Graphene: Relativistic transport in a nearly perfect quantum liquid

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Outline

• Relativistic physics in graphene, quantum critical systems and conformal field theories

• Strong coupling features in collision-dominated transport

• Comparison with strongly coupled fluids (via AdS-CFT)

• Graphene: an almost perfect quantum liquid
Dirac fermions in graphene
(Semenoff ’84, Haldane ‘88)

Honeycomb lattice of C atoms

Tight binding dispersion
Dirac fermions in graphene

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Honeycomb lattice of C atoms

Tight binding dispersion

2 massless Dirac cones in the Brillouin zone:
(Sublattice degree of freedom ↔ pseudospin)

Close to the two Fermi points $K$, $K'$:

$$H \approx v_F \left( \vec{p} - \vec{K} \right) \cdot \vec{\sigma}_{\text{sublattice}}$$

$$\rightarrow E_p = v_F |\vec{p} - \vec{K}|$$
Dirac fermions in graphene

(Honeycomb lattice of C atoms)

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Fermi velocity (speed of light”)

\( v_F \approx 1.1 \times 10^6 \text{ m/s} \approx \frac{c}{300} \)

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Close to the two Fermi points \(\mathbf{K}, \mathbf{K}'\):

Fermi velocity (speed of light"")

Coulomb interactions: Fine structure constant

\[
H \approx v_F (\mathbf{p} - \mathbf{K}) \cdot \mathbf{\sigma}_{\text{sublattice}}
\]

\[
E_p = v_F |\mathbf{p} - \mathbf{K}|
\]

\[
v_F \approx 1.1 \cdot 10^6 \text{ m/s} \approx \frac{c}{300}
\]

\[
\alpha \equiv \frac{e^2}{\varepsilon \hbar v_F} = O(1)
\]
Relativistic fluid at the Dirac point


- Relativistic plasma physics of interacting particles and holes!
Relativistic fluid at the Dirac point


- Relativistic plasma physics of interacting particles and holes!
- Strongly coupled, nearly quantum critical fluid at $\mu = 0$
Relativistic fluid at the Dirac point


- Relativistic plasma physics of interacting particles and holes!
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Very similar as at quantum criticality (with $z=1$, e.g. SIT) and the associated CFT's
Graphene – Fermi liquid?

1. Tight binding kinetic energy
   → massless Dirac quasiparticles

\[ H_0 = \sum_{\lambda=\pm} \sum_{a=1}^{N} \int \frac{d^2 k}{(2\pi)^2} \lambda v_F k \gamma_{\lambda a}^\dagger(k) \gamma_{\lambda a}(k) \]

2. Coulomb interactions:
   Unexpectedly strong!
   → nearly quantum critical!

\[ V(q) = \frac{2\pi e^2}{\varepsilon |q|} \]

\[ H_1 = \frac{1}{2} \int \frac{d^2 k_1}{(2\pi)^2} \frac{d^2 k_2}{(2\pi)^2} \frac{d^2 q}{(2\pi)^2} \Psi_{a}^\dagger(k_2 - q) \Psi_{a}(k_2) V(q) \Psi_{b}^\dagger(k_1 + q) \Psi_{b}(k_1) \]
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Coulomb only marginally irrelevant for $\mu = 0$!

RG: $\mu = 0$

\[
\frac{d\alpha}{d\ell} = -\frac{\alpha^2}{4} + \mathcal{O}(\alpha^3)
\]

\[
\alpha(T) = \alpha^0 \frac{4}{1 + (\alpha^0/4) \ln(\Lambda/T)} \sim \frac{4}{\ln(\Lambda/T)}
\]

Strong coupling!

Cb marginal!
Graphene – Fermi liquid?

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RG:
(\( \mu = 0 \))
(\( \mu > 0 \) \( T < \mu \): Screening kicks in, short ranged \( C_b \) irrelevant

Strong coupling!
Quantum critical liquid
Hole Fermi liquid
Electron Fermi liquid
Interactive-dominated (hydrodynamic)
Disorder dominated
Strong coupling in undoped graphene

MM, L. Fritz, and S. Sachdev, PRB ‘08.

Inelastic scattering rate
(Electron-electron interactions)

\[ \tau_{ee}^{-1} \sim \alpha^2 \frac{k_B T^2}{\hbar \mu} \]

\( \mu > T \): standard 2d Fermi liquid
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Relaxation rate \( \sim T \),
like in quantum critical systems!
Fastest possible rate!

\( \mu > T \): standard 2d Fermi liquid

\( \mu < T \): strongly coupled relativistic liquid
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“Heisenberg uncertainty principle for well-defined quasiparticles”

\[ E_{qp} (\sim k_B T) \geq \Delta E_{\text{int}} = \hbar \tau_{ee}^{-1} \sim \alpha^2 k_B T \]
Strong coupling in undoped graphene

*MM, L. Fritz, and S. Sachdev, PRB ‘08.*

Inelastic scattering rate
(Electron-electron interactions)

\[ \tau_{ee}^{-1} \sim \alpha^2 \frac{k_B T^2}{\hbar \mu} \]

\[ \alpha \tau_h \ll T \rightarrow \text{Nearly universal strong coupling features in transport,} \]

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\[ \mu < T \]: strongly coupled relativistic liquid

“Heisenberg uncertainty principle for well-defined quasiparticles”

\[ E_{qp} \sim k_B T \geq \Delta E_{\text{int}} = \hbar \tau_{ee}^{-1} \sim \alpha^2 k_B T \]

As long as \( \alpha(T) \sim 1 \), energy uncertainty is saturated, scattering is maximal

\( \rightarrow \) Nearly universal strong coupling features in transport,

similarly as at the 2d superfluid-insulator transition [Damle, Sachdev (1996, 1997)]
Consequences for transport

1. Collisionlimited conductivity $\sigma$ in clean undoped graphene

2. Graphene - a perfect quantum liquid: very small viscosity $\eta$!
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2. Graphene - a perfect quantum liquid: very small viscosity $\eta$!

3. Emergent relativistic invariance at low frequencies!
   Despite the breaking of relativistic invariance by
   • finite $T$,
   • finite $\mu$,
   • instantaneous $1/r$ Coulomb interactions
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Collision-dominated transport $\rightarrow$ relativistic hydrodynamics:

a) Response fully determined by covariance, thermodynamics, and $\sigma$, $\eta$

b) Collective cyclotron resonance in small magnetic field (low frequency)

Hydrodynamic regime: (collision-dominated)

$\tau_{ee}^{-1} \gg \tau_{imp}^{-1}, \omega_c^{\text{typ}}, \omega_{AC}$
Collision-limited conductivity

Fritz et al. (2008), Kashuba (2008)

Finite conductivity in a pure system at particle-hole symmetry ($\rho = 0$)!
Collision-limited conductivity

Finite conductivity in a pure system at particle-hole symmetry ($\rho = 0$)!

- Key: Charge current without momentum

\[ \vec{J} \neq 0, \quad \vec{P} = 0 \]

- Finite collision-limited conductivity:
  \[ \sigma(\mu = 0) < \infty \quad ; \quad \sigma(\mu \neq 0) = \infty \]

and

- Infinite thermal conductivity:
  \[ \kappa(\mu = 0) = \infty \quad ; \quad \kappa(\mu \neq 0) < \infty \]

(true also in pure semiconductors)

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Fritz et al. (2008), Kashuba (2008)

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- Only marginal irrelevance of Coulomb:
  Maximal possible relaxation rate, set only by temperature

\[ \tau_{ee}^{-1} \approx \alpha^2 \frac{k_B T}{\hbar} \]
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→ Nearly universal conductivity at strong coupling
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Pair creation/annihilation leads to current decay

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\[ \sigma_{Drude} = \frac{e}{m} \rho \tau \rightarrow \sigma(\mu = 0) \sim \frac{e}{k_B T} \frac{e}{\sqrt{\hbar v^2}} \left( \frac{k_B T^2}{(\hbar v)^2} \right) \frac{\hbar}{\alpha^2 k_B T} \sim \frac{1}{\alpha^2} \frac{e^2}{\hbar} \]
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Marginal irrelevance of Coulomb:

\[ \alpha \approx \frac{4}{\log(\Lambda/T)} < 1 \]
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\]

Marginal irrelevance of Coulomb:

\[
\alpha \approx \frac{4}{\log(\Lambda/T)} < 1
\]

Expect saturation as $\alpha \rightarrow 1$, and eventually phase transition to insulator
Boltzmann approach


Boltzmann equation in Born approximation

\[
\left( \partial_t + e \mathbf{E} \cdot \frac{\partial}{\partial \mathbf{k}} \right) f_{\pm}(\mathbf{k}, t) = I_{\text{coll}}^{\text{Cb}}[\mathbf{k}, t | \{ f_{\pm}(\mathbf{k}', t) \}] \propto \alpha^2(T)
\]

Collision-limited conductivity:

\[
\sigma(\mu = 0) = \frac{0.76 e^2}{\alpha^2(T) \hbar}
\]
Beyond weak coupling approximation:

Graphene

⇔

Very strongly coupled, critical relativistic liquids?

AdS – CFT!
Quark-gluon plasma is described by QCD (nearly conformal, critical theory)

Low viscosity fluid!
Compare graphene to Strongly coupled relativistic liquids


Obtain exact results via string theoretical AdS–CFT correspondence

→ Response functions for particular strongly coupled relativistic fluids
  (for maximally supersymmetric SU(N) Yang Mills theory with $N \rightarrow \infty$ colors)
By mapping to weakly coupled gravity problem:

$$\text{AdS} \quad - \quad \text{CFT} [\text{SU}(N\gg 1)]$$

weak coupling \quad - \quad strong coupling
**SU(N) transport from AdS/CFT**

Gravitational dual to SUSY SU(N)-CFT\(_{2+1}\): Einstein-Maxwell theory

\[
I = \frac{1}{g^2} \int d^4x \sqrt{-g} \left[ -\frac{1}{4} R + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{3}{2} \right].
\]

(embedded in M theory as $AdS_4 \times S^7$: $1/g^2 \sim N^{3/2}$)

It has a black hole solution (with electric and magnetic charge):

\[
ds^2 = \frac{\alpha^2}{z^2} [-f(z) dt^2 + dx^2 + dy^2] + \frac{1}{z^2} \frac{dz^2}{f(z)},
\]

\[
F = h \alpha^2 dx \wedge dy + q \alpha dz \wedge dt,
\]

\[
f(z) = 1 + (h^2 + q^2) z^4 - (1 + h^2 + q^2) z^3.
\]

Electric charge $q$ and magnetic charge, $h$

$\leftrightarrow \mu$ and $B$ for the CFT
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Background \(\leftrightarrow\) Equilibrium

Transport \(\leftrightarrow\) Perturbations in \(g_{tx,ty}, A_{x,y}\).

Response via Kubo formula from \(\delta^2 I/\delta(g, A)^2\).
Compare graphene to
Strongly coupled relativistic liquids


Obtain exact results via string theoretical AdS–CFT correspondence

- Confirm the structure of the hydrodynamic response functions $\sigma(\omega)$ etc.
- Allow to calculate the transport coefficients for a strongly coupled theory!

\[
\text{SUSY - SU(N): } \sigma(\mu = 0) = \sqrt{\frac{2}{9}} N^{3/2} \frac{e^2}{\hbar}
\]
Compare graphene to
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\[
\text{SUSY - SU(N): } \sigma(\mu = 0) = \sqrt{\frac{2}{9}} N^{3/2} \frac{e^2}{\hbar} ; \eta_{\text{shear}}^{s}(\mu = 0) = \frac{1}{4\pi} \frac{\hbar}{k_B}
\]
Graphene – a nearly perfect liquid!

Anomalously low viscosity (like quark-gluon plasma)

\[ \frac{\eta}{s} \sim E_{qp} \tau \geq 1 \]

“Heisenberg”

Measure of strong coupling:

shear viscosity \[
\eta > \frac{\hbar}{k_B} \frac{1}{4\pi}
\]

entropy density \[
\frac{s}{\eta} > \frac{\hbar}{k_B} \frac{1}{4\pi}
\]

Conjecture from AdS-CFT:

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\text{shear viscosity} \quad \frac{\eta}{s} \quad > \quad \frac{\hbar}{k_B} \frac{1}{4\pi}
\]

\[
\text{entropy density} \quad \frac{s}{\eta} \quad \propto \quad \frac{1}{k_B T}
\]

Doped Graphene & Fermi liquids:

\[
\eta \sim n \cdot mv^2 \cdot \tau \quad \rightarrow \quad n \cdot E_F \cdot \frac{\hbar E_F}{(k_B T)^2}
\]

\[
\eta \sim k_B n \frac{T}{E_F}
\]

\[
\eta \sim \frac{\hbar}{k_B} \left( \frac{E_F}{T} \right)^3
\]
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“Heisenberg” Measure of strong coupling:

\[ \frac{\eta}{s} > \frac{\hbar}{k_B} \frac{1}{4\pi} \]

Conjecture from AdS-CFT:

\[ \frac{\eta}{s} \propto \frac{T}{E_F} \]

Doped Graphene & Fermi liquids: (Khalatnikov etc)

\[ \eta \propto n \cdot m v^2 \cdot \tau \rightarrow n \cdot E_F \cdot \frac{\hbar E_F}{(k_B T)^2} \]

\[ s \propto k_B n T \]

\[ \eta \propto \frac{\eta}{s} \sim \frac{\hbar}{k_B} \left( \frac{E_F}{T} \right)^3 \]

Undoped Graphene:

\[ \eta \propto n \cdot m v^2 \cdot \tau \rightarrow n_{th} \cdot \frac{k_B T}{\alpha^2 k_B T} \cdot \frac{\hbar}{\alpha^2} = \frac{\hbar}{\alpha^2} n_{th} \]

\[ s \propto k_B n_{th} \]

Exact (Boltzmann-Born Approx):

\[ \frac{\eta}{s} = \frac{\hbar}{k_B} \cdot \frac{0.449\pi}{9\zeta(3)\alpha^2(T)} = \frac{\hbar}{k_B} \cdot \frac{0.13}{\alpha^2(T)} \]
Electronic consequences of low viscosity?

Electronic turbulence in clean, strongly coupled graphene?
(or at quantum criticality!)

Reynolds number:

$$\text{Re} = \frac{s/k_B}{\eta/\hbar} \times \frac{k_B T}{\hbar v/L} \times \frac{u_{\text{typ}}}{v}$$
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$$ Re = \frac{s/k_B}{\eta/\hbar} \times \frac{k_BT}{\hbar v/L} \times \frac{u_{typ}}{v} $$

Strongly driven mesoscopic systems: (Kim’s group [Columbia])

- $L = 1\mu m$
- $u_{typ} = 0.1v$
- $T = 100K$

$Re \sim 10 - 100$

Complex fluid dynamics!
(pre-turbulent flow)

New phenomenon in an electronic system!
Summary

• Undoped graphene is strongly coupled in a large temperature window!

• Nearly universal strong coupling features in transport; many similarities with strongly coupled critical fluids (described by AdSCFT)

• Emergent relativistic hydrodynamics at low frequency

• Graphene: Nearly perfect quantum liquid!
  → Possibility of complex (turbulent?) current flow at high bias