Graphene: Relativistic transport in a nearly perfect quantum liquid

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in collaboration with

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(AdS/CFT)

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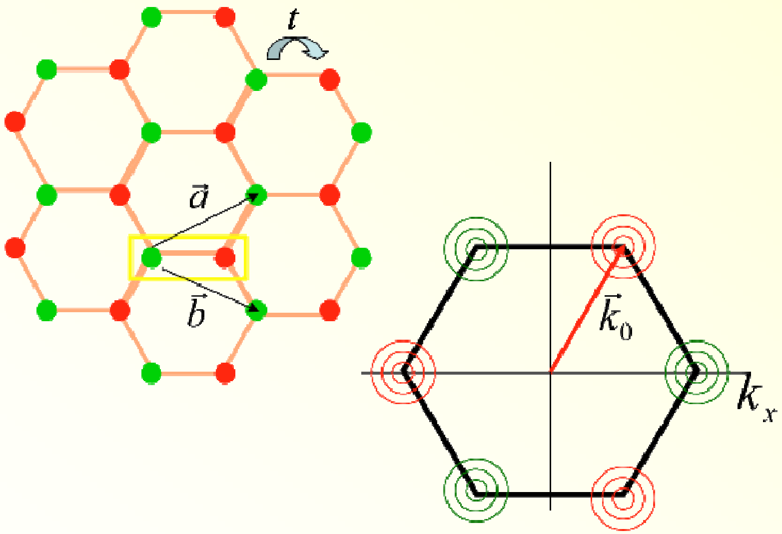
Outline

- Relativistic physics in graphene, quantum critical systems and conformal field theories
- Strong coupling features in collision-dominated transport
- Comparison with strongly coupled fluids (via AdS-CFT)
- Graphene: an almost perfect quantum liquid

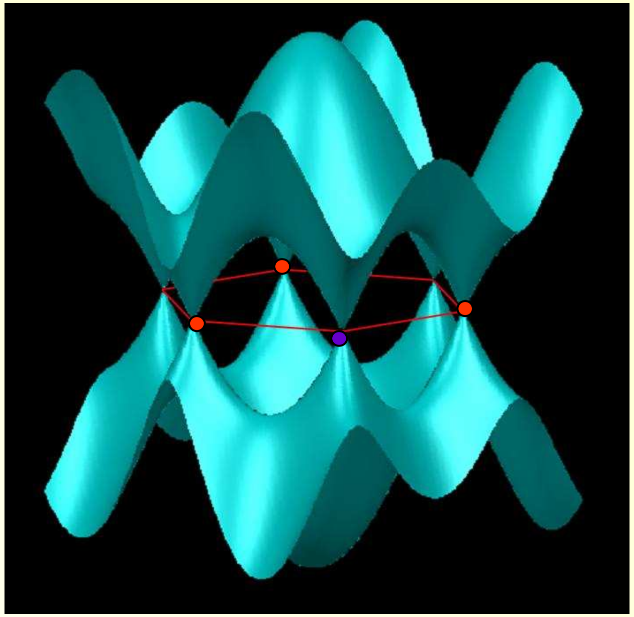
Dirac fermions in graphene

(Semenoff '84, Haldane '88)

Honeycomb lattice of C atoms



Tight binding dispersion



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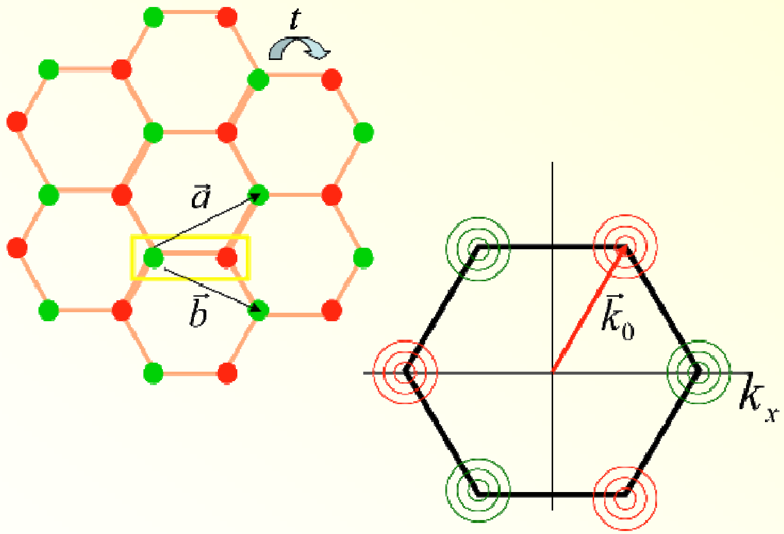

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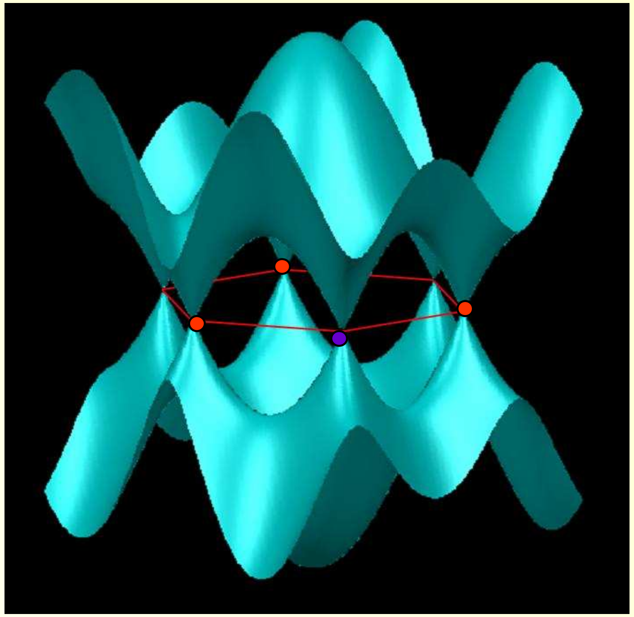

2 massless Dirac cones in the Brillouin zone: (Sublattice degree of freedom ↔ pseudospin)

Close to the two Fermi points **K**, **K'**:

$$H \approx \mathrm{v}_F \left(\vec{\mathbf{p}} - \vec{\mathbf{K}}\right) \cdot \vec{\sigma}_{\text{sublattice}}$$

$$\rightarrow \quad E_{\mathbf{p}} = \mathrm{v}_F \left|\vec{\mathbf{p}} - \mathbf{K}\right|$$

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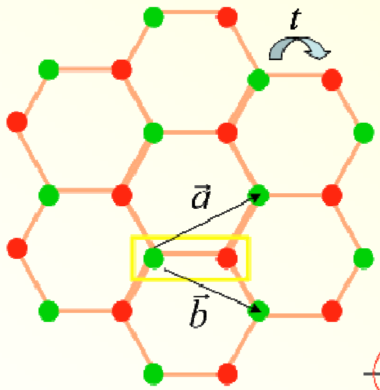

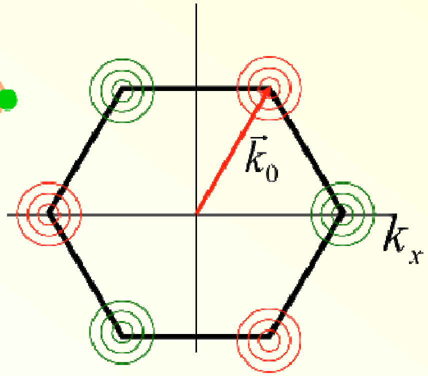

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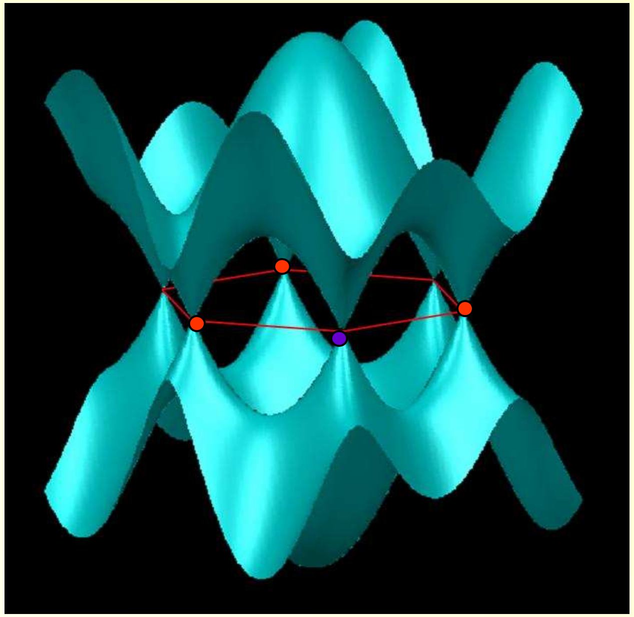

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Fermi velocity (speed of light")

$$v_F \approx 1.1 \cdot 10^6 \text{ m/s} \approx \frac{c}{300}$$

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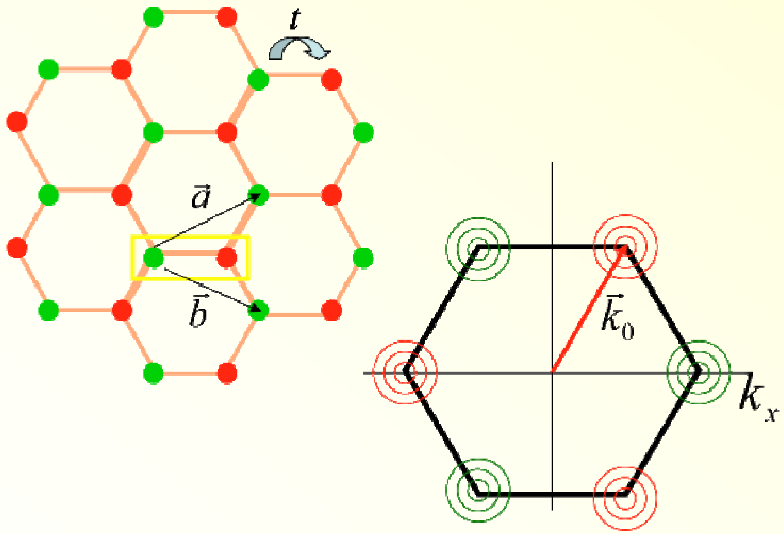

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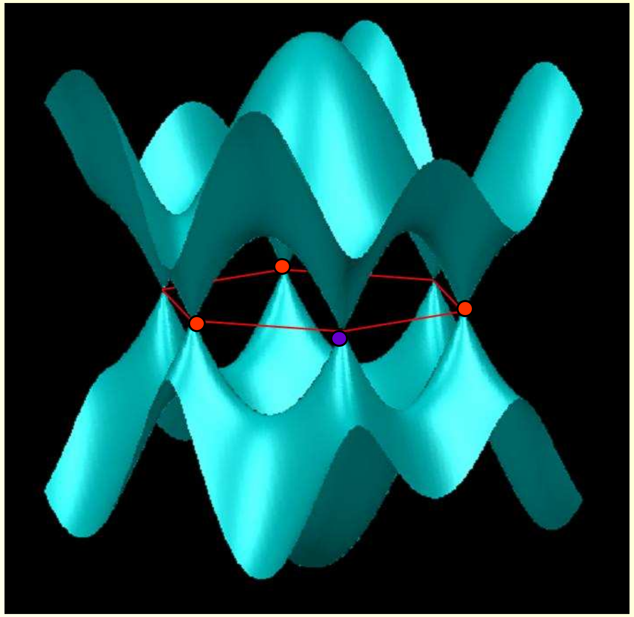

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Coulomb interactions: Fine structure constant

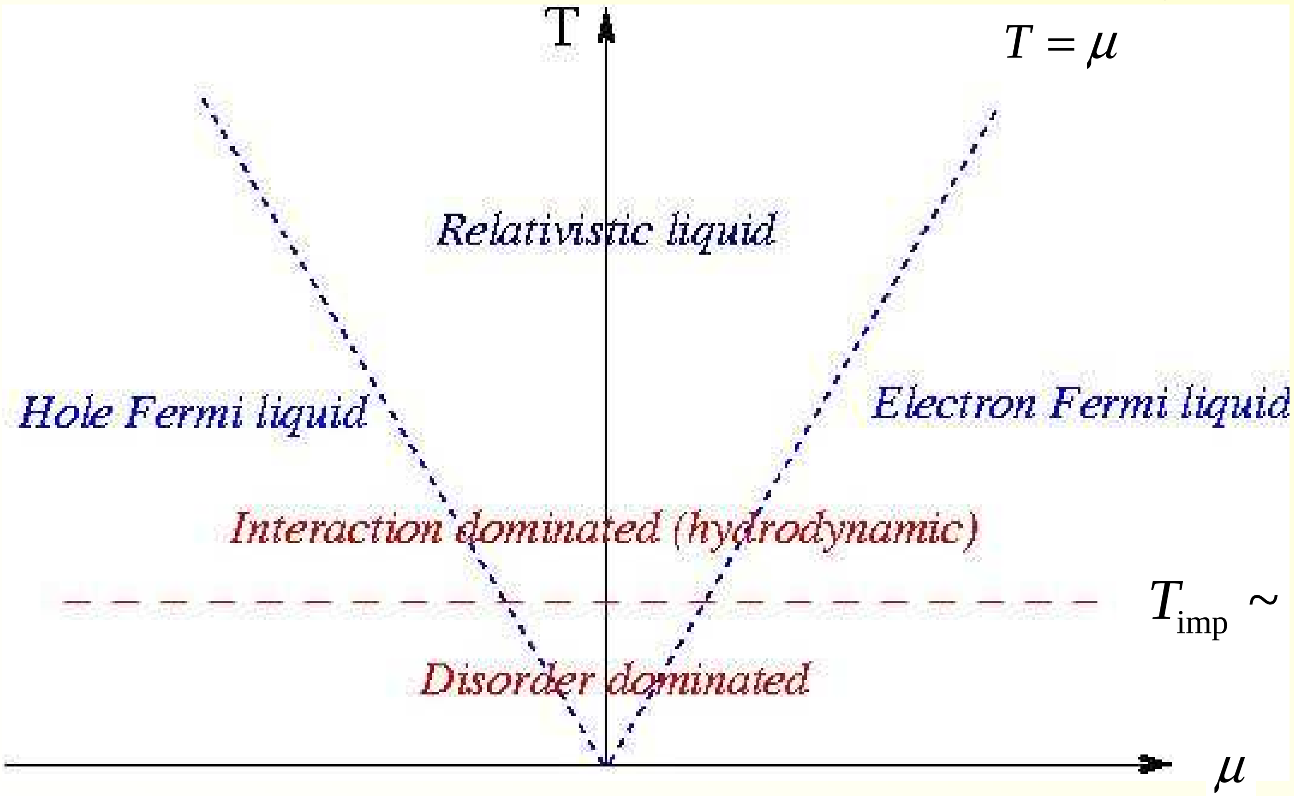
$$\alpha \equiv \frac{e^2}{\varepsilon \hbar \mathrm{v}_F} = O(1)$$

Relativistic fluid at the Dirac point

D. Sheehy, J. Schmalian, Phys. Rev. Lett. ***99****, 226803 (2007).*

- Relativistic plasma physics of interacting particles and holes!

Crossover:
$T = \mu$



$T_{\mathrm{imp}} \sim \rho_{\mathrm{imp}}$

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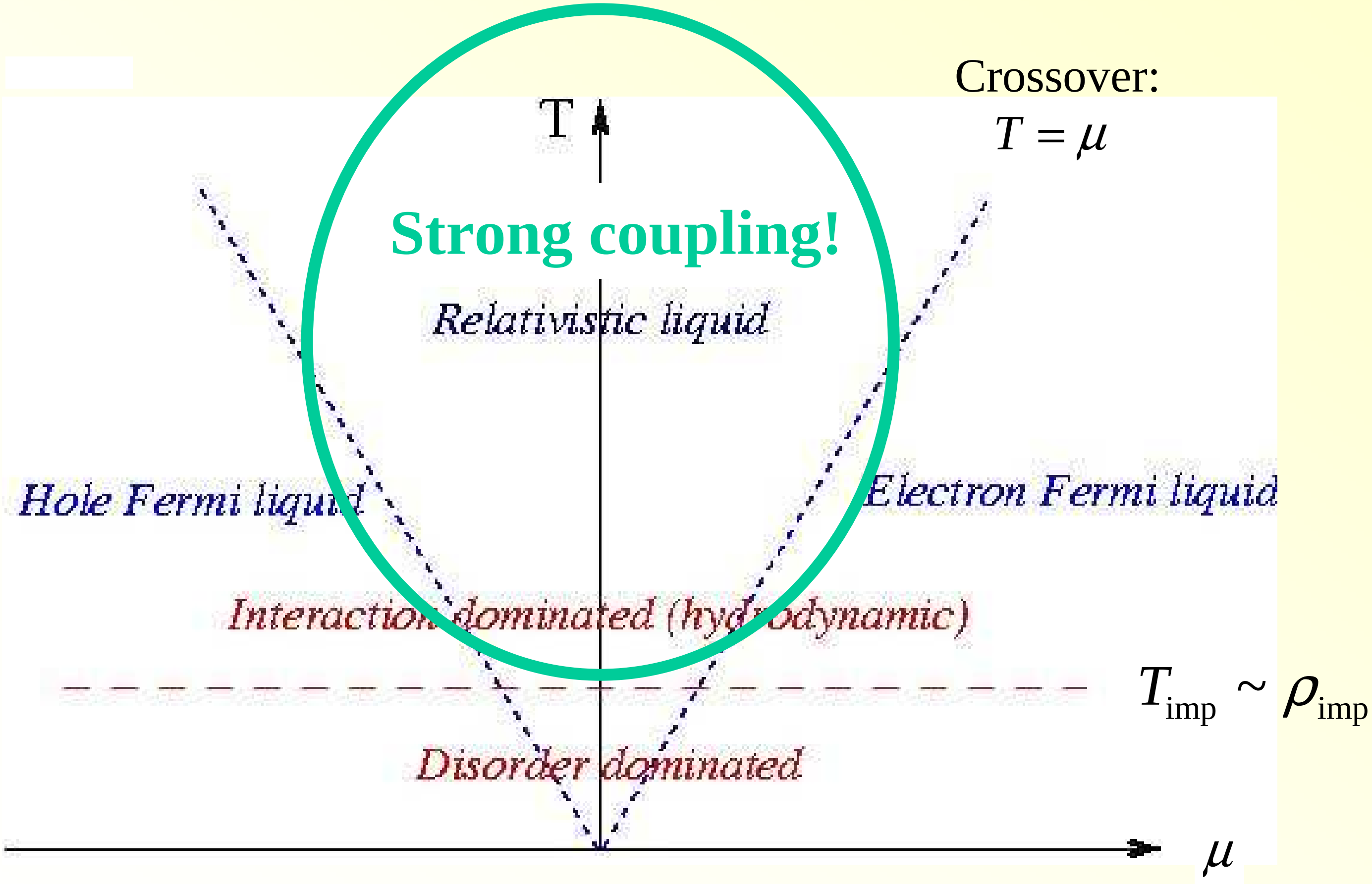

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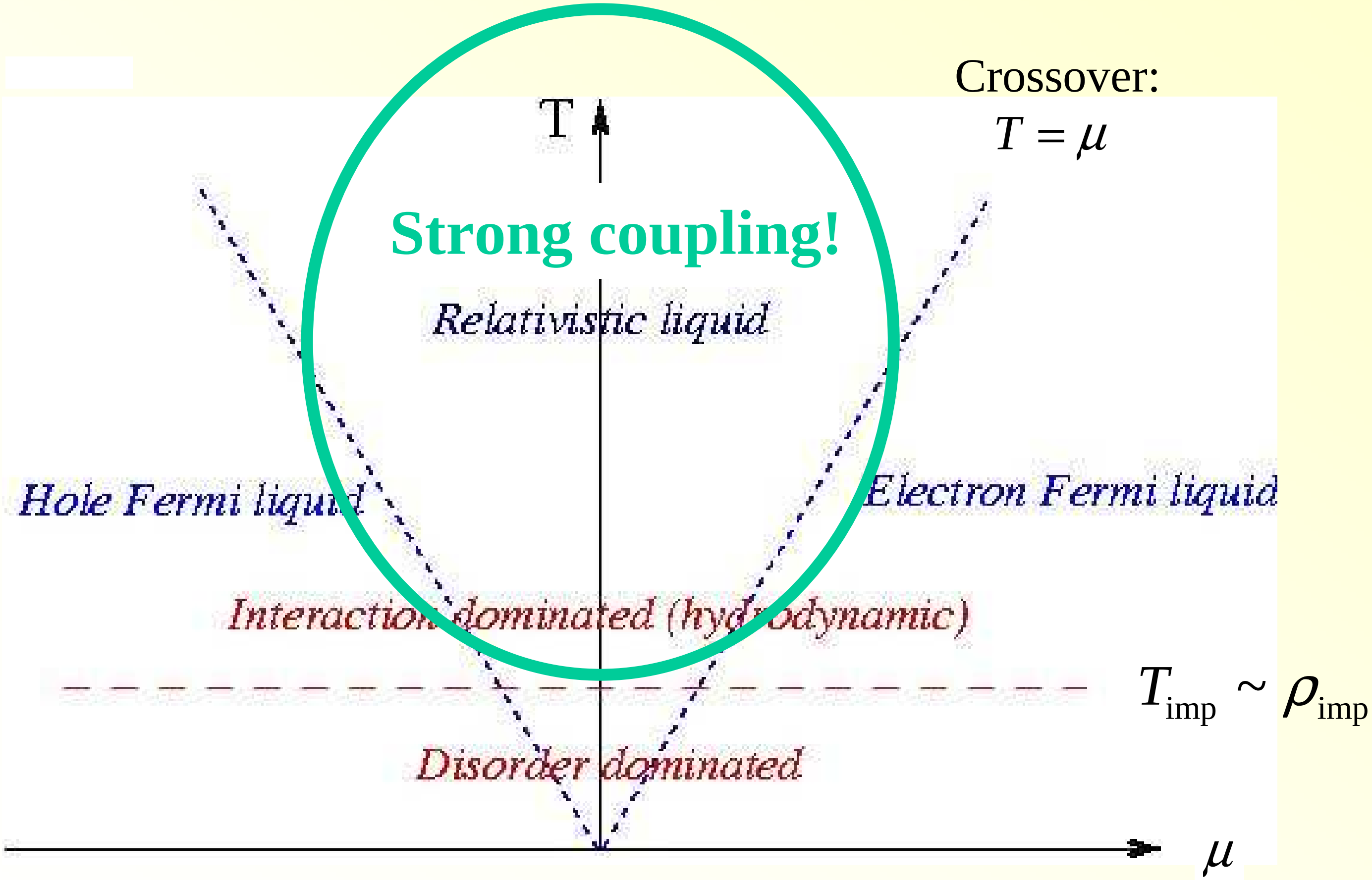

Very similar as at quantum criticality (with z=1, e.g. SIT) and the associated CFT's

Graphene – Fermi liquid?

1. Tight binding kinetic energy
$\rightarrow$ massless Dirac quasiparticles

$$H_0 = \sum_{\lambda=\pm} \sum_{a=1}^{N} \int \frac{d^2k}{(2\pi)^2} \lambda v_F k \, \gamma^\dagger_{\lambda a}(\mathbf{k}) \gamma_{\lambda a}(\mathbf{k})$$

2. Coulomb interactions:
Unexpectedly strong!
$\rightarrow$ nearly quantum critical!

$$\boxed{V(\mathbf{q}) = \frac{2\pi e^2}{\varepsilon |\mathbf{q}|}}$$

$$H_1 = \frac{1}{2} \int \frac{d^2k_1}{(2\pi)^2} \frac{d^2k_2}{(2\pi)^2} \frac{d^2q}{(2\pi)^2} \Psi^\dagger_a(\mathbf{k}_2 - \mathbf{q}) \Psi_a(\mathbf{k}_2) V(\mathbf{q}) \Psi^\dagger_b(\mathbf{k}_1 + \mathbf{q}) \Psi_b(\mathbf{k}_1)$$

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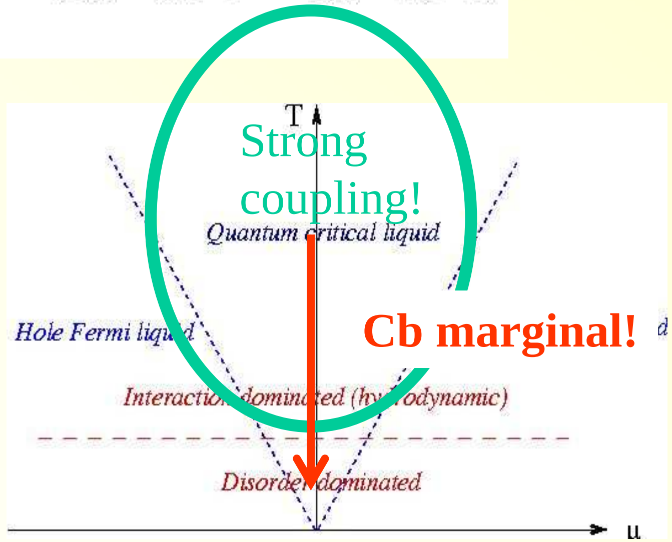

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(μ = 0)

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$$\boxed{\alpha \equiv \frac{e^2}{\varepsilon\hbar v_F} = O(1)}$$

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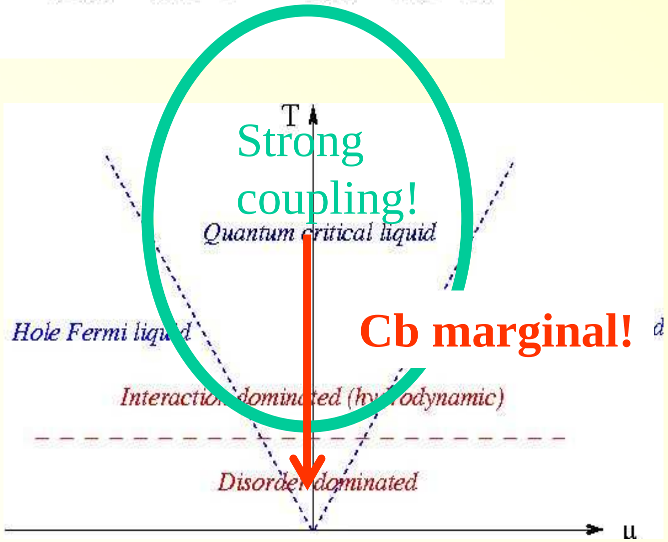

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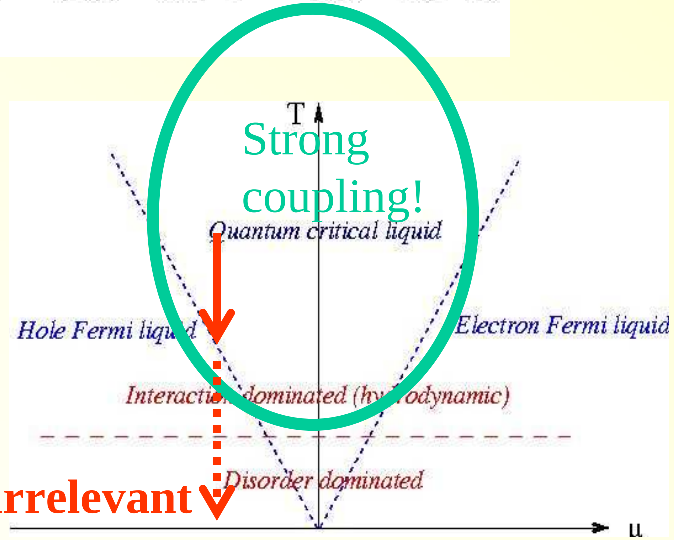
(μ > 0) $T < \mu$: **Screening kicks in, short ranged Cb irrelevant**



Strong coupling in undoped graphene

MM, L. Fritz, and S. Sachdev, PRB '08.

Inelastic scattering rate
(Electron-electron interactions)

μ > T: standard 2d
Fermi liquid

$$\boxed{\tau_{\text{ee}}^{-1} \sim \alpha^2 \frac{k_B T^2}{\hbar\mu}}$$

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Fastest possible rate!

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As long as $\alpha(T) \sim 1$, energy uncertainty is saturated, scattering is maximal
$\rightarrow$ Nearly universal strong coupling features in transport,
similarly as at the 2d superfluid-insulator transition *[Damle, Sachdev (1996, 1997)]*

Consequences for transport

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Collision-dominated transport → relativistic hydrodynamics:

a) Response fully determined by covariance, thermodynamics, and σ, η

b) Collective cyclotron resonance in small magnetic field (low frequency)

Hydrodynamic regime:
(collision-dominated)

$$\tau_{\mathrm{ee}}^{-1} >> \tau_{\mathrm{imp}}^{-1}, \omega_{\mathrm{c}}^{\mathrm{typ}}, \omega_{\mathrm{AC}}$$

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Damle, Sachdev, (1996).
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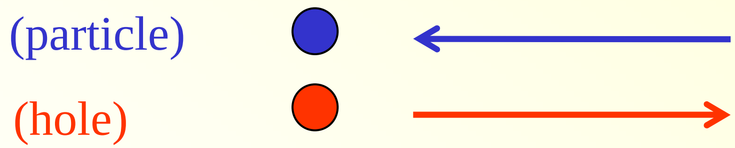

$$\vec{J} \neq 0, \quad \vec{P} = 0$$

Pair creation/annihilation leads to current decay

- Finite collision-limited conductivity! $\sigma(\mu = 0) < \infty \quad ; \quad \sigma(\mu \neq 0) = \infty$

and

- Infinite thermal conductivity! $\kappa(\mu = 0) = \infty \quad ; \quad \kappa(\mu \neq 0) < \infty$

(true also in pure semiconductors)

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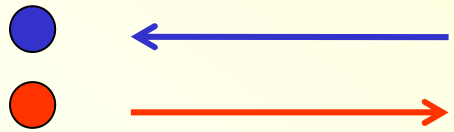

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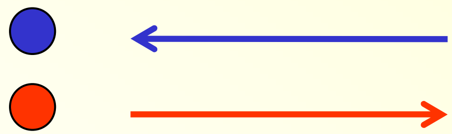

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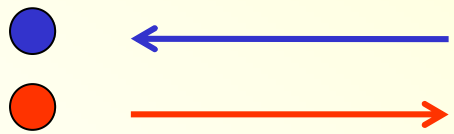

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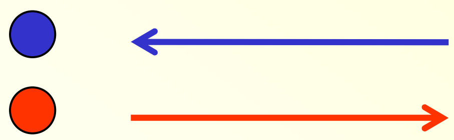

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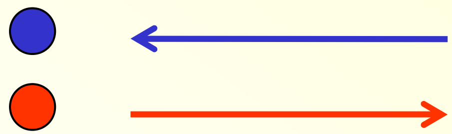

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Expect saturation as α →1, and eventually phase transition to insulator

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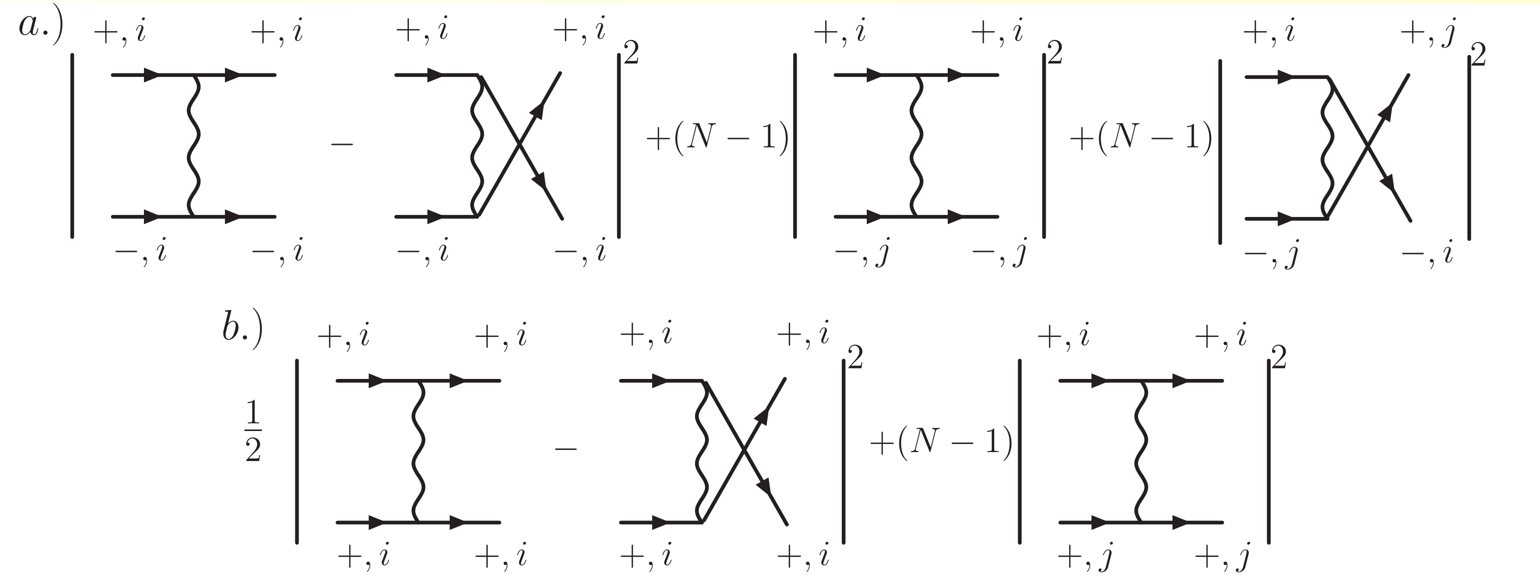
Boltzmann approach

L. Fritz, J. Schmalian, MM, and S. Sachdev, PRB 2008

Boltzmann equation in Born approximation

$$\left(\partial_t + e\mathbf{E}\cdot\frac{\partial}{\partial\mathbf{k}}\right) f_\pm(\mathbf{k},t) = I_{\text{coll}}^{Cb}\left[\mathbf{k},t \mid \{f_\pm(\mathbf{k}',t)\}\right] \propto \alpha^2(T)$$



→ Collision-limited conductivity: $\boxed{\sigma(\mu=0) = \frac{0.76}{\alpha^2(T)}\frac{e^2}{h}}$

Beyond weak coupling approximation:

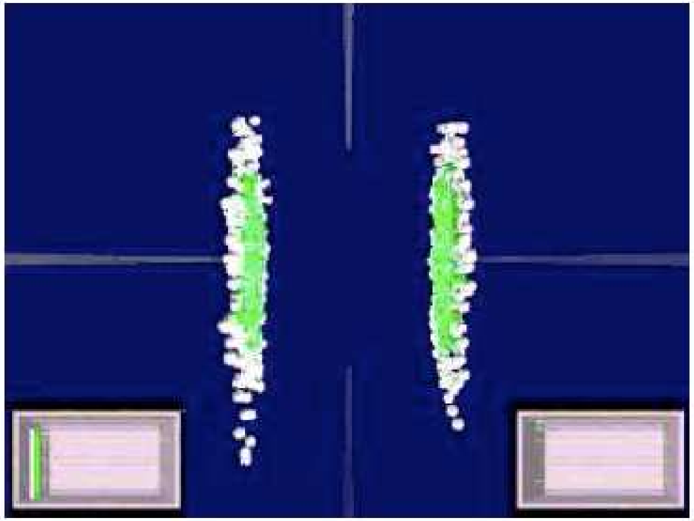
Graphene

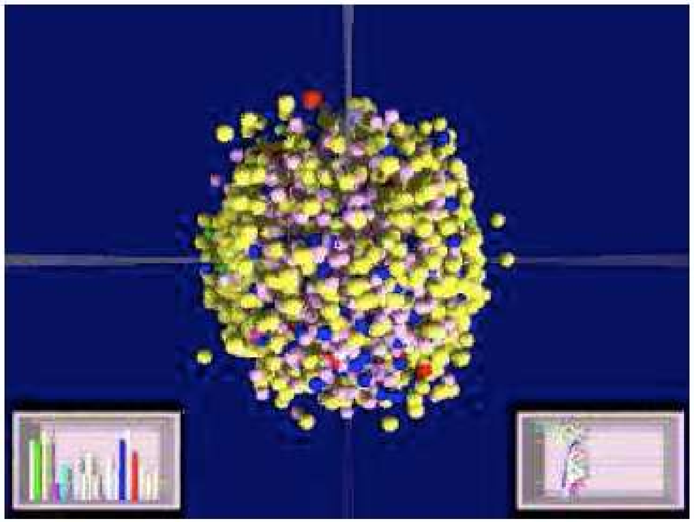
$\leftrightarrow$

Very strongly coupled, critical relativistic liquids?

AdS – CFT !

Au+Au collisions at RHIC





Quark-gluon plasma is described by QCD (nearly conformal, critical theory)

–

Low viscosity fluid!

Compare graphene to Strongly coupled relativistic liquids

S. Hartnoll, P. Kovtun, MM, S. Sachdev (2007)

Obtain exact results via string theoretical AdS–CFT correspondence

→ Response functions for particular strongly coupled relativistic fluids
(for maximally supersymmetric SU(N) Yang Mills theory with $N \to \infty$ colors)
By mapping to weakly coupled gravity problem:

AdS - CFT [SU(N>>1)]

weak coupling - strong coupling

SU(N) transport from AdS/CFT

Gravitational dual to SUSY SU(N)-CFT$_{2+1}$: Einstein-Maxwell theory

$$I = \frac{1}{g^2}\int d^4x\sqrt{-g}\left[-\frac{1}{4}R + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{3}{2}\right].$$

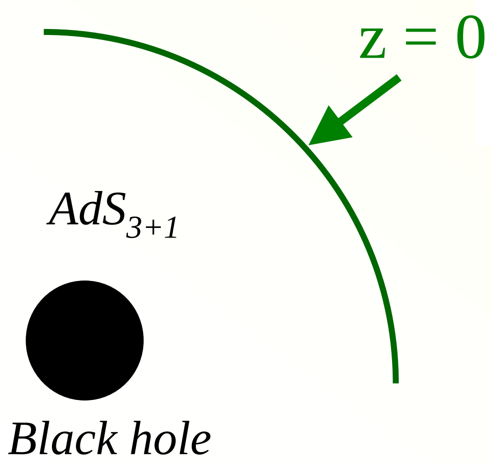
(embedded in M theory as $AdS_4 \times S^7$: $1/g^2 \sim N^{3/2}$)

It has a black hole solution (with electric and magnetic charge):

$$ds^2 = \frac{\alpha^2}{z^2}\left[-f(z)dt^2 + dx^2 + dy^2\right] + \frac{1}{z^2}\frac{dz^2}{f(z)},$$

$$F = h\alpha^2 dx \wedge dy + q\alpha dz \wedge dt,$$

$$f(z) = 1 + (h^2 + q^2)z^4 - (1 + h^2 + q^2)z^3.$$



Electric charge q and magnetic charge, h

$\leftrightarrow$ μ and B for the CFT

SU(N) transport from AdS/CFT

Gravitational dual to SUSY SU(N)-CFT$_{2+1}$: Einstein-Maxwell theory

$$I = \frac{1}{g^2}\int d^4x\sqrt{-g}\left[-\frac{1}{4}R + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{3}{2}\right].$$

(embedded in M theory as $AdS_4 \times S^7$: $1/g^2 \sim N^{3/2}$)

It has a black hole solution (with electric and magnetic charge):

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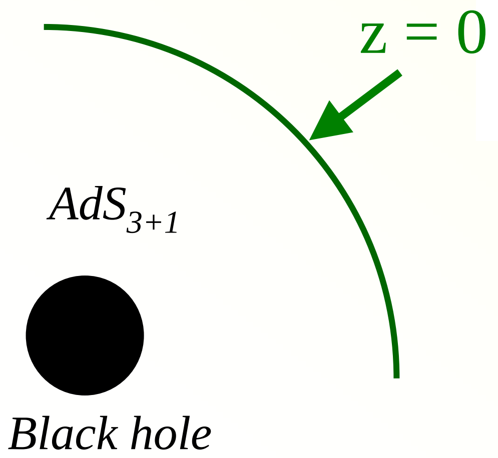

Background $\leftrightarrow$ Equilibrium

Transport $\leftrightarrow$ Perturbations in $g_{tx,ty}, A_{x,y}$.

Response via Kubo formula from $\delta^2 I/\delta(g,A)^2$.

Compare graphene to Strongly coupled relativistic liquids

S. Hartnoll, P. Kovtun, MM, S. Sachdev (2007)

Obtain exact results via string theoretical AdS–CFT correspondence

→

- Confirm the structure of the hydrodynamic response functions σ(ω) etc.

- Allow to calculate the transport coefficients for a strongly coupled theory!

SUSY - SU(N): $\sigma(\mu=0)=\sqrt{\frac{2}{9}}N^{3/2}\frac{e^2}{h}$

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Graphene – a nearly perfect liquid!

MM, J. Schmalian, and L. Fritz, (PRL 2009)

Anomalously low viscosity (like quark-gluon plasma)

"Heisenberg" $\boxed{\frac{\eta}{s} \sim E_{qp}\tau \geq 1}$ → Measure of strong coupling:

Conjecture from AdS-CFT:

$$\boxed{\frac{\text{shear viscosity}}{\text{entropy density}} = \frac{\eta}{s} > \frac{\hbar}{k_B}\frac{1}{4\pi}}$$

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Doped Graphene & Fermi liquids:
(Khalatnikov etc)

$$\boxed{\eta \propto n \cdot m\mathrm{v}^2 \cdot \tau \rightarrow n \cdot E_F \cdot \frac{\hbar E_F}{(k_B T)^2}} \quad \boxed{s \propto k_B n \frac{T}{E_F}} \quad \boxed{\frac{\eta}{s} \sim \frac{\hbar}{k_B}\left(\frac{E_F}{T}\right)^3}$$

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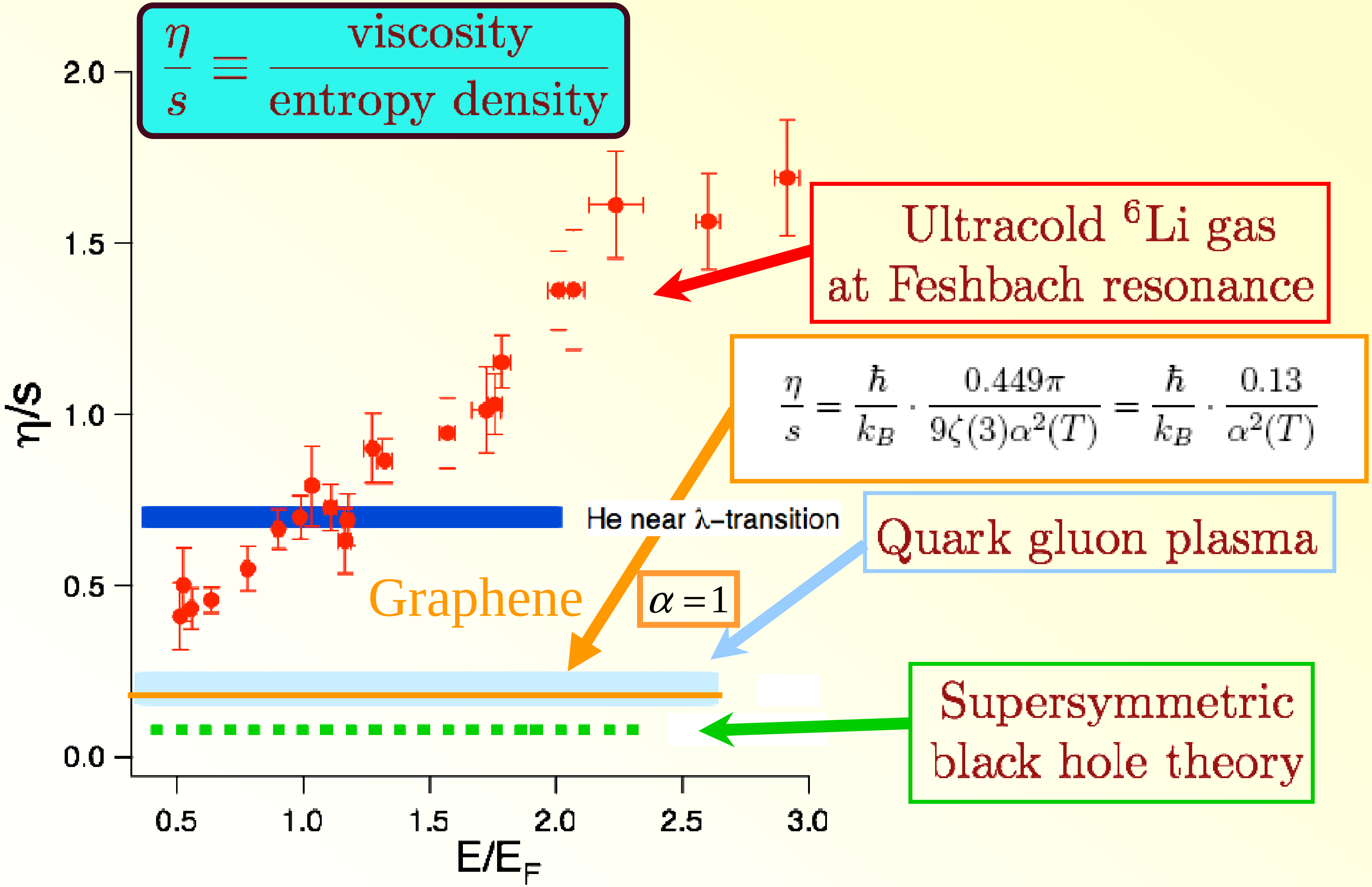
Undoped Graphene:

$$\eta \propto n \cdot mv^2 \cdot \tau \;\rightarrow\; n_{\mathrm{th}} \cdot k_B T \cdot \frac{\hbar}{\alpha^2 k_B T} = \frac{\hbar}{\alpha^2} n_{\mathrm{th}}$$

$$s \propto k_B n_{\mathrm{th}}$$

Exact (Boltzmann-Born Approx):

$$\frac{\eta}{s} = \frac{\hbar}{k_B} \cdot \frac{0.449\pi}{9\zeta(3)\alpha^2(T)} = \frac{\hbar}{k_B} \cdot \frac{0.13}{\alpha^2(T)}$$



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Electronic consequences of low viscosity?

MM, J. Schmalian, L. Fritz, (PRL 2009)

Electronic turbulence in clean, strongly coupled graphene?
(or at quantum criticality!)
Reynolds number:

$$\boxed{\mathrm{Re} = \frac{s/k_B}{\eta/\hbar} \times \frac{k_B T}{\hbar v/L} \times \frac{u_{\mathrm{typ}}}{v}}$$

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Strongly driven mesoscopic systems: (Kim's group [Columbia])

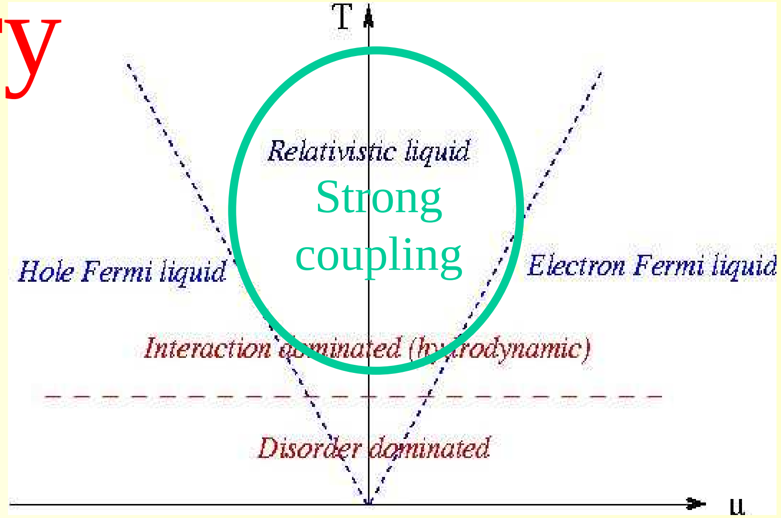
$$L = 1\mu m$$
$$u_{typ} = 0.1\mathrm{v}$$
$$T = 100K$$

$$\longrightarrow \quad \mathrm{Re} \sim 10 - 100$$

Complex fluid dynamics!
(pre-turbulent flow)

New phenomenon in an electronic system!

Summary



- Undoped graphene is strongly coupled in a large temperature window!
- Nearly universal strong coupling features in transport; many similarities with strongly coupled critical fluids (described by AdSCFT)
- Emergent relativistic hydrodynamics at low frequency
- Graphene: Nearly perfect quantum liquid!
  → Possibility of complex (turbulent?) current flow at high bias