Transport properties of the electron-hole plasma in graphene

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in collaboration with
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Subir Sachdev (Harvard)
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WHERE DISCOVERIES BEGIN

Harvard University, January 29, 2009

Outline

- Relativistic physics in graphene, quantum critical systems and conformal field theories
  Relativistic signatures in (magneto-)transport

- Hydrodynamic description

- Boltzmann theory
  Hydrodynamics from microscopics

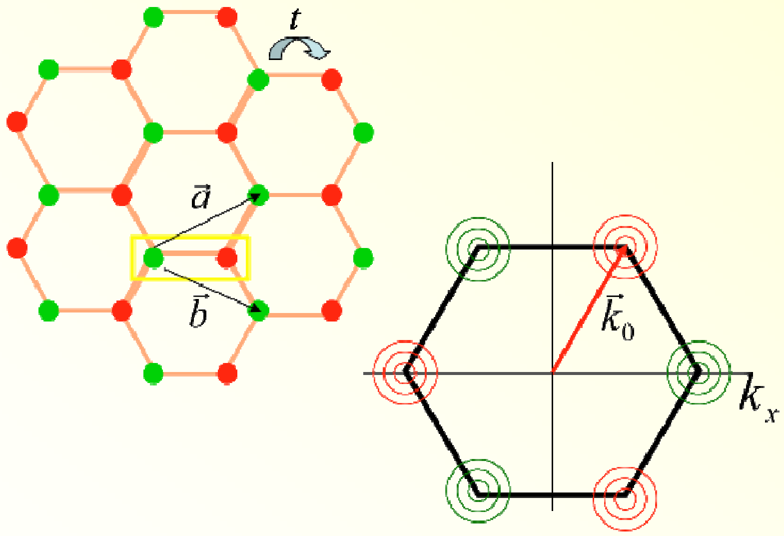
- Strongly coupled relativistic fluids via AdS-CFT

- Graphene as an almost perfect liquid?
  Nearly minimal viscosity (like quark gluon plasma)

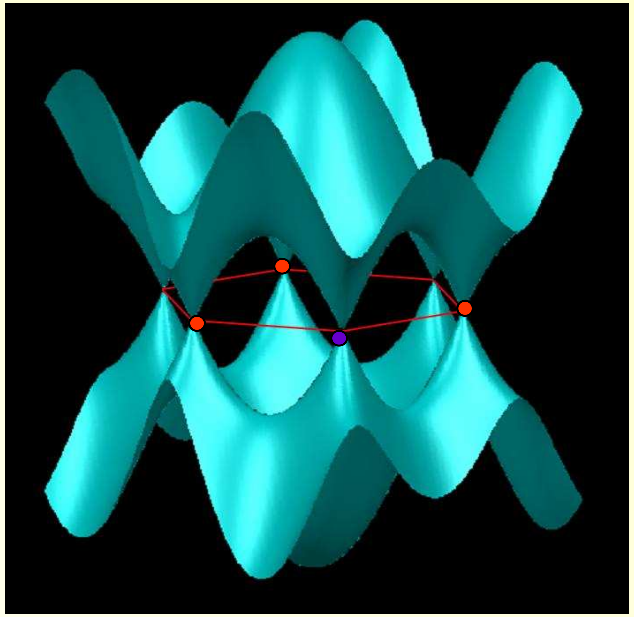
Dirac fermions in graphene

(Semenoff '84, Haldane '88)

Honeycomb lattice of C atoms



Tight binding dispersion



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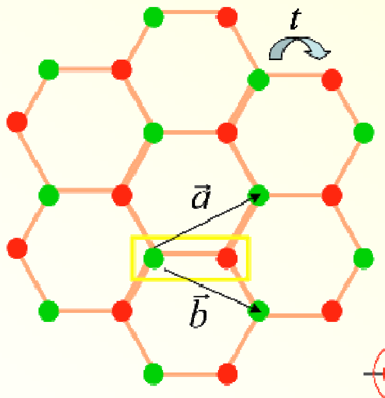

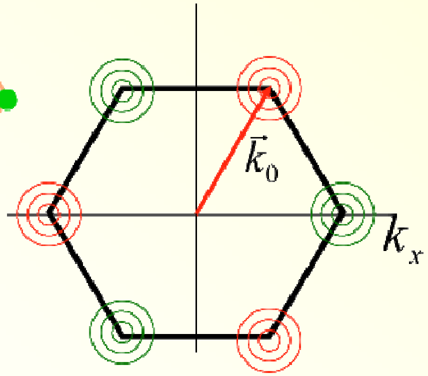

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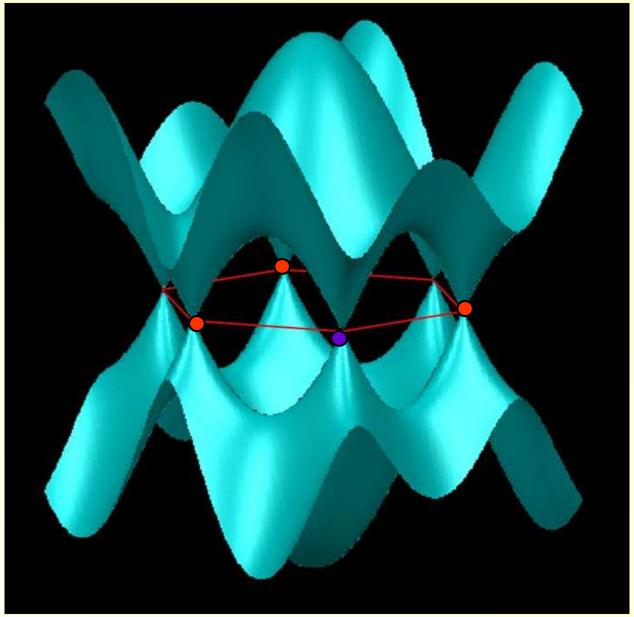

2 massless Dirac cones in the Brillouin zone: (Sublattice degree of freedom ↔ pseudospin)

Close to the two Fermi points **K**, **K'**:

$$H \approx v_F\ (\mathbf{p}-\mathbf{K})\cdot\sigma_{\text{sublattice}}$$

$$\rightarrow\quad E_{\mathbf{k}} = v_F\left|\mathbf{k}-\mathbf{K}\right|$$

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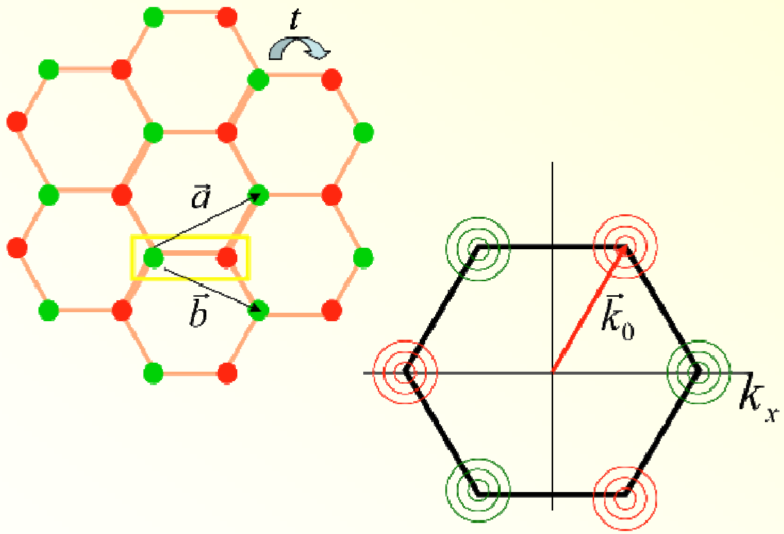

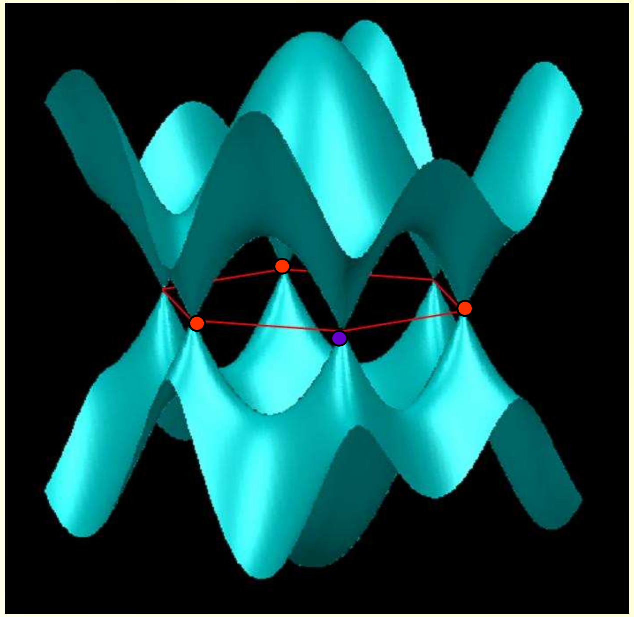

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Fermi velocity (speed of light")

$$v_F \approx 1.1\cdot 10^6 \text{ m/s} \approx \frac{c}{300}$$

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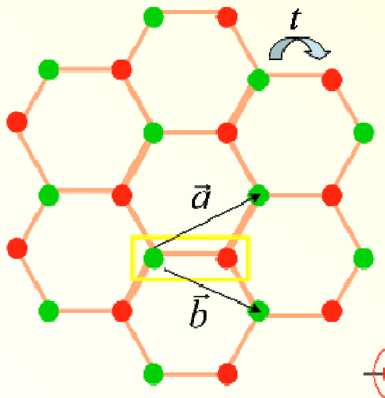

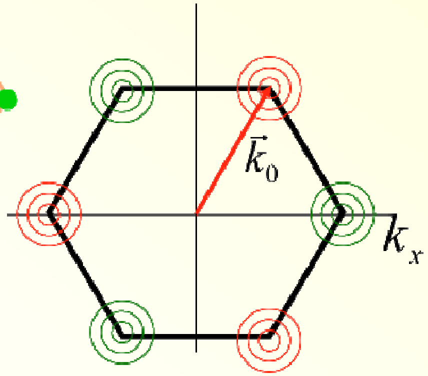

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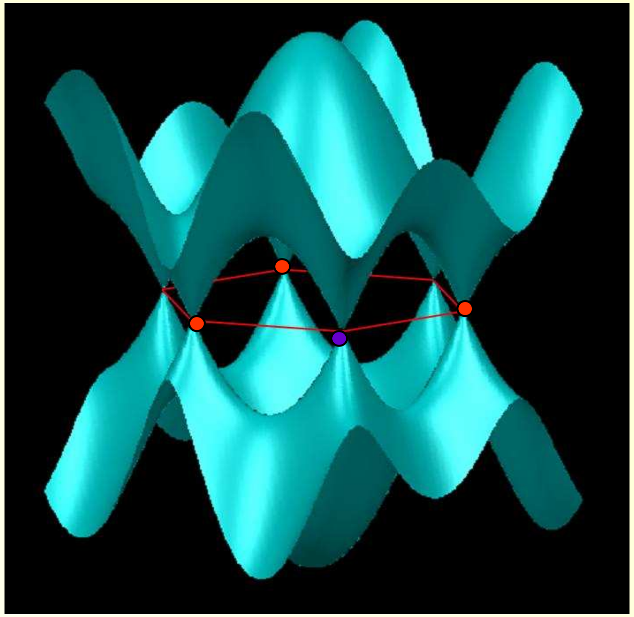

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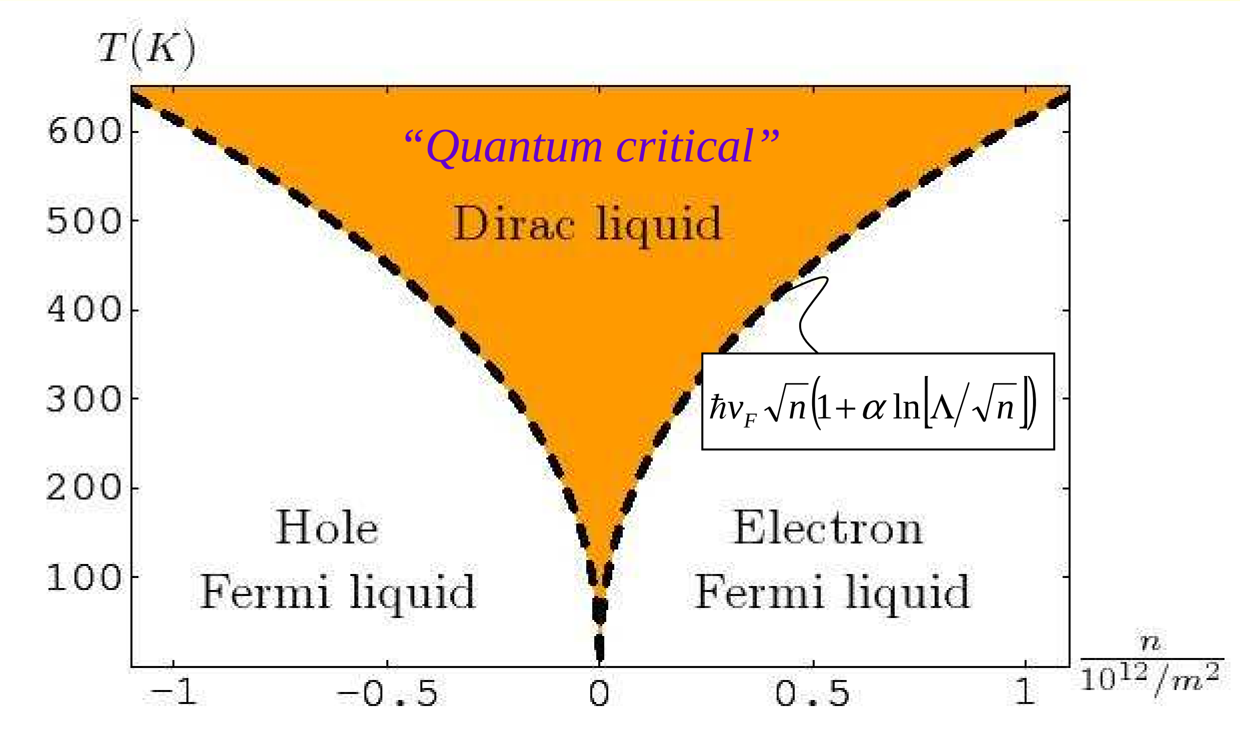
Coulomb interactions: Fine structure constant

$$\alpha \equiv \frac{e^2}{\varepsilon\, \hbar v_F} = O(1)$$

Relativistic fluid at the Dirac point

Expect relativistic plasma physics of interacting particles and holes!



D. Sheehy, J. Schmalian, Phys. Rev. Lett. ***99****, 226803 (2007).*

Other relativistic fluids:

- Bismuth (3d Dirac fermions with very small mass)

- Systems close to quantum criticality (with z = 1)

Example: Superconductor-insulator transition

Maximal possible relaxation rate!

$$\tau_{rel}^{-1} \approx \frac{\hbar}{k_B T}$$

Damle, Sachdev (1996)
Bhaseen, Green, Sondhi (2007).
Hartnoll, Kovtun, MM, Sachdev (2007)

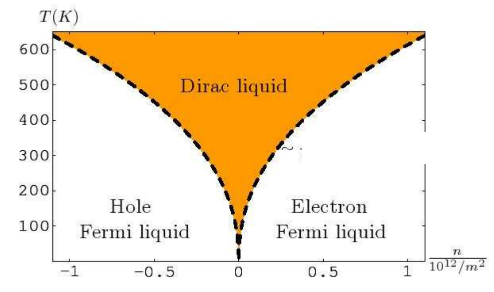
- Conformal field theories

E.g.: strongly coupled Non-Abelian gauge theories (QCD):
→ Exact treatment via AdS-CFT correspondence!

C. P. Herzog, P. Kovtun, S. Sachdev, and D. T. Son (2007)
Hartnoll, Kovtun, MM, Sachdev (2007)

Transport and phase diagram

Expect relativistic plasma physics of interacting particles and holes!



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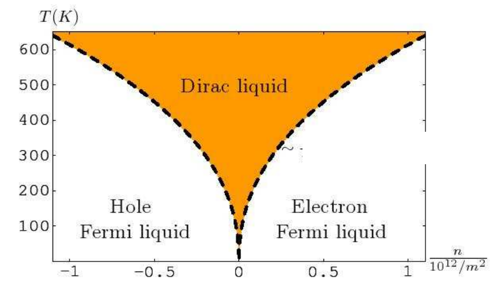

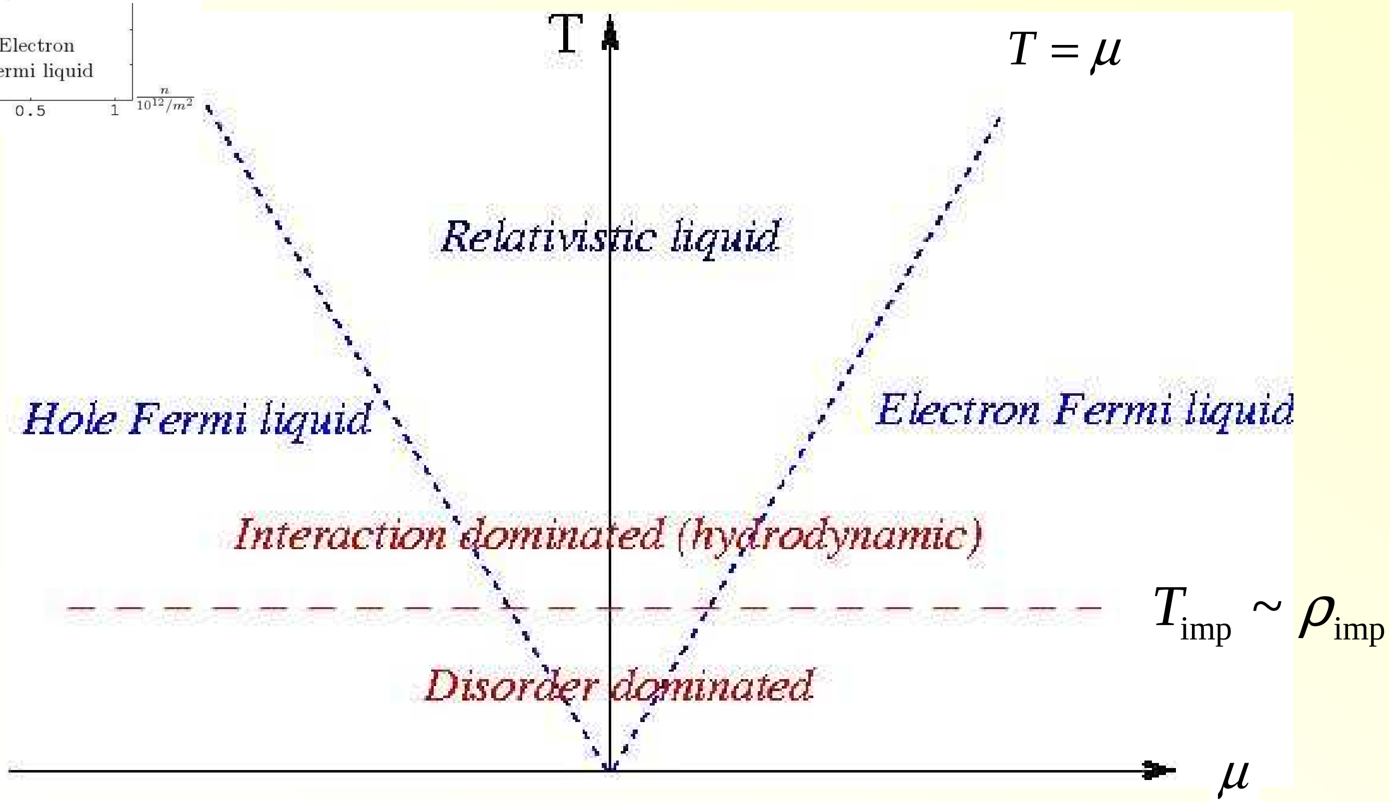

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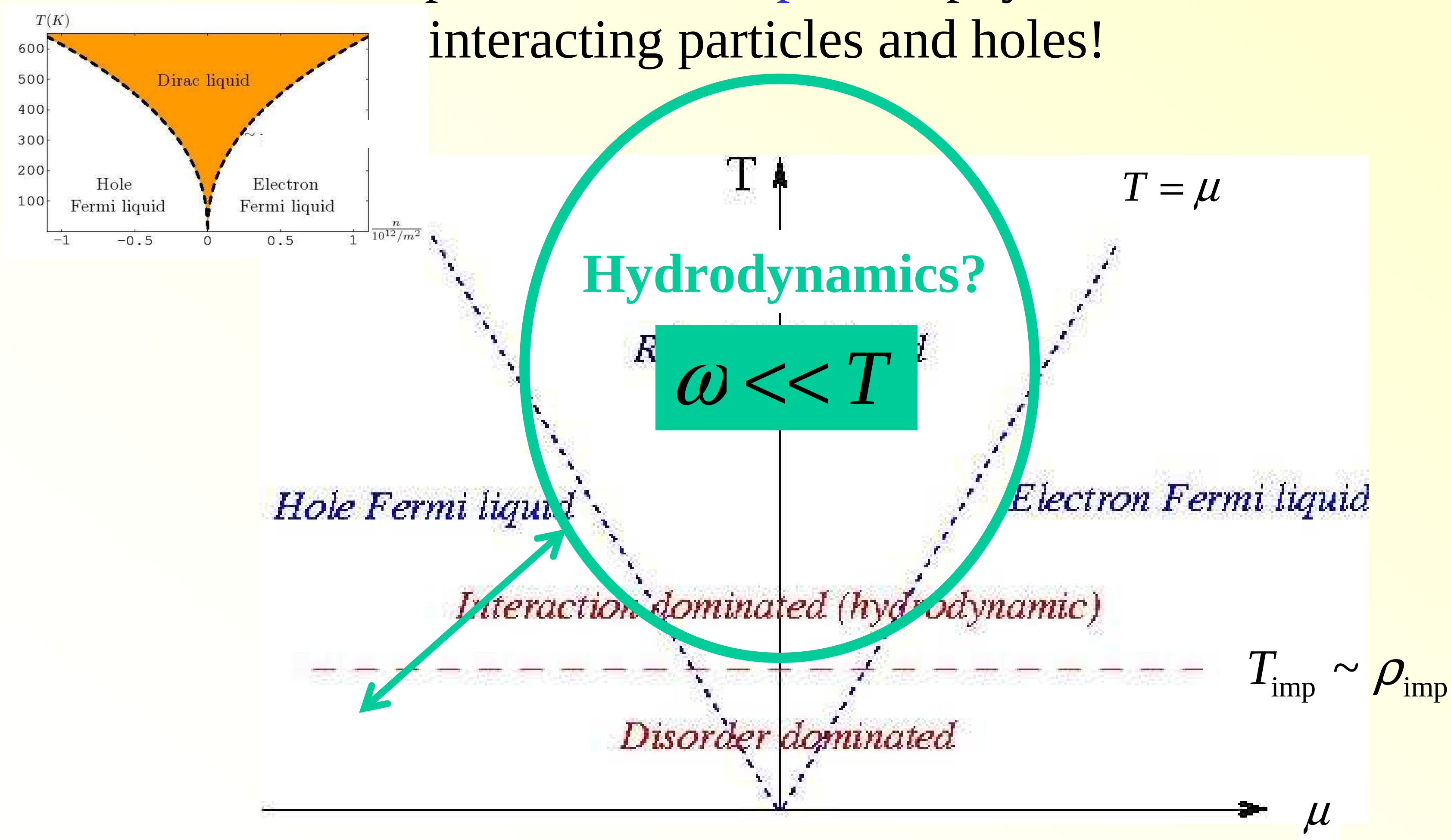
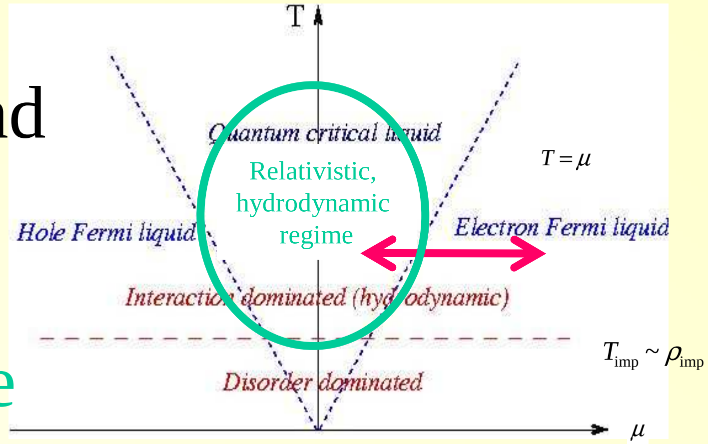


Questions

- Transport characteristics of the relativistic plasma in graphene and at quantum criticality?

- Connecting the relativistic regime to the electron Fermi liquid at large doping?

- Graphene as a nearly perfect fluid (like the quark-gluon plasma)?



Model of graphene

Graphene with Coulomb interactions and disorder

$$H = H_0 + H_1 + H_{\text{dis}}$$

1. Tight binding kinetic energy

$$H_0 = -\sum_{a=1}^{N} \int d\mathbf{x} \left[\Psi_a^\dagger \left(i v_F \vec{\sigma} \cdot \vec{\nabla} + \mu \right) \Psi_a \right]$$

$$H_0 = \sum_{\lambda=\pm} \sum_{a=1}^{N} \int \frac{d^2k}{(2\pi)^2} \lambda v_F k \, \gamma_{\lambda a}^\dagger(\mathbf{k}) \gamma_{\lambda a}(\mathbf{k})$$

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2. Coulomb interactions

$$H_1 = \frac{1}{2} \int \frac{d^2k_1}{(2\pi)^2} \frac{d^2k_2}{(2\pi)^2} \frac{d^2q}{(2\pi)^2} \Psi_a^\dagger(\mathbf{k}_2 - \mathbf{q}) \Psi_a(\mathbf{k}_2) V(\mathbf{q}) \Psi_b^\dagger(\mathbf{k}_1 + \mathbf{q}) \Psi_b(\mathbf{k}_1)$$

$$V(\mathbf{q}) = \frac{2\pi e^2}{\varepsilon |\mathbf{q}|}$$

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$$V(\mathbf{q}) = \frac{2\pi e^2}{\varepsilon |\mathbf{q}|}$$

Coulomb marginally irrelevant!

$$\alpha \equiv \frac{e^2}{\varepsilon \hbar v_F} = O(1)$$

RG:

$$\frac{d\alpha}{d\ell} = -\frac{\alpha^2}{4} + \mathcal{O}(\alpha^3)$$

$$\alpha(T) = \frac{\alpha^0}{1 + (\alpha^0/4)\ln(\Lambda/T)} \overset{T \to 0}{\sim} \frac{4}{\ln(\Lambda/T)}$$

Model of graphene

Graphene with Coulomb interactions and disorder

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Coulomb marginally irrelevant!

3. Disorder: charged impurities

$$H_{dis} = \int d\mathbf{x} V_{dis}(\mathbf{x}) \Psi_a^\dagger(\mathbf{x}) \Psi_a(\mathbf{x})$$

$$V_{dis}(\mathbf{x}) = \sum_i \delta(\mathbf{x} - \mathbf{x}_i) \frac{Ze^2}{\varepsilon |\mathbf{x} - \mathbf{x}_i|}.$$

Time scales

MM, L. Fritz, and S. Sachdev, cond-mat 0805.1413.

1. Inelastic scattering rate
(Electron-electron interactions)

$$\tau_{\mathrm{ee}}^{-1} \sim \alpha^2 \frac{k_B T}{\hbar} \frac{1}{\max\left[1, \mu/T\right]}$$

Relativistic regime ($\mu < T$):
Relaxation rate set by temperature,
like in quantum critical systems!

$$\tau_{\mathrm{ee}}^{-1} \sim \alpha^2 \frac{k_B T}{\hbar}$$

Fastest possible rate!

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Subdominant at high T, low disorder

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3. Deflection rate due to magnetic field
(Cyclotron frequency of non-interacting particles with thermal energy)

$$\tau_{\mathrm{B}}^{-1} \sim \omega_c^{\mathrm{typ}} \sim \frac{eB\mathrm{v}_F^2}{\max[T, \mu]}$$

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Hydrodynamic regime:
(collision-dominated)

$$\tau_{\mathrm{ee}}^{-1} >> \tau_{\mathrm{imp}}^{-1}, \tau_{\mathrm{B}}^{-1}, \omega$$

Hydrodynamic Approach

Hydrodynamics

Hydrodynamic collision-dominated regime

$$\tau_{ee}^{-1} >> \tau_{imp}^{-1}, \tau_{B}^{-1}, \omega$$

Long times,
Large scales

$$t >> \tau_{ee}$$

Hydrodynamics

Hydrodynamic collision-dominated regime $\tau_{ee}^{-1} >> \tau_{imp}^{-1}, \tau_{B}^{-1}, \omega$

Long times,
Large scales $t >> \tau_{ee}$

- Local equilibrium established: $T_{loc}(r)\ ,\ \mu_{loc}(r)\ ;\ \vec{u}_{loc}(r)$
- Study relaxation towards global equilibrium
- Slow modes: Diffusion of the density of conserved quantities:
  - Charge
  - Momentum
  - Energy

Relativistic Hydrodynamics

*S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B **76**, 144502 (2007).*

Energy-momentum tensor $T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$

$$\begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{pmatrix}$$

Current 3-vector $J^\mu = \rho u^\mu + \nu^\mu$

$$\begin{pmatrix} \rho \\ \rho u_x + \nu_x \\ \rho u_y + \nu_y \end{pmatrix}$$

u^μ : 3-velocity: $u^\mu = (1,0,0) \rightarrow$ No energy current

ν^μ : Dissipative current

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+ Thermodynamic relations

$$\varepsilon + P = Ts + \mu\rho, \quad d\varepsilon = Tds + \mu d\rho,$$

Gibbs-Duhamel 1st law of thermodynamics

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Conservation laws (equations of motion):

$\partial_{\mu} J^{\mu} = 0$ Charge conservation

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Energy/momentum conservation

$$\partial_\nu T^{\mu\nu} = F^{\mu\nu} J_\nu + \frac{1}{\tau_{\mathrm{imp}}} T^{0\nu}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y \\ -E_x & 0 & B \\ -E_y & -B & 0 \end{pmatrix}$$

$$\vec{E} = -i\vec{k}\frac{2\pi}{|k|}\rho_{\vec{k}}$$ Coulomb interaction

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Dissipative current and viscous tensor?

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Landau-Lifschitz,
Relat. plasma physics

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Heat current $Q^{\mu} = (\varepsilon + P)u^{\mu} - \mu J^{\mu}$

→ Entropy current $S^{\mu} = Q^{\mu} / T$

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Positivity of entropy production (Second law):

$$\partial_\mu S^\mu \equiv A_\alpha(\partial T, \partial\mu, F^{\mu\nu})\nu^\alpha + B_{\alpha\beta}(\partial T, \partial\mu, F^{\mu\nu})\tau^{\alpha\beta} \geq 0$$

$$\Rightarrow \nu^\mu = \text{const.} \times A^\mu(\partial T, \partial\mu, \partial u; F^{\mu\nu})$$

$$\tau^{\mu\nu} = \text{const.} \times B^{\mu\nu} + \text{const.} \times \delta^{\mu\nu} B^\alpha_\alpha$$

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$\rightarrow$

$$\nu^\mu = \sigma_Q(g^{\mu\nu} + u^\mu u^\nu)\left[(-\partial_\nu\mu + F_{\nu\lambda}u^\lambda) + \mu\frac{\partial_\mu T}{T}\right]$$

$$\tau^{\mu\nu} = -(g^{\mu\lambda} + u^\mu u^\lambda)[\eta(\partial_\lambda u^\nu + \partial^\nu u_\lambda) + (\zeta - \eta)\delta^\nu_\lambda \partial_\alpha u^\alpha]$$

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$$\tau^{\mu\nu} = \text{const.} \times B^{\mu\nu} + \text{const.} \times \delta^{\mu\nu} B^{\alpha}_{\alpha}$$

$\rightarrow$

$$\nu^{\mu} = \sigma_Q (g^{\mu\nu} + u^{\mu}u^{\nu})\left[(-\partial_{\nu}\mu + F_{\nu\lambda}u^{\lambda}) + \mu\frac{\partial_{\mu}T}{T}\right]$$

$$\tau^{\mu\nu} = -(g^{\mu\lambda} + u^{\mu}u^{\lambda})[\eta(\partial_{\lambda}u^{\nu} + \partial^{\nu}u_{\lambda}) + (\zeta - \eta)\delta^{\nu}_{\lambda}\partial_{\alpha}u^{\alpha}]$$

B small!

Relativistic Hydrodynamics

*S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B **76**, 144502 (2007).*

Landau-Lifschitz,
Relat. plasma physics

Dissipative current and viscous tensor?

Heat current $Q^\mu = (\varepsilon + P)u^\mu - \mu J^\mu$

$\to$ Entropy current $S^\mu = Q^\mu / T$

Positivity of entropy production (Second law):

$$\partial_\mu S^\mu \equiv A_\alpha\left(\partial T, \partial\mu, F^{\mu\nu}\right)\nu^\alpha + B_{\alpha\beta}\left(\partial T, \partial\mu, F^{\mu\nu}\right)\tau^{\alpha\beta} \geq 0$$

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$\to$

$$\nu^\mu = \sigma_Q(g^{\mu\nu} + u^\mu u^\nu)\left[(-\partial_\nu\mu + F_{\nu\lambda}u^\lambda) + \mu\frac{\partial_\mu T}{T}\right]$$

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Irrelevant for response at $k \to 0$

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Irrelevant for response at k $\rightarrow$ 0

One single transport coefficient (instead of two)!

Meaning of σ_Q ?

- At zero doping (particle-hole symmetry):

$$\sigma_Q = \sigma_{xx}\left(\rho_{\text{imp}} = 0\right) < \infty \ !$$

→ Interaction-limited conductivity of the pure system!

Why is $\sigma_{xx}\left(\rho_{\text{imp}} = 0\right)$ finite ??

Universal conductivity σ_Q

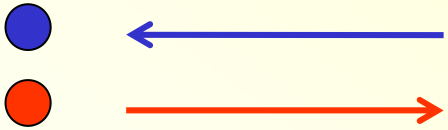
K. Damle, S. Sachdev, (1996).

Particle-hole symmetry ($\rho = 0$)

- Key: Charge current without momentum!

(particle)

(hole)



Pair creation/annihilation leads to current decay

$$\vec{J} \neq 0, \quad \vec{P} = 0$$

- Finite "quantum critical" conductivity!

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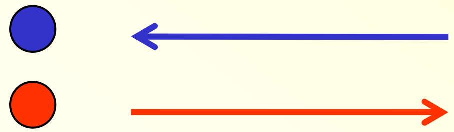

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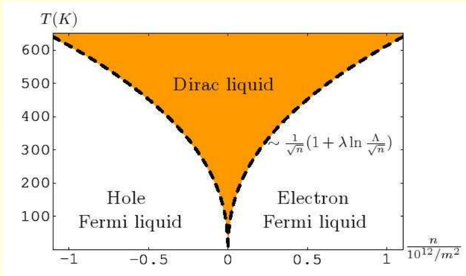
- As in quantum criticality: Maximal possible relaxation rate, set by temperature alone

$$\tau_{ee}^{-1} \approx \alpha^2 \frac{\hbar}{k_B T}$$



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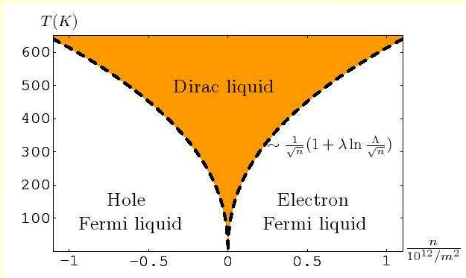

→ Nearly universal conductivity

$$\sigma_{Drude} = \frac{e}{m}\rho\tau \rightarrow \sigma_Q \sim \frac{e}{k_B T / \mathrm{v}^2}\left(e\frac{(k_B T)^2}{(\hbar \mathrm{v})^2}\right)\frac{\hbar}{\alpha^2 k_B T} \sim \frac{1}{\alpha^2}\frac{e^2}{h}$$

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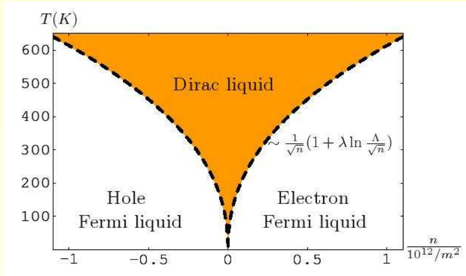

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Only possible result: no other dimensionless parameter than α!

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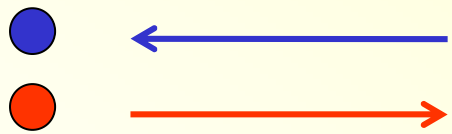

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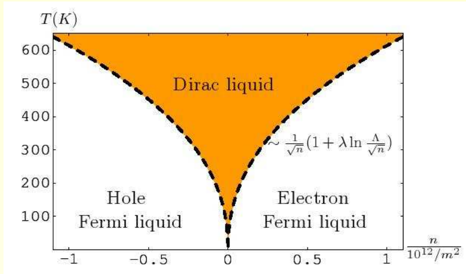

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Exact (Boltzmann)

$$\sigma_Q(\mu = 0) = \frac{0.76}{\alpha^2}\frac{e^2}{h}$$

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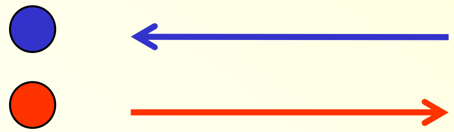

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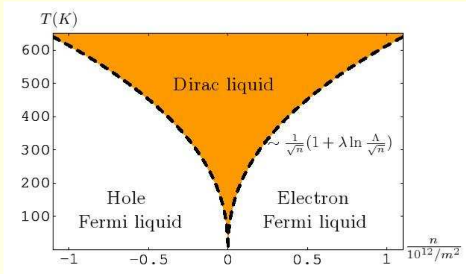

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$$\sigma_Q(\mu = 0) = \frac{0.76}{\alpha^2}\frac{e^2}{h}$$

Marginal irrelevance of Coulomb:

$$\alpha \approx \frac{4}{\log(\Lambda/T)}$$

Thermoelectric response

S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B ***76****, 144502 (2007).*

Charge and heat current:

$$J^\mu = \rho u^\mu - \nu^\mu$$

$$Q^\mu = (\varepsilon + P)u^\mu - \mu J^\mu$$

Thermo-electric response

$$\begin{pmatrix} \vec{J} \\ \vec{Q} \end{pmatrix} = \begin{pmatrix} \hat{\sigma} & \hat{\alpha} \\ T\hat{\alpha} & \hat{\bar{\kappa}} \end{pmatrix} \begin{pmatrix} \vec{E} \\ -\vec{\nabla} T \end{pmatrix}$$

$$\hat{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix}$$

etc.

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$$\hat{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix}$$ etc.

i) Solve linearized hydrodynamic equations
ii) Read off the response functions *(Kadanoff & Martin 1960)*

Results from Hydrodynamics

Response functions at B=0

Symmetry z → -z : $\sigma_{xy} = \alpha_{xy} = \kappa_{xy} = 0$

Longitudinal conductivity:

$$\sigma_{xx}(\omega, k; B=0) = \left(\sigma_Q + \frac{\rho^2}{P+\varepsilon}\frac{\tau}{1-i\omega\tau}\right)$$

Universal conductivity at the quantum critical point ρ = 0

Drude-like conductivity, divergent for $\tau \to \infty, \omega \to 0, \rho \neq 0$

Momentum conservation (ρ≠0)!

Response functions at B=0

Symmetry z $\longrightarrow$ -z : $\sigma_{xy} = \alpha_{xy} = \kappa_{xy} = 0$

Longitudinal conductivity:

$$\sigma_{xx}(\omega, k; B=0) \;=\; \left(\sigma_Q + \frac{\rho^2}{P+\varepsilon}\frac{\tau}{1-i\omega\tau}\right)$$

Thermal conductivity:

$$\kappa_{xx}(\omega, k; B=0) \;=\; \sigma_Q\frac{\mu^2}{T} + \frac{s^2\,T}{P+\varepsilon}\frac{\tau}{1-i\omega\tau} + \mathcal{O}(k^2).$$

Relativistic Wiedemann-Franz-like relations between σ and κ in the quantum critical window!

Response functions at B=0

Symmetry z ⟶ -z : $\sigma_{xy} = \alpha_{xy} = \kappa_{xy} = 0$

Longitudinal conductivity:

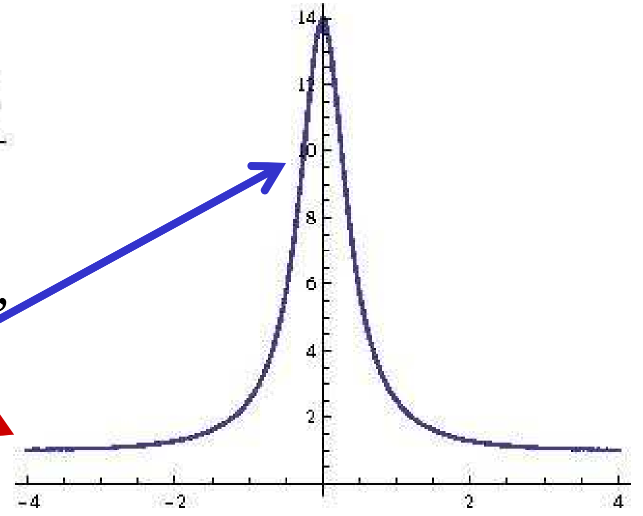
$$\sigma_{xx}(\omega, k; B=0) = \left(\sigma_Q + \frac{\rho^2}{P+\varepsilon}\frac{\tau}{1-i\omega\tau}\right)$$

Thermopower:

$$\alpha_{xx}(\mu, \omega=0) = -\frac{\pi^2}{3e}k_B^2 T \frac{d\sigma(\mu, \omega=0)}{d\mu}$$

Only valid in the degenerate e-gas regime,
but violated in the relativistic window.

$$-\frac{3e}{\pi^2}\frac{1}{k_B^2 T}\frac{\alpha_{xx}}{d\sigma_{xx}/d\mu}$$



$$\frac{\mu}{T}$$

B > 0 : Cyclotron resonance

E.g.: Longitudinal conductivity

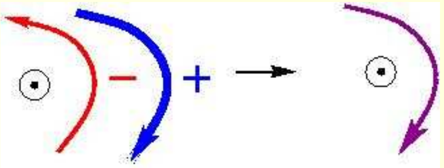
$$\sigma_{xx}(\omega) = \sigma_Q \frac{\omega\left(\omega + i\gamma + i\omega_c^2/\gamma\right)}{\left(\omega + i\gamma\right)^2 - \omega_c^2}$$

Pole in the response

$$\omega = \pm\omega_c^{\mathrm{QC}} - i\gamma - i/\tau$$

Collective cyclotron frequency of the relativistic plasma

$$\omega_c^{\mathrm{QC}} = \frac{\rho B}{(\varepsilon + P)/\mathrm{v}_{\mathrm{F}}^2} \leftrightarrow \omega_c^{\mathrm{FL}} = \frac{e B}{m}$$



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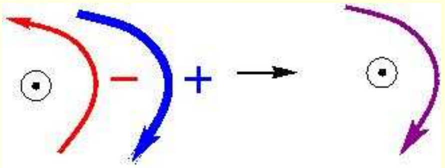

Intrinsic, interaction-induced broadening
(↔ Galilean invariant systems:
No broadening due to Kohn's theorem)

$$\gamma = \sigma_{\mathrm{Q}} \frac{B^2}{(\varepsilon + P)/\mathrm{v}_{\mathrm{F}}^2}$$

B > 0 : Cyclotron resonance

Longitudinal conductivity

$$\sigma_{xx}(\omega,k) = \sigma_Q \frac{(\omega+i/\tau)(\omega+i/\tau+i\gamma+i\omega_c^2/\gamma)}{(\omega+i/\tau+i\gamma)^2-\omega_c^2}$$

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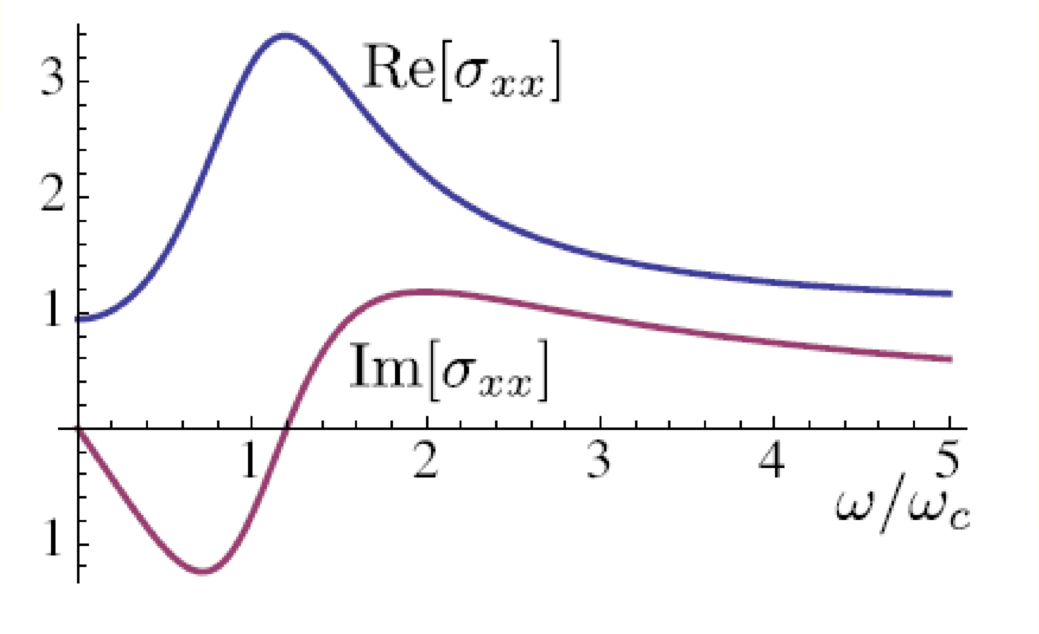

Observable at room temperature!

$T \approx 300K$

$B \approx 0.1T$

$\rho \approx 10^{11} cm^{-2}$

$\omega_c^{QC} \approx 10^{13} s^{-1}$

Does relativistic hydrodynamics apply?

- Do T and μ break relativistic invariance?
- Validity at large chemical potential?
- Larger magnetic field?

Boltzmann Approach

MM, L. Fritz, and S. Sachdev, PRB 2008

→ Recover and refine the hydrodynamic description

→ Describe relativistic-to-Fermi-liquid crossover

Boltzmann approach

L. Fritz, J. Schmalian, MM, and S. Sachdev, PRB 2008

Boltzmann equation in Born approximation

$$\left(\partial_t + e\left[\mathbf{E} + \mathbf{v}\times\mathbf{B}\right]\cdot\frac{\partial}{\partial\mathbf{k}}\right) f_{\pm}\left(\mathbf{k},t\right) = \alpha^2 I_{\mathrm{coll}}^{Cb}\left[\mathbf{k},t \mid \left\{ f_{\pm}\left(\mathbf{k}',t\right)\right\}\right] + \Delta\, I_{\mathrm{coll}}^{dis}\left[\mathbf{k},t \mid \left\{ f_{\pm}\left(\mathbf{k}',t\right)\right\}\right]$$

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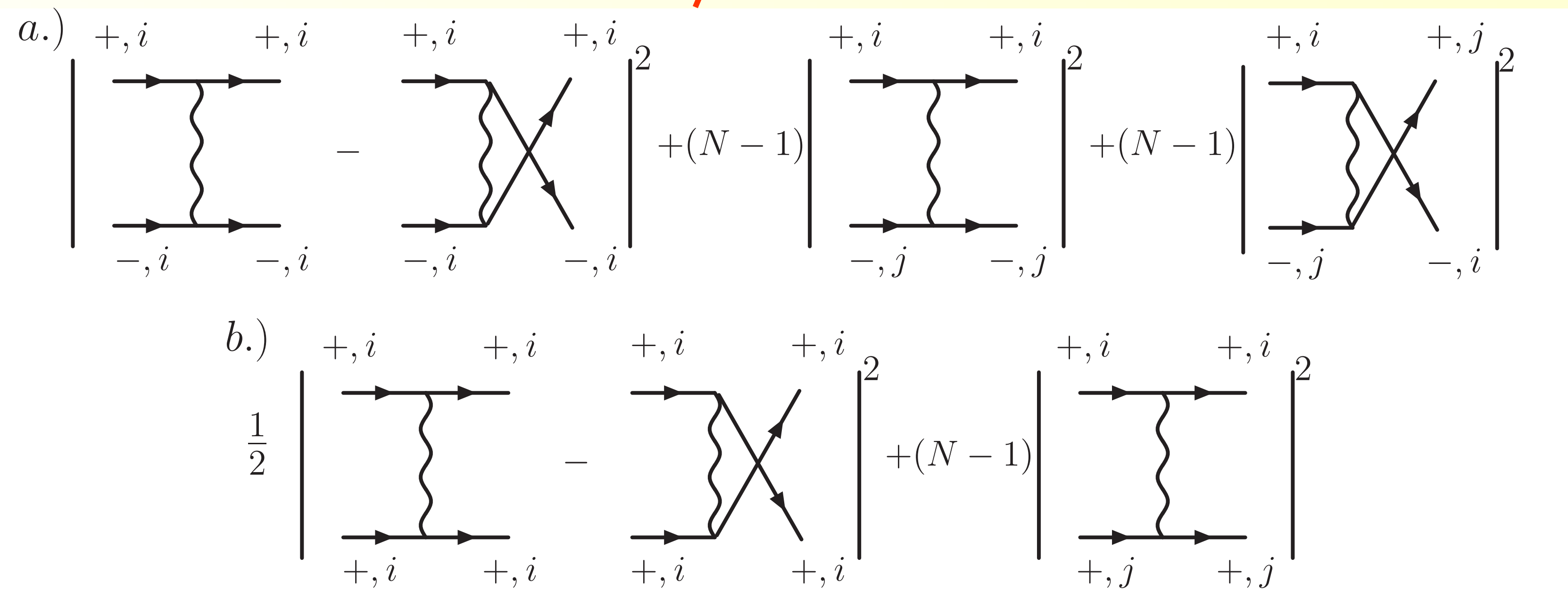

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1. Linearization: $f_\pm(\mathbf{k},t) = f_\pm^{eq}(\mathbf{k},t) + \delta f_\pm(\mathbf{k},t)$

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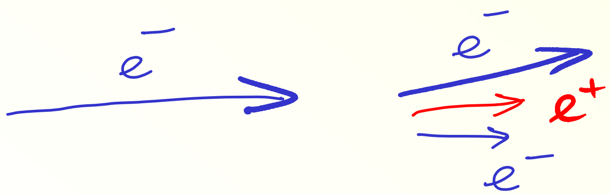
1. Linearization: $f_\pm(\mathbf{k},t) = f_\pm^{eq}(\mathbf{k},t) + \delta f_\pm(\mathbf{k},t)$

2. Forward scattering diverges logarithmically in 2d! (Cutoff at $\theta \approx \alpha \ll 1$)



Wilkins et al. (1971)

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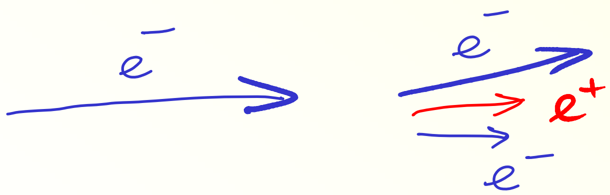
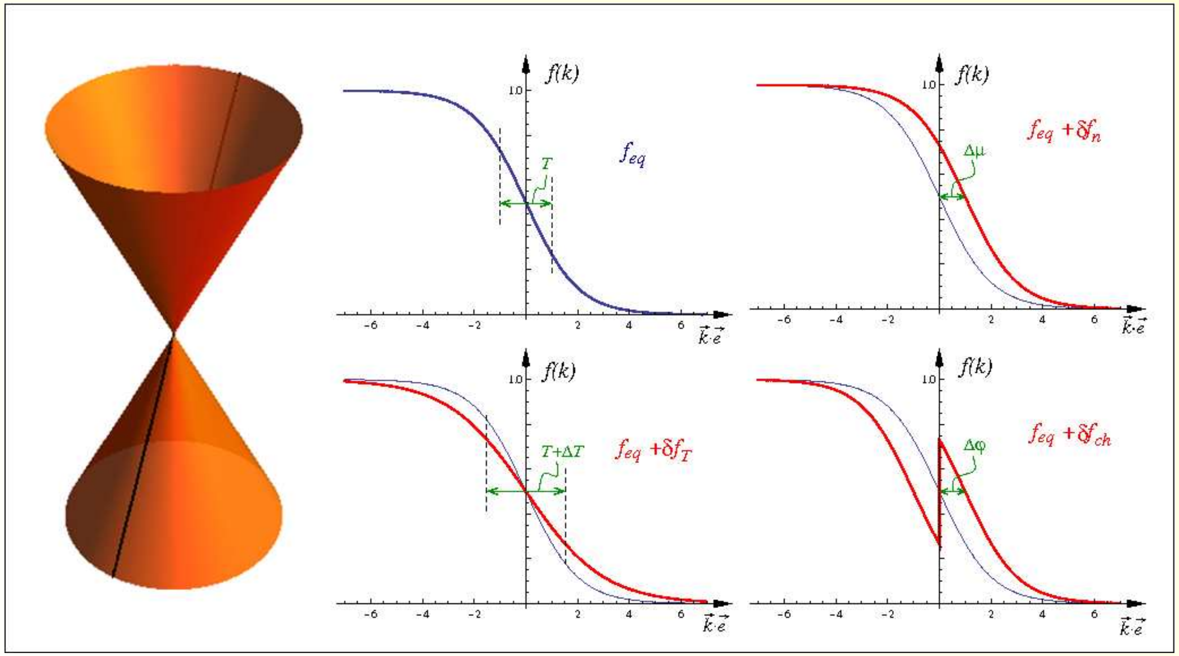


Wilkins et al. (1971)

→ Equilibration among particles with same group velocity



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L. Fritz, J. Schmalian, MM, and S. Sachdev, PRB 2008

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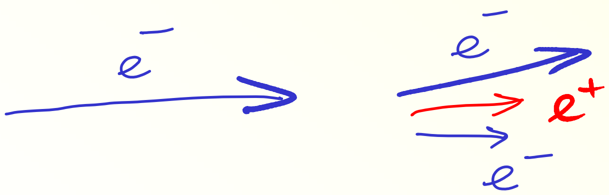

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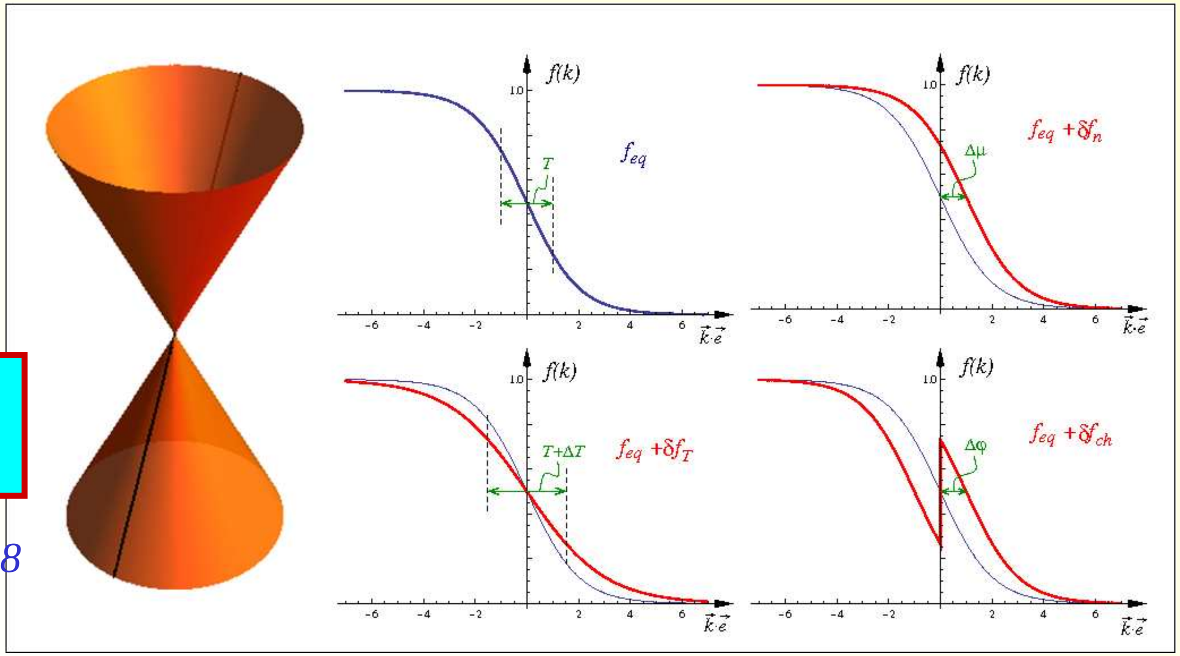


Reduced to simple optimization problem for c_μ, c_T, c_φ !

MM, L. Fritz, and S. Sachdev, PRB 2008

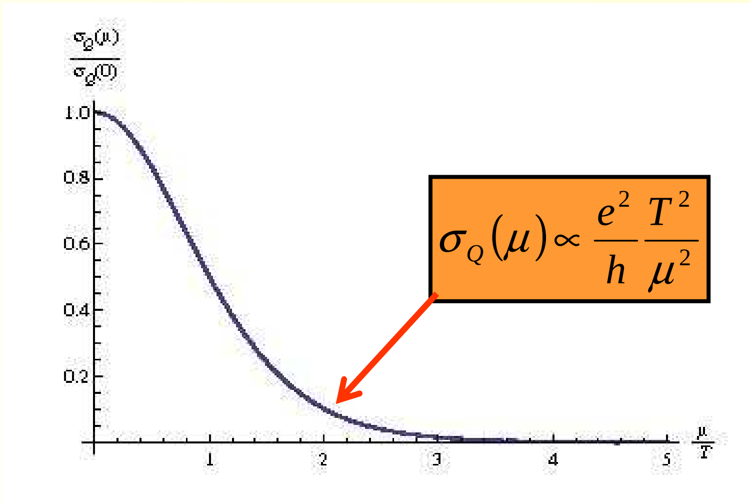
Boltzmann approach

MM, L. Fritz, and S. Sachdev, PRB 2008
Kashuba, PRB 2008

Collision-dominated conductivity

$$\sigma_Q(\mu=0) \approx \frac{0.76}{\alpha^2(T)} \frac{e^2}{h}$$

Gradual disappearance of relativistic physics



Recovering magnetohydrodynamics

MM, L. Fritz, and S. Sachdev, PRB 2008

Momentum conservation →
Exact zero mode of the Coulomb collision integral! (↔ Boost!)

$$\delta f_{\pm}^{(0)}(\mathbf{k}) = \pm\, c_T \mathbf{k}\cdot\mathbf{E}\;\; f_{\pm}^{eq}(\mathbf{k})\left[1 - f_{\pm}^{eq}(\mathbf{k})\right]$$

Recover Magnetohydrodynamics:

Study the dynamics of the momentum mode under
- Deflection from B
- Impurities
- Coupling to all other modes

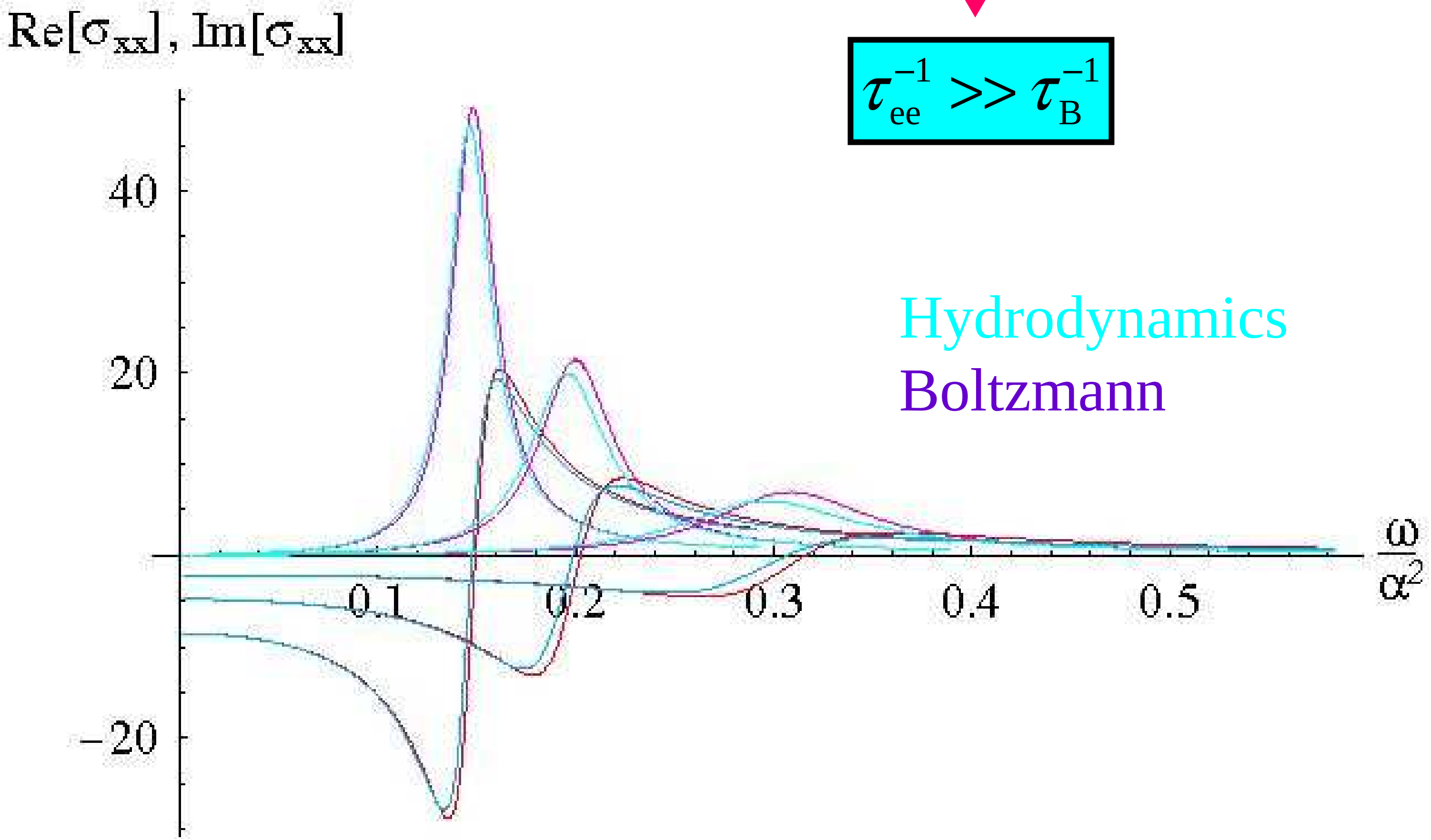
$$\sigma_{xx}(\omega, B) = \sigma_{xx}^{\mathrm{MHD}}(\omega, B) + \mathcal{O}(b/\alpha^2, \omega/\alpha^2)$$

Corrections small if τ_{ee}^{-1} is dominant.

Cyclotron resonance revisited

$$\sigma_{xx}(\omega, B) = \sigma_{xx}^{\mathrm{MHD}}(\omega, B) + \mathcal{O}(b/\alpha^2, \omega/\alpha^2)$$

$$\tau_{ee}^{-1} >> \tau_{B}^{-1}$$



Cyclotron resonance revisited II

Crossover to the electron Fermi liquid:

1. Semiclassical ω_c recovered at $\mu >> T$

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Cyclotron resonance revisited II

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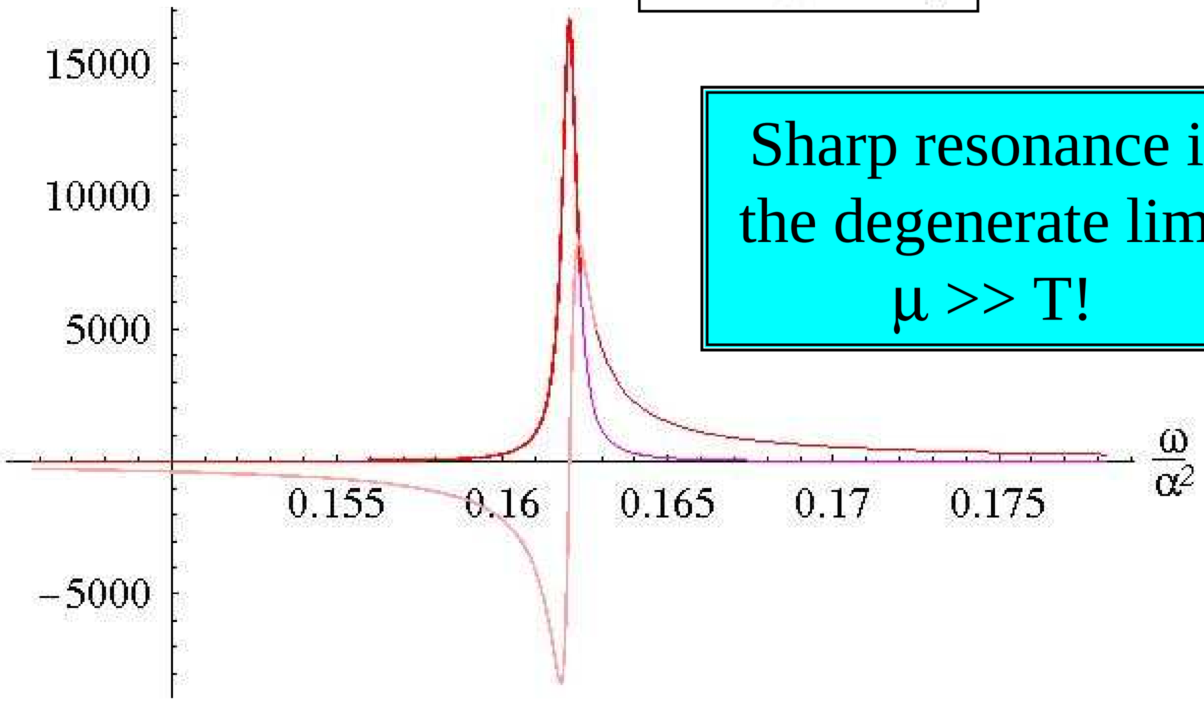
2. Kohn's theorem recovered:
   No broadening of the resonance for a single parabolic band!

$$\gamma \equiv \frac{\sigma_Q B^2 v_F^2}{(\varepsilon + P)}$$

$$\gamma \propto \sigma_Q(\mu) \xrightarrow{\mu>>T} 0$$



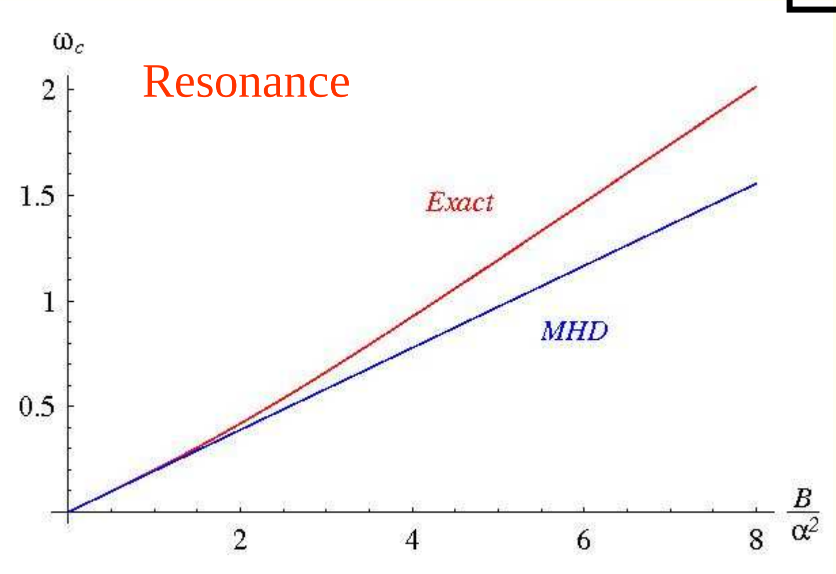
Sharp resonance in the degenerate limit $\mu >> T$!

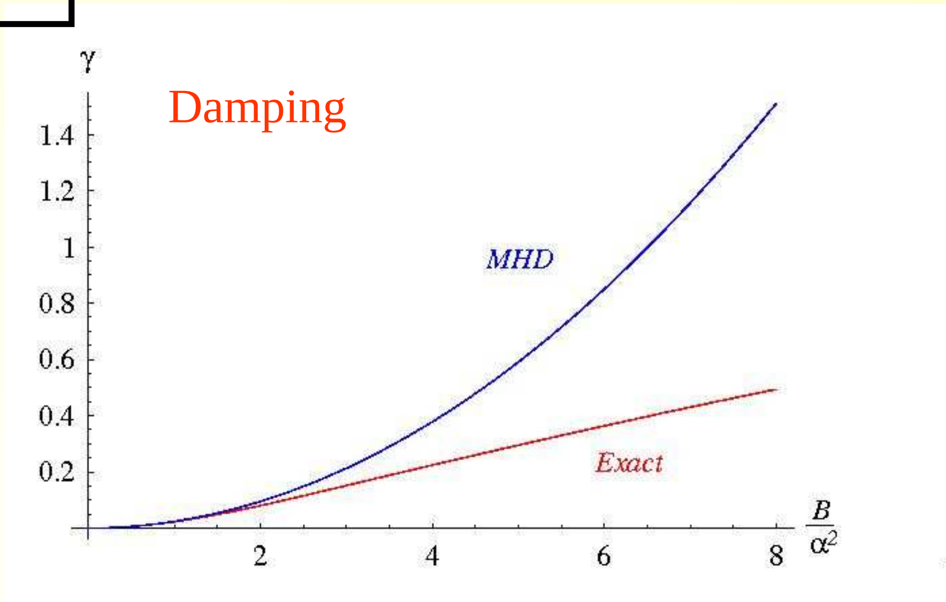
Cyclotron resonance revisited

Beyond hydrodynamics: Large fields

$$\tau_{\mathrm{B}}^{-1} > \tau_{\mathrm{ee}}^{-1} >> \tau_{\mathrm{imp}}^{-1}, \omega$$

$$\mu = T$$





Strongly coupled liquids

S. Hartnoll, P. Kovtun, MM, S. Sachdev (2007)

Exact results via string theoretical AdS–CFT correspondence

→ Response functions in special strongly coupled relativistic fluids
(for N=4 supersymmetric Yang Mills theories):

• Confirms the structure of the hydrodynamic response functions $\sigma(\omega)$ etc.

• Allows to calculate the transport coefficient σ_Q for a strongly coupled theory!

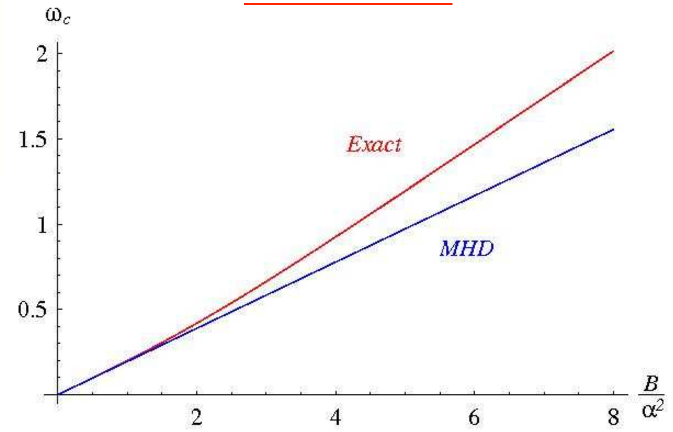
Strongly coupled liquids

Same trends as in exact (AdS-CFT) results for strongly coupled relativistic fluids!

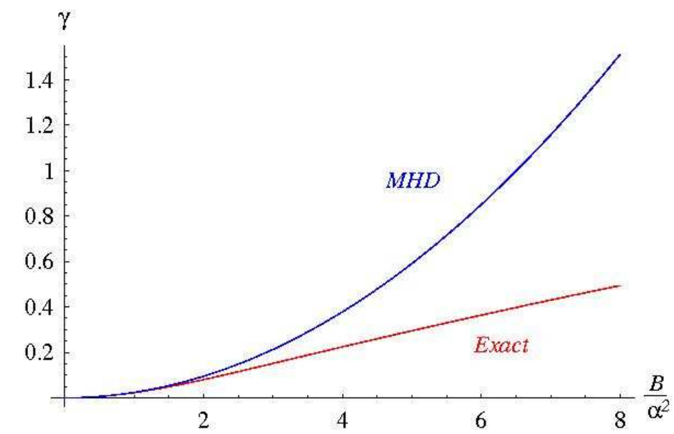
S. Hartnoll, C. Herzog (2007)

Graphene

Resonance



Damping



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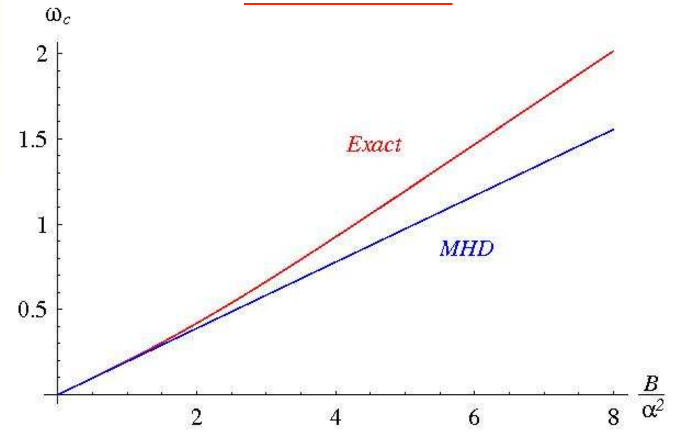

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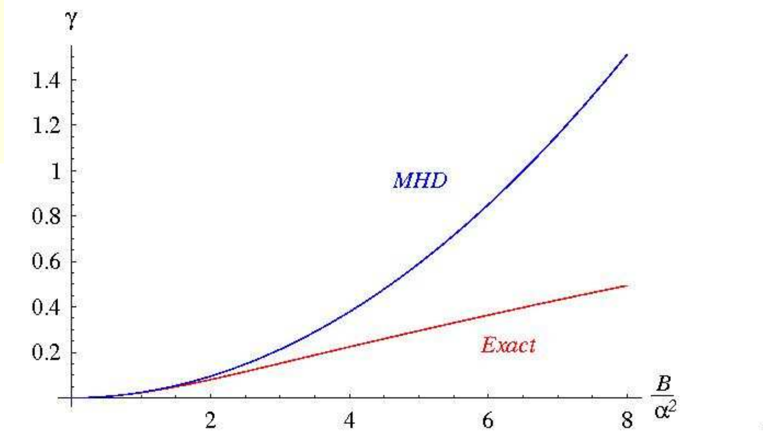
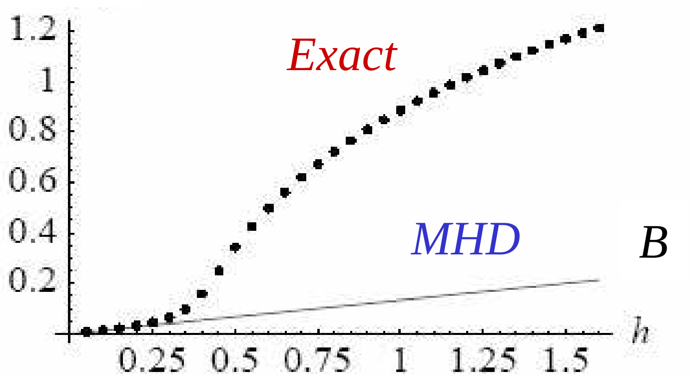
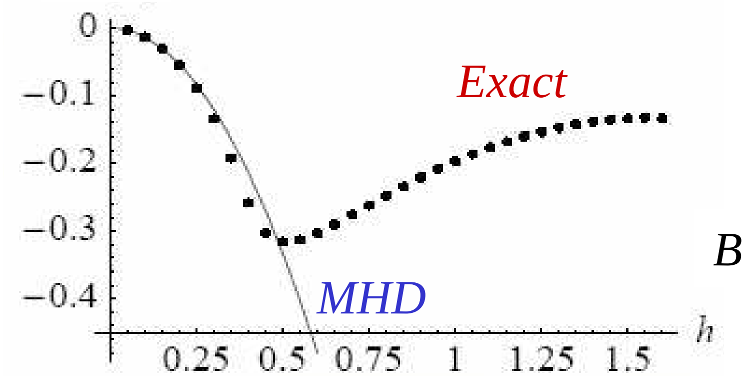


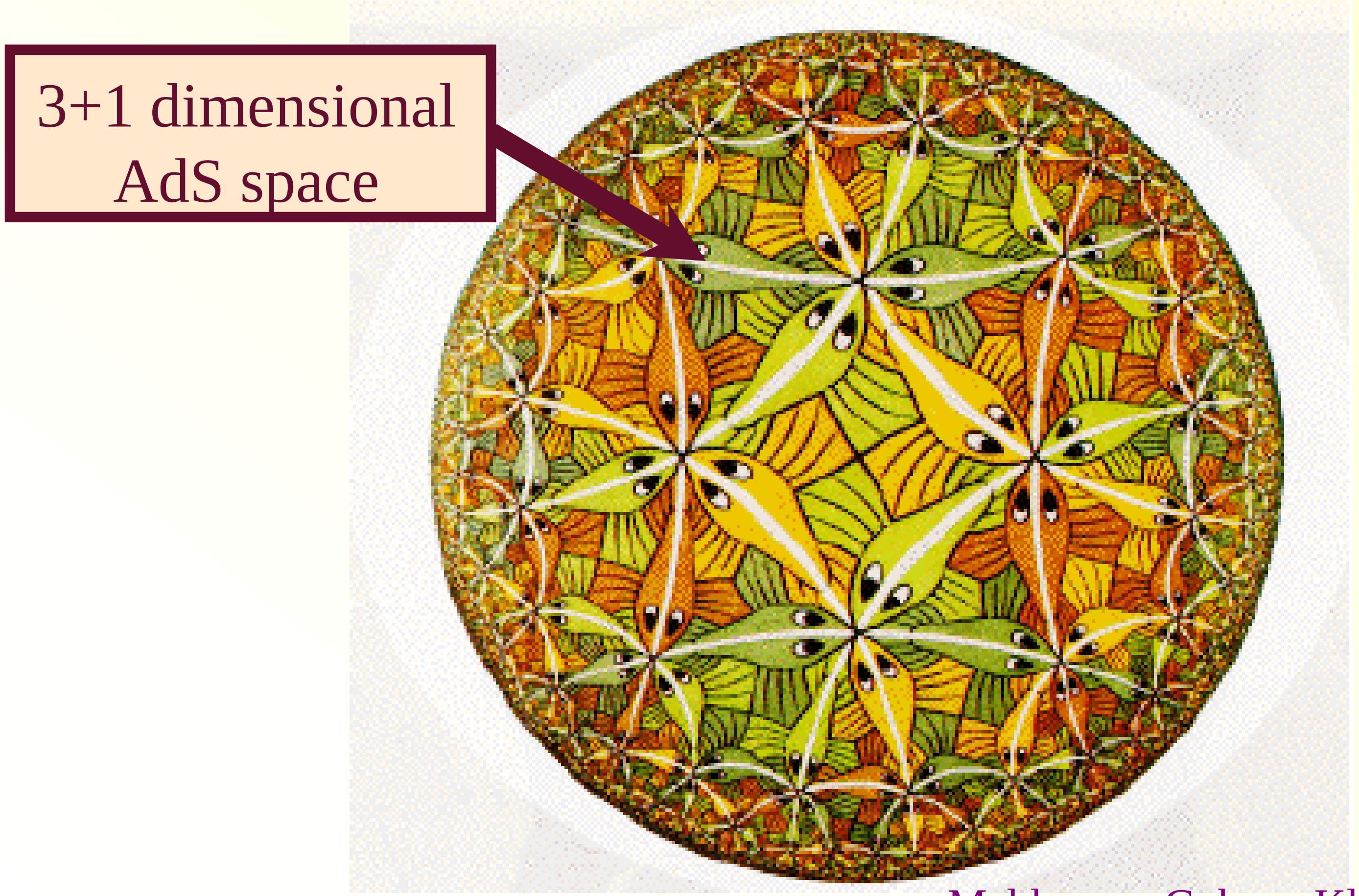
$\mathcal{N}$= 4 SUSY SU(N) gauge theory [flows to CFT at low energy]





AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions



Maldacena, Gubser, Klebanov, Polyakov, Witten

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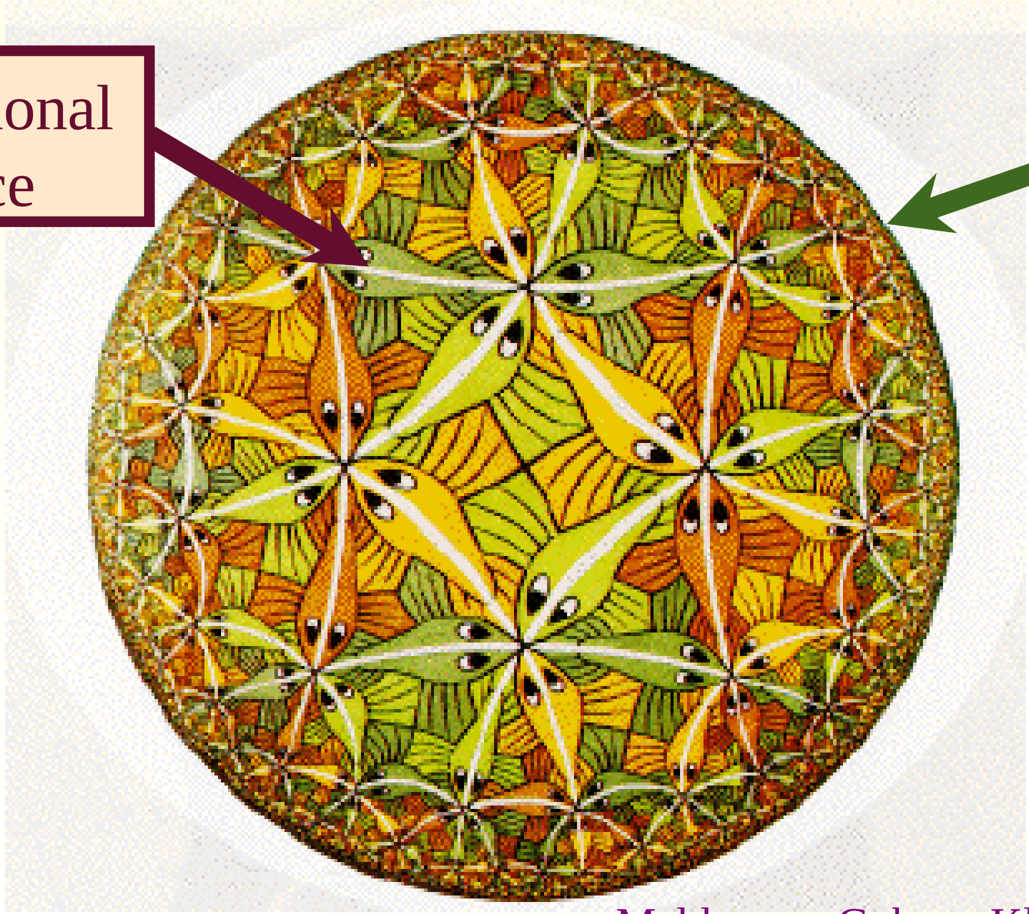

3+1 dimensional AdS space

A 2+1 dimensional system at its quantum critical point

Maldacena, Gubser, Klebanov, Polyakov, Witten

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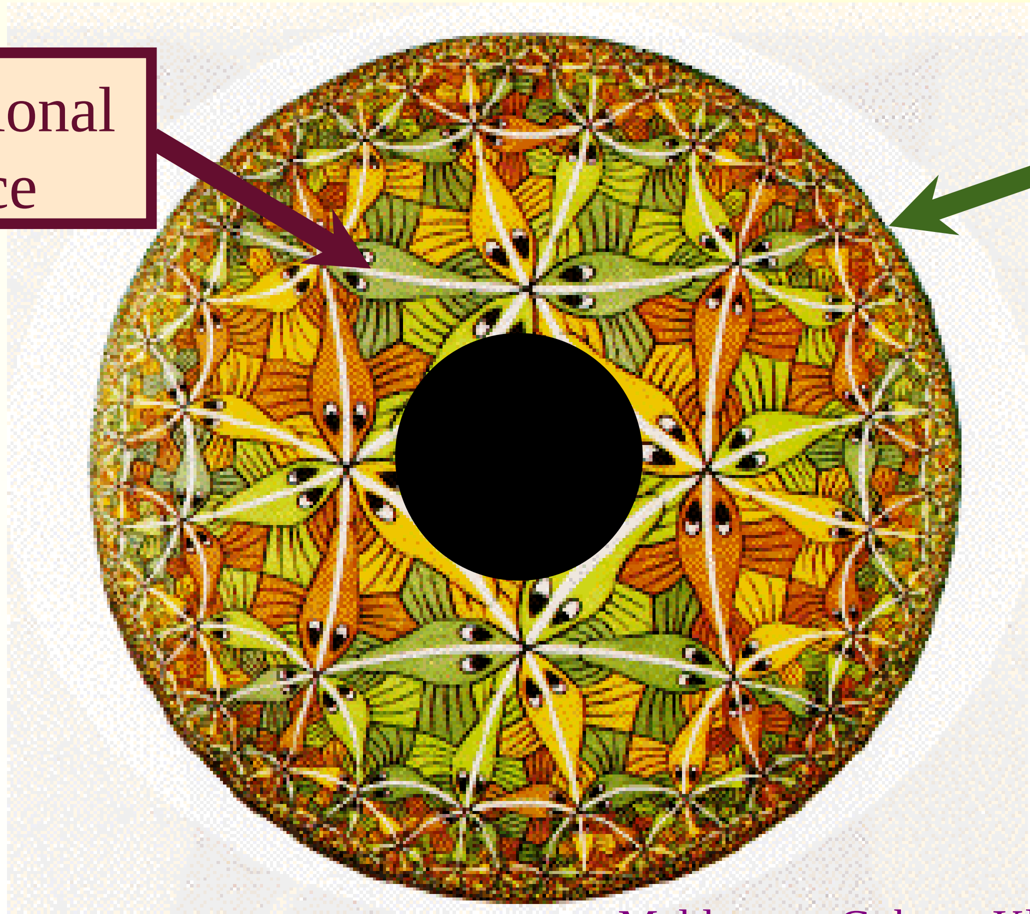

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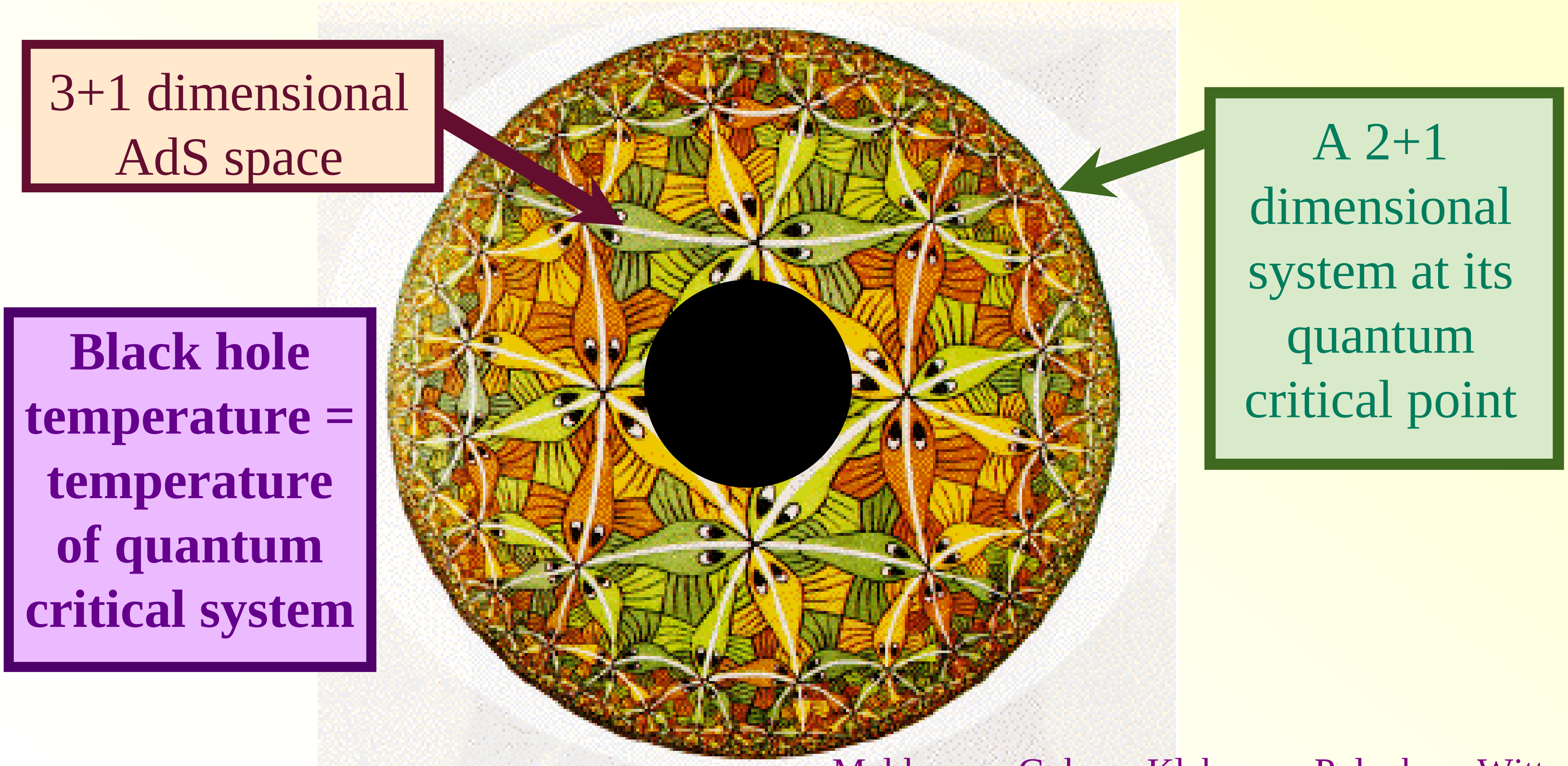

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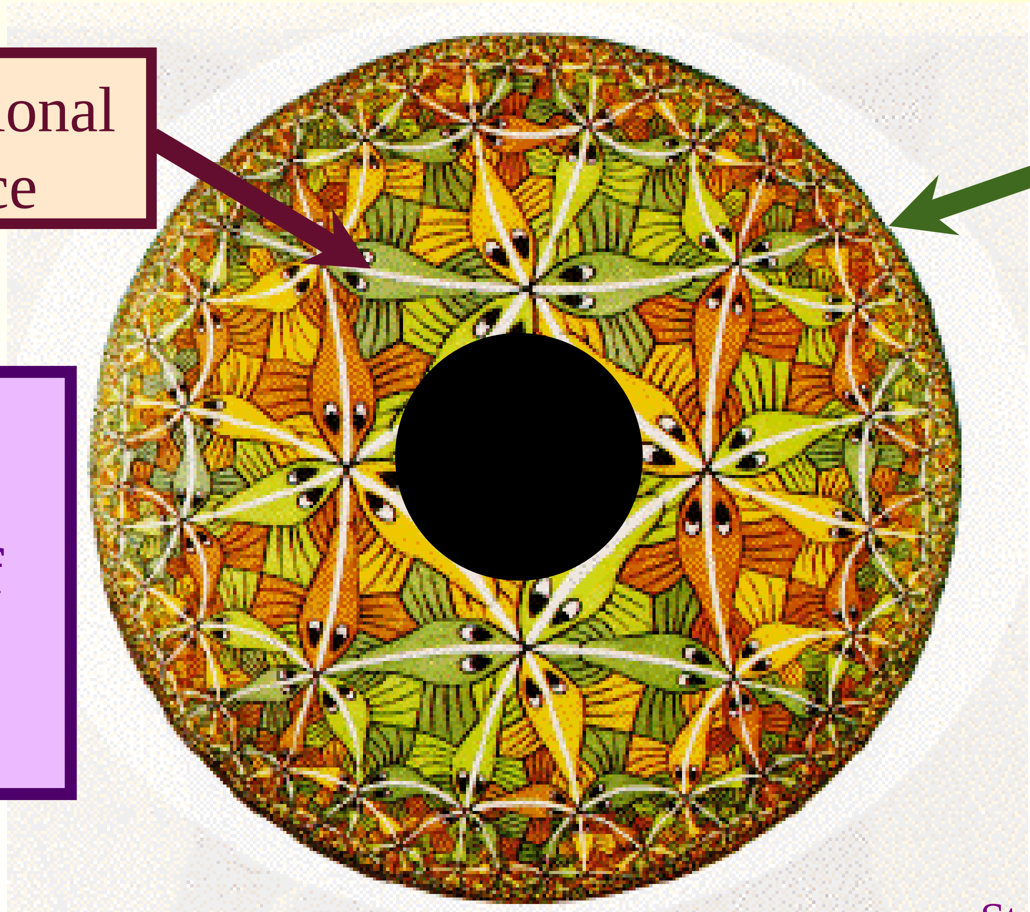

A 2+1 dimensional system at its quantum critical point

Black hole entropy = Entropy of quantum criticality

Strominger, Vafa

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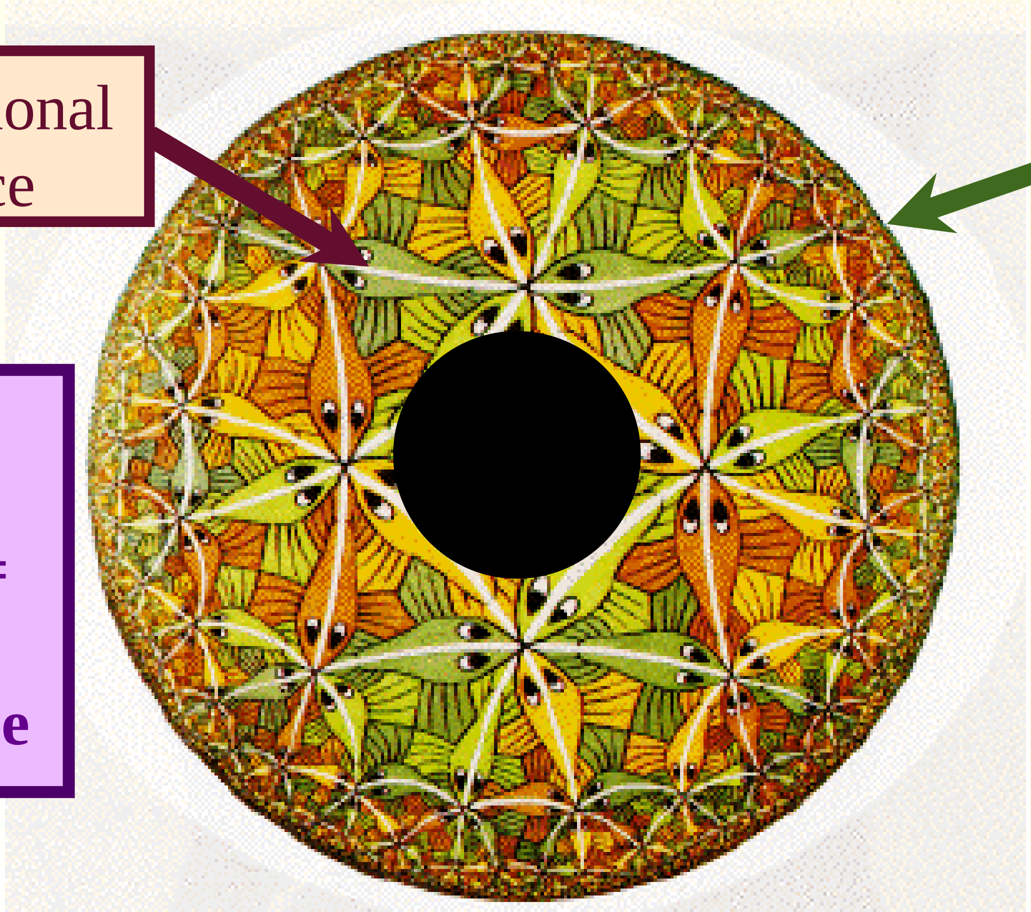

A 2+1 dimensional system at its quantum critical point

Quantum critical dynamics = waves in curved space

Maldacena, Gubser, Klebanov, Polyakov, Witten

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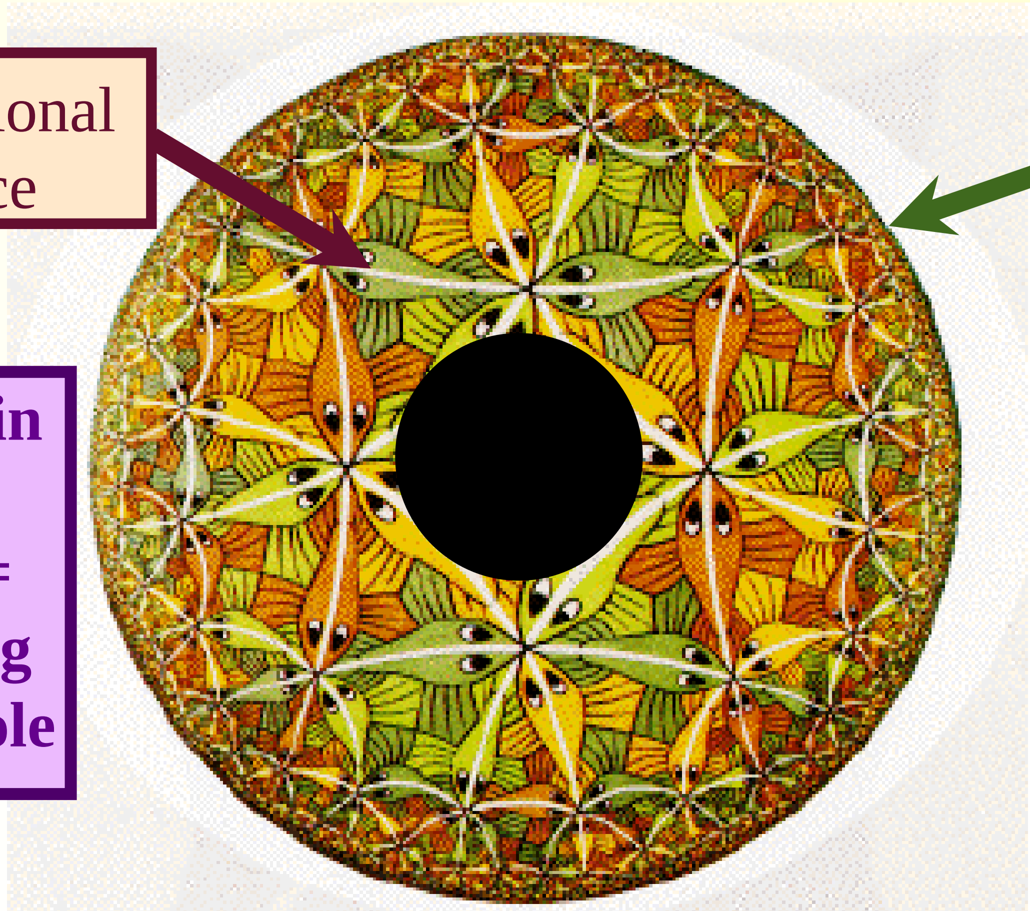

A 2+1 dimensional system at its quantum critical point

Dissipation in quantum criticality = waves falling into black hole

Kovtun, Policastro, Son

Further analogy with AdS-CFT

MM and J. Schmalian, (2008)

Is quantum critical graphene a nearly perfect fluid?

$\longleftrightarrow$

Anomalously low viscosity? – Yes!

Conjecture from black hole physics:

$$\frac{\text{shear viscosity}}{\text{entropy density}} = \frac{\eta}{s} > \frac{\hbar}{k_B}\frac{1}{4\pi}$$

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Undoped Graphene:

$$\eta \propto n \cdot m\mathrm{v}^2 \cdot \tau \;\rightarrow\; n_{\mathrm{th}} \cdot k_B T \cdot \frac{\hbar}{\alpha^2 k_B T} = \frac{\hbar}{\alpha^2} n_{\mathrm{th}}$$

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Doped Graphene:

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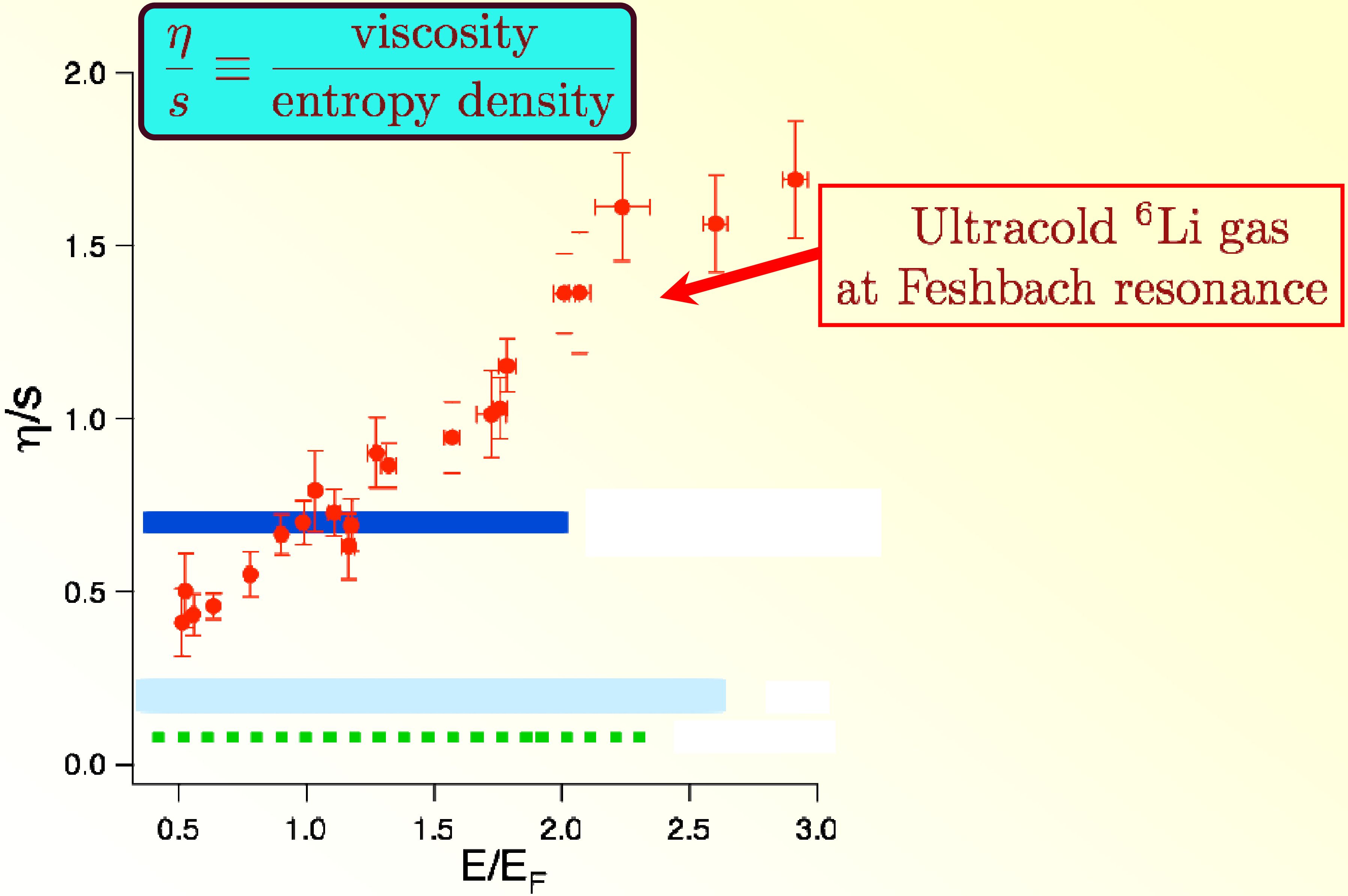
$$s \propto k_B n \frac{T}{E_F}$$

$$\frac{\eta}{s} \sim \frac{\hbar}{k_B}\left(\frac{E_F}{T}\right)^3$$



T. Schäfer, Phys. Rev. A ***76****, 063618 (2007).*

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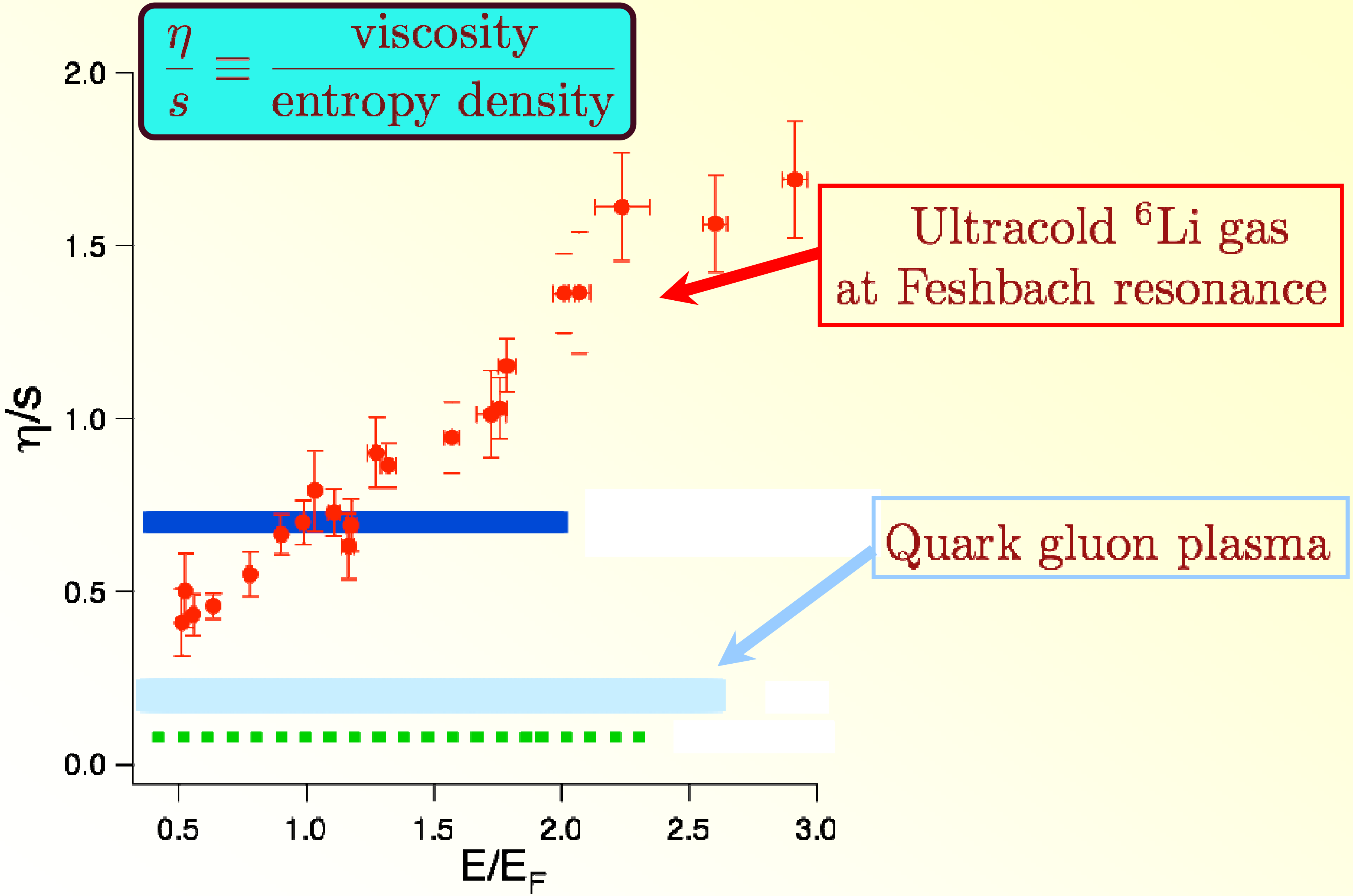

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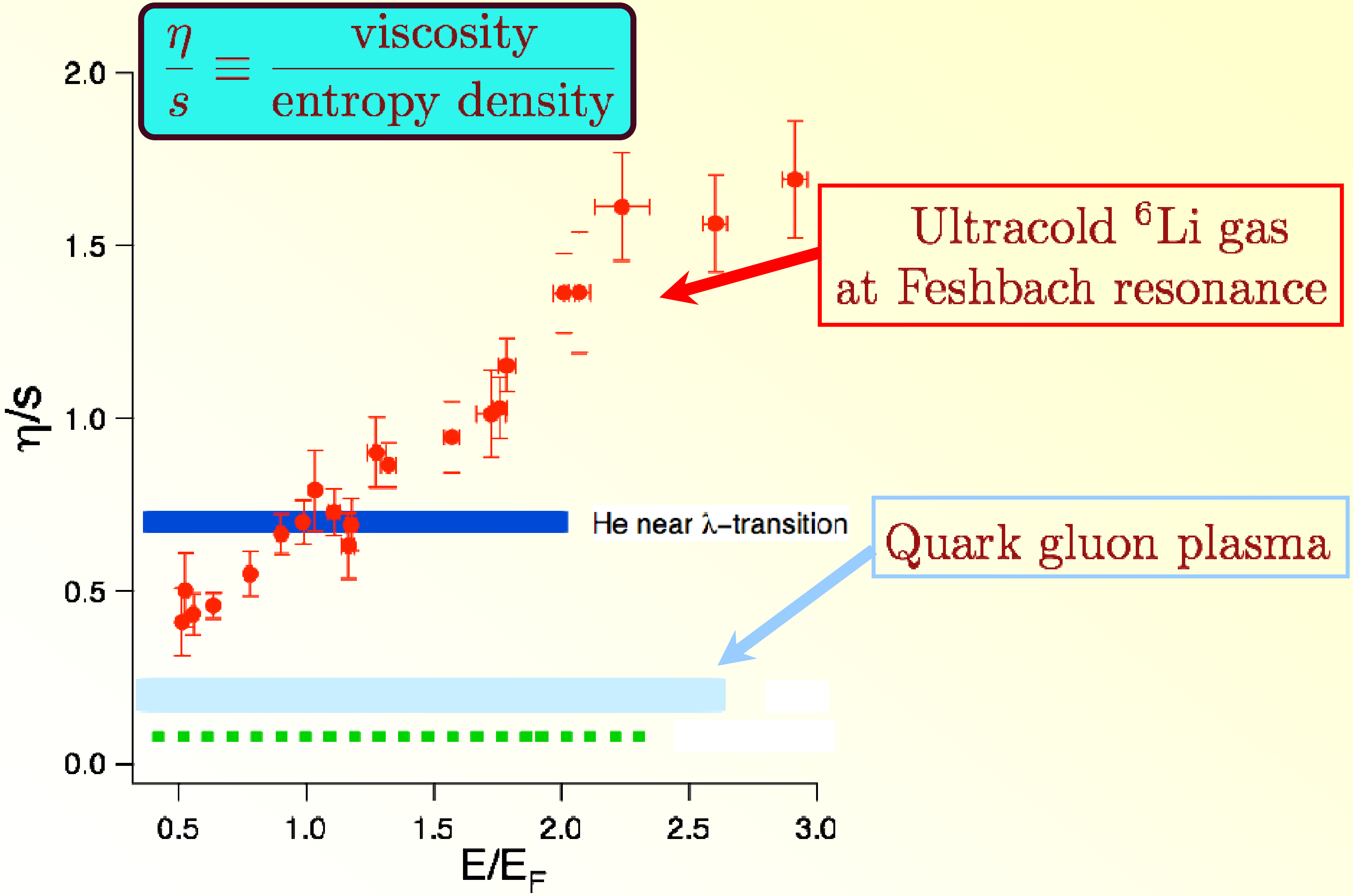

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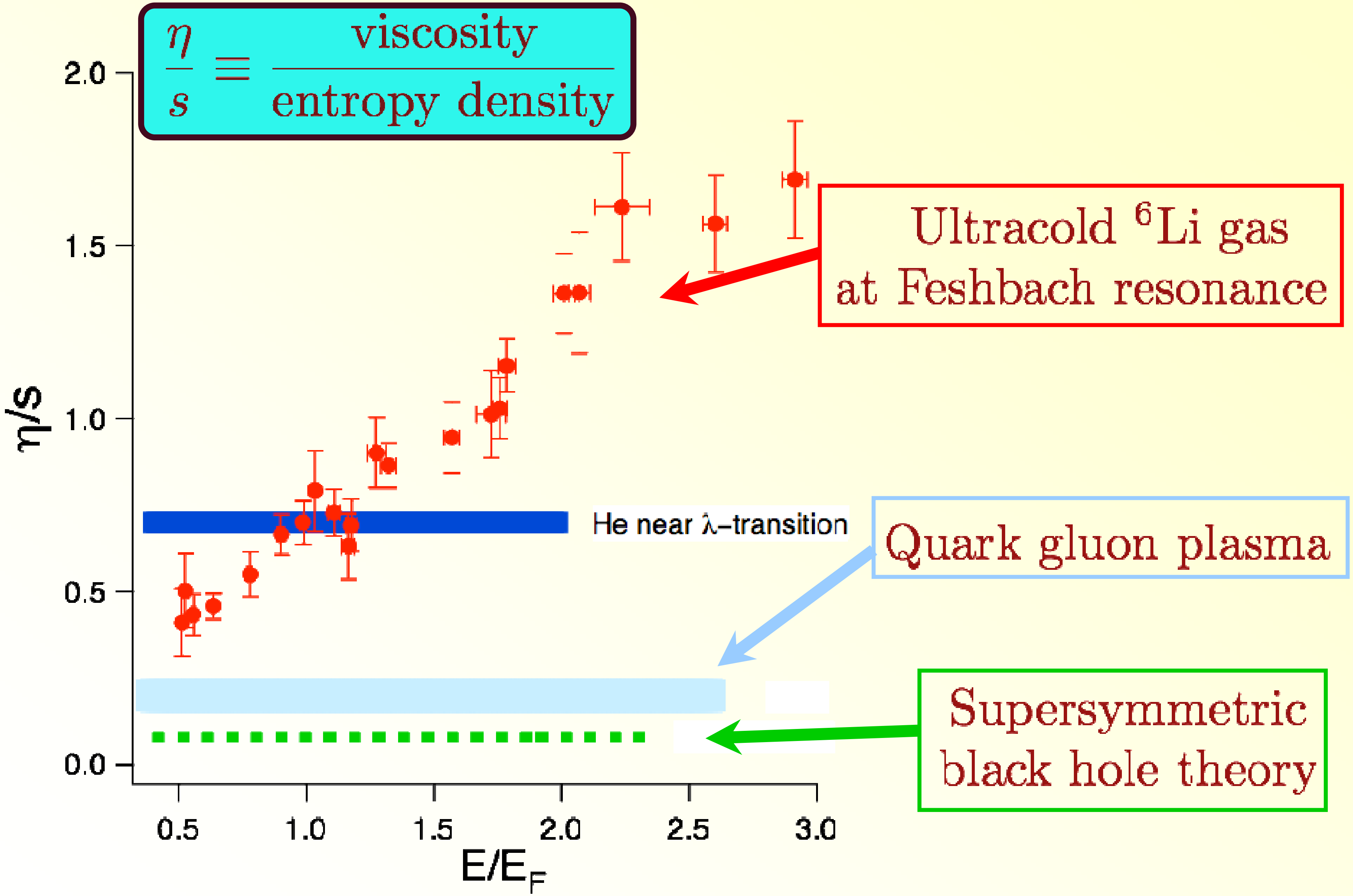

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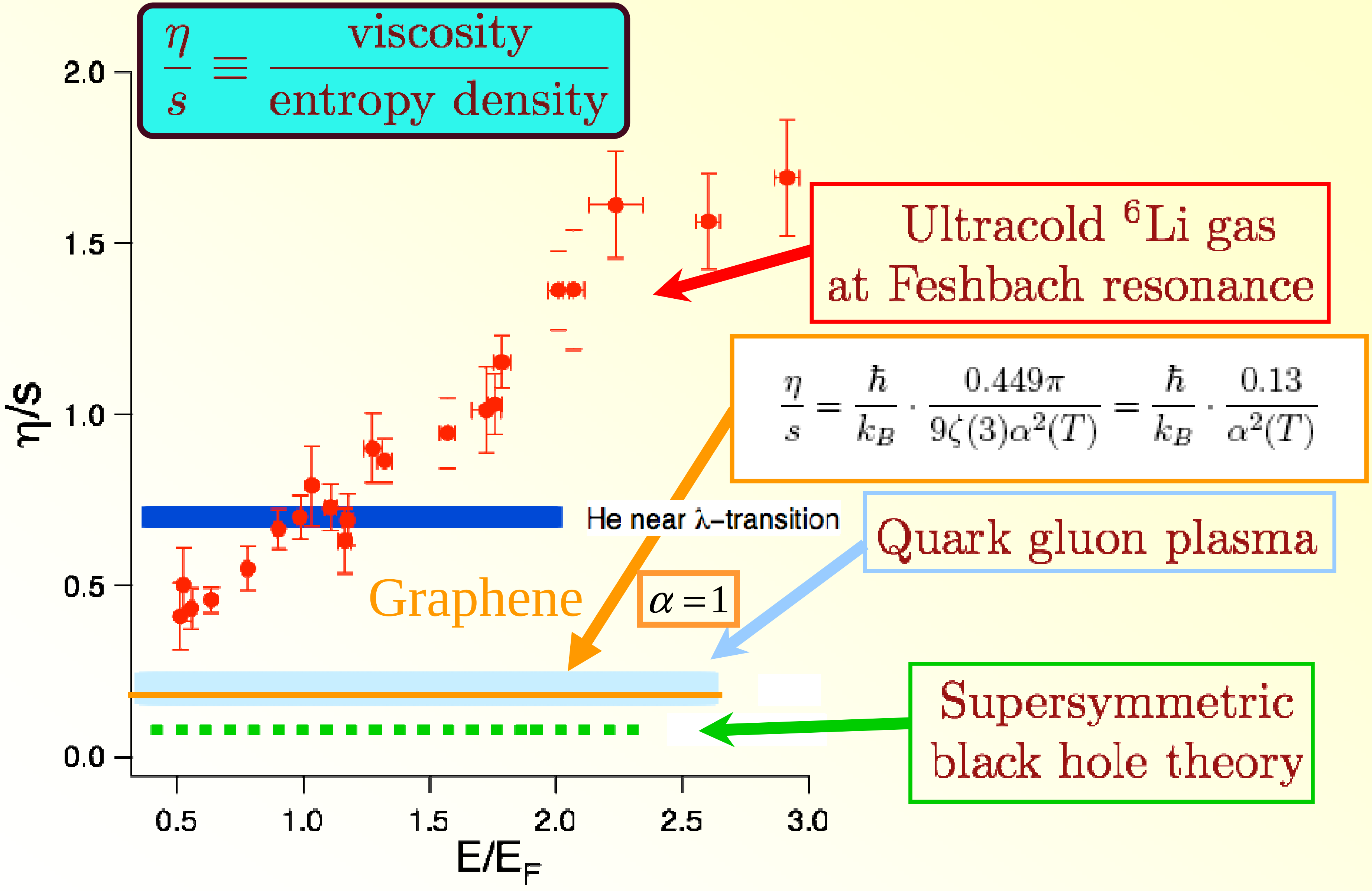

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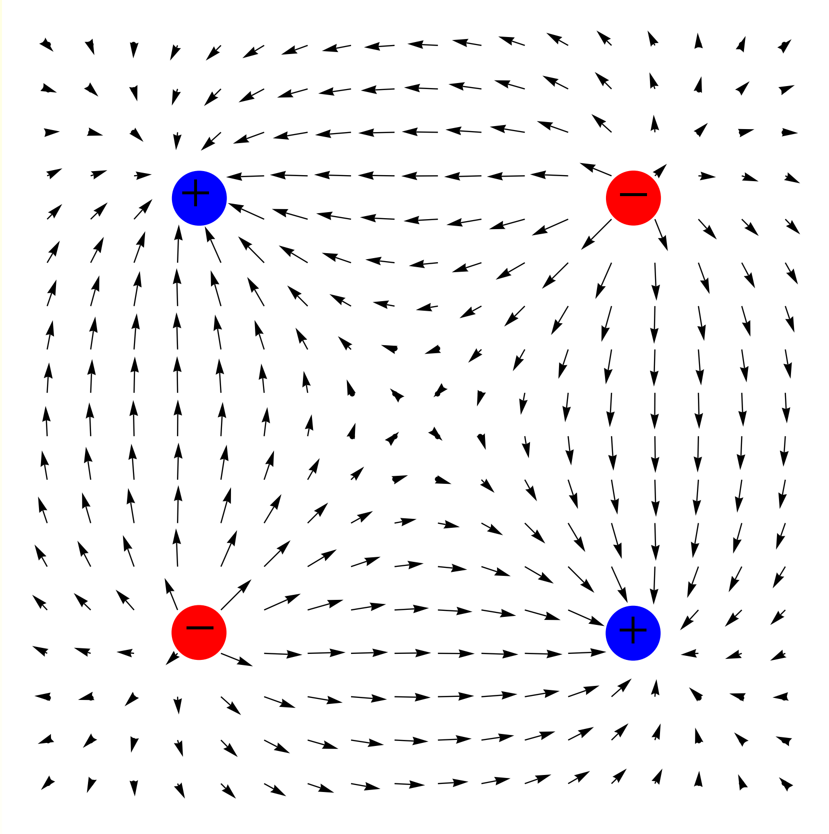
Electronic consequences of low viscosity?

MM, J. Schmalian, L. Fritz, (2008)

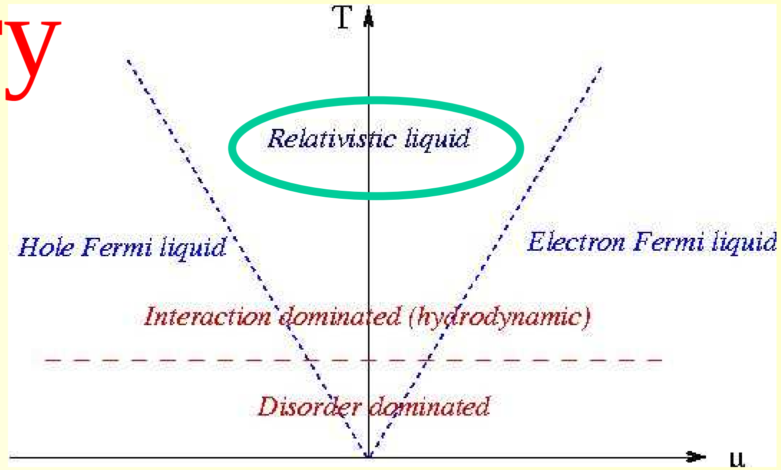
• Viscous effects on conductance in non-uniform current flow?
(Decrease with length scale)

• Electronic turbulence in clean graphene?
Reynolds number:

$$\mathrm{Re} \propto \frac{s}{\eta} >> 1$$



Summary



- Hydrodynamic description of relativistic transport

  → Collective cyclotron resonance

  → Lorentz invariance → thermodynamics and *only one* parameter σ_Q determine all response

- Boltzmann theory

  → Hydrodynamics confirmed and refined

- Toy example of relativistic/quantum critical systems

  → Graphene as a nearly perfect fluid, like the quark gluon plasma!