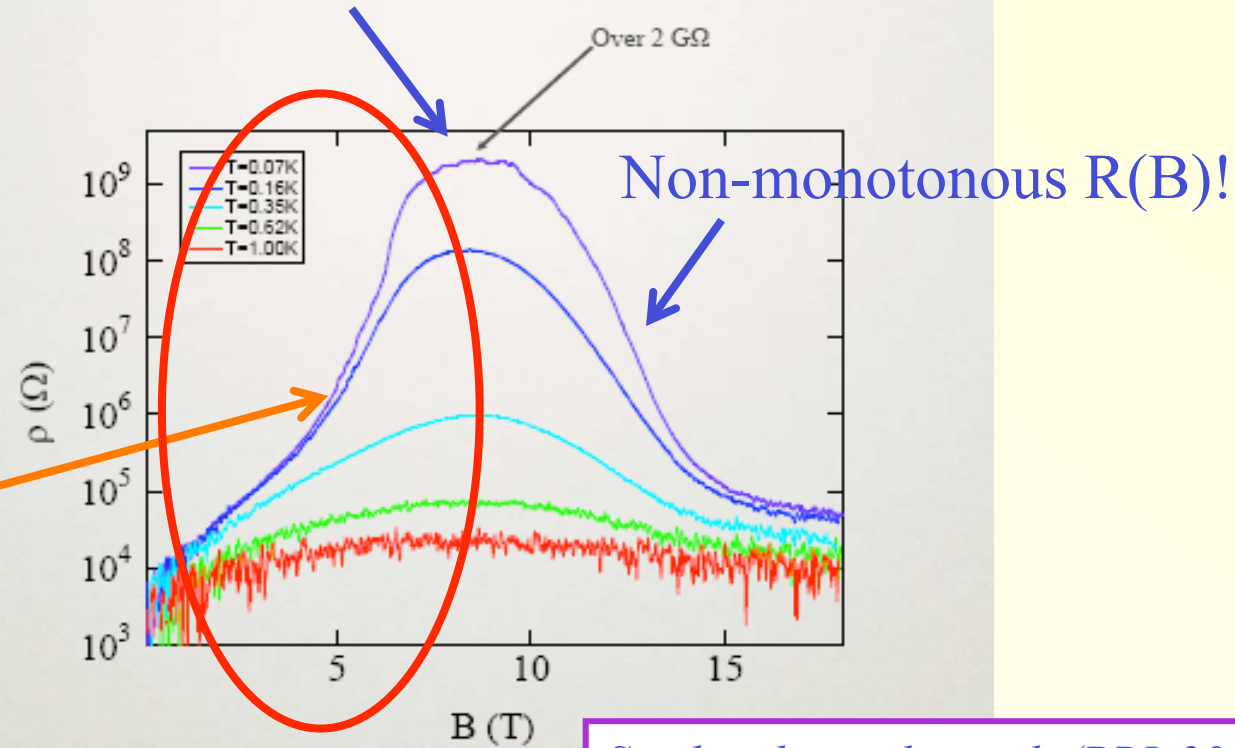
Giant magnetoresistance



Insulating behavior **enhanced** by local superconductivity!

Insulator: Giant magnetoresistance

Giant magnetoresistance



Common belief:
Pairs (bosons)
survive in the
insulator:
→ **Bose glass**

Evidence also
from tunneling

Sambandamurthy et al. (PRL 2005)

Insulating behavior **enhanced** by local superconductivity!

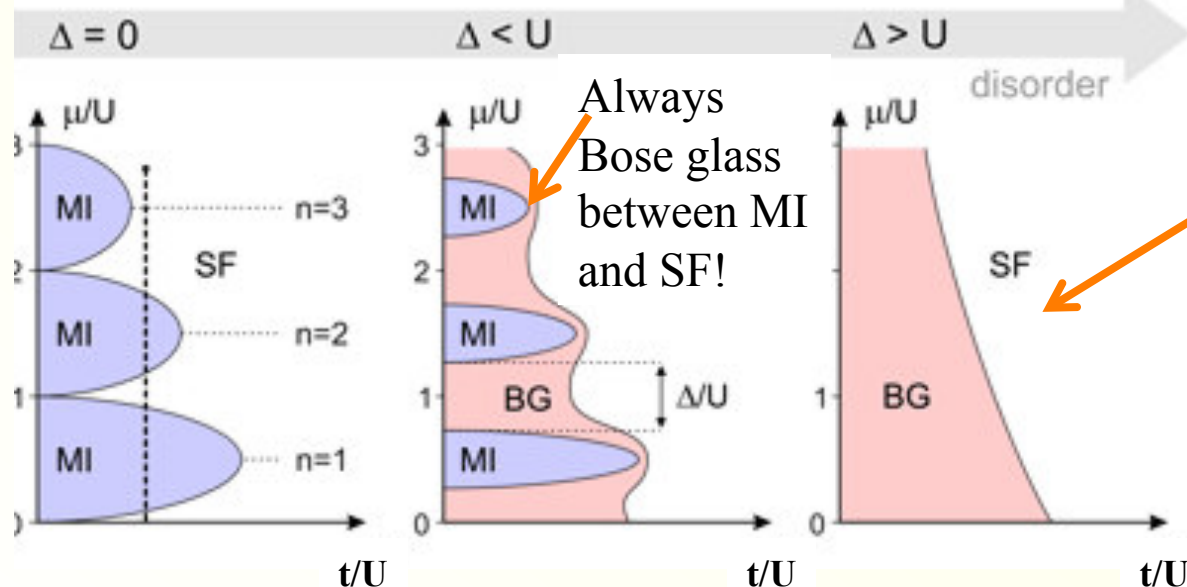
Bose-Hubbard model and Bose glass

Fisher et al., Phys. Rev. B 40, 546 (1989) --- Altland et al, Gurarie et al. (2009)

- Assume “preformed Cooper pairs”: bosons without global superconductivity
- Dirty boson model (Bose-Hubbard model with disorder):

$$H = t \sum_{\langle i,j \rangle} b_i^\dagger b_j + U \sum_i n_i (n_i - 1) + \sum_i (\varepsilon_i - \mu) n_i$$

Disorder: $\varepsilon_i \in [-\Delta, \Delta]$



Two puzzling features in transport in strongly disordered samples

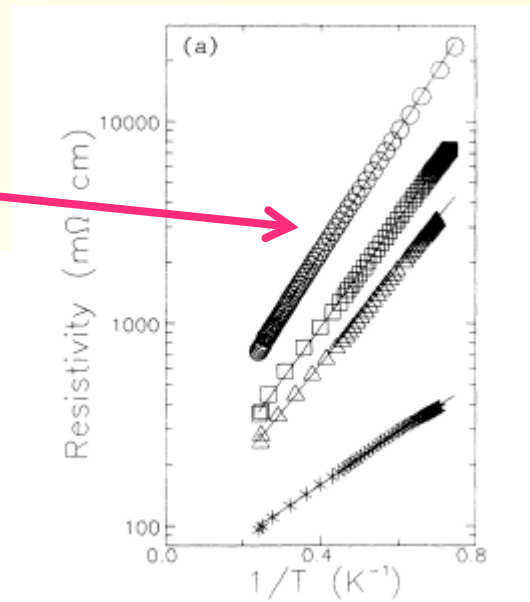
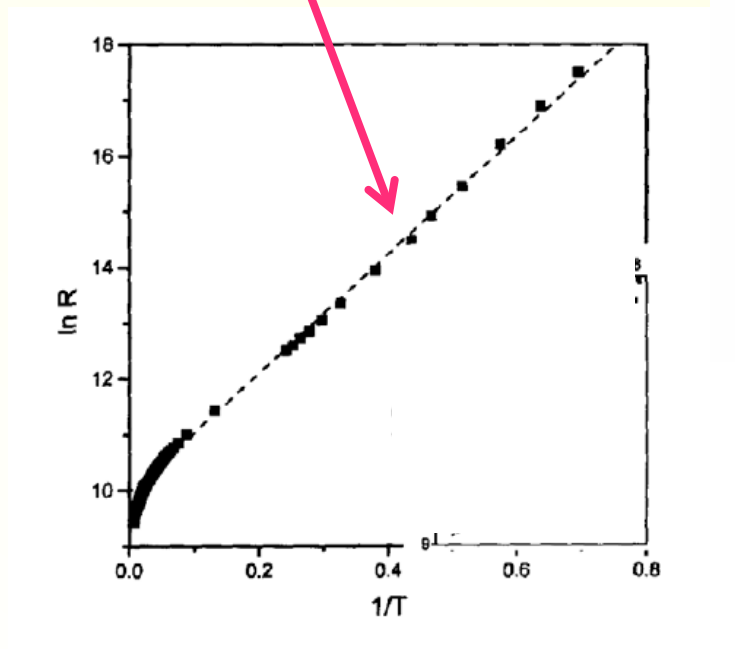
1. Simple activation in $R(T)$
2. Evidence for purely electronic mechanism

Activated transport near the SIT

D. Shahar, Z. Ovadyahu, PRB 46, 10971 (1992).

Insulating InO_x

Simple activation!?



$$R(T) = R_0 \exp\left[\frac{\Delta}{T}\right]$$

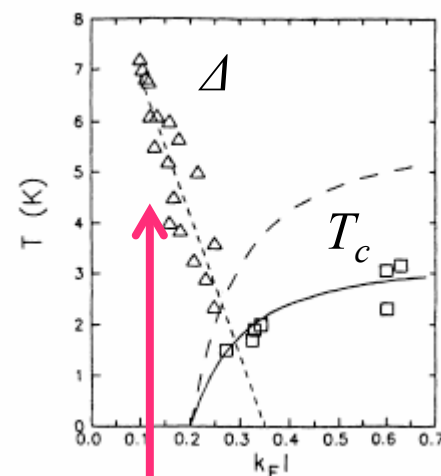
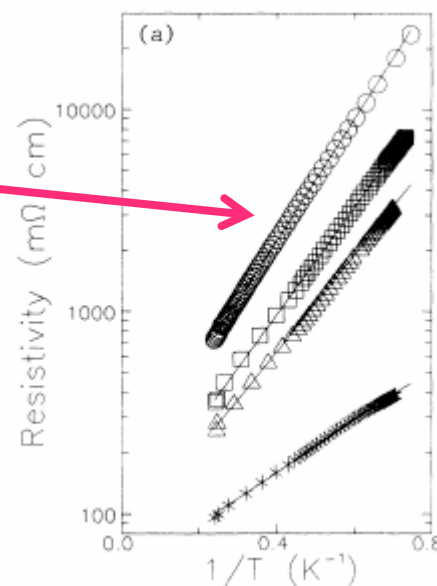
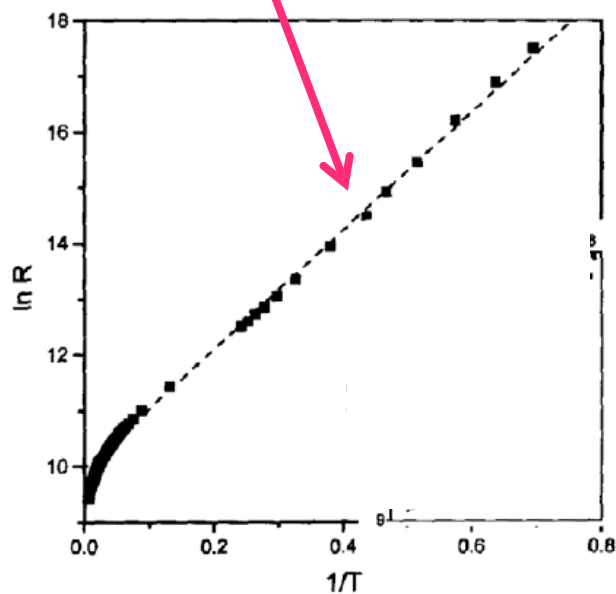
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Simple activation!?



Natural aspect:
Activation energy
increases with
distance to SIT

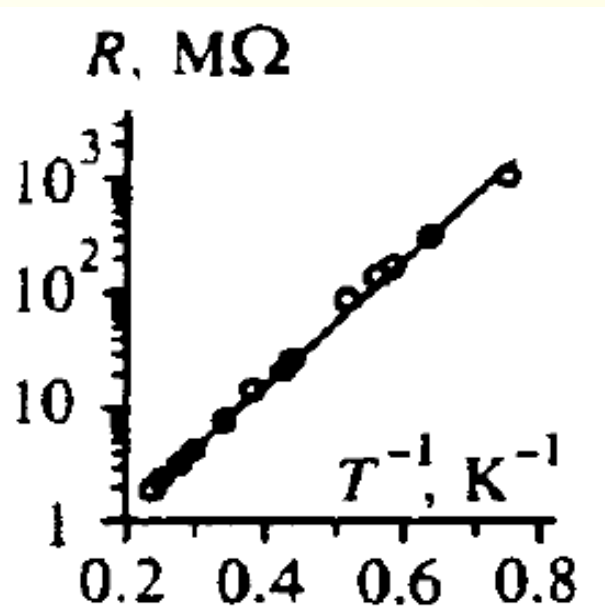
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Activated transport near the SIT

V. F. Gantmakher, M. V. Golubkov, J. Lok, A. K. Geim, Sov. Phys. JETP, 82, 951 (1996).

Insulating InO_x



Origin of simple activation?

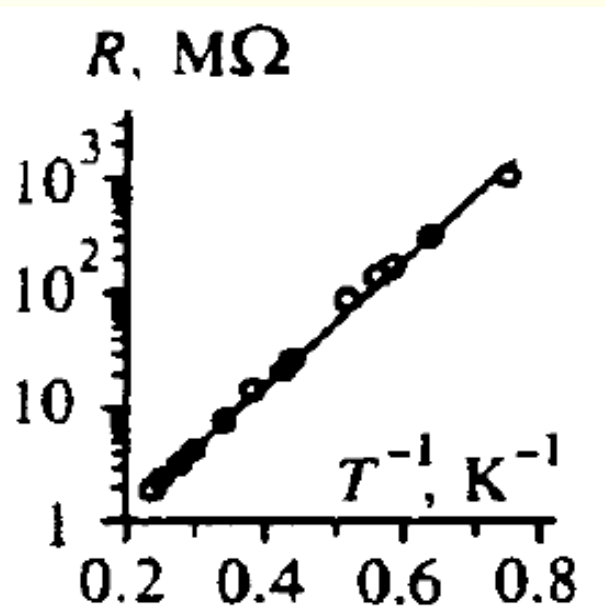
- Gap in the density of states?
A: NO! Too disordered systems!
There is no (Mott) gap!

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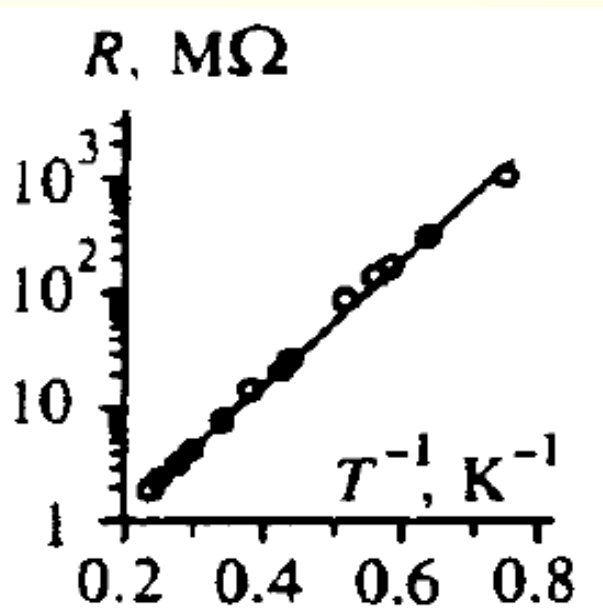
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- Why no variable range hopping?
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Also: Would give too large prefactor R_0 .

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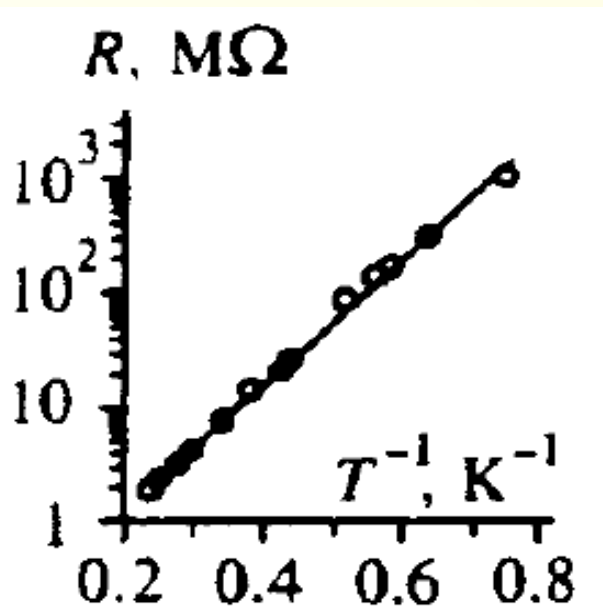
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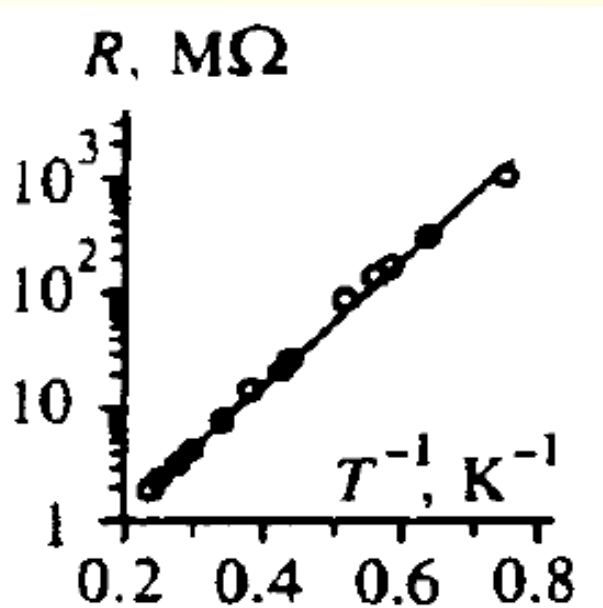
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- No depairing of bosons (positive MR!)
[Feigel'man et al.]

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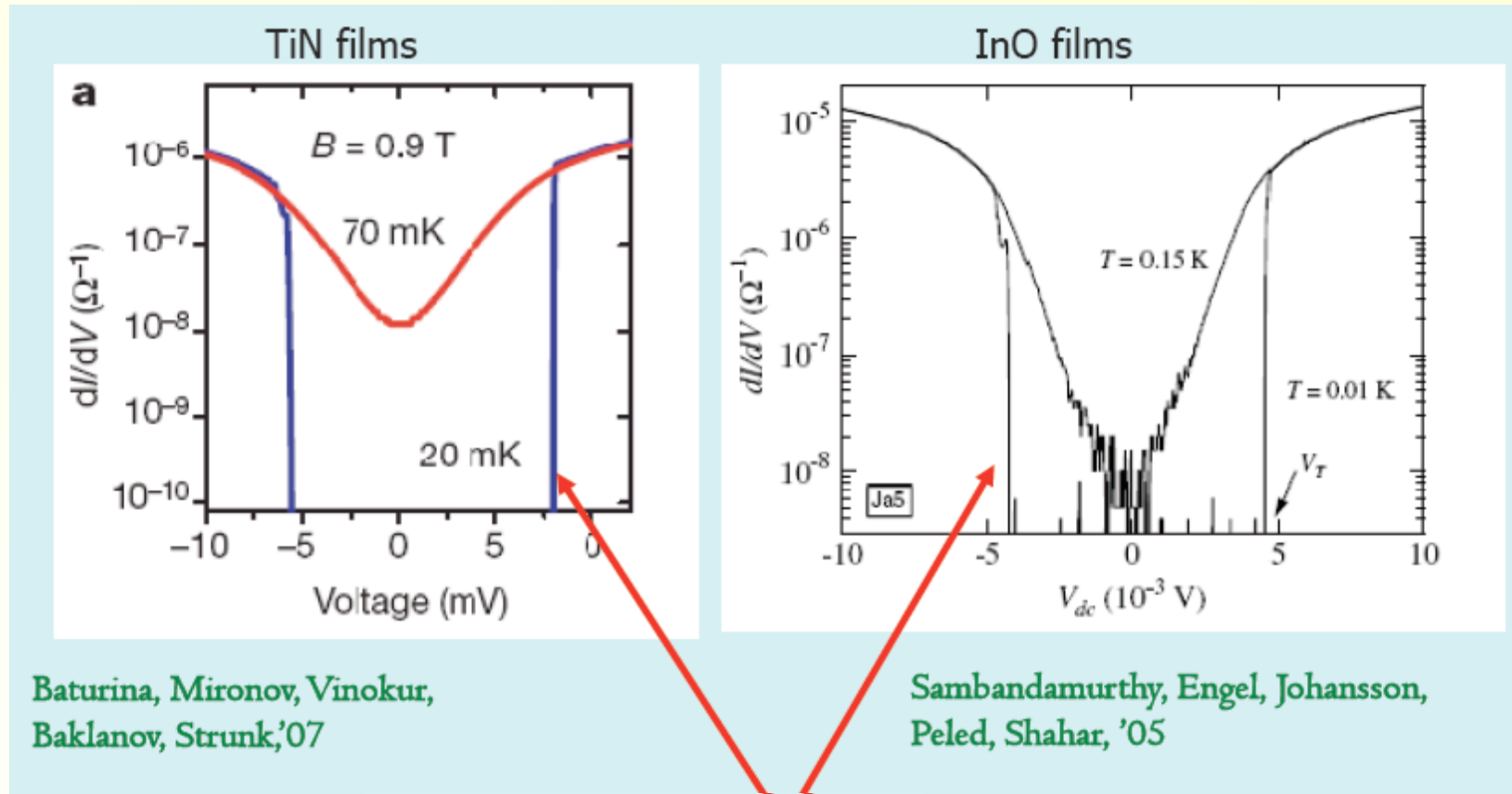


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- Nearest neighbor hopping?
A: NO! Inconsistent with the experimental prefactor of Arrhenius
- No depairing of bosons (positive MR!)
- **Instead: (Multi-)boson mobility edge!**
(similar to Anderson localization)

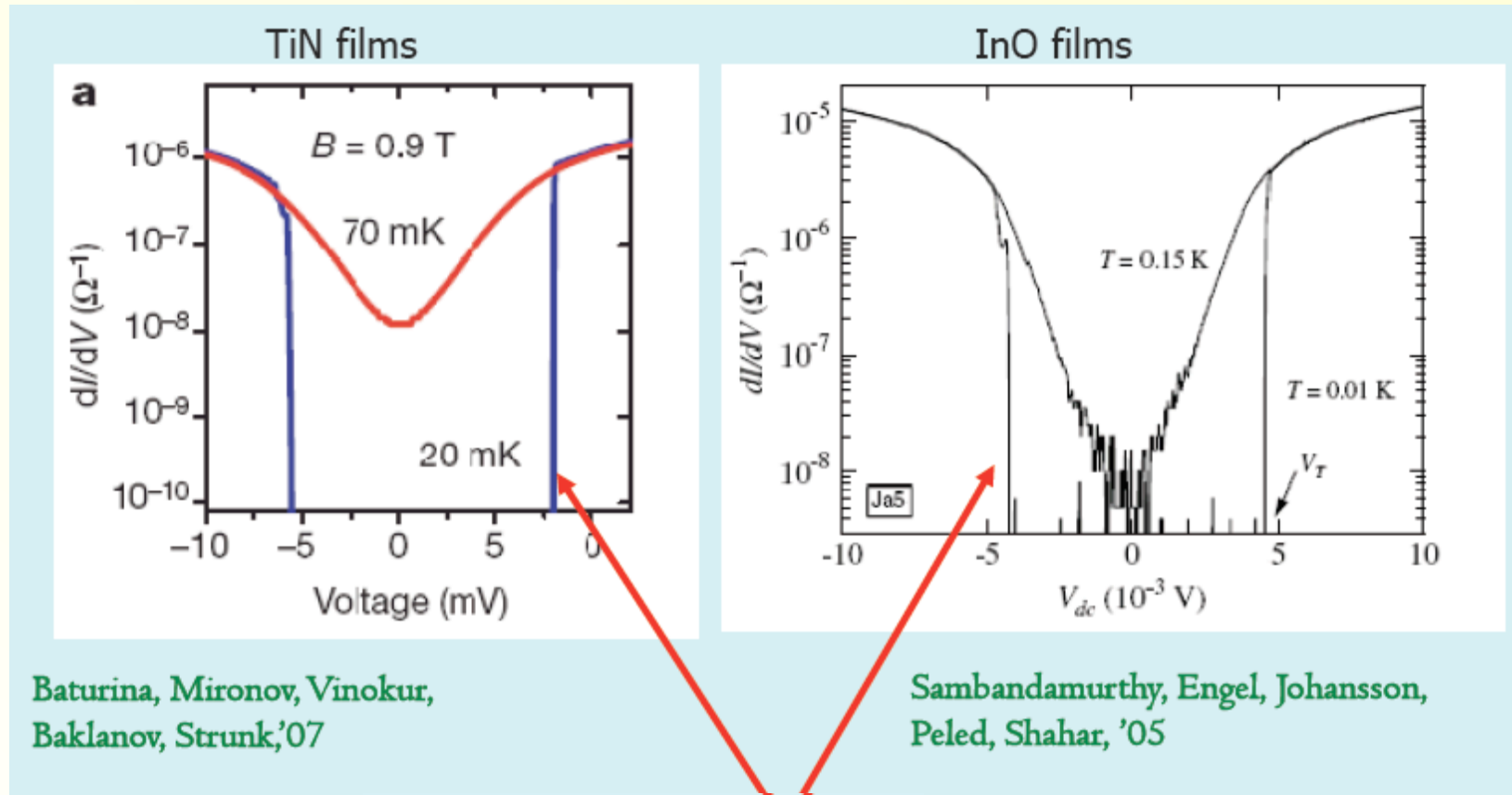
Purely electronic transport mechanism!



Giant jumps in resistance
from $k\Omega$ to $G\Omega$ regime

Non-Ohmic resistance in the insulator!

Purely electronic transport mechanism!

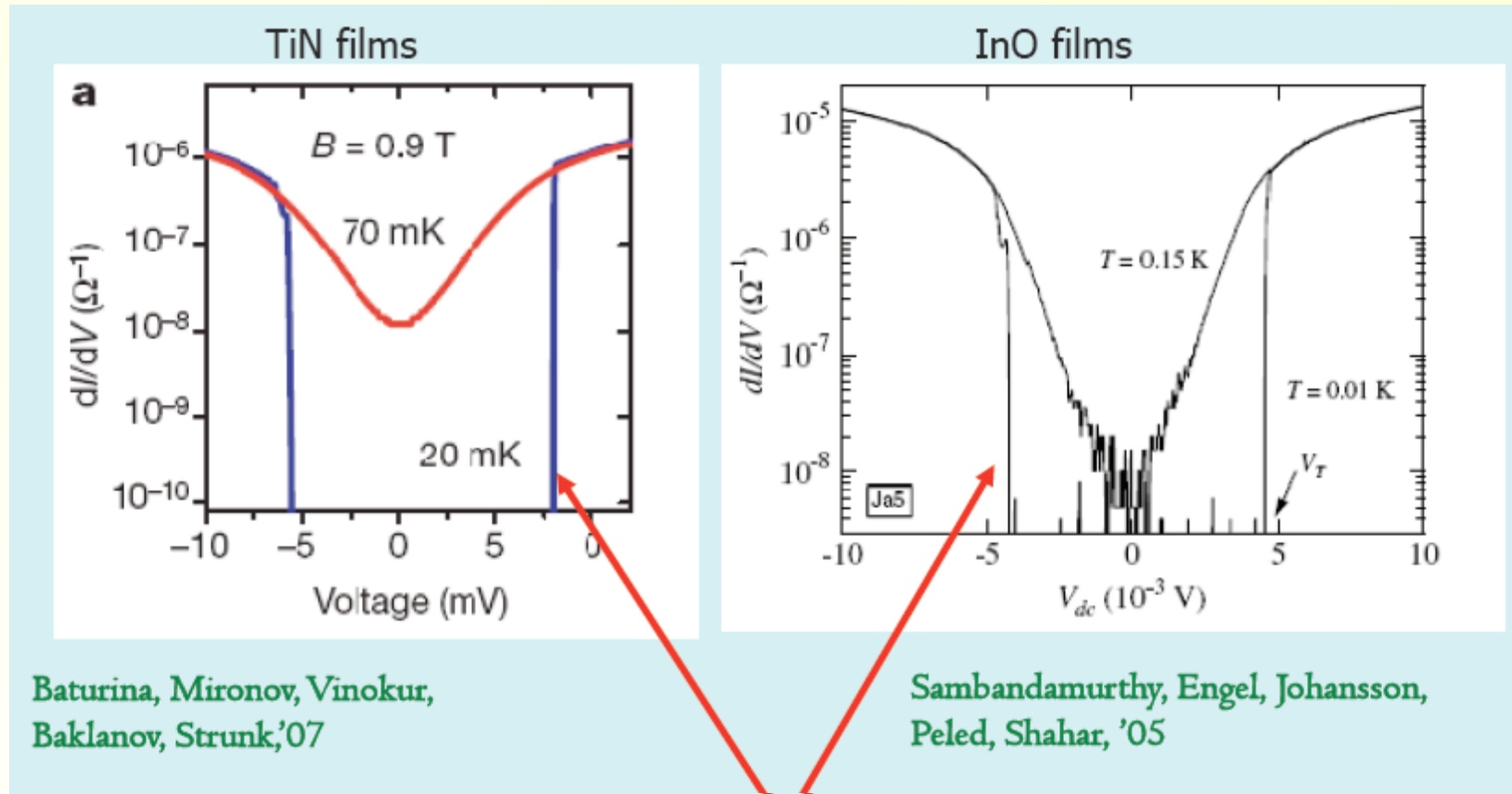


Giant jumps in resistance
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Simple explanation: instability from low T /high R state to overheated state.

Altshuler, Kravtsov, Lerner, Aleiner (09)

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Crucial conclusion: transport is not phonon- but electron-activated! - Mechanism???

Purely electronic transport mechanism!

Transport is **electron- not phonon**-activated in the insulators!
Mechanism ???

Electron-activation is an old, puzzling phenomenon:

Electronic mechanism close to the metal-insulator transition
experimentally inferred from non-linear transport:

$R = R(T_{el}(V))$ -- **not** $R = R(T_{ph})$ (*West, Pfeifer; Gershenson, Pepper*)

Mechanism???

Proposal for the MIT: (*Müller & Ioffe (2007)*)

Idea: Quantum glassiness of the electrons leads to low energy collective modes and yields electron activated variable range hopping.

This does **not work for the SIT** (simple activation, weak Coulomb interactions)

Summary of puzzles at the SIT

1. Close to the SI transition the **transport** is essentially simply activated (**Arrhenius**):

How come?

2. Evidence for **purely electronic** transport in the Bose insulator

What is its origin?

From dirty superconductor to Bose glass: model

Models

$$H = -t \sum_{\langle i,j \rangle} b_i^+ b_j + U \sum_i n_i (n_i - 1) + \sum_i (\varepsilon_i - \mu) n_i$$

Disorder: $\varepsilon_i \in [-\Delta, \Delta]$

Easier to think about: $U = \infty$ limit, i.e., hard core bosons

→ equivalent to **Anderson pseudospins** ($s=1/2$) Interactions (e.g. Coulomb)

*(Anderson, Ma+Lee,
Kapitulnik+Kotliar)*

$$H = -t \sum_{\langle i,j \rangle} s_i^+ s_j^- + \sum_i (\varepsilon_i - \mu) s_i^z + \sum_{\langle i,j \rangle} J_{ij} s_i^z s_j^z$$


Equivalent to an XY Ising ferromagnet with random transverse fields (z-direction)

Similar considerations of disorder can be interesting in “quantum spin ice” (cf Gingras)

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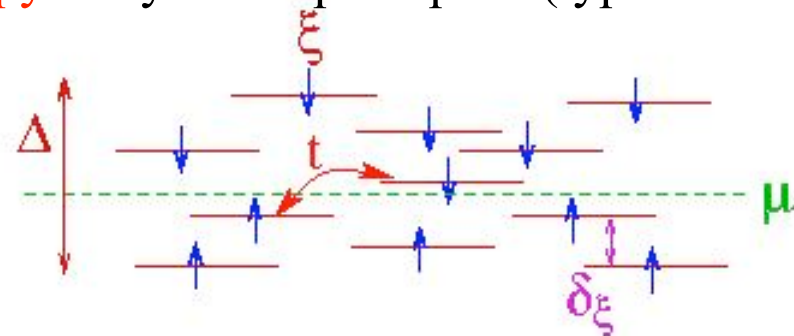
• “Sites” i : **localized states for a pair to occupy**. May overlap in space (typical size: ξ)

• Relevant scale characterizing disorder:

Level spacing δ_ξ between close levels

Disorder strength:

$$g \equiv \delta_\xi / t$$



From dirty superconductor to Bose glass: the phases

- **Superconducting phase:** Bose condensation into delocalized mode in the presence of self-consistently screened disorder
 - finite phase stiffness
 - infinite conductivity for $T < T_c$
- **Bose glass:** No delocalized bosonic mode at $E = 0$ (otherwise condensation would occur)
 - role of disorder: no homogeneous gap, still compressible phase

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 - **Note:** “Bose glass” := disordered Bose insulator without spectral gap
 - It is an **insulator**, i.e. $\sigma(T \rightarrow 0) = 0$ [no Bose metal!]

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Nature of transport in the Bose glass?

From dirty superconductor to Bose glass: the spectrum

SIT = Localization transition of the
bosons !

Analyze the evolution of the
manybody spectrum!

*Berkovits and Shklovskii
Basko, Aleiner, Altshuler
Huse, Oganesyan*

Warm up: Clean case

- Superconductor: gapless excitations (phonons)

- Mott insulator of bosons:

Spectrum:

Finite gap

But: No discrete spectrum!

All excitations are delocalized and

disperse with well-defined momenta \mathbf{k}



With disorder:

Much more complex and more interesting!

The spectrum as indicator of localization

Local spectrum of operator O
at $T = 0$

$$\rho_O(\omega) = \int_0^{\infty} \langle O(x,t)O(x,0) \rangle_{GS} e^{i\omega t} dt$$

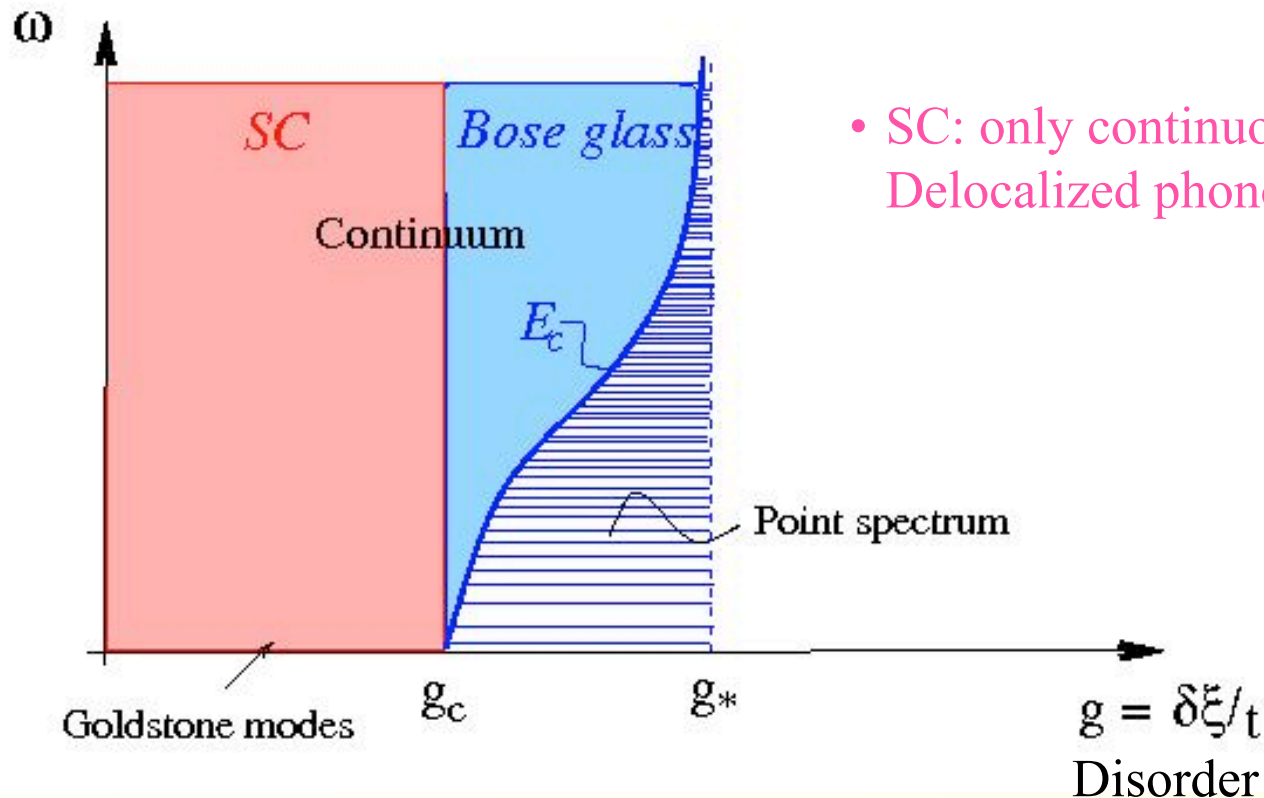
2 possibilities:

- Continuous spectrum
(\leftrightarrow delocalized excitations)
- Point spectrum: “locally discrete”
(bunch of delta functions in local correlation functions \leftrightarrow localized excitations)

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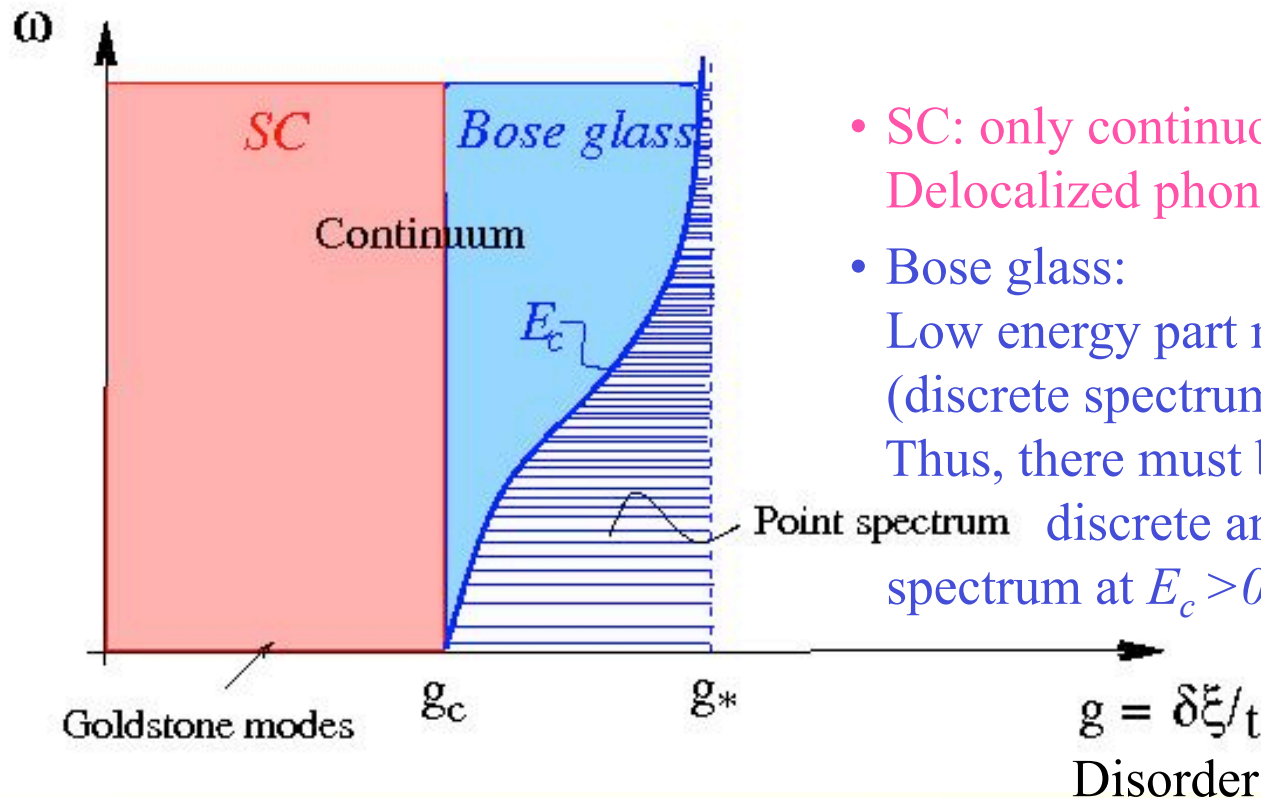


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Delocalized phonons at low ω

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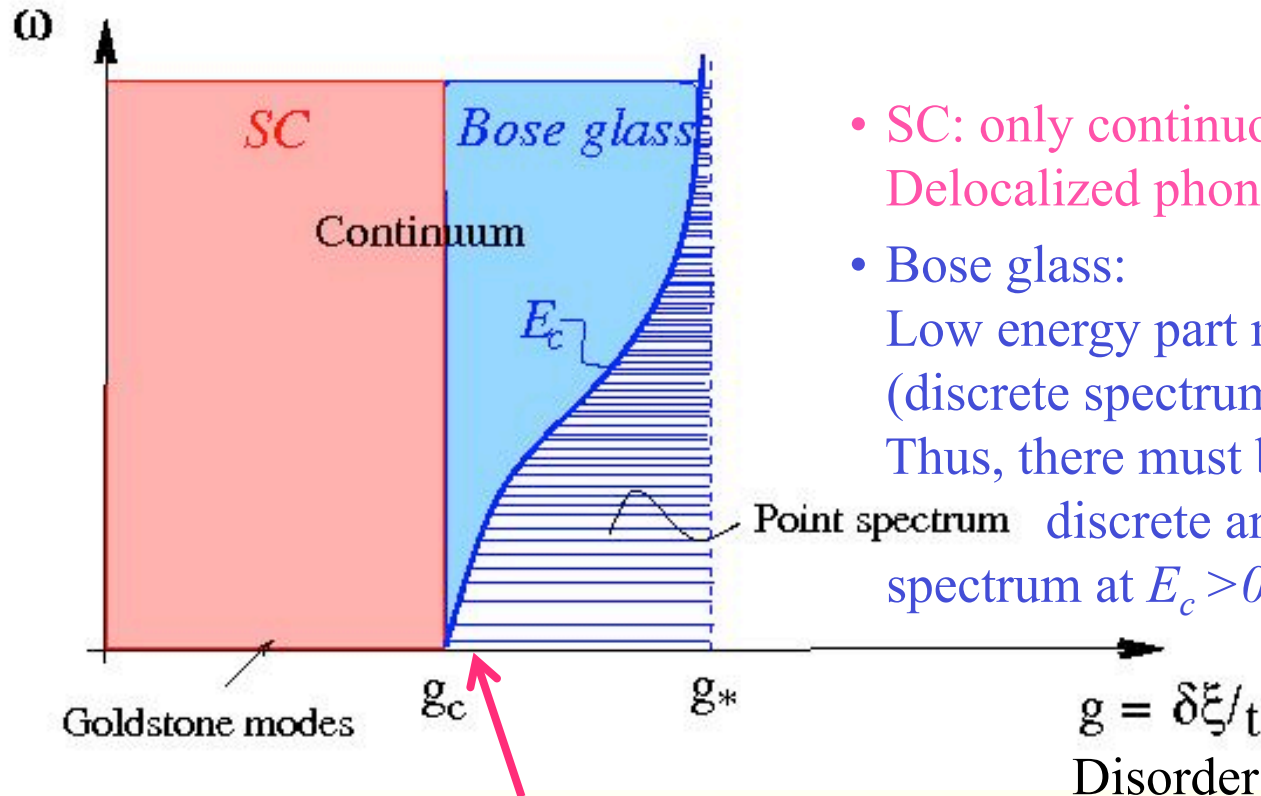


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- Bose glass:
Low energy part must be localized (discrete spectrum).
Thus, there must be a border between discrete and continuous spectrum at $E_c > 0$

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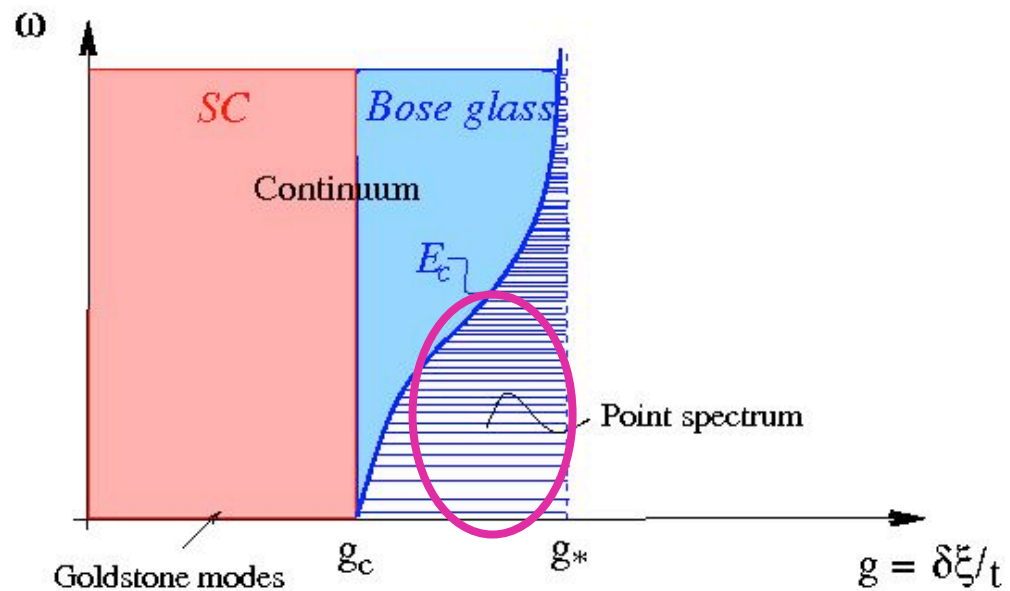


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Could there be a non-superfluid with continuum everywhere? – Probably not, see later!

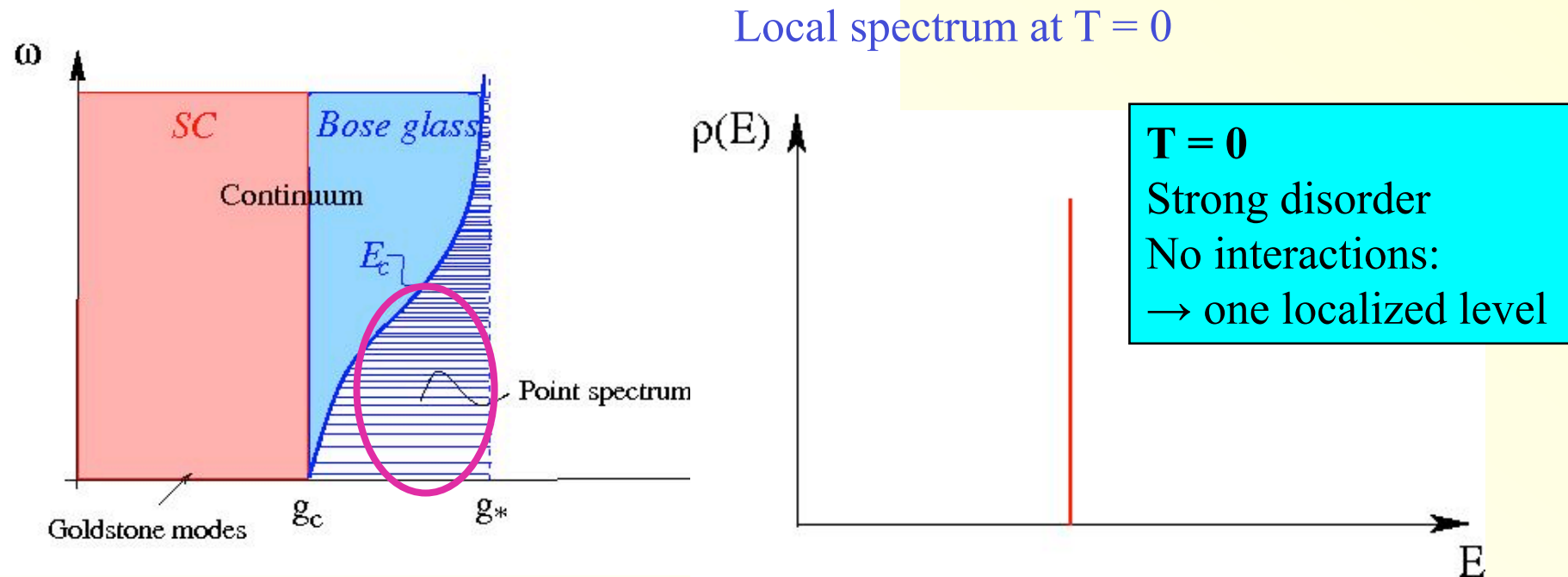
Spectrum at $T = 0$

The point spectrum at low energies



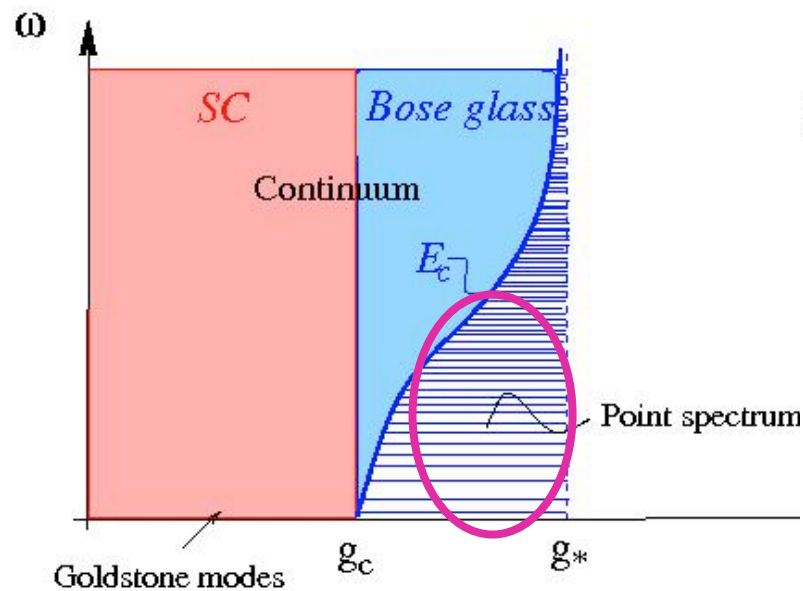
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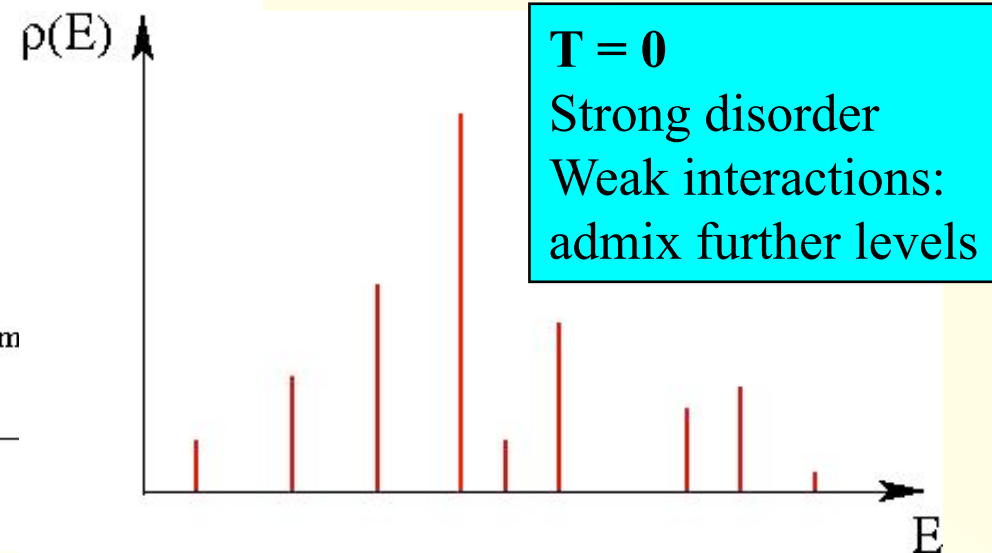


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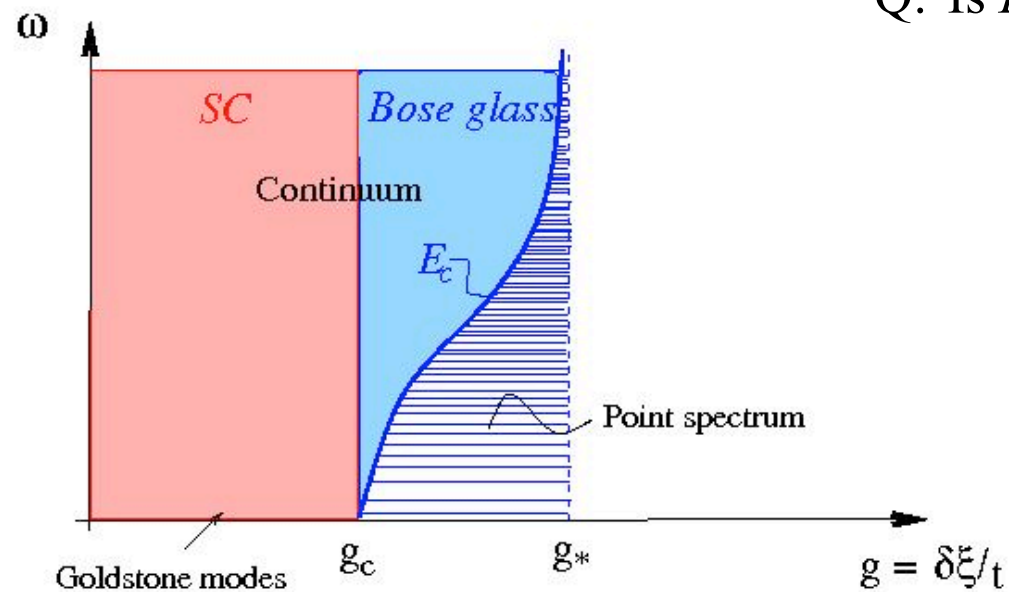


- • Discrete levels: no transport, no current!
 $\sigma(T=0) = 0$
- • Genuine glass at $T=0$: perturbations don't relax
Reason: Transition probabilities are zero because energy conservation can never be satisfied!

Mobility edge

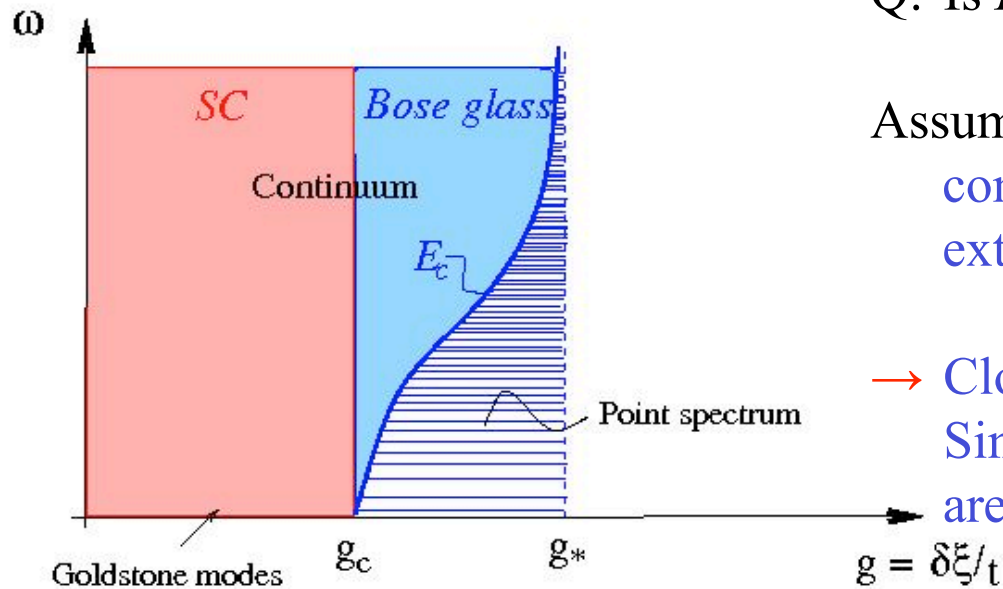
Many-body “mobility edge” in the Bose glass

Q: Is E_c finite or extensive? (\sim Volume)



Mobility edge

Many-body “mobility edge” in the Bose glass



Q: Is E_c finite or extensive? (\sim Volume)

Assumption: The SIT is essentially a condensation of a self-consistent, extended single boson mode.

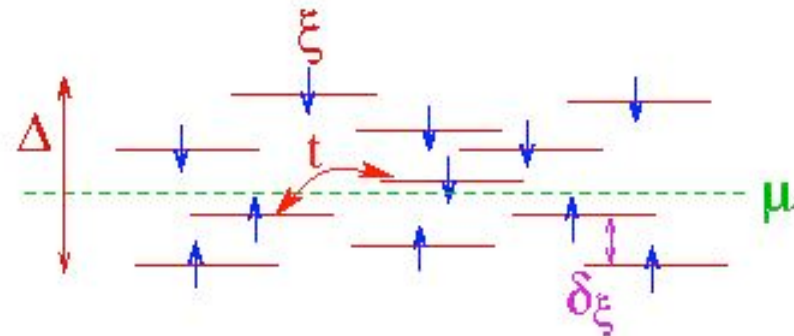
→ Close to the SIT ($g = g_c$) E_c is bounded: Single boson excitations at $E - \mu \gg t$ are still delocalized (for $d > 2$) → $E_c < \infty$

$$H = -t \sum_{\langle i,j \rangle} s_i^+ s_j^- + \sum_i (\varepsilon_i - \mu) s_i^z$$

Excited eigenstates
(only good if $E \gg t$)

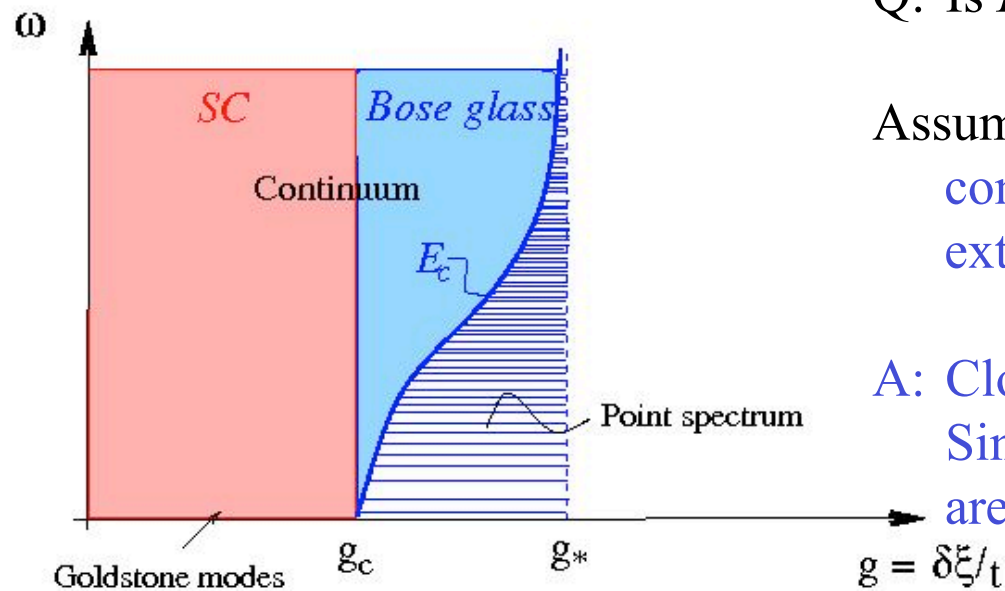
$$|E\rangle_{\text{single boson}} = \sum_i a_i s_i^+ |GS\rangle$$

$$(\varepsilon_i - \mu) a_i - t \sum_{j \in \partial i} a_j = E a_i$$



Mobility edge

Many-body “mobility edge” in the Bose glass



Q: Is E_c finite or extensive? (\sim Volume)

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A: Close to the SIT ($g = g_c$) E_c is bounded: Single boson excitations at $E - \mu \gg t$ are still delocalized (for $d > 2$) $\rightarrow E_c < \infty$

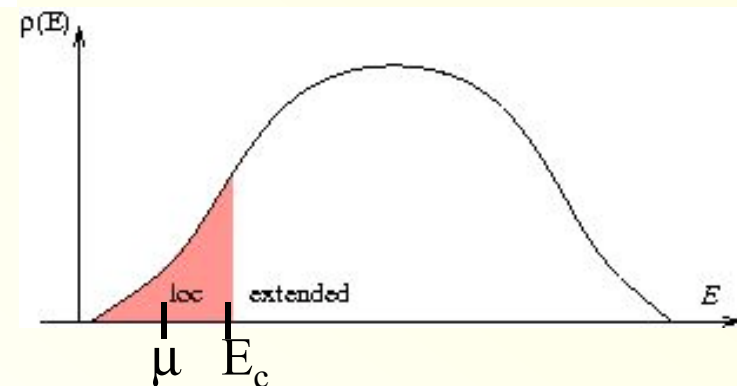
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Close analogon:

Localization at band edge (Anderson model)

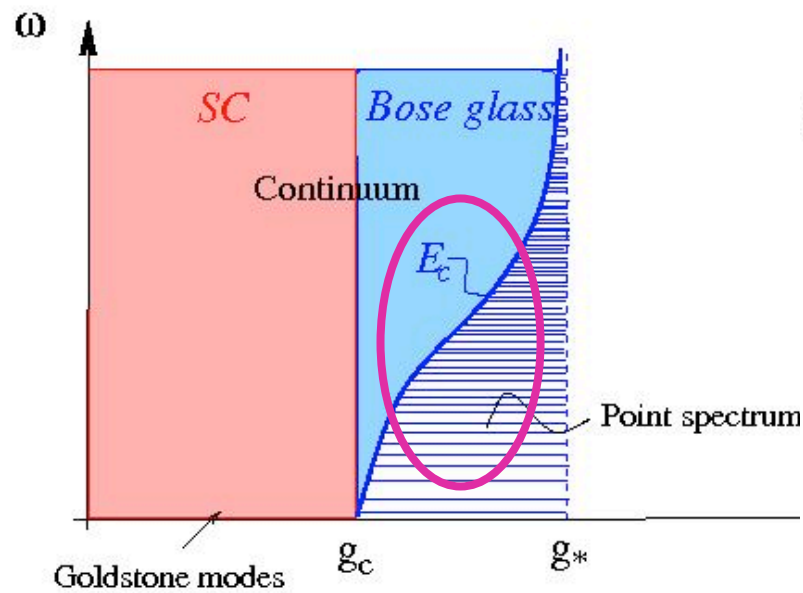
(see also Feigelman, Ioffe, Mézard, arXiv:1006.5767)

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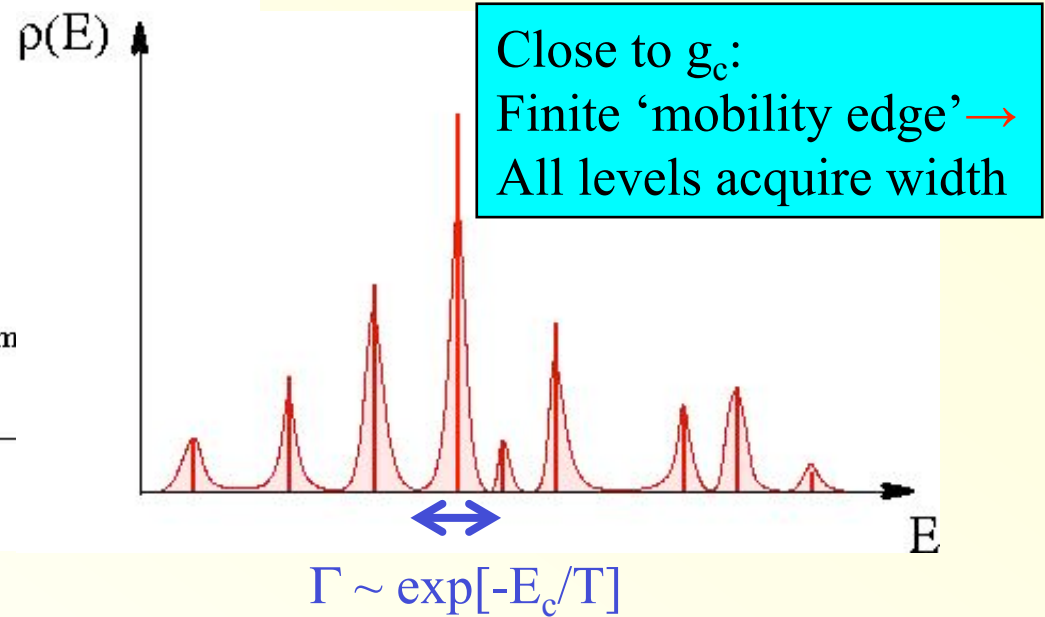


Finite T

The point spectrum at low energies

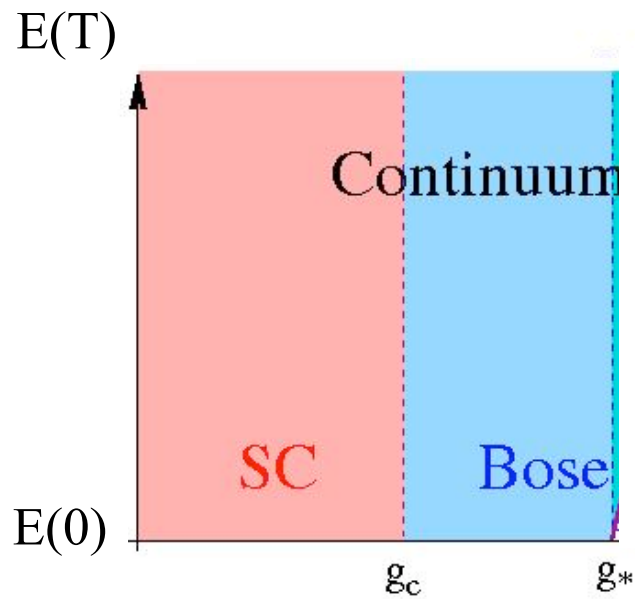


Local spectrum at $T > 0$

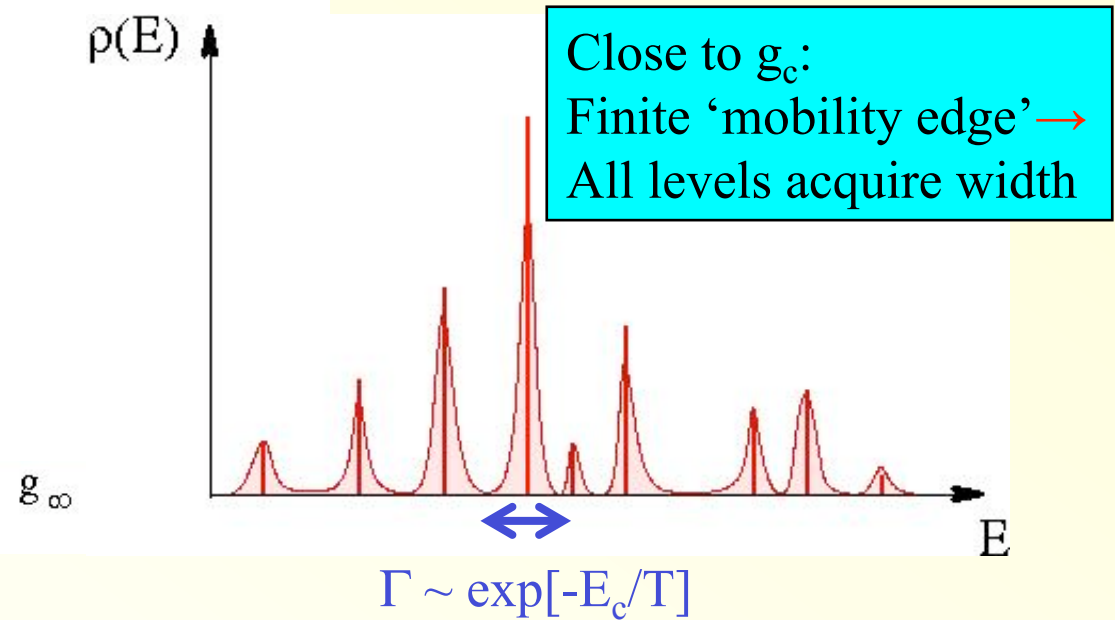


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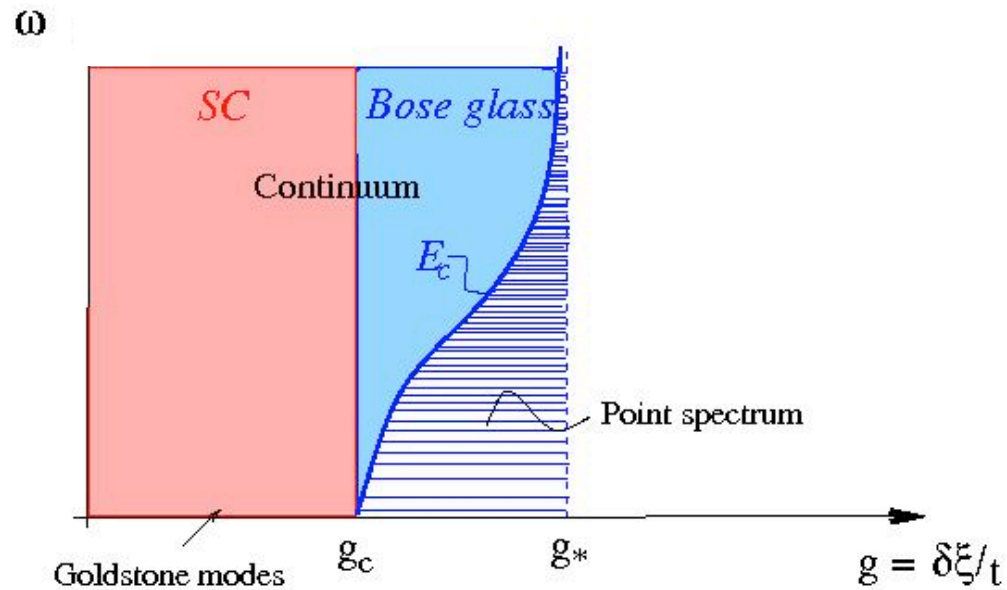


- Continuum everywhere! $\sigma(T > 0) \neq 0$ for $g < g_*$ where $E_c(g) < \infty$

Electronic activated conduction

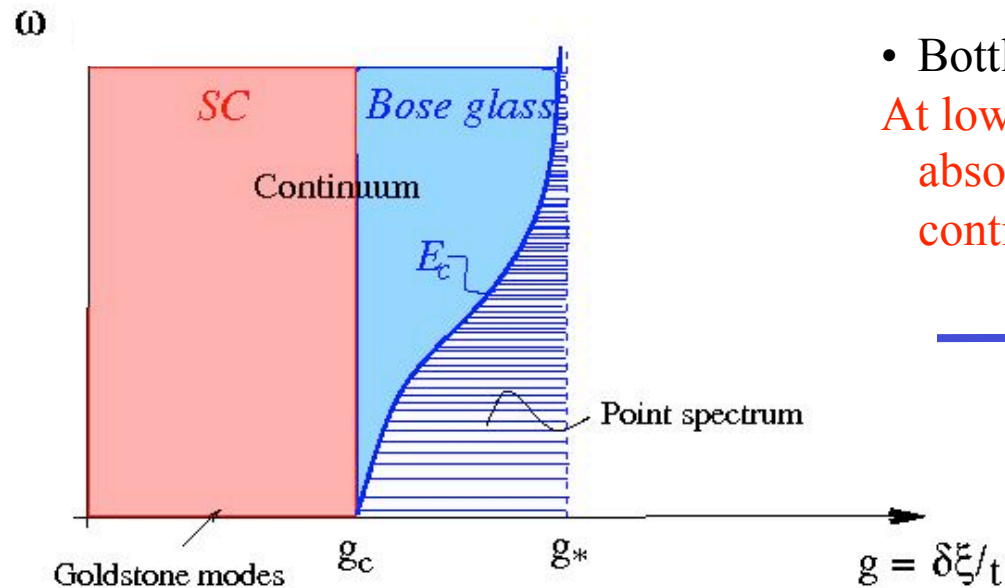
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- Bottle neck for conduction:

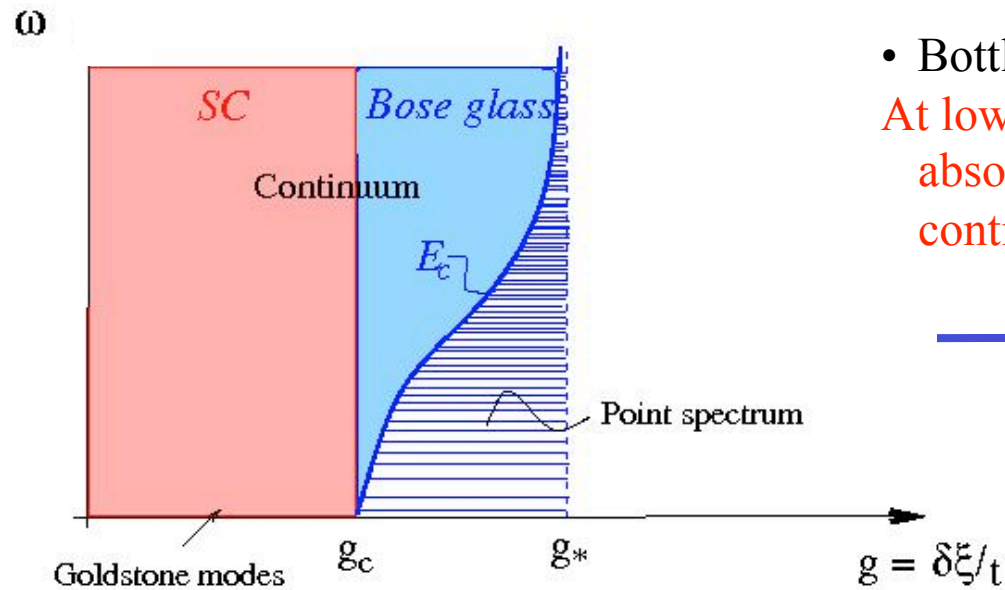
At low T: Transitions allowed only due to the absorption of modes from the (T=0) continuum or diffusion above E_c



$$\sigma(T) \sim \sigma_0 \exp[-E_c/T]$$

Electronic activated conduction

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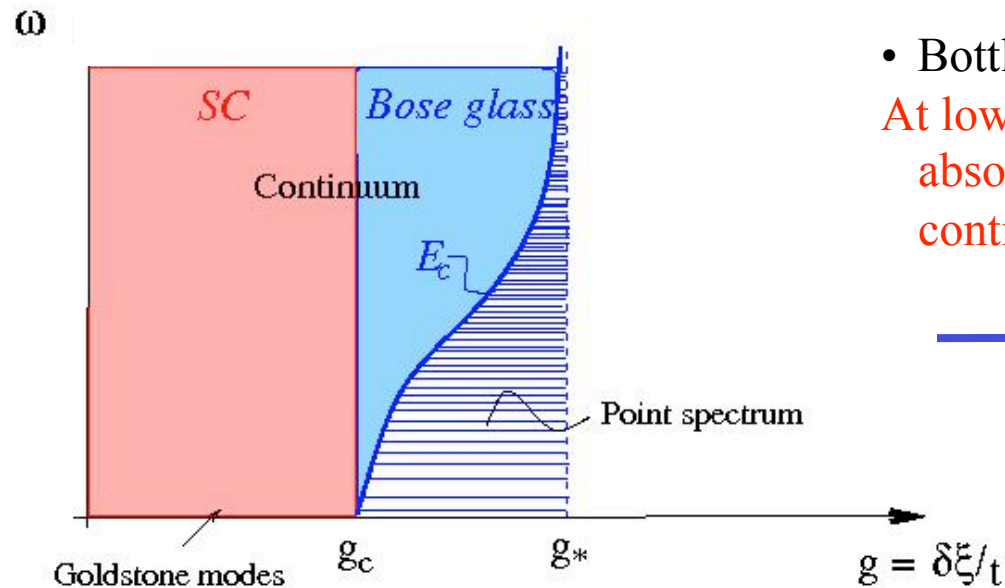


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Simple activation (Arrhenius) law in a compressible, gapless system!
No variable range hopping e^{1/T^α} !

Electronic activated conduction

$$g < g_* : E_c(g) < \infty$$



- Continuum at finite T! $\rightarrow \sigma(T>0) \neq 0$

- Bottle neck for conduction:

At low T: Transitions allowed only due to the absorption of modes from the (T=0) continuum or diffusion above E_c



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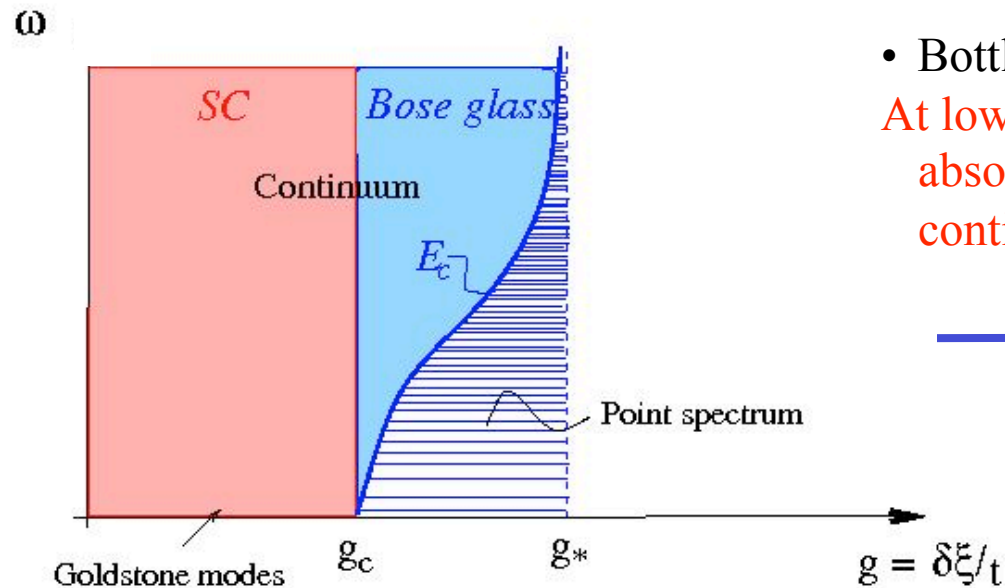


- No phonons needed! (they are anyway very inefficient at low T)
- Purely electronic transport mechanism
 \rightarrow crucial ingredient to explain the overheating in the non-Ohmic regime
- Prefactor: $\sigma_0 \sim e^2/h\xi^{d-2}$ nearly universal in quasi 2d, similar to experiment!



Electronic activated conduction

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! T effect:

Inelastic scattering rate at $T > 0$ lowers the activation energy needed to get diffusion! $\rightarrow E_{\text{act}} = E_c - \Delta E(T)$! \rightarrow superactivation!

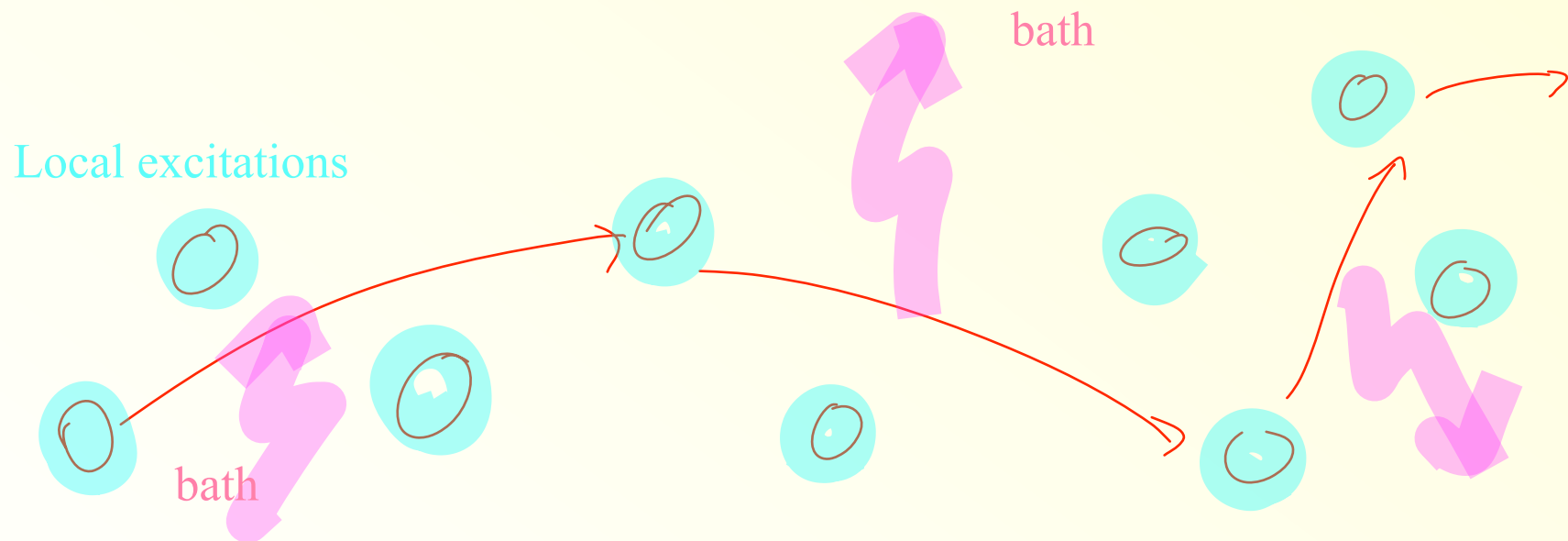
Why no standard
variable range
hopping transport?

--

Is there many body
localization??

? Transport and thermalization in insulators ?

Essential ingredient into variable range hopping:
Continuous bath which activates the hops!

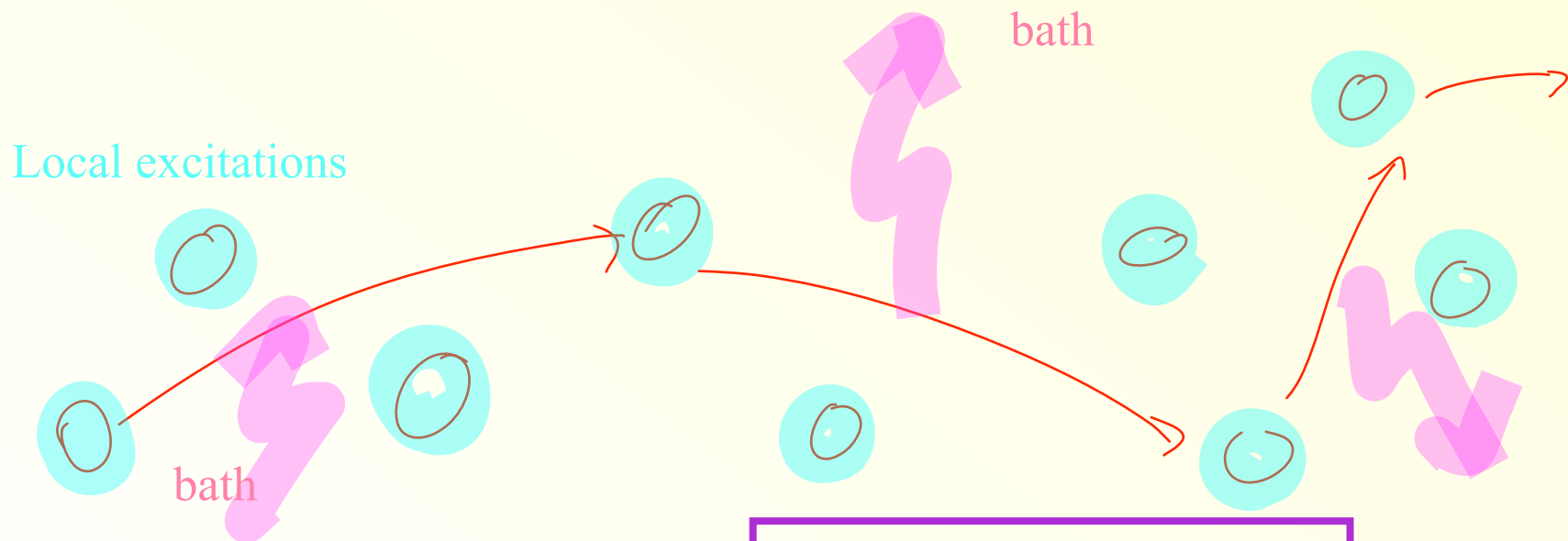


Candidates for the bath:

- Phonons: at low T for pair hopping are very inefficient!

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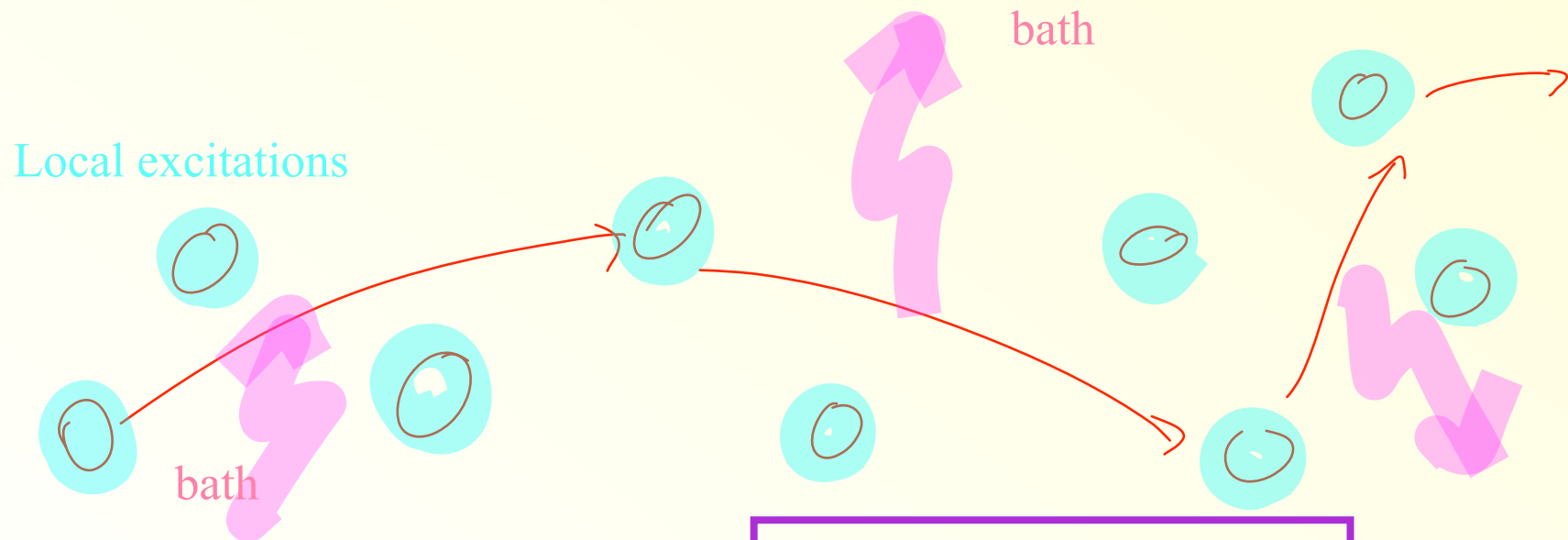
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? Transport and thermalization in insulators ?

Essential ingredient into variable range hopping:
Continuous bath which activates the hops!



Candidates for the bath:

- ~~Phonons: at low T for pair hopping are very inefficient!~~
- bosonic excitations **above the many body(!) mobility edge**

Too weak → not considered

What if there is no
bath whatsoever?

Strong disorder

$g > g_*$: $E_c(g) = \infty$ (\sim Volume)

- If disorder is strong ($g = \delta_\xi/t > g_*$) *all single (and few) boson excitations* above the GS (at $T = 0$) are localized: $E_c \rightarrow \infty$. **Then there is NO bath!**

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When inelastic rate \sim level spacing $\delta_\xi \rightarrow$ self-consistent level broadening

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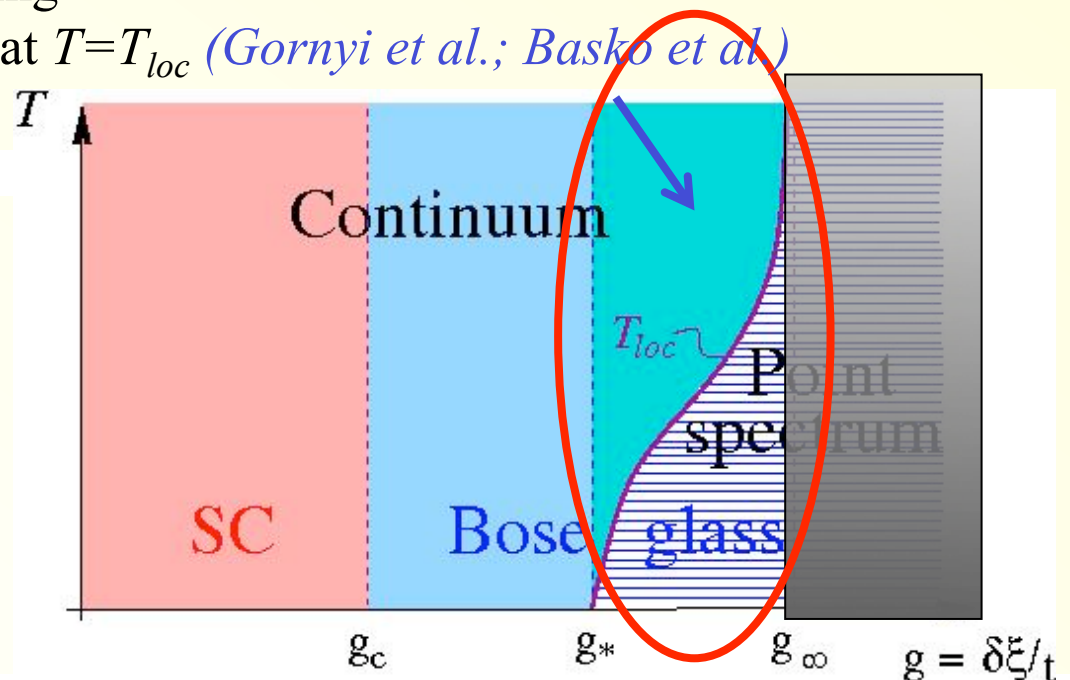
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delocalization in Fock space at $T = T_{loc}$ (*Gornyi et al.; Basko et al.*)

\rightarrow **Finite T transition from**
 $\sigma = 0$ to $\sigma > 0$ state!



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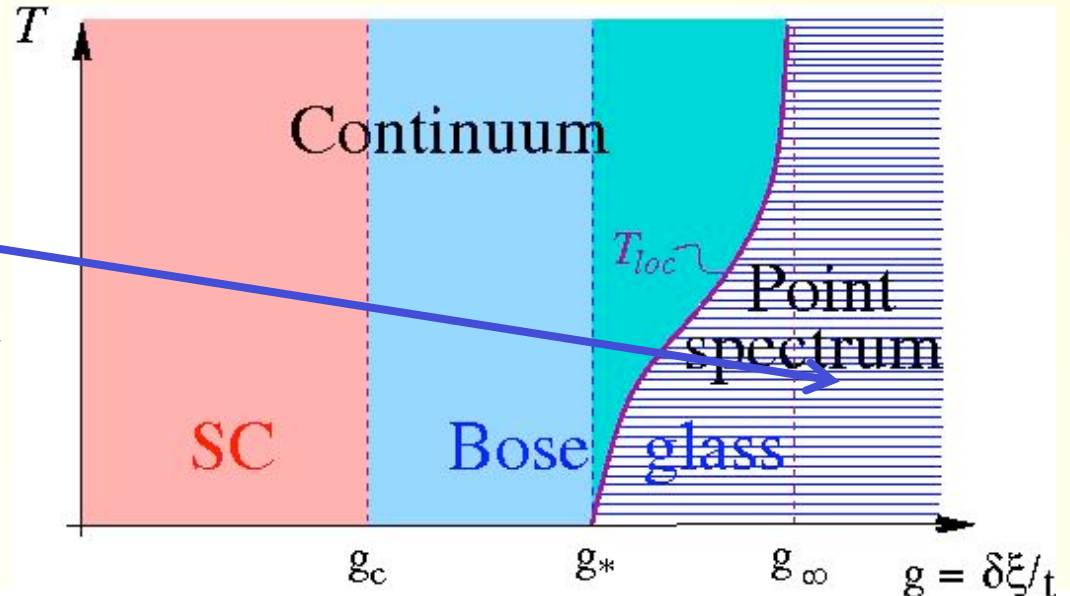
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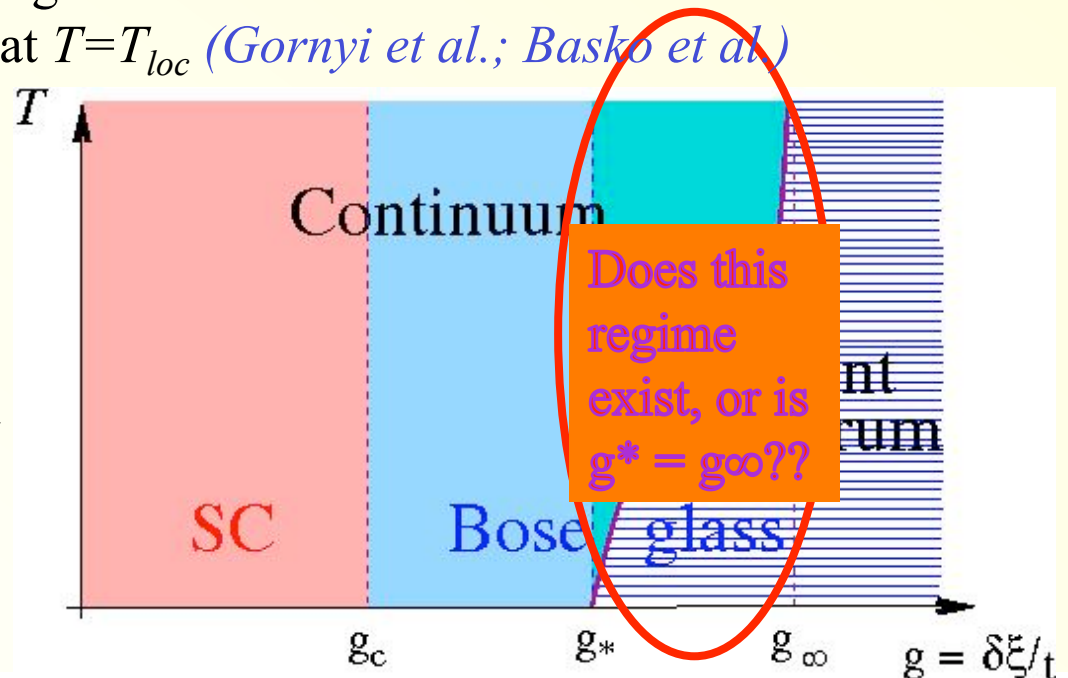
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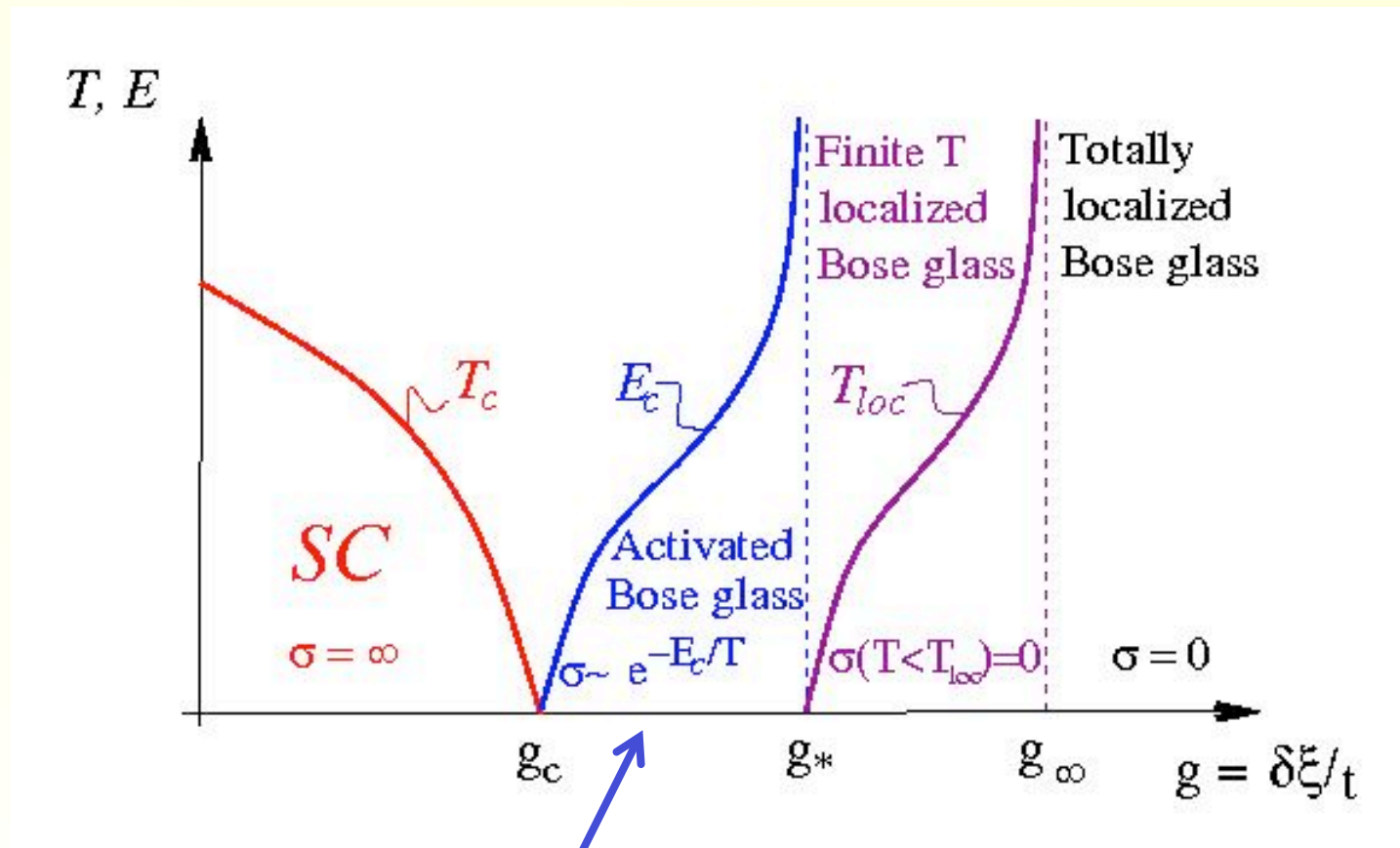
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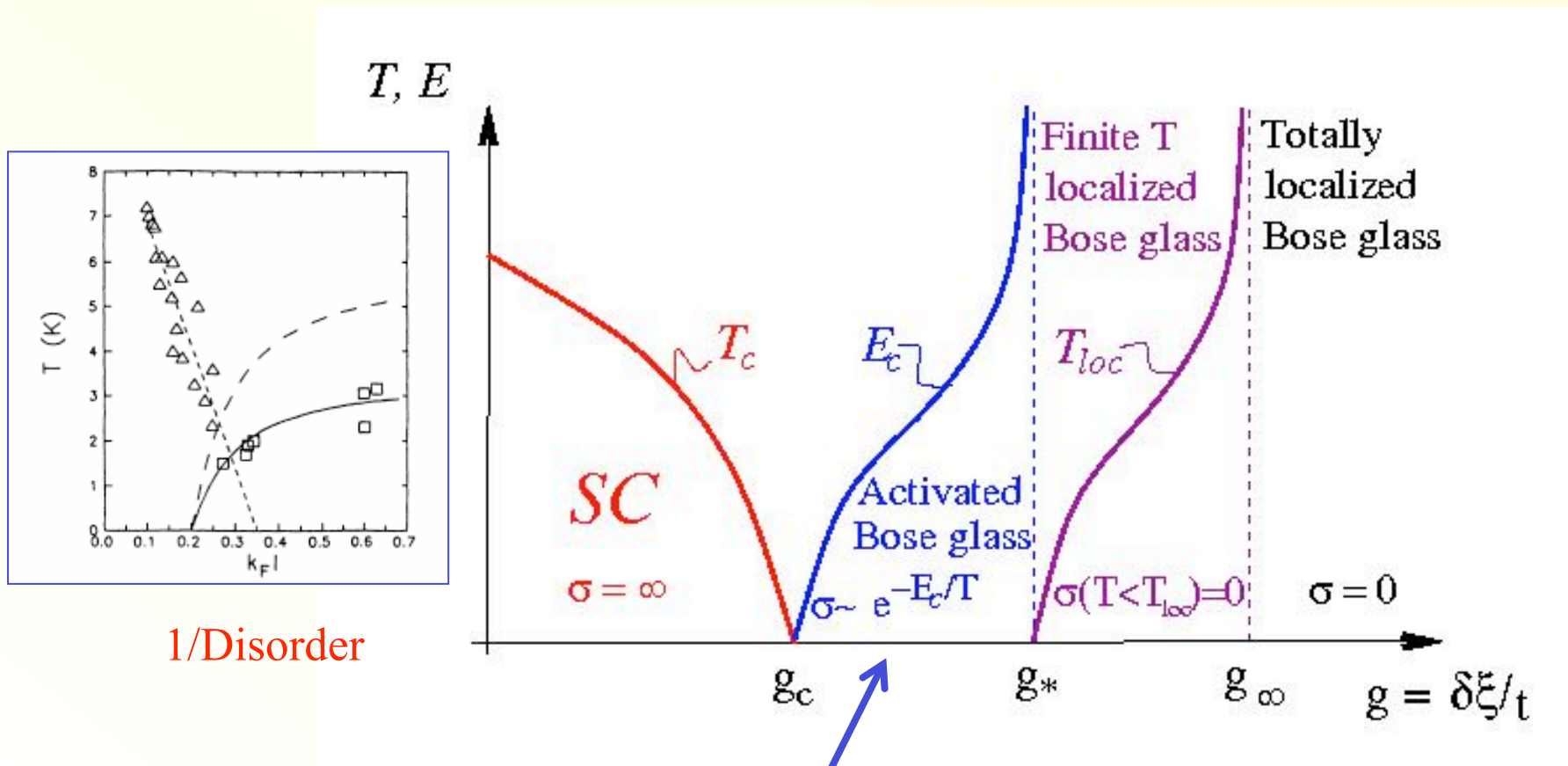


Summary: Bose-Hubbard model and Bose glass



Purely electronic transport at low T: **Asymptotically** Arrhenius law!

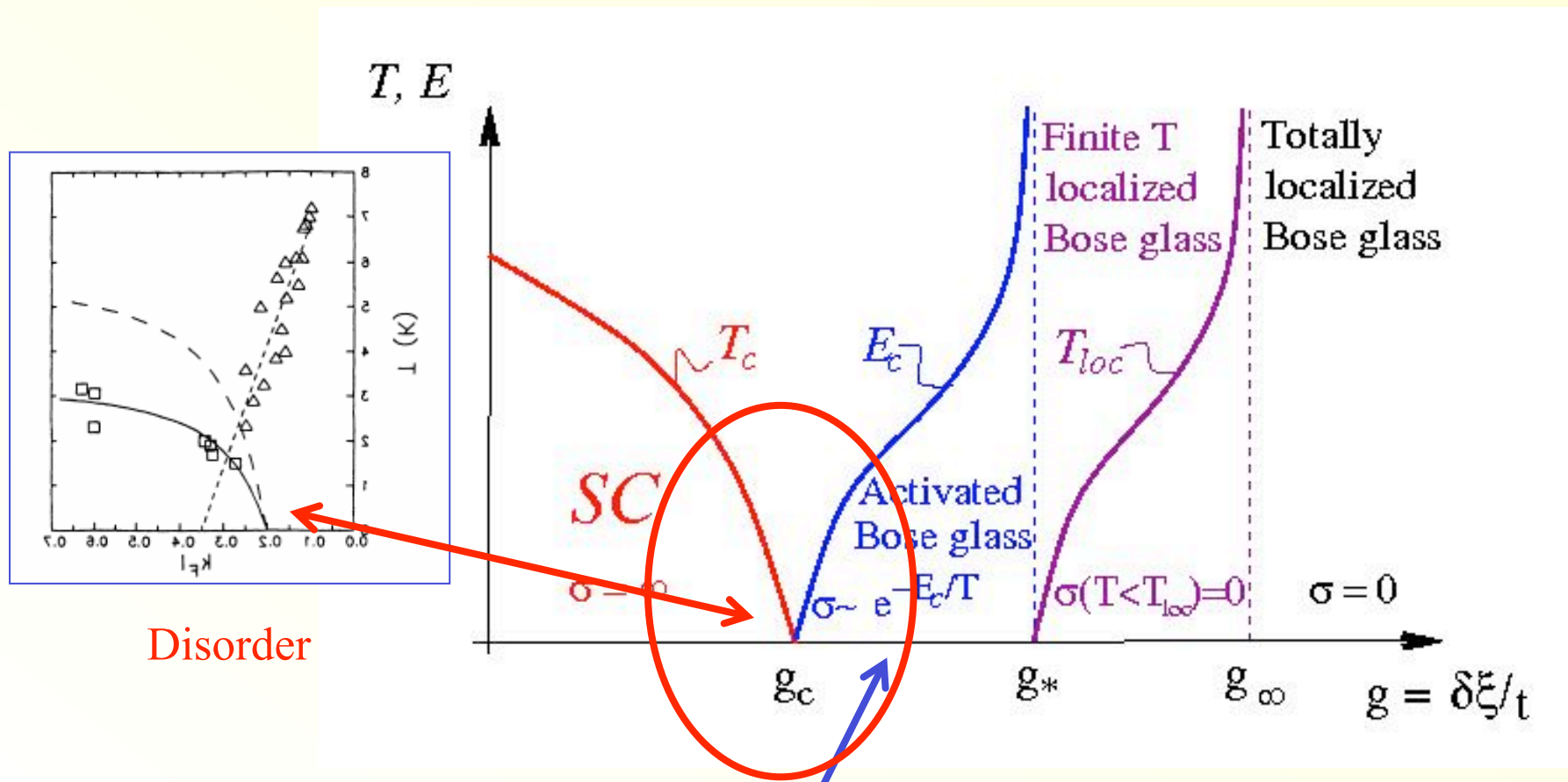
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1/Disorder

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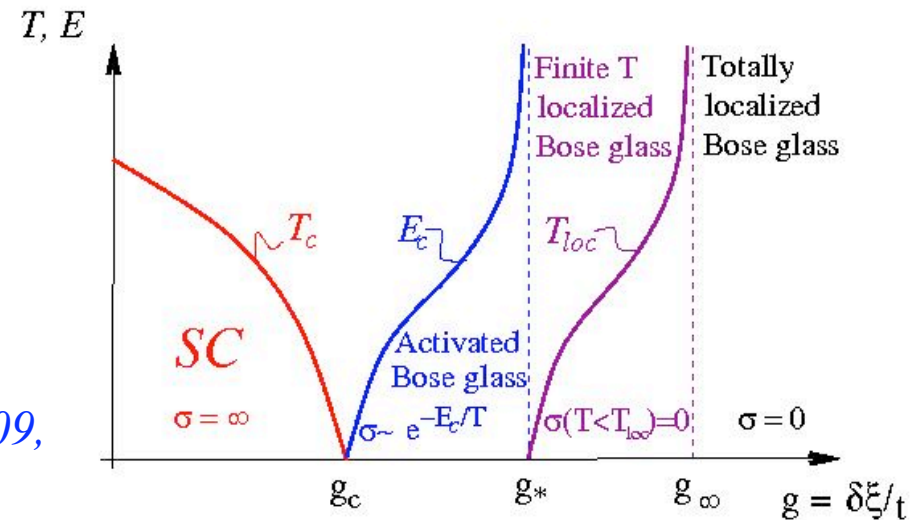
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Summary: Bose-Hubbard model and Bose glass

Can this scenario be proved?

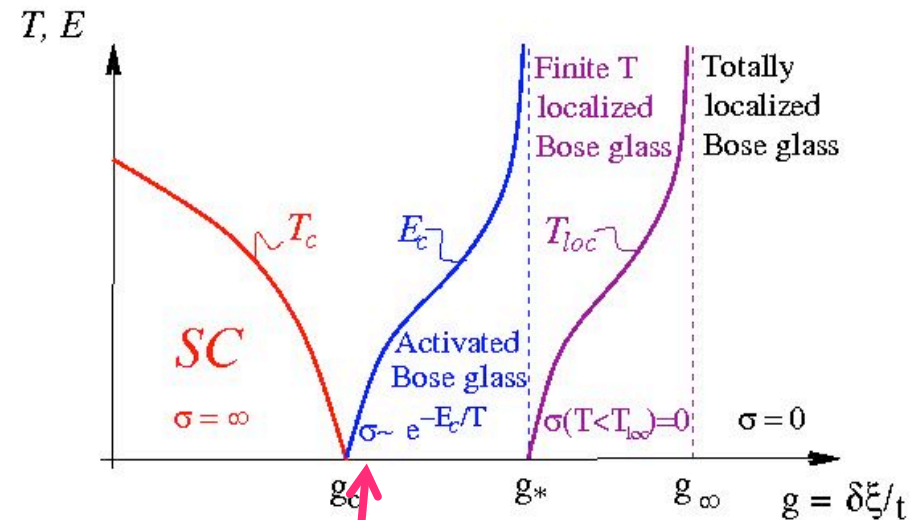
- Total localization: similar to Mirlin et al. and Basko et al. (*Aleiner et al.*, '09);
- Finite mobility edge: Approximative solution for hard core bosons on high connectivity Bethe lattice (*Ioffe & Mézard '09*, *Feigelman, Ioffe, Mézard '10*): mobility edge and finite T broadening, as discussed
- Open Q: Is the scenario true in $d = 1$ and 2 ?
 - Aleiner, Altshuler & Shlyapnikov conjecture: direct transition from SC to manybody localization
 - My conjecture: intermediate phase also in $d < 3$, or at most a very weakly volume-dependent E_c



Non-superfluid = localization at low energies?

Hints from the model on the
Bethe lattice:

1. Without frustration:
Easier for bosons to condense
(establish order parameter)
than to delocalize and (decay to
infinity)
2. On the other hand:
Condensate implies delocalized
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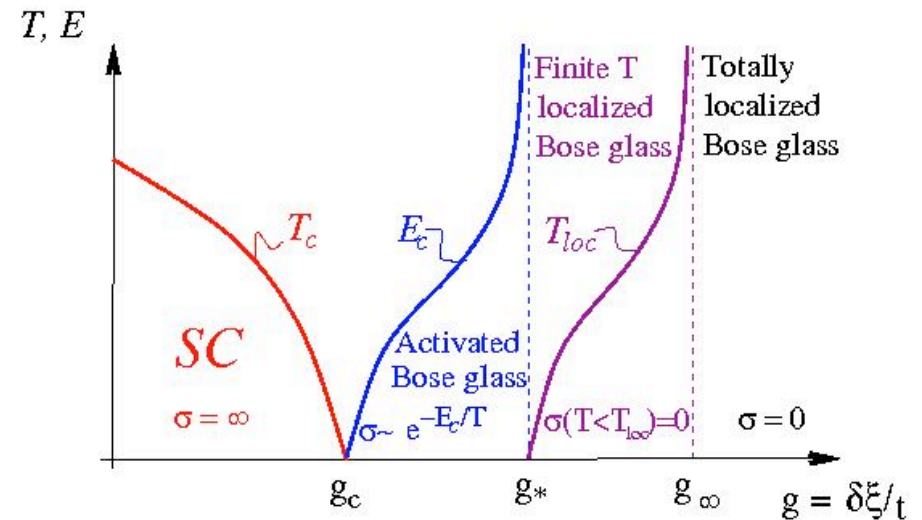


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→ SIT and spectral delocalization at $E=0$
transition occur usually at the same
point!
Possible exception: in magnetic field

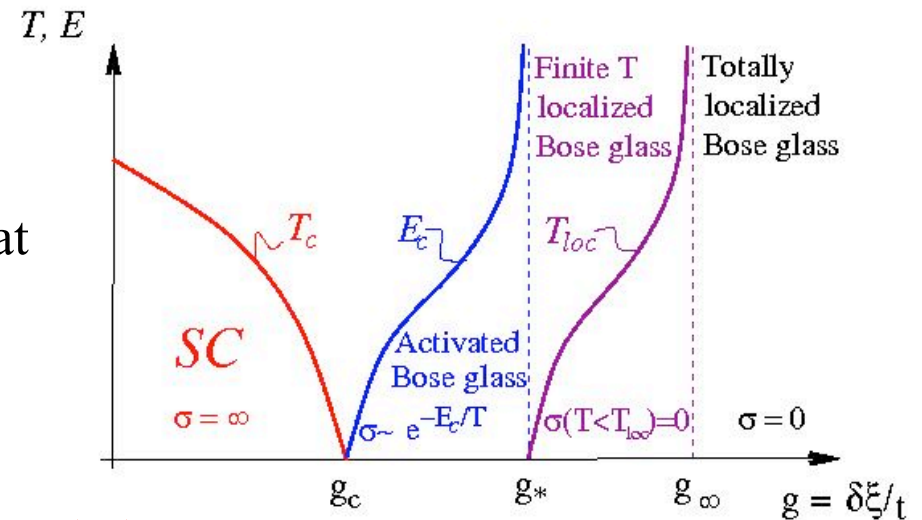


How to test the activated Bose glass scenario?

Predictions:

- Hard gap for single electrons
(→ as observed in tunneling)
- Absence of delocalized electronic modes at low energy! Experimental consequences:

- **discrete** low energy spectrum
- very low **microwave absorption**
- only imaginary (**non-dissipative**) part of $\sigma(\omega)$
- very **inefficient electron-phonon coupling**
(as observed in InOx → strong heating)
- **energy/charge diffusion may set in after a minimal, finite energy injection!**



Conclusion

- Transport in the Bose glass is a rich problem due to manyparticle localization (quantum interference) phenomena
- SI transition: promising system to observe those and their precursors
- Similar ideas apply to disordered magnetic quantum phase transitions and the metal-insulator transition

