

Purely electronic transport and localization in the Bose glass

Markus Müller

Discussions with

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B. Sacépé
D. Shahar



The Abdus Salam
ICTP Trieste

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Outline

- The dirty **superconductor-insulator transition (SIT)**
- Brief review of various **puzzling transport** experiments on the insulating side of the SIT (in the supposed **Bose glass** phase)
- Proposed resolution:
Study of spectral properties in the Bose glass in the absence of phonons. Implications for
 - **Transport** as a function of temperature
 - **Many-body localization and its precursors**

SI transition in thin films

M. Strongin, et al., Phys. Rev. B1, 1078 (1970).

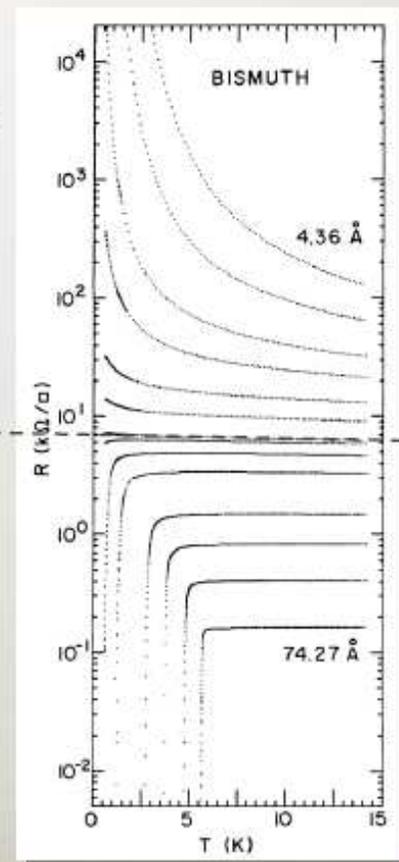
D. B. Haviland, Y. Liu, and A. M. Goldman, Phys. Rev. Lett. 62, 2180 (1989)...

Thickness tuned transition

T = 0 transition

Review: Finkl'stein ('94),
Markovic and Goldman ('98).

2D

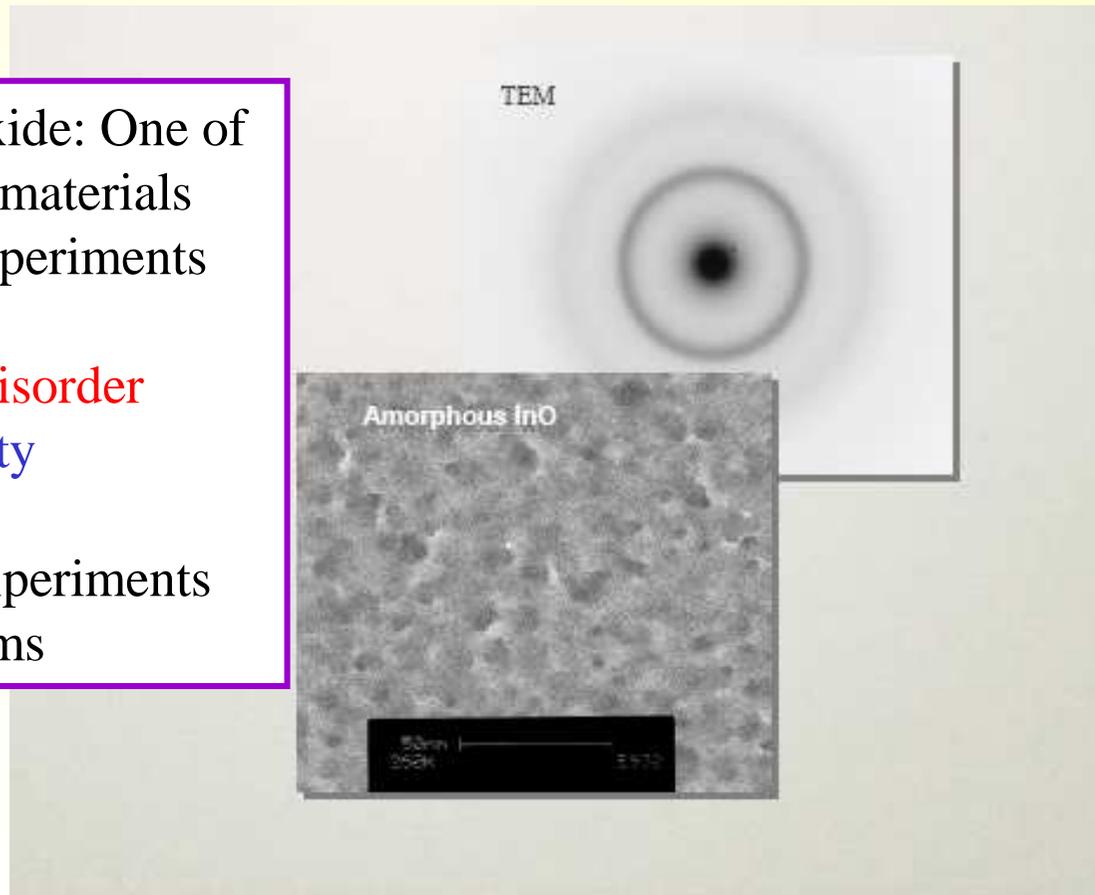


Indium-oxide (InO_x)

Indium-oxide: One of the major materials used in experiments

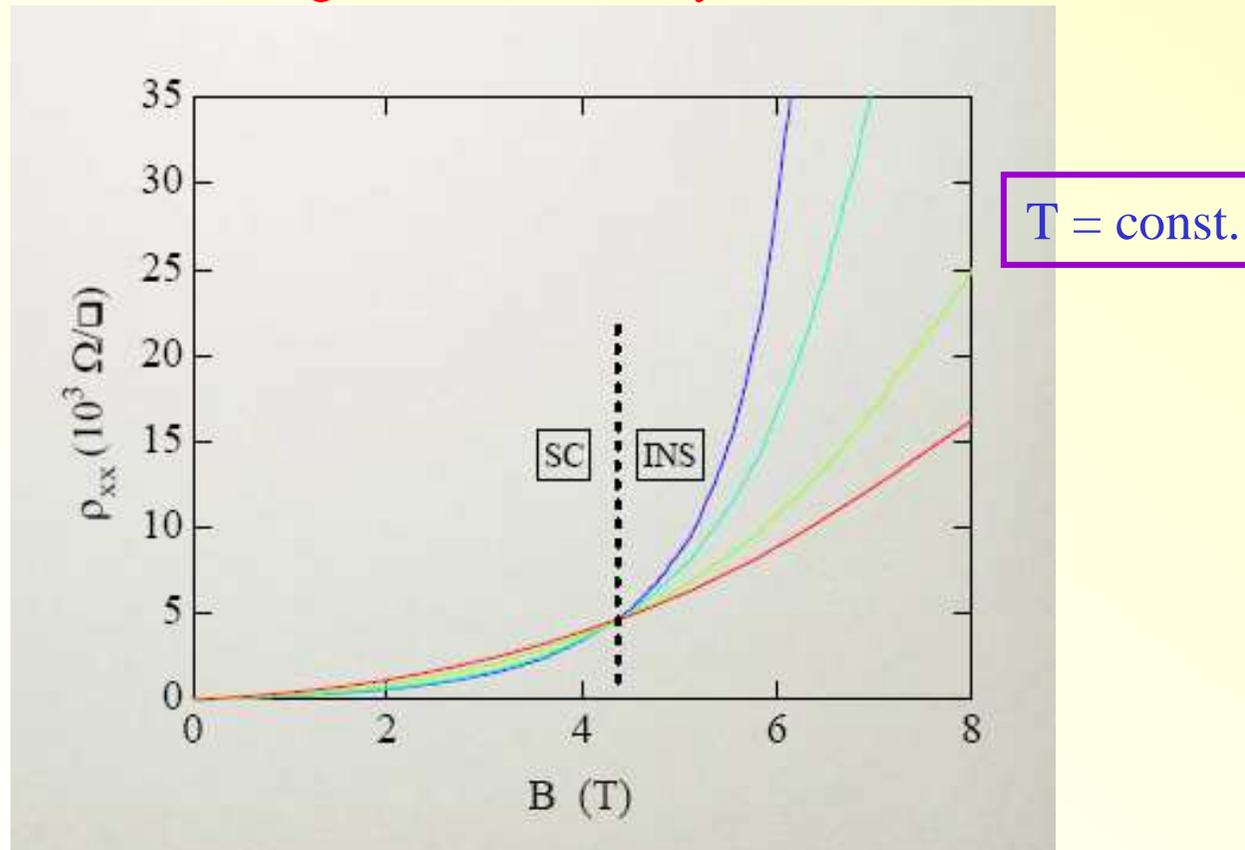
- Strong disorder
- Tunability

Similar experiments in TiN films



Field driven transition

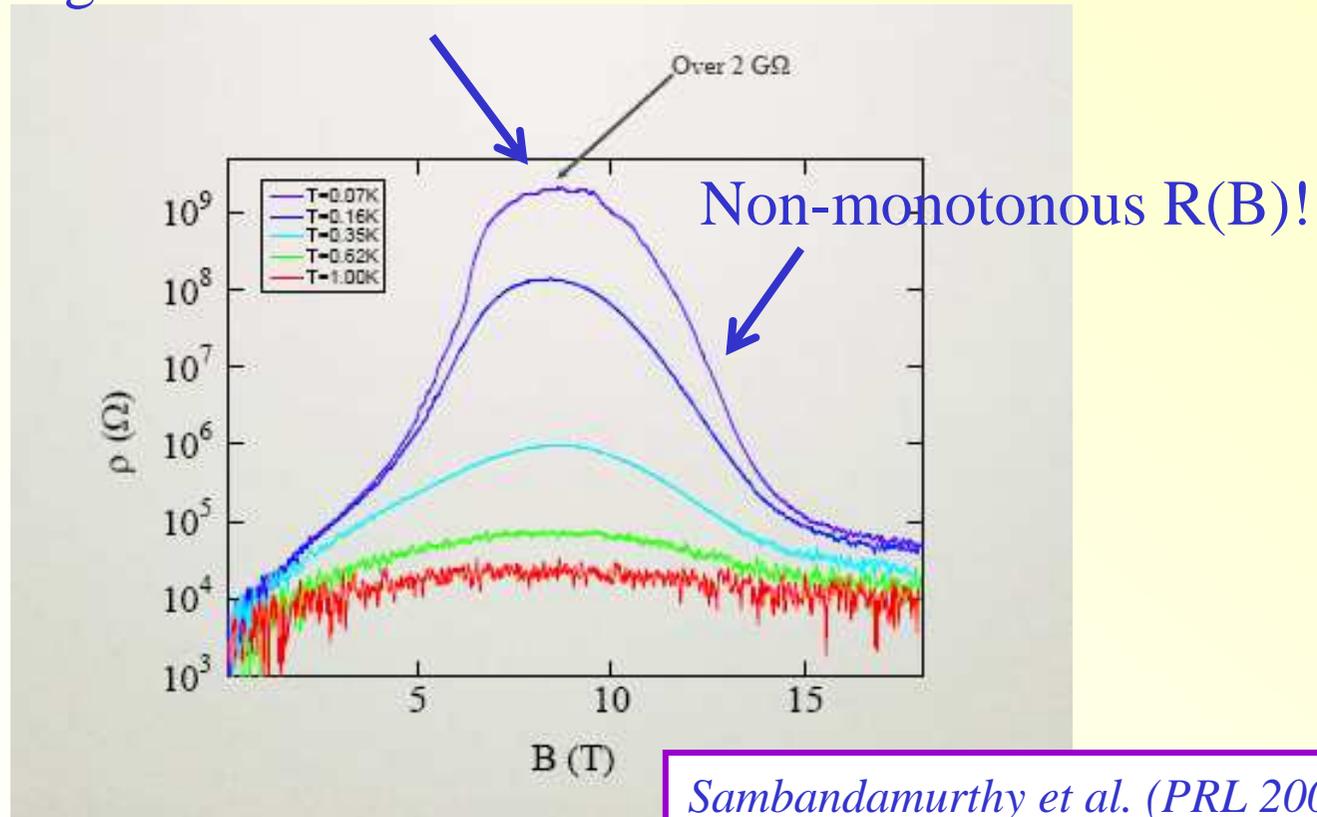
Magnetic field destroys SC!



Gantmakher, Shahar, Kapitulnik, Goldman, Baturina

Insulator: Giant magnetoresistance

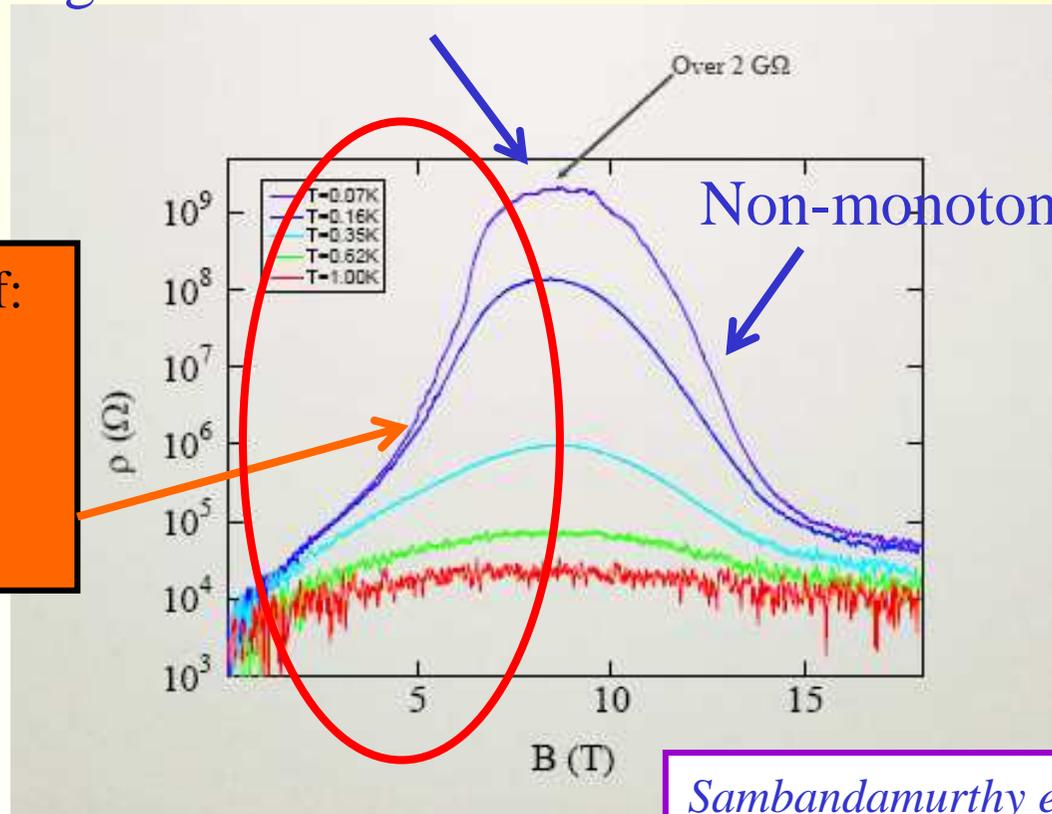
Giant magnetoresistance



Insulating behavior **enhanced** by local superconductivity!

Insulator: Giant magnetoresistance

Giant magnetoresistance



Common belief:
Pairs (bosons)
survive in the
insulator:
Bose glass

Sambandamurthy et al. (PRL 2005)

Insulating behavior **enhanced** by local superconductivity!

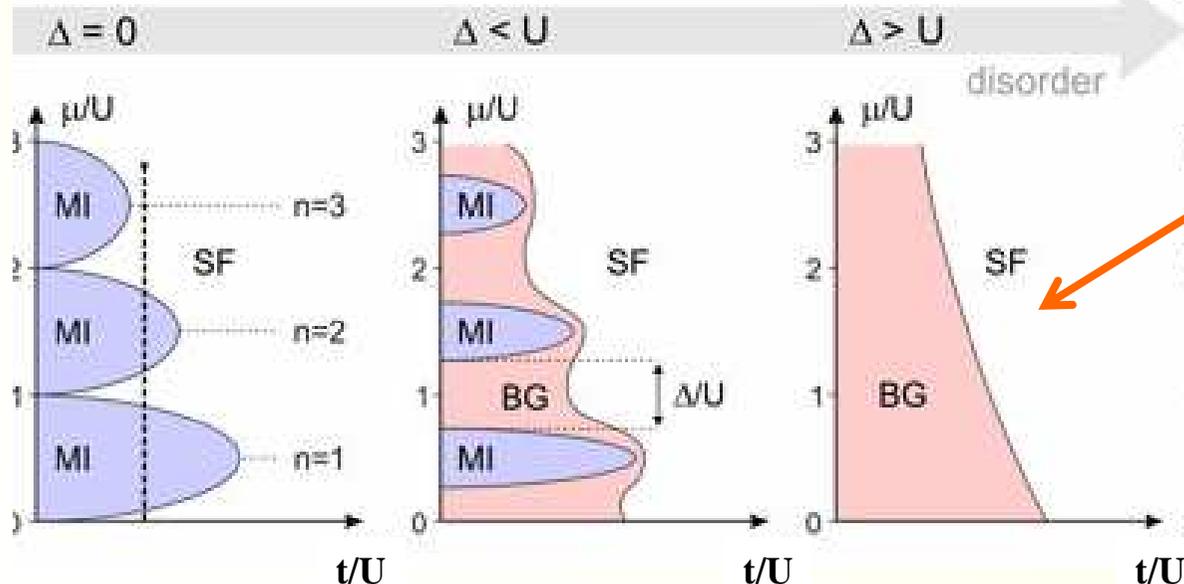
Bose-Hubbard model and Bose glass

Fisher et al., Phys. Rev. B 40, 546 (1989)

- Assume “preformed Cooper pairs”: bosons without global superconductivity
- Dirty boson model (Bose-Hubbard model with disorder):

$$H = t \sum_{\langle i,j \rangle} b_i^\dagger b_j + U \sum_i n_i (n_i - 1) + \sum_i (\varepsilon_i - \mu) n_i$$

Disorder: $\varepsilon_i \in [-\Delta, \Delta]$



Most likely scenario for experiments:
Strong disorder, no Mott gap!

Two puzzling features in transport

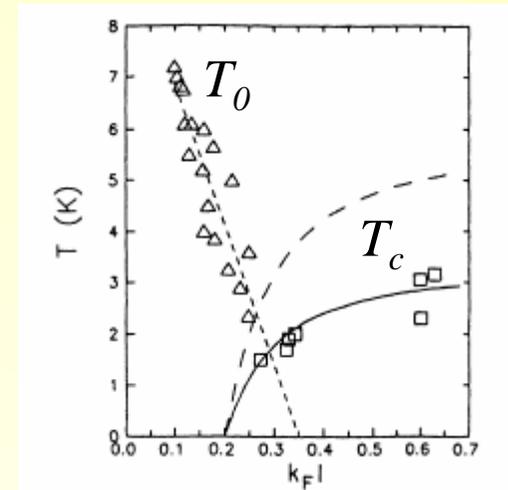
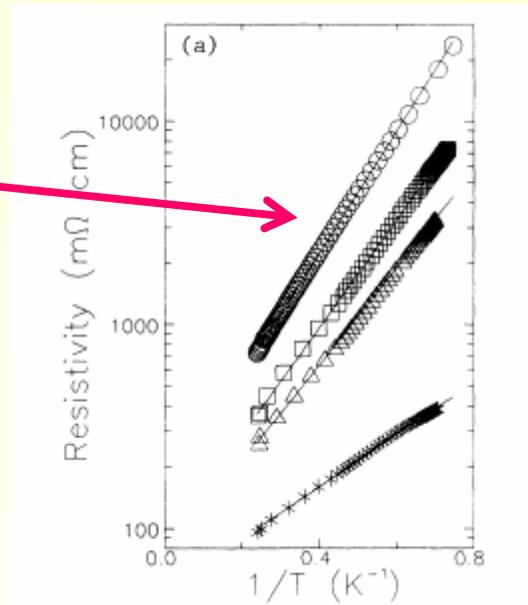
1. Simple activation in $R(T)$
2. Evidence for purely electronic mechanism

Activated transport near the SIT

D. Shahar, Z. Ovadyahu, PRB 46, 10971 (1992).

Insulating InO_x

Simple activation!



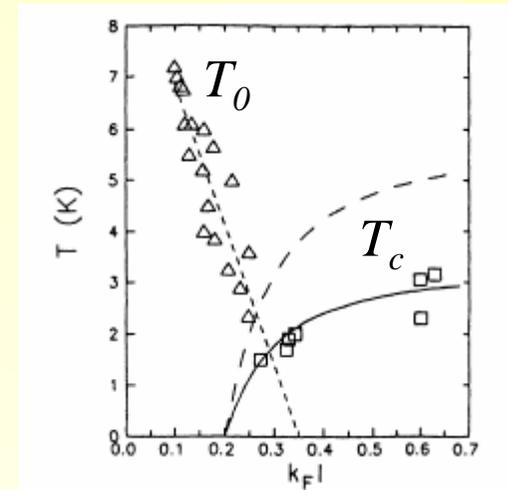
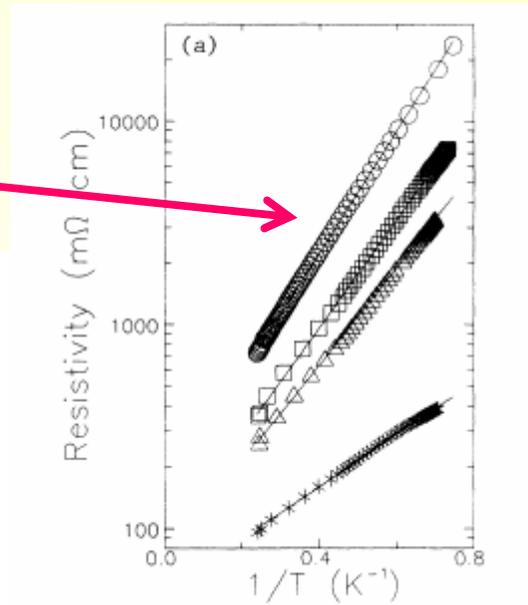
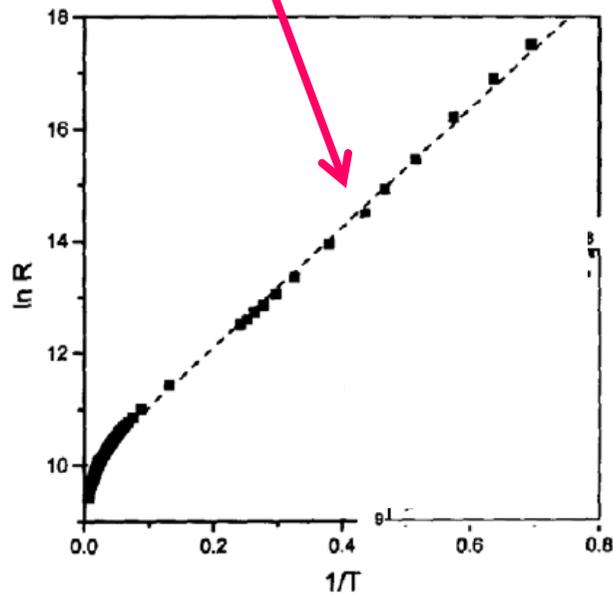
Activation energy
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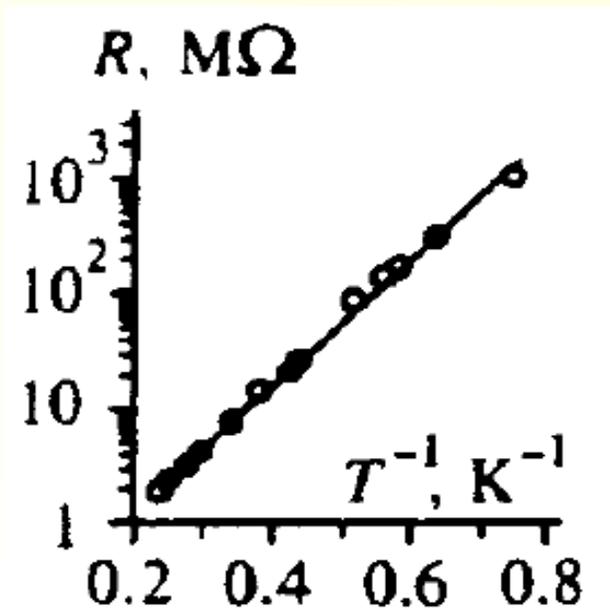
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D. Kowal and Z. Ovadyahu, Sol. St. Comm. 90, 783 (1994).

Activated transport near the SIT

V. F. Gantmakher, M. V. Golubkov, J. Lok, A. K. Geim, Sov. Phys. JETP, 82, 951 (1996).

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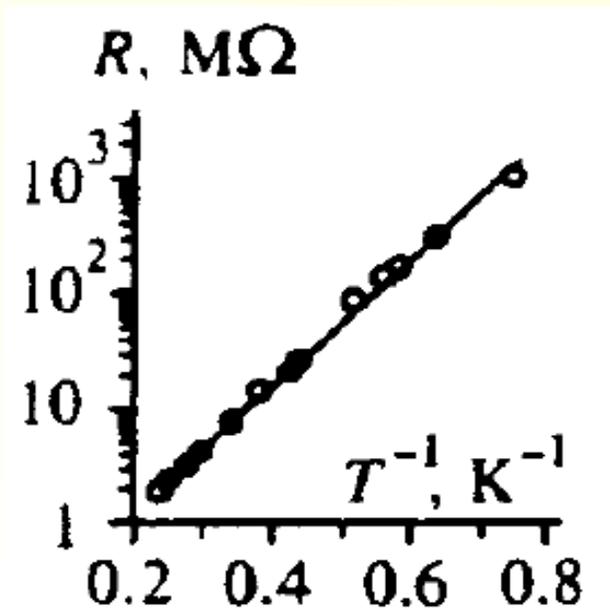
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A: NO! Too disordered systems!
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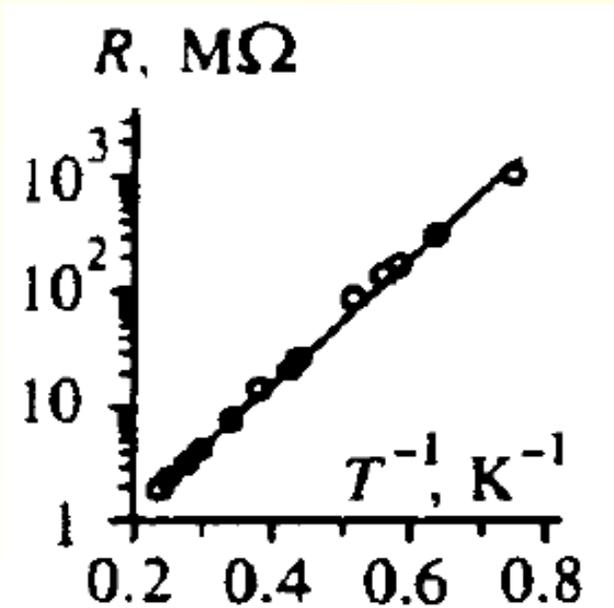
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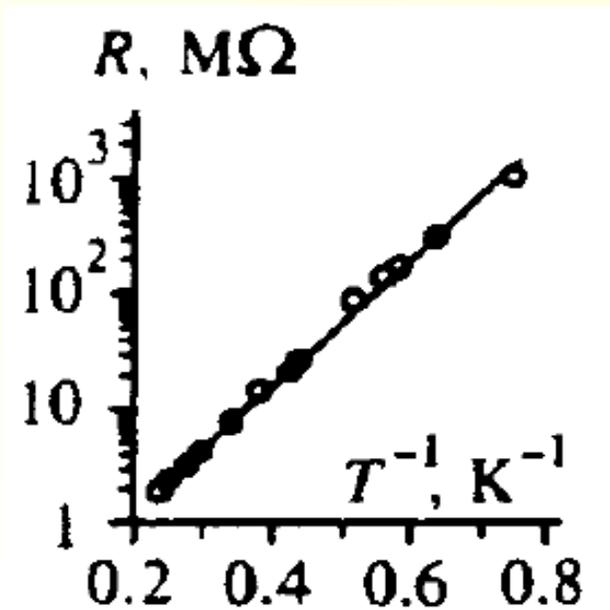
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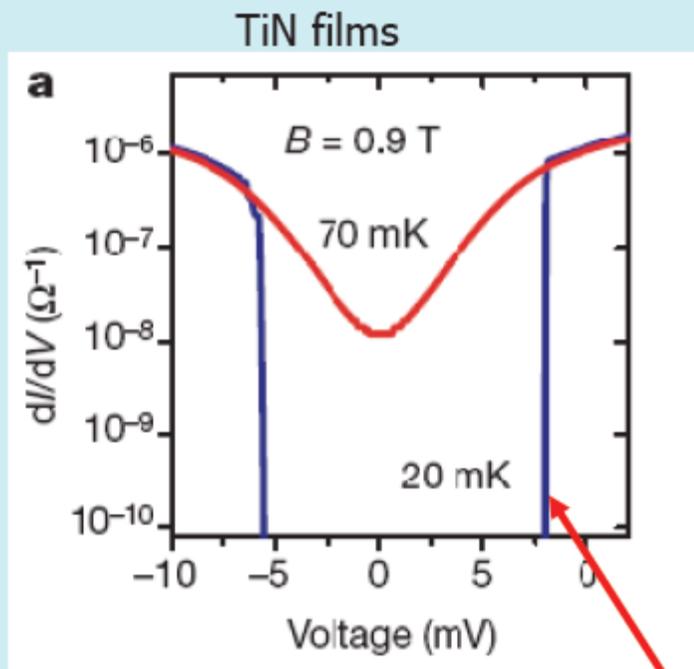
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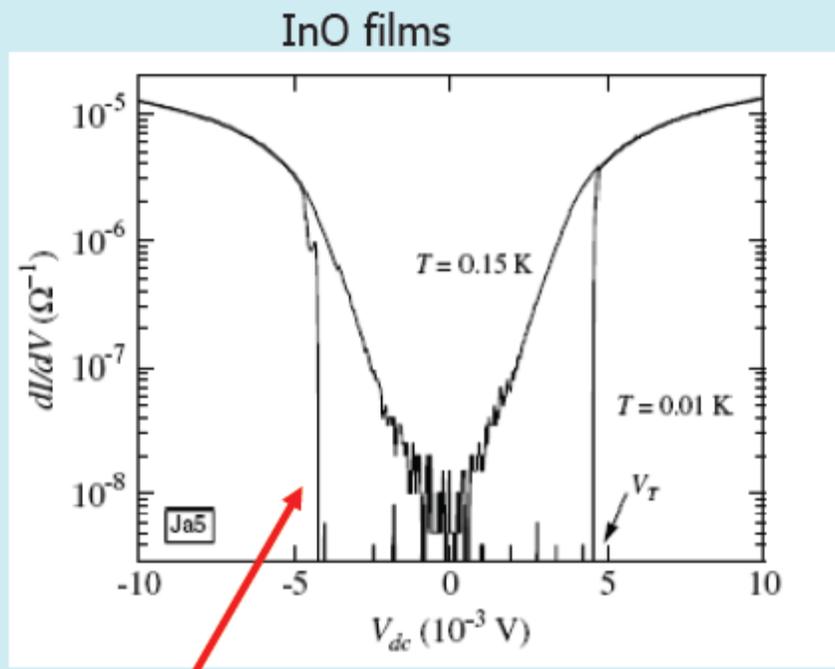
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- Boson mobility edge !
(Similar to Anderson localisation)

Purely electronic transport mechanism!



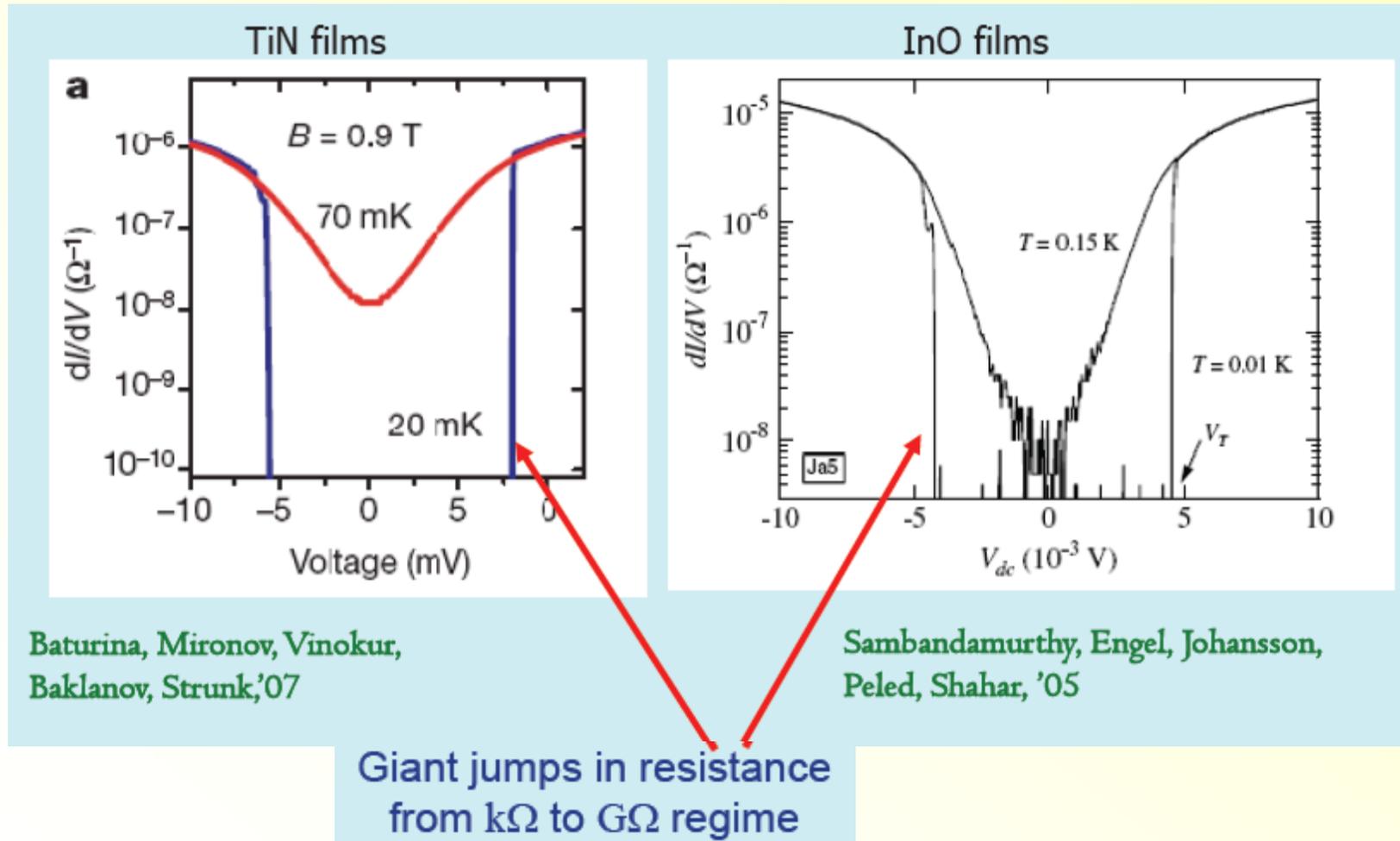
Baturina, Mironov, Vinokur,
Baklanov, Strunk, '07



Sambandamurthy, Engel, Johansson,
Peled, Shahar, '05

Giant jumps in resistance
from $k\Omega$ to $G\Omega$ regime

Purely electronic transport mechanism!



Simple but effective explanation: bistability from low T to overheated state.

Altshuler, Kravtsov, Lerner, Aleiner (09)

Crucial ingredient: transport is not phonon- but electron-activated! - Mechanism???

Summary

- 1. Close to the SI transition the transport is essentially simply activated (Arrhenius):
How come?**
- 2. Evidence for purely electronic transport from heating instability in non-Ohmic regime:
What is its origin?**

From dirty superconductor to Bose glass

Models

$$H = t \sum_{\langle i,j \rangle} b_i^+ b_j + U \sum_i n_i (n_i - 1) + \sum_i (\varepsilon_i - \mu) n_i$$

Disorder: $\varepsilon_i \in [-\Delta, \Delta]$

Easier to think about: $U = \infty$ limit, i.e., hard core bosons
→ bosons equivalent to pseudospins ($s=1/2$)

Interactions (e.g. Coulomb)

(Anderson, Ma+Lee,
Kapitulnik+Kotliar)

$$H = t \sum_{\langle i,j \rangle} s_i^+ s_j^- + \sum_i (\varepsilon_i - \mu) s_i^z + \sum_{\langle i,j \rangle} J_{ij} s_i^z s_j^z$$


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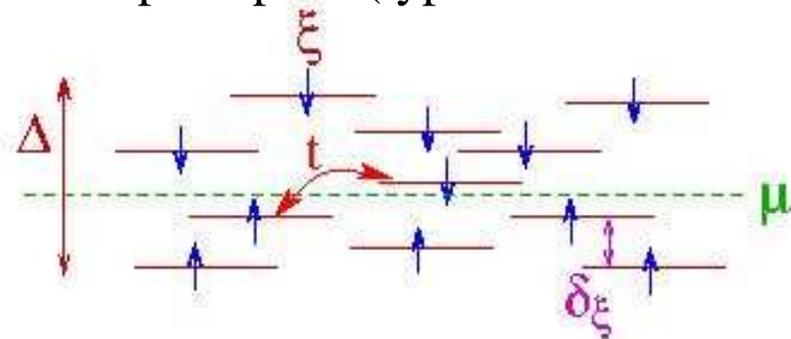
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• “Sites” i : states for bosons to occupy. May overlap in space (typical size of a state: ξ)

• Relevant scale characterizing disorder:
 Level spacing δ_ξ between close levels
 Disorder strength:

$$g \equiv \delta_\xi / t$$



From dirty superconductor to Bose glass

- Superconducting phase: Bose condensation into delocalized mode
 - finite phase stiffness
 - infinite conductivity for $T < T_c$
- Bose glass: No delocalized bosonic mode anymore (otherwise condensation would occur)
 - role of disorder: no homogeneous gap, still compressible phase
 - (**Note:** “Bose glass”: **unfrustrated** but disordered Bose insulator)
 - but: insulator: $\sigma(T \rightarrow 0) = 0$ [no Bose metals in non-exotic models!]

Nature of transport in the Bose glass?

From dirty superconductor to Bose glass

Localization of the bosons?

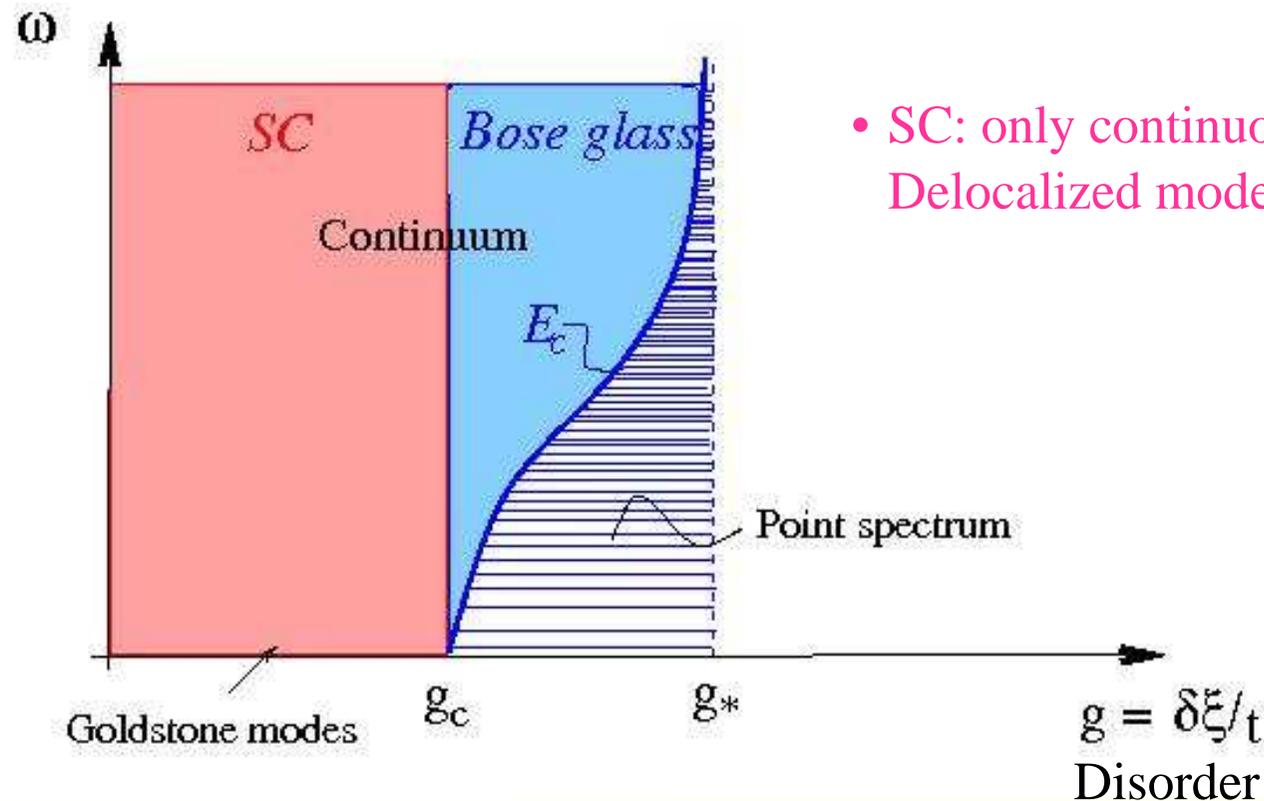
Look at changes in the spectral
properties!

From dirty superconductor to Bose glass

Local spectrum at $T = 0$ $\rho_o(\omega) = \int_0^\infty \langle O(x,t)O(x,0) \rangle_{GS} e^{i\omega t}$

From dirty superconductor to Bose glass

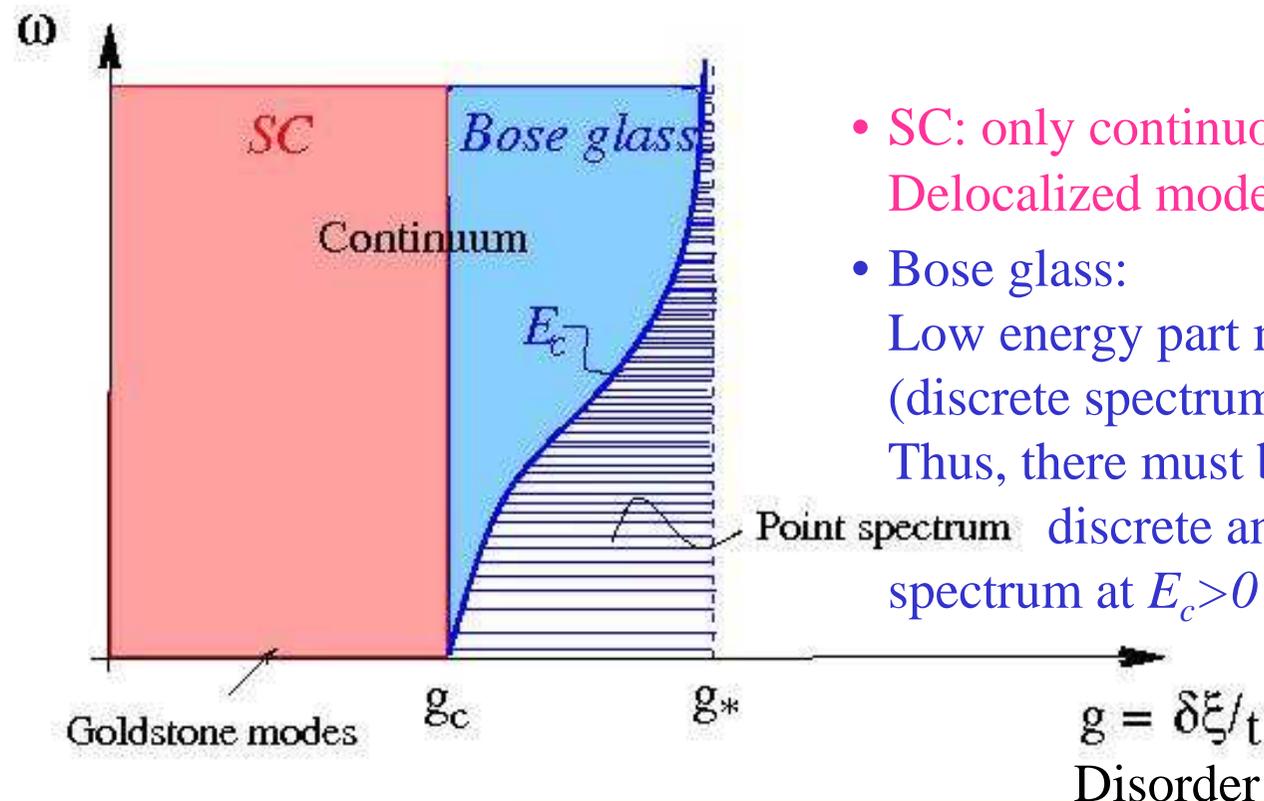
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- SC: only continuous spectrum!
Delocalized modes down to $\omega=0$

From dirty superconductor to Bose glass

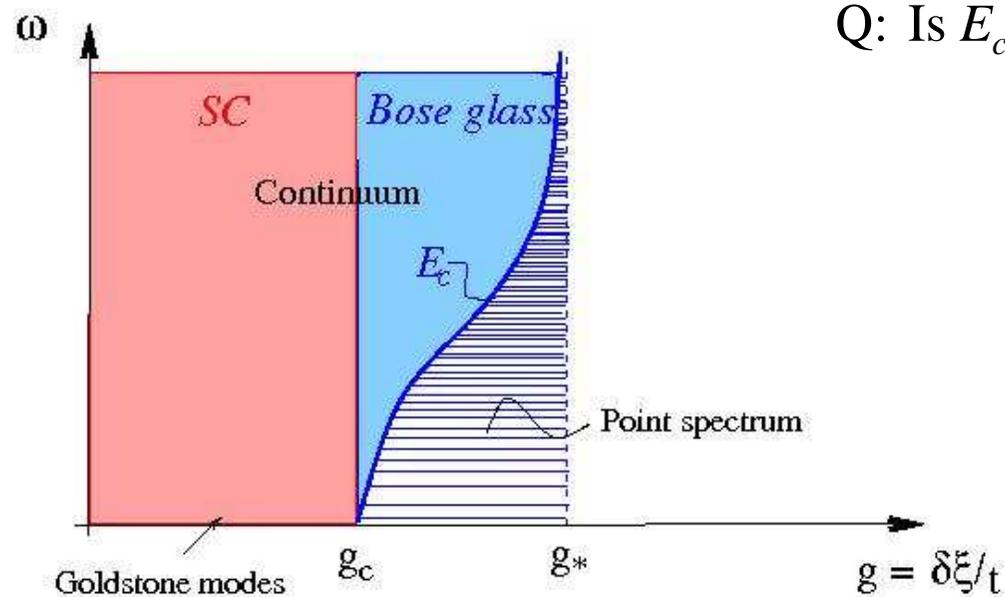
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- SC: only continuous spectrum!
Delocalized modes down to $\omega=0$
- Bose glass:
Low energy part must be localized (discrete spectrum).
Thus, there must be a border between discrete and continuous spectrum at $E_c > 0$

Spectral properties of the Bose glass

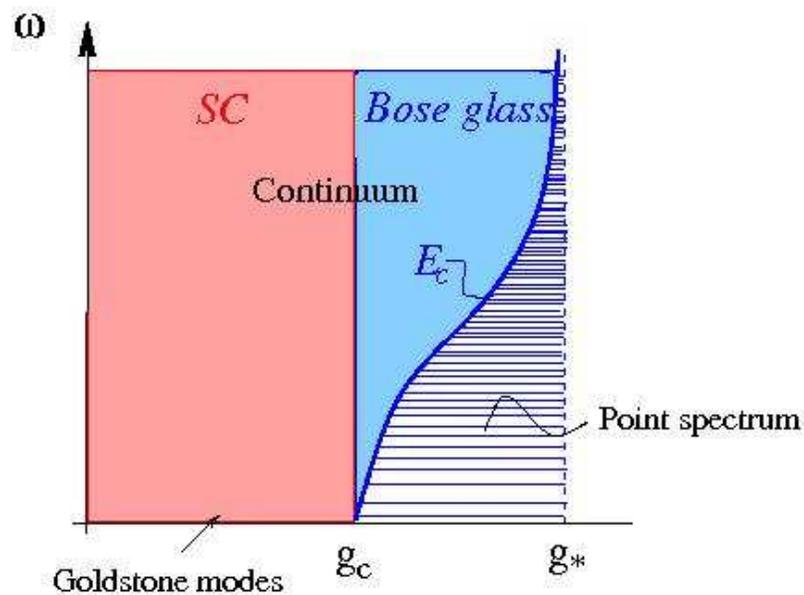
Many-body “mobility edge” in the Bose glass



Q: Is E_c finite or extensive? ($\sim \text{Vol}$)

Spectral properties of the Bose glass

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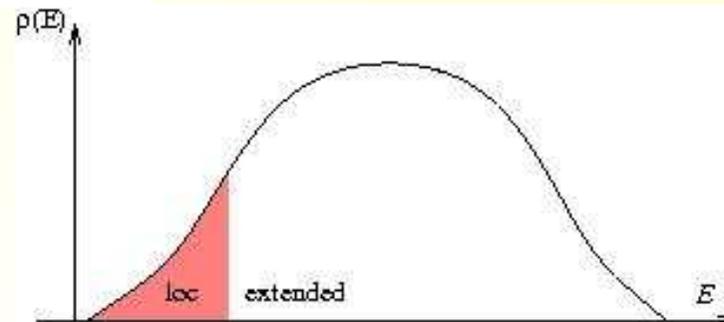


Q: Is E_c finite or extensive? ($\sim \text{Vol}$)

A: Close to the SIT ($g = g_c$) E_c is finite:
Single boson excitations at $E - \mu \gg t$
are delocalized $\rightarrow E_c < \infty$
(while at low energies bosons localize
due to the hard core constraints)

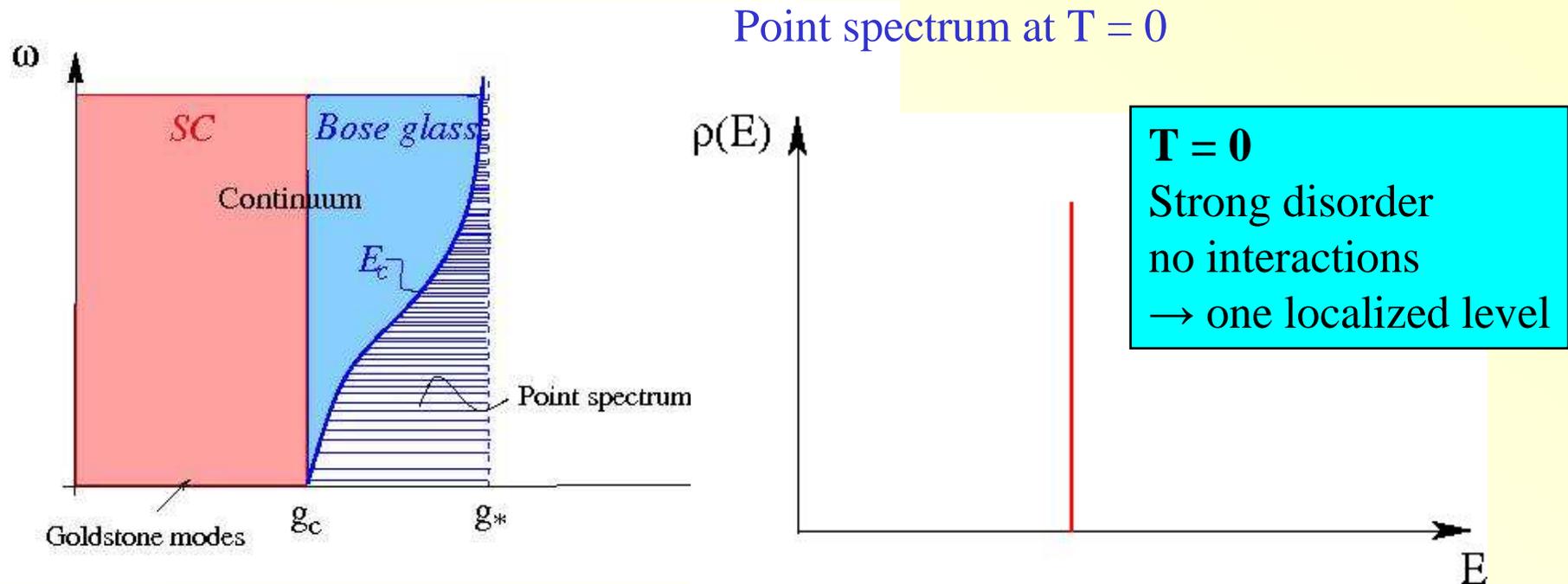
Analogue:

Localization at band edge (Anderson)



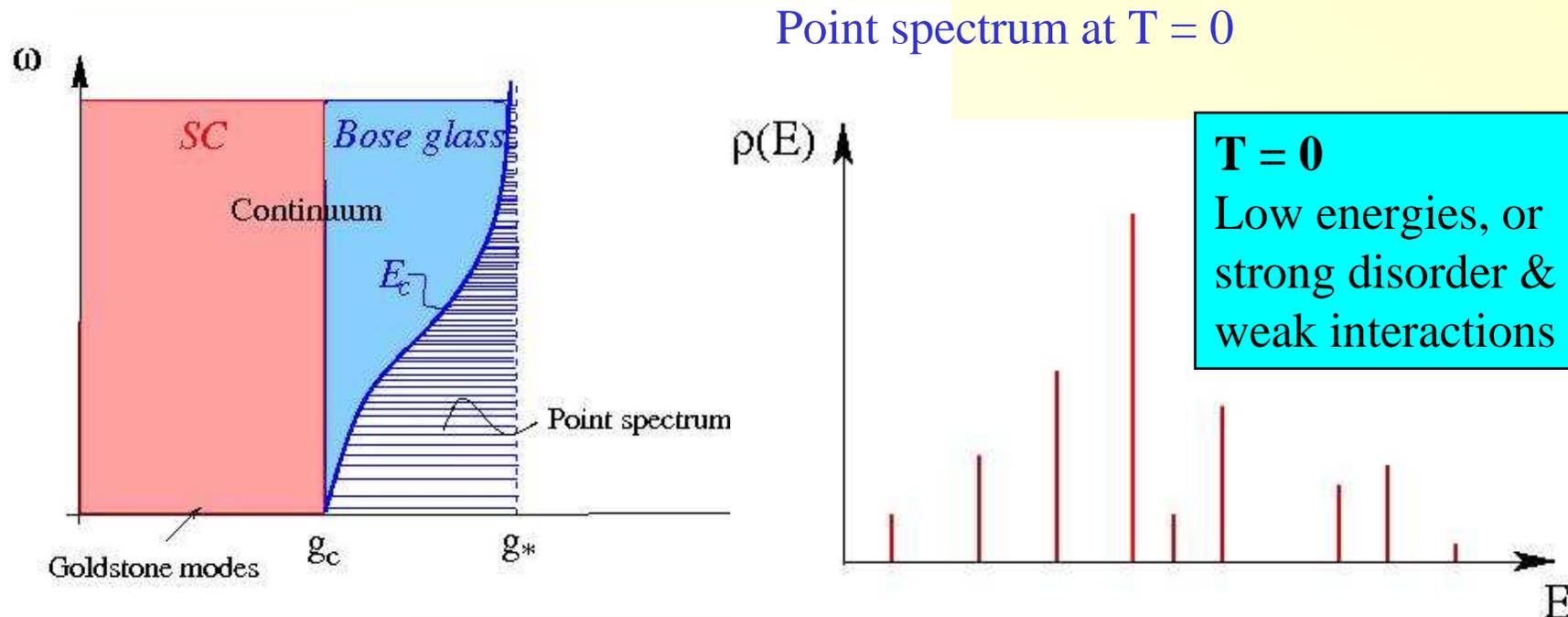
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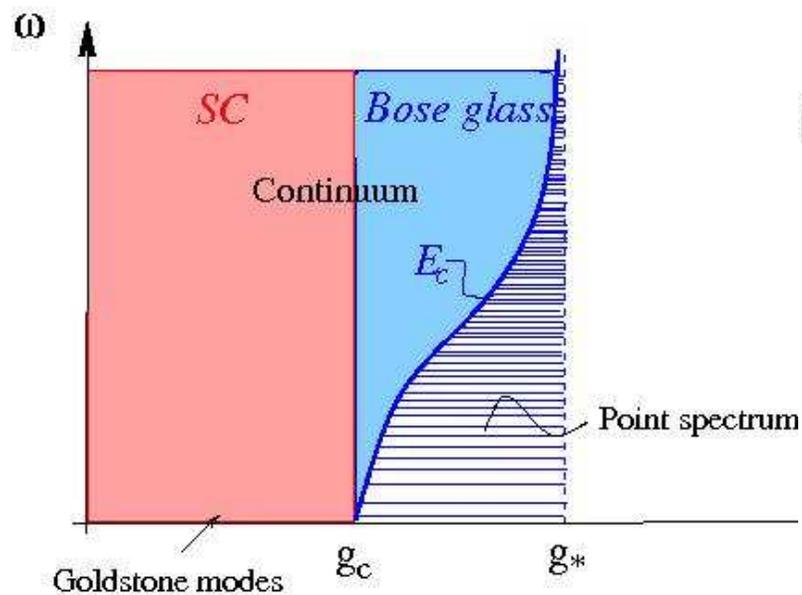
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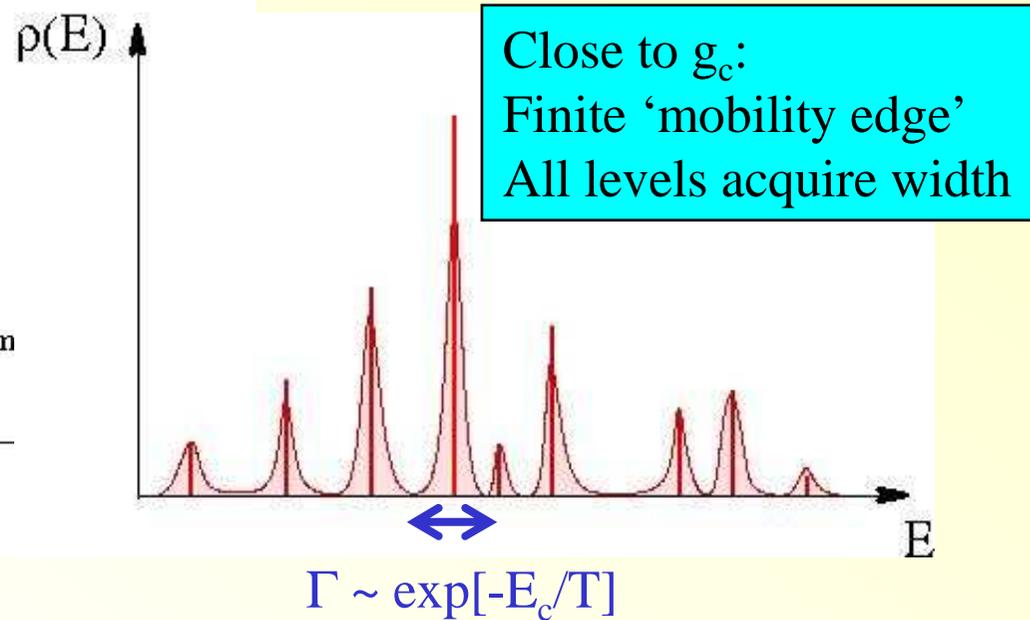
- Discrete levels: no transport, no current!
 $\sigma(T=0) = 0$
- Genuine glass at $T=0$: perturbations don't relax
Reason: Transition probabilities are zero because energy conservation can never be satisfied!

Mobility edge

Many-body “mobility edge” in the Bose glass

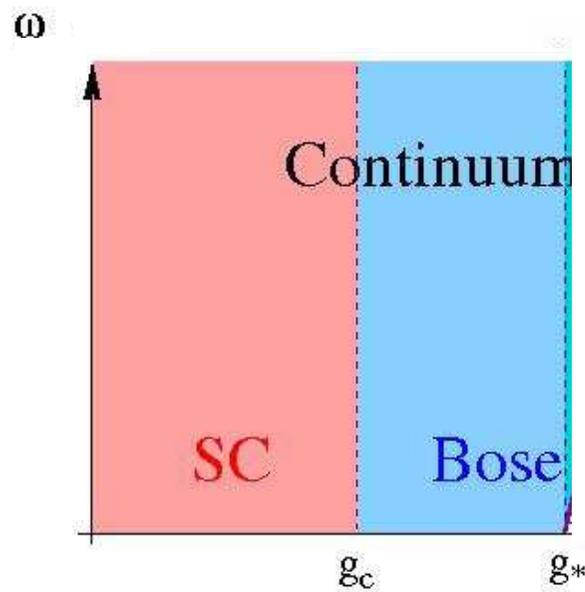


Q: What happens at $T > 0$?



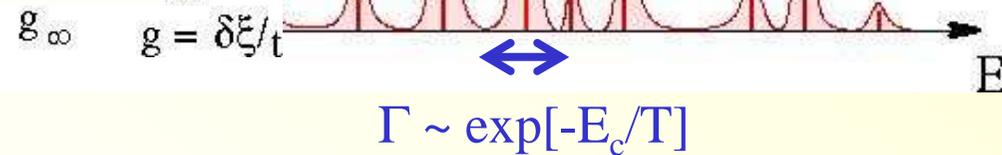
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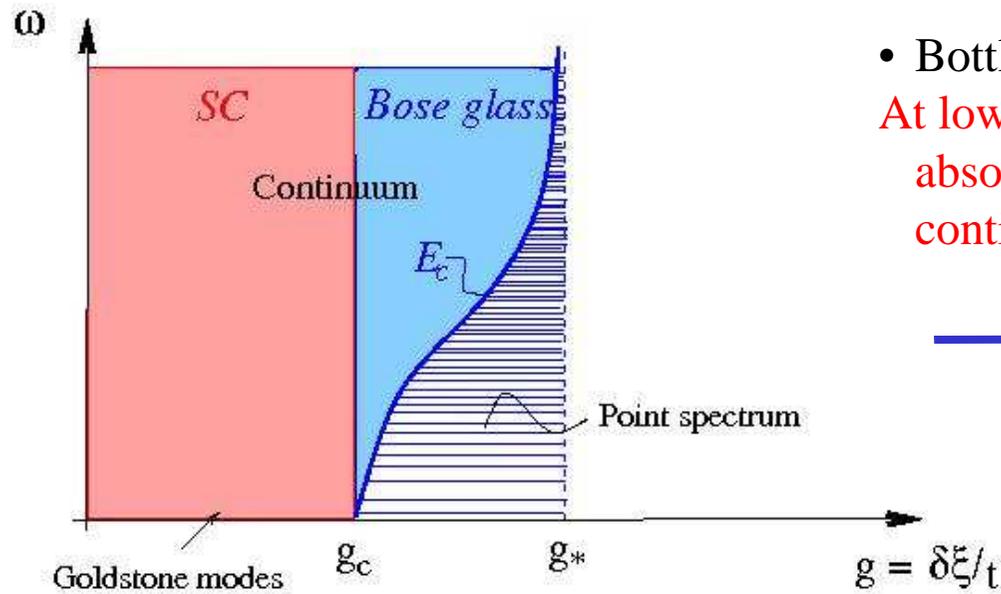
Close to g_c :
Finite ‘mobility edge’
All levels acquire width



- Continuum everywhere! $\sigma(T > 0) \neq 0$
for $g < g_*$ where $E_c(g) < \infty$

Electronic activated conduction

$$g < g_* : E_c(g) < \infty$$



- Continuum everywhere! $\sigma(T>0) \neq 0$

- Bottle neck for conduction:

At low T: Transitions allowed only due to the absorption of modes from the (T=0) continuum

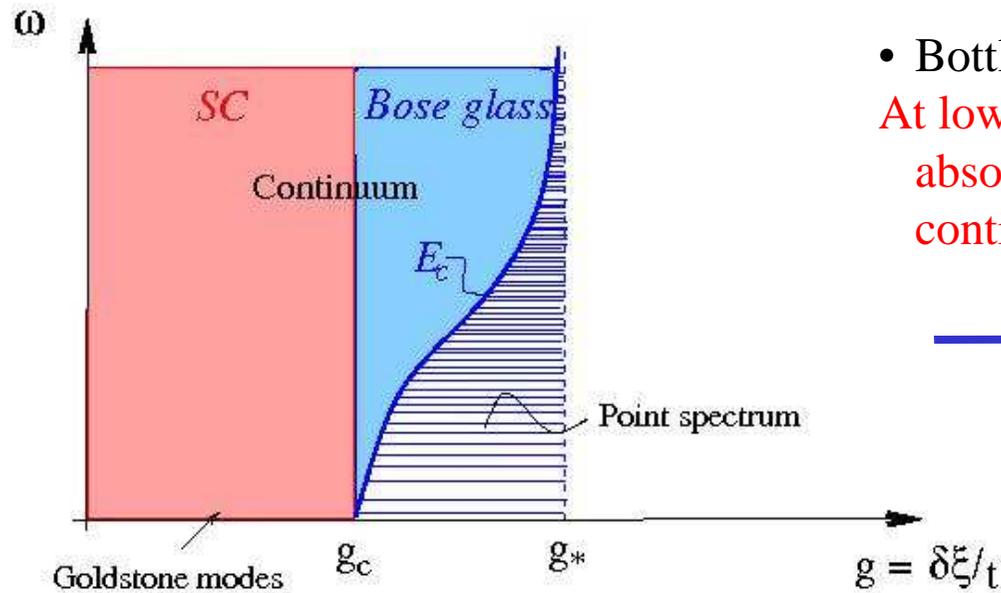


$$\sigma(T) \sim \sigma_0 \exp[-E_c/T]$$

Simple activation (Arrhenius) law in a compressible, gapless system!
No variable range hopping $e^{T^{-\alpha}}$!

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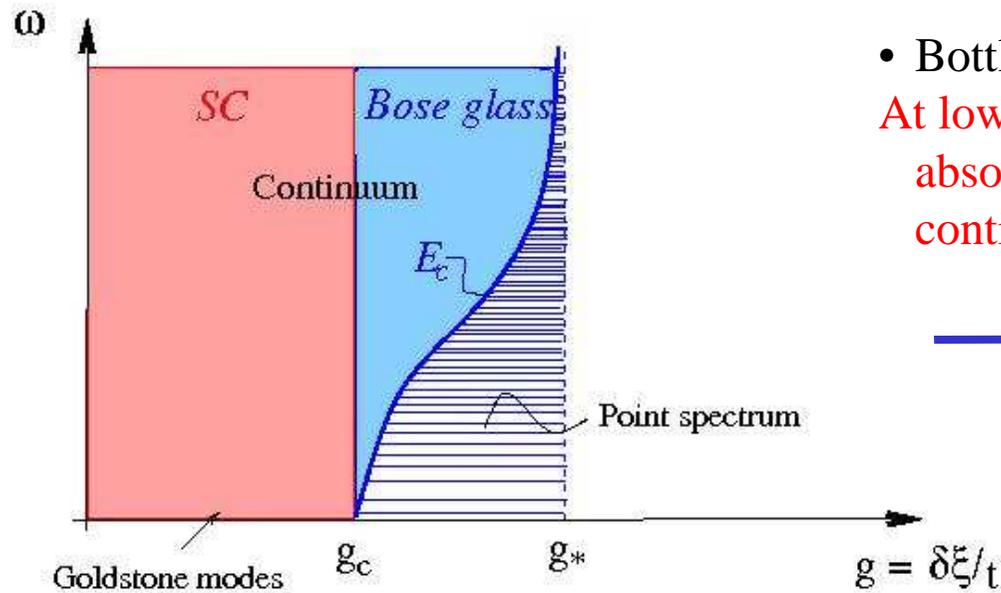
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Simple activation (Arrhenius) law in a compressible, gapless system!
No variable range hopping $e^{T^{-\alpha}}$!

- No phonons needed! (Would anyway be very inefficient at this low T)
- Purely electronic transport mechanism
→ crucial ingredient to explain the overheating in the non-Ohmic regime
- Prefactor: $\sigma_0 \sim e^2/h\xi^{d-2}$ nearly universal in $d=2$, similar to experiment!
- “Conductivity at the mobility edge” more robust than for electrons:
Relevant energy scale $t \sim T_c \sim$ few K, instead of E_F ; no fine-tuning of E_c over sample!

Electronic activated conduction

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Simple activation (Arrhenius) law in a compressible, gapless system!
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1. Note: Arrhenius law is only asymptotic at lowest T :

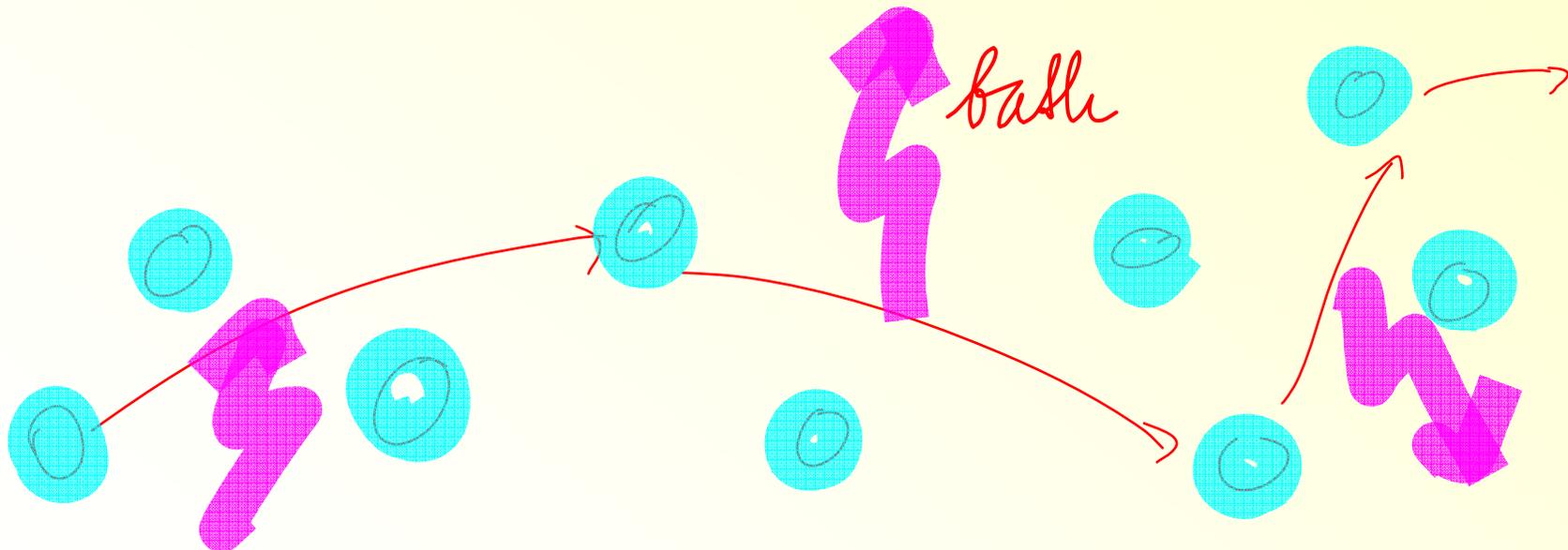
Finite inelastic scattering rate at $T > 0$ lowers the activation energy needed to get diffusion! $\rightarrow E_{\text{act}} = E_c - \Delta E(T)$! \rightarrow superactivation!

2. In reality: E_{act} is bounded from above by depairing energy!

Bosonic description breaks down too far from SIT (or in high B field)

? How to understand that variable range hopping is not seen, but instead activation? ?

Essential ingredient into variable range hopping:
Continuous bath which activates the hops!

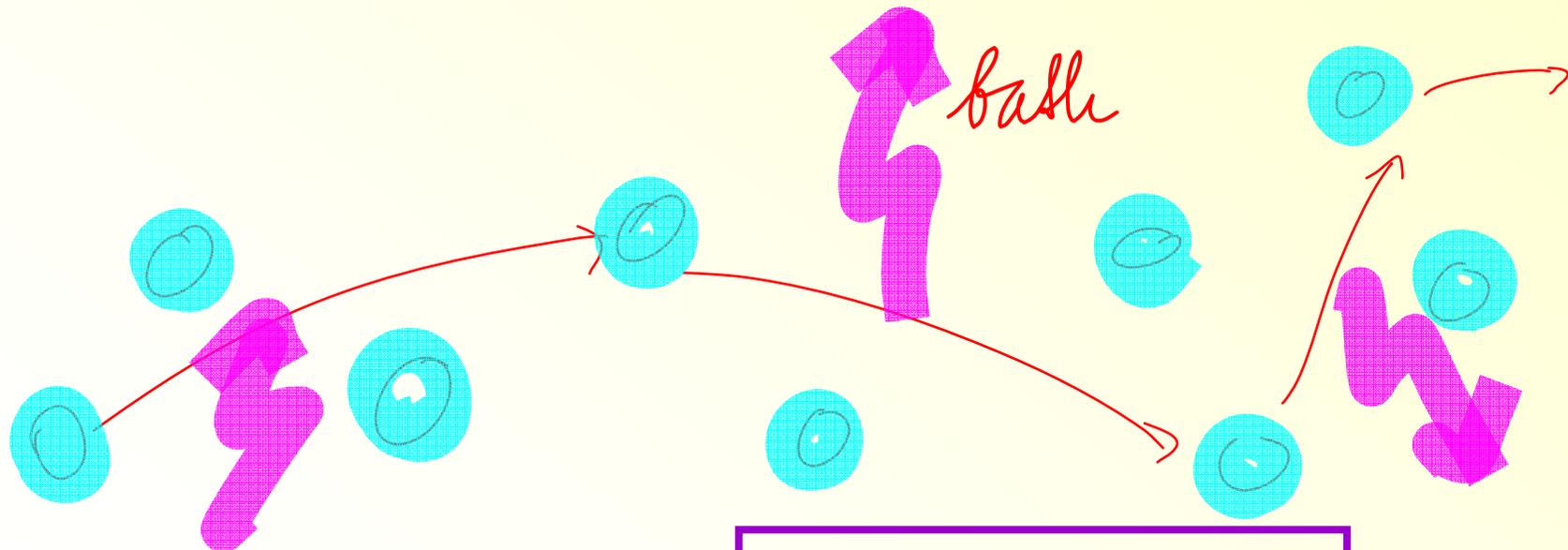


Candidates for the bath:

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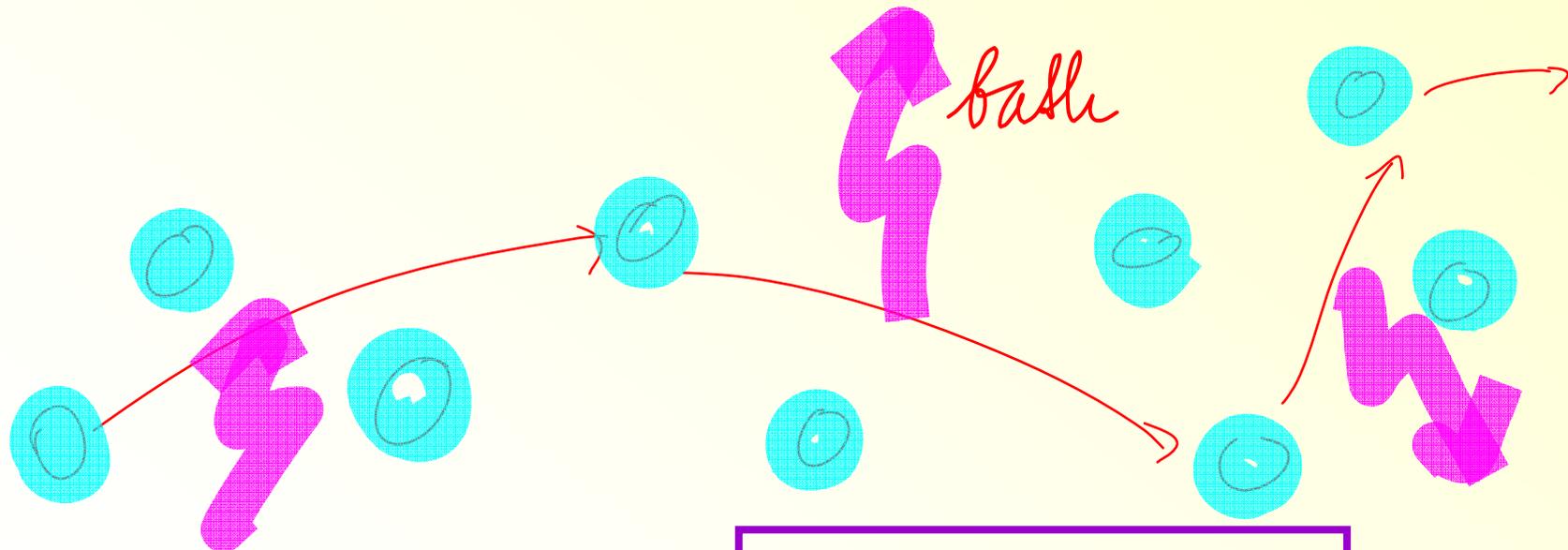


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Candidates for the bath:

- ~~Phonons: at low T for pair hopping are very inefficient!~~
- (possibly collective) pair excitations above the mobility edge

Strong disorder

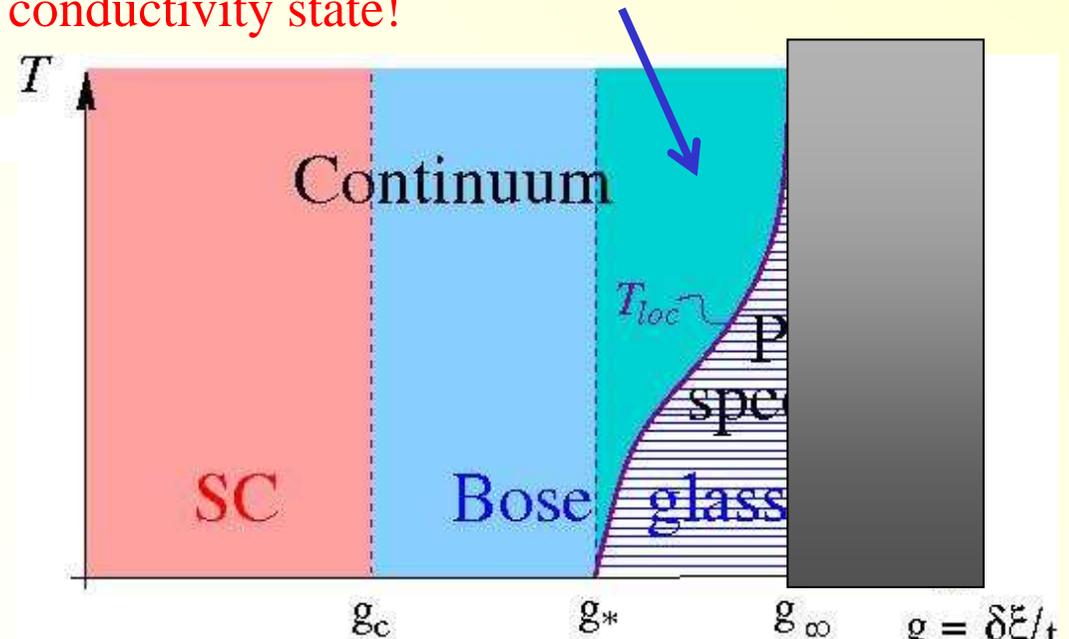
$g > g_*$: $E_c(g) = \infty$ (\sim Volume)

- If disorder is strong ($g = \delta_\xi/t > g_*$) high energy single boson excitations above the GS (at $T = 0$) are localized as well: $E_c \rightarrow \infty$

- But at finite T : finite density of excited bosons \rightarrow increased inelastic scattering \rightarrow localization tendency reduced:

Available boson-boson scattering phase space $\sim T/\delta_\xi$ sets connectivity in Fock space larger \rightarrow delocalization in Fock space at $T = T_{loc}$ (Basko et al.)

\rightarrow **Finite T transition to zero conductivity state!**

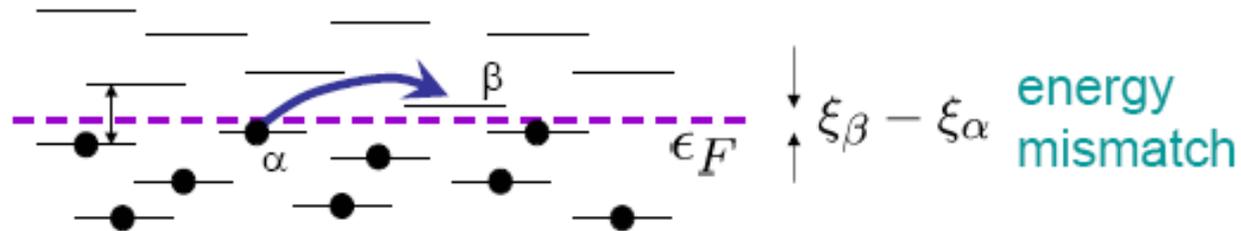


Localization despite interactions?

Fleishman, Anderson, Licciardello (1980, 1982)

Basko et al., Gornyi et al. (2005, 2006)

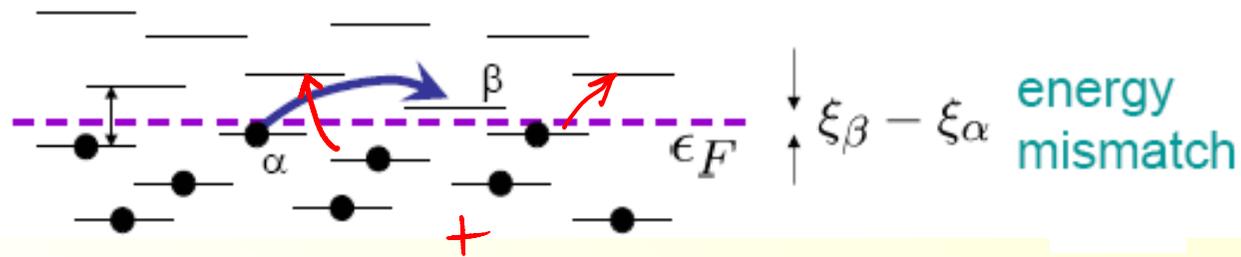
Is there **many-body localization** (localization in Hilbert space) \leftrightarrow **absence of diffusion**; even at finite **T**?



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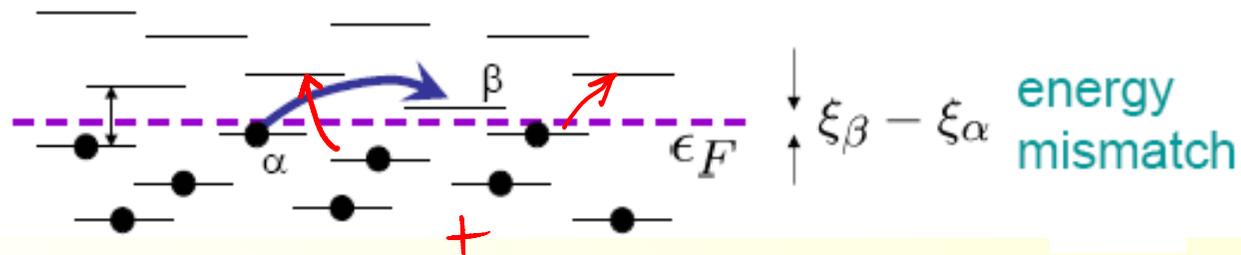
Can multi-particle arrangements
bridge the energy mismatch?

NO: not always!

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Is there **many-body localization** (localization in Hilbert space) \leftrightarrow **absence of diffusion**; even at finite **T**?



Assumptions:

1. Low dimensions \rightarrow all single particle states are localized
2. Weak short range interactions
3. No phonons

Answer: For $T < \delta_\xi / \lambda$ ($\lambda \ll 1$: interaction parameter)

- **Energy conservation impossible**: electrons do not constitute a continuous bath!
- All many body excitations remain **discrete** in energy!
- **Conductivity = 0** even at finite T – and **no thermal equilibration** either!

Strong disorder

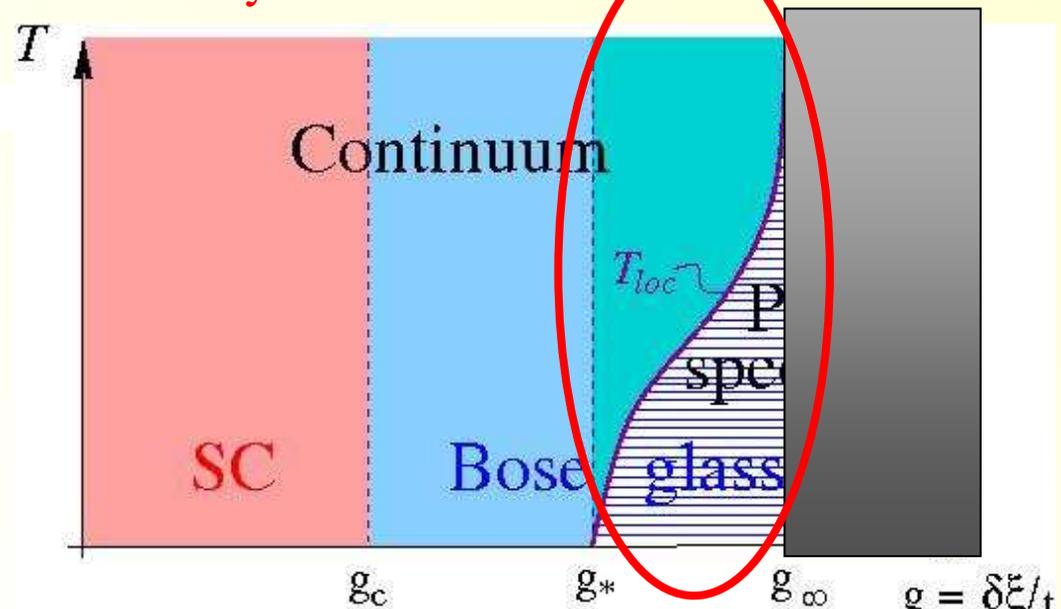
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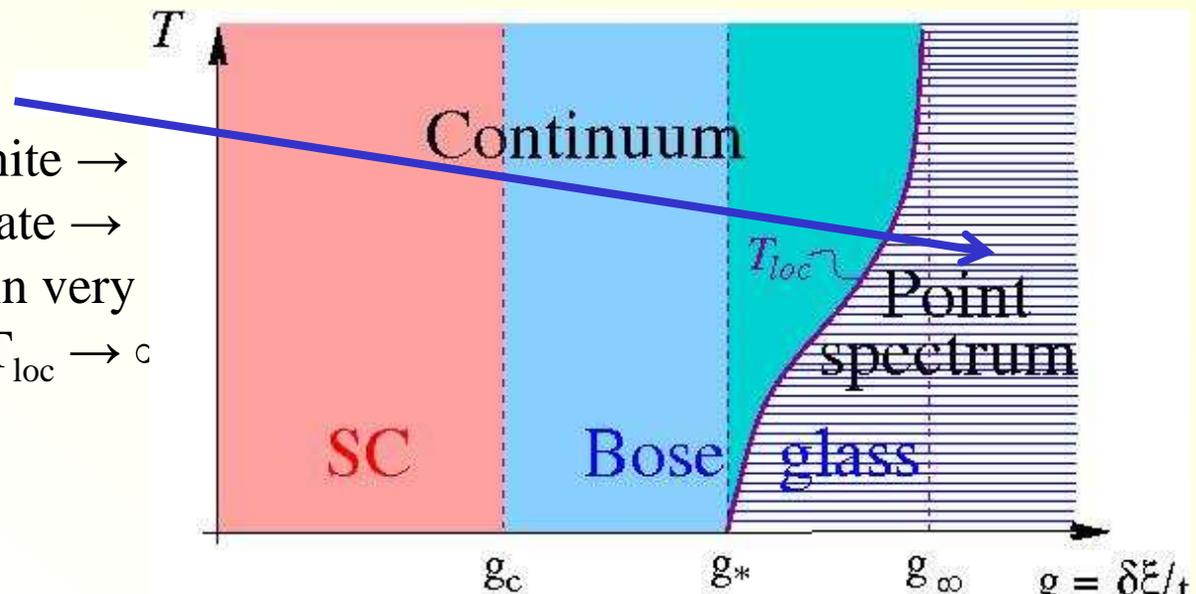


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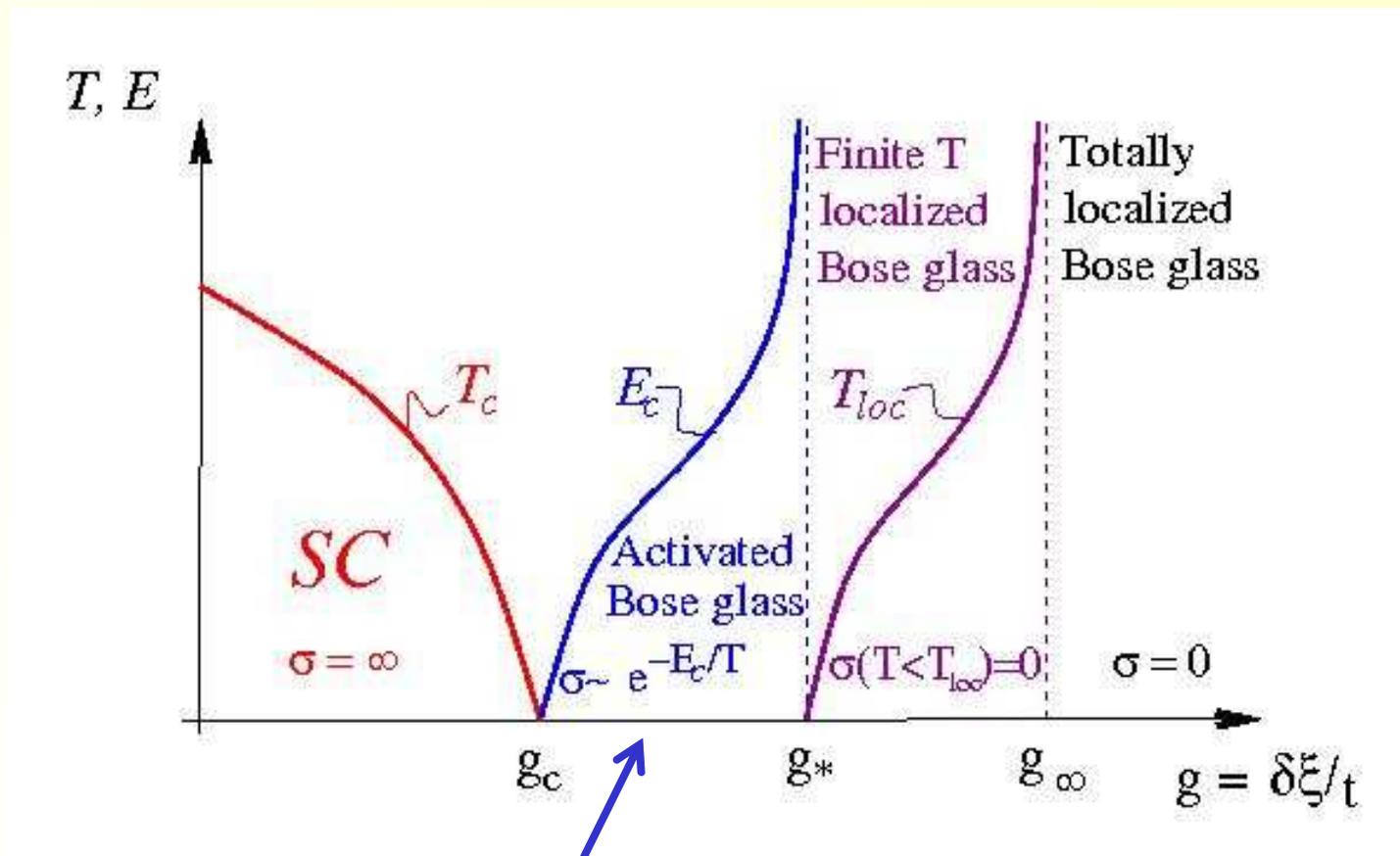
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- At biggest $g > g_\infty$:
If energy range Δ is finite \rightarrow
 maximal scattering rate \rightarrow
complete localization in very strong disorder when $T_{loc} \rightarrow 0$

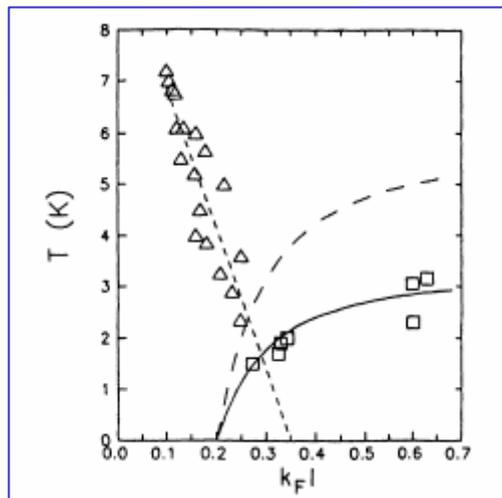


Summary: Bose-Hubbard model and Bose glass

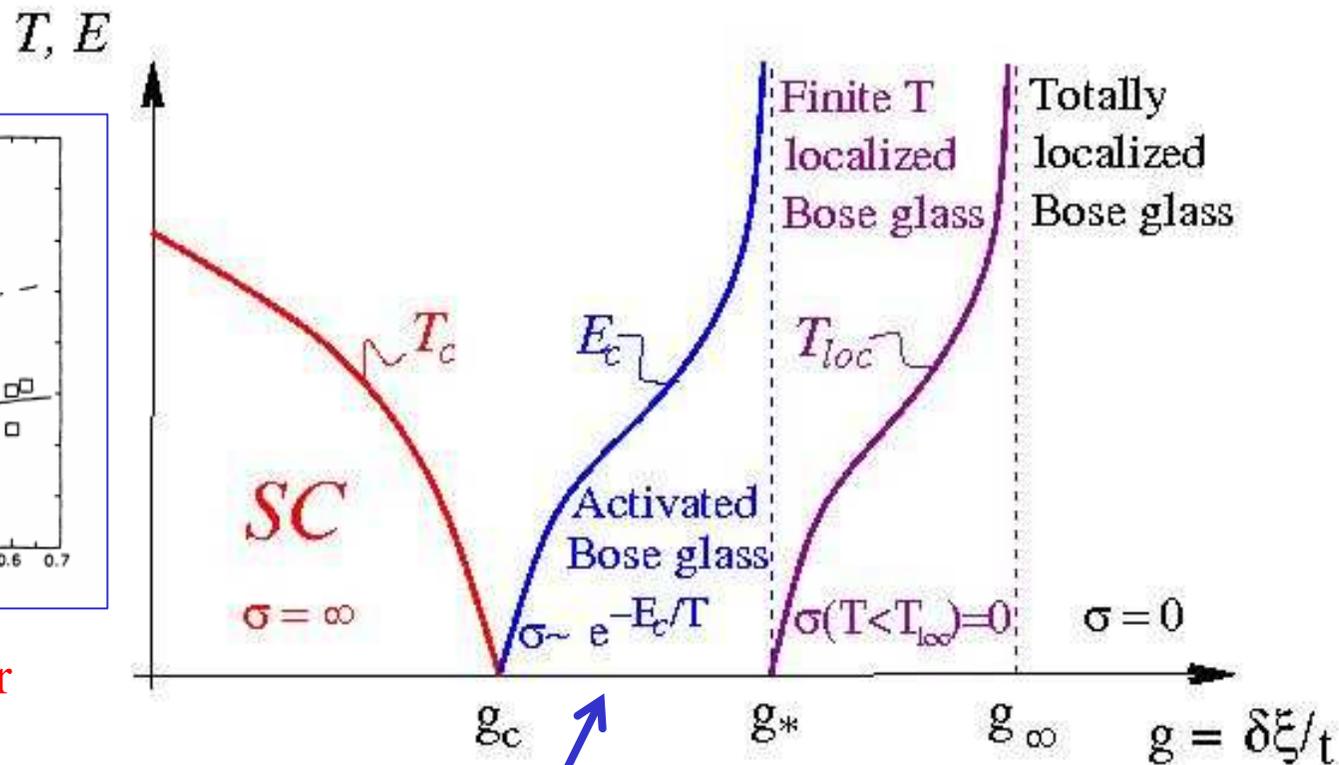


Purely electronic transport at low T: **Asymptotically** Arrhenius law!

Summary: Bose-Hubbard model and Bose glass

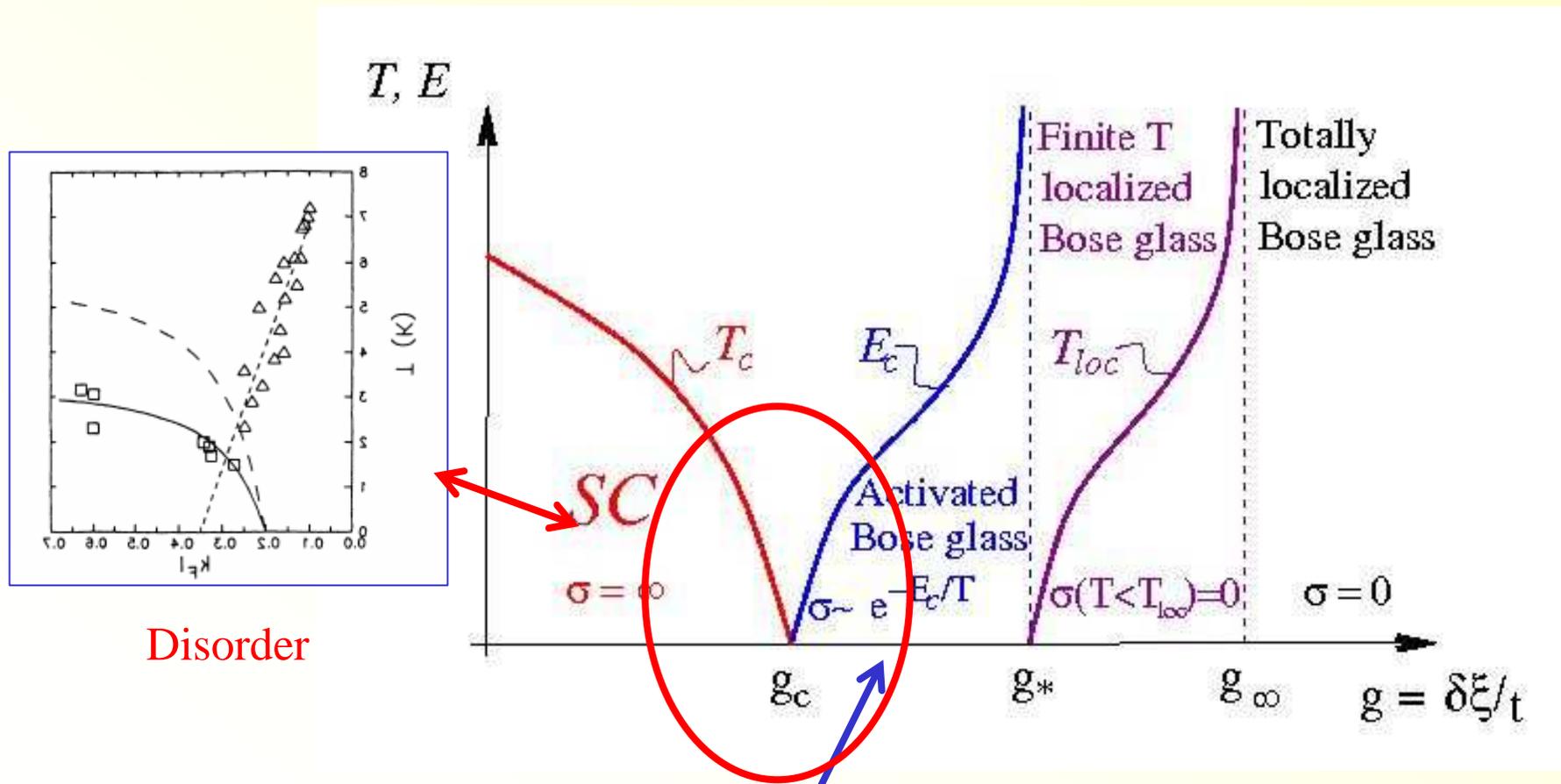


1/Disorder



Purely electronic transport at low T: **Asymptotically Arrhenius law!**

Summary: Bose-Hubbard model and Bose glass



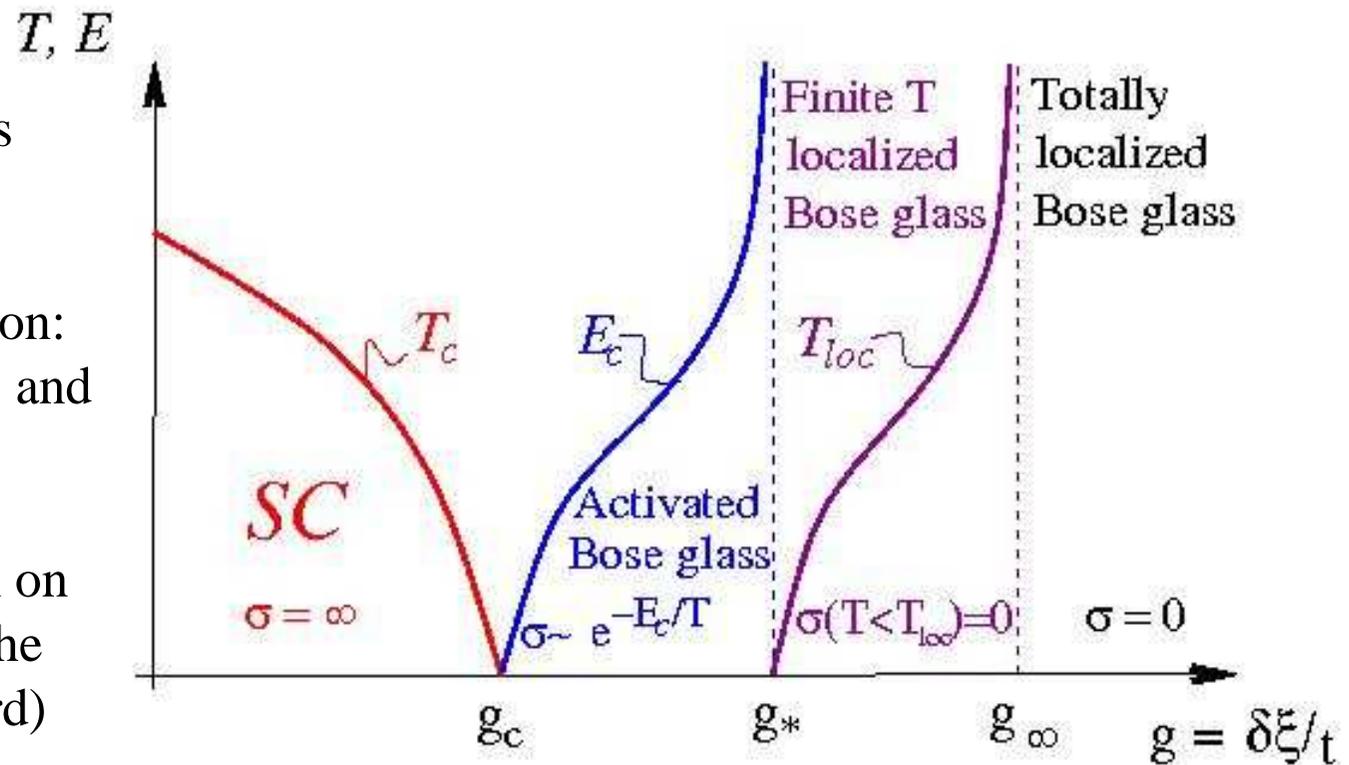
Disorder

Purely electronic transport at low T: **Asymptotically** Arrhenius law!

Summary: Bose-Hubbard model and Bose glass

Can this scenario
be proved?

- $E_c < \infty$ regime seems unavoidable
- T_{loc} & total localization: similar to Mirlin et al. and Basko et al.
- static approximation on high connectivity Bethe lattice (Ioffe & Mézard)
- total localization: might be possible to prove rigorously



Conclusion

- Transport in the Bose glass (without phonons) is a very rich problem due to various localization phenomena
- Phase diagram generic for disorder-driven delocalization transitions quantum phase transitions. Essentially similar picture close to the Metal-Insulator transition with interactions

- **Note: Quantum glassiness WITHOUT frustration!**

Is localization easier or harder to achieve in frustrated systems?

Is delocalization and equilibration the same concept? (I believe NO...)

Two different sub-notions of glassiness!?

