Self-organized criticality and avalanches in spin glasses

Markus Müller

The Abdus Salam International Center of Theoretical Physics

In collaboration with
Pierre Le Doussal (LPT-ENS Paris)
Kay Wiese (LPT-ENS Paris)

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Outline

- Crackling, avalanches, and "shocks" in disordered, non-linear systems; Self-organized criticality

- Avalanches in the magnetizing process ("Barkhausen noise")

- The criticality of spin glasses at equilibrium – why to expect scale free avalanches

- Magnetization avalanches in the Sherrington-Kirkpatrick spin glass – an analytical study.

- Outlook: electron glasses, finite dimensions…

Crackling

*Review: Sethna, Dahmen, Myers, Nature **410**, 242 (2001).*

Crackling = Response to a slow driving which occurs in a discrete set of avalanches, spanning a wide range of sizes.

Occurs often but not necessarily only out-of equilibrium.

Examples:

- Earthquakes
- Crumpling paper
- Vortices and vortex lattices in disordered media etc.
- Disordered magnet in a changing external field magnetizes in a series of jumps

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But: Not everything crackles!

It is intermediate between snapping (e.g., twigs, chalk, weakly disordered ferromagnets, nucleation in clean systems) and popping (e.g., popcorn, strongly disordered ferromagnets)

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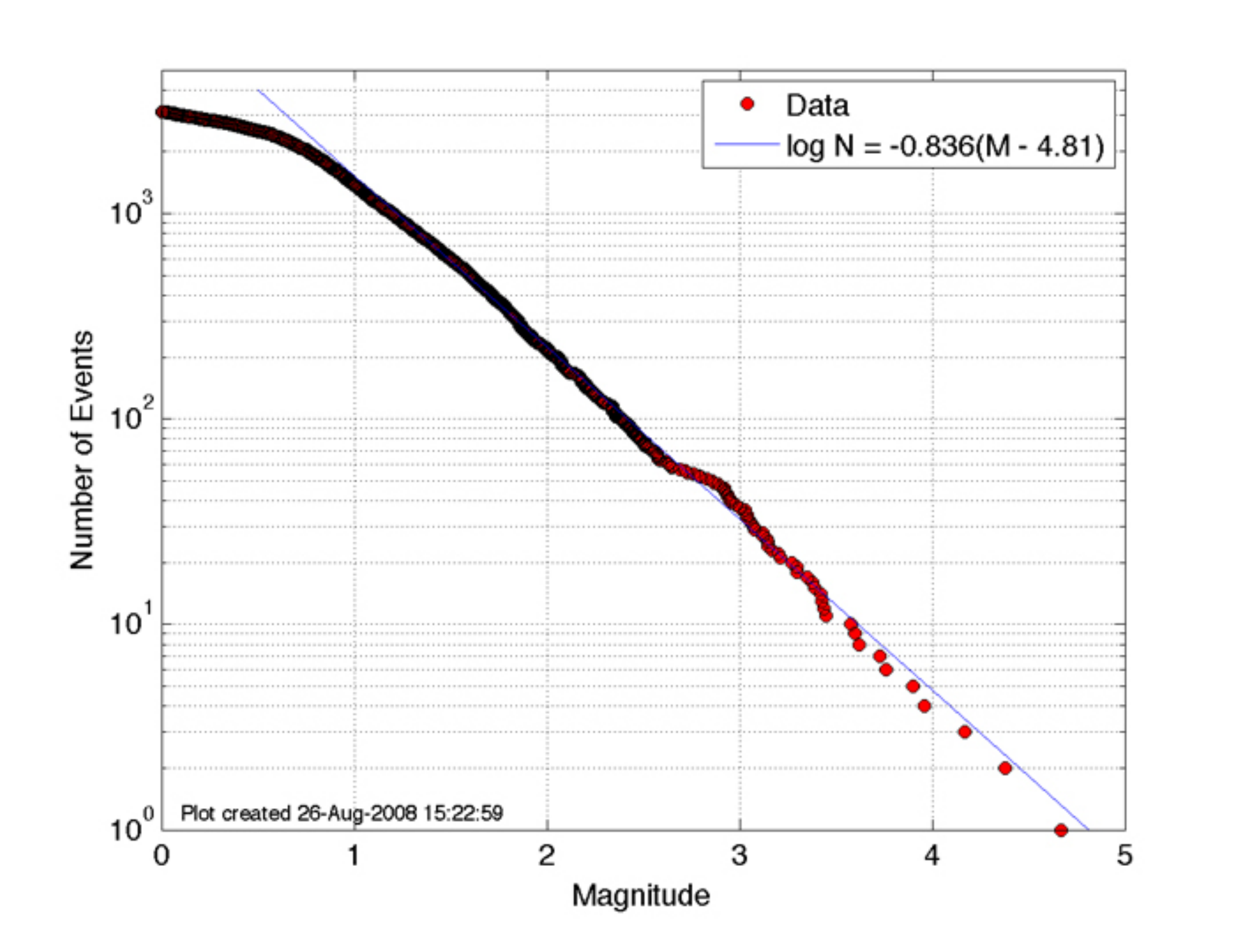
and popping (e.g., popcorn, strongly disordered ferromagnets)

Crackling on all scales is generally a signature of a critical state in driven, non-linear systems. It can thus be an interesting *diagnostic*.

Examples of crackling I

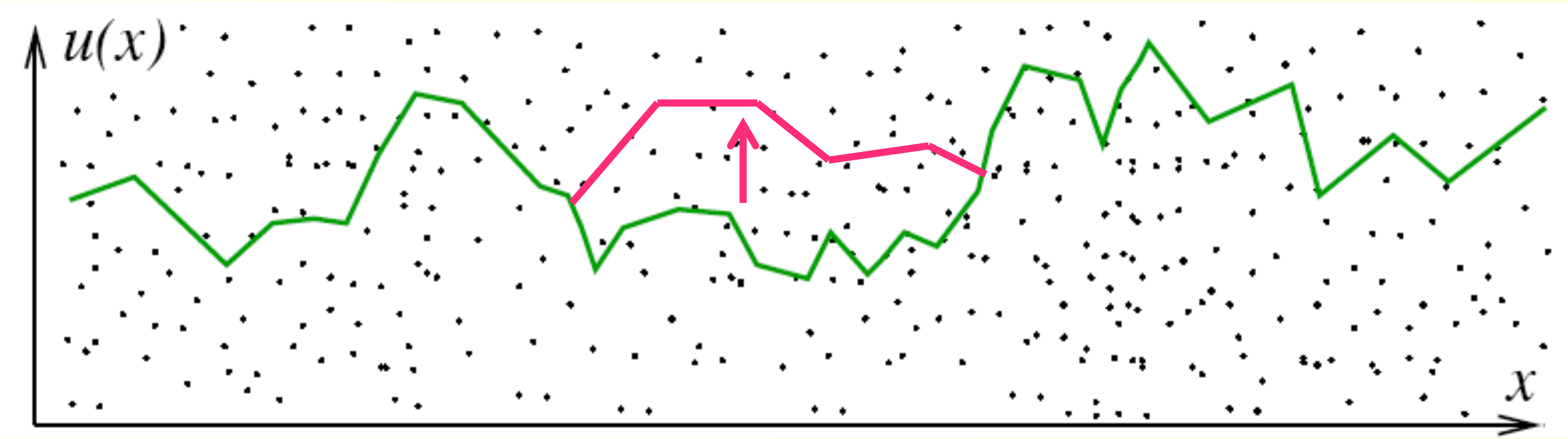
- Gutenberg-Richter law for strength of earthquakes
  (jumps of driven tectonic plates)



Examples of crackling II

- Depinning of elastic interfaces

Liquid fronts, domain walls, charge density waves, vortex lattices:



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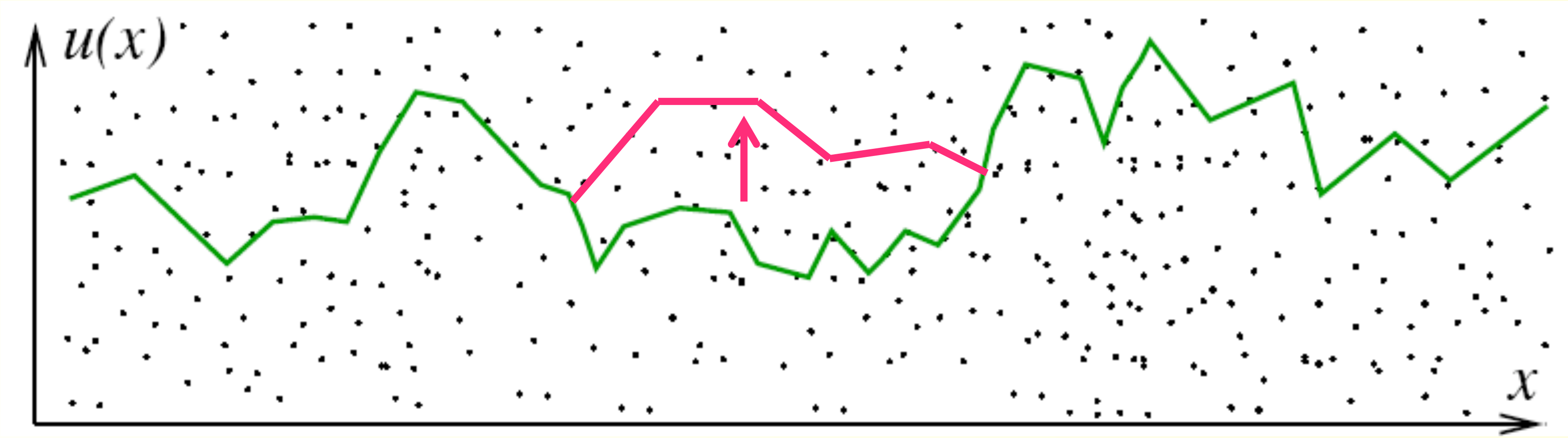


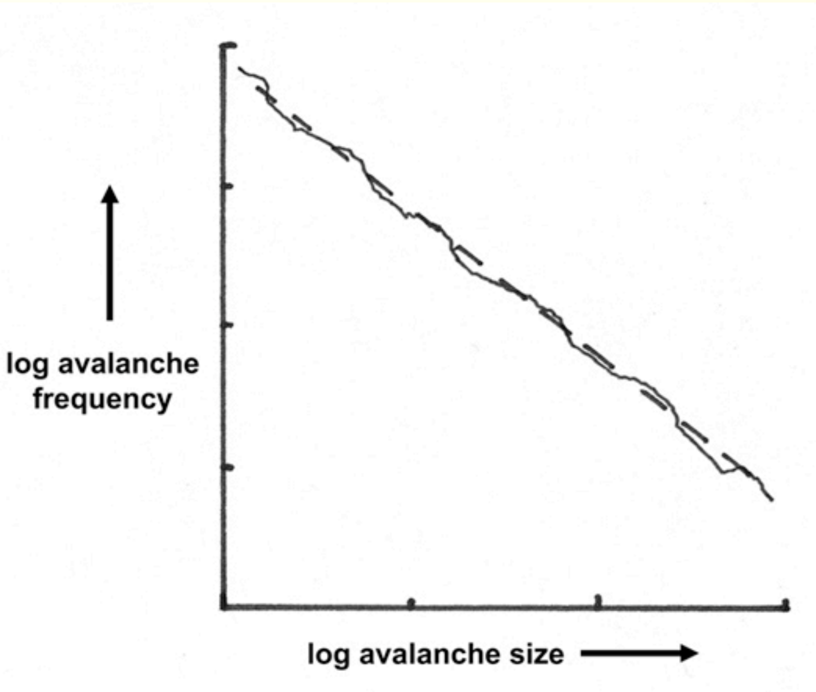
Depinning as a dynamical critical phenomenon in disordered glassy systems
Sophisticated theory approach: functional RG “FRG” [D. Fisher, LeDoussal, ...]

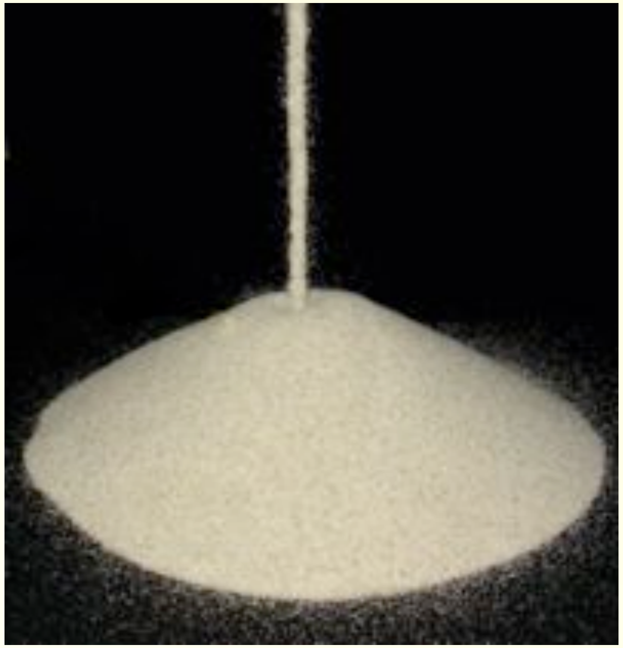
Statistics of avalanches: - mean field theory
- recent first steps and successes with FRG
→ find non-trivial critical power laws (without scale)

Examples of crackling III

• Power laws due to self-organized criticality:
  Dynamics is attracted to a critical state, without fine-tuning of parameters

Example: sandpile model by Bak, Tang, and Wiesenfeld





Magnetic systems

- Crackling noise in the hysteresis loop: "Barkhausen noise"
- When does crackling occur in random magnets, and why?
- What happens in frustrated spin glasses
  (as opposed to just dirty ferromagnets)?

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Equilibrium avalanches in the hysteresis reflect criticality of the glass phase! Noise as a diagnostic of a critical glass state!

Avalanches in ferromagnetic films

Direct Observation of Barkhausen Avalanche in (ferro) Co Thin Films

Kim, Choe, and Shin (PRL 2003)

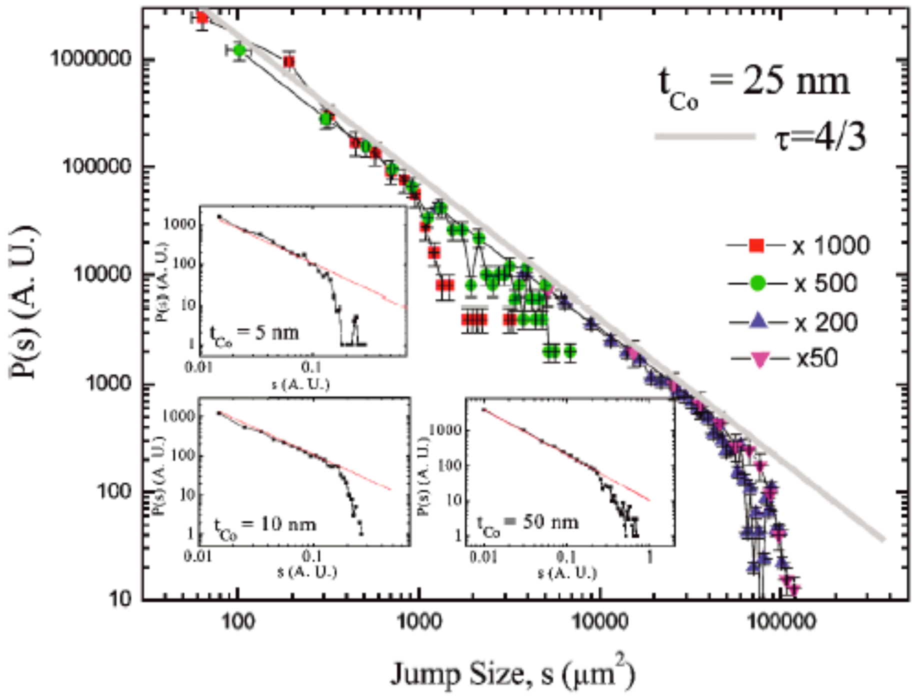

FIG. 3 (color). Distributions of the Barkhausen jump size in 25 and 50-nm Co samples. Distributions in 5, 10, and 50-nm Co samples are shown in the insets. Fitting curve with $\tau = 1.33$ is denoted at each graph.

Distribution of magnetization jumps

$$\boxed{P(s) = \frac{A}{s^{\tau}} \qquad \tau = \frac{4}{3}}$$

Cizeau et al.:

Theoretical model with **dipolar long range interactions**

(believed to be crucial to get criticality)

Model ferromagnets

Dahmen, Sethna
Vives, Planes

Random field Ising model (short range):

$$H = -J \sum_{<ij>} s_i s_j - \sum_i h_i s_i - h_{ext} \sum_i s_i$$

- Generically non-critical
- Scale free avalanches require fine tuning of disorder $\Delta = \left\langle h_i^2 \right\rangle$ *and* field $h_{ext,crit}$

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Experiment:

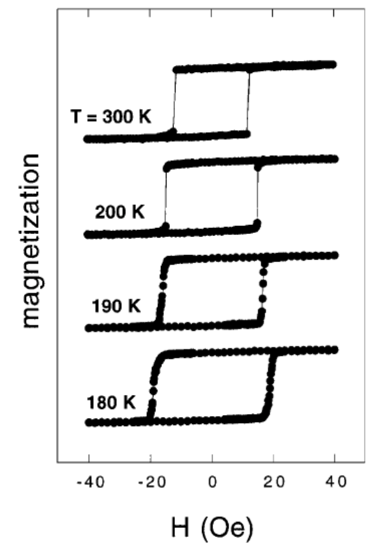

FIG. 1. $M(H)$ loops measured on a Co/CoO-bilayer structure for the temperatures indicated. The thin lines are guides to the eye.

(T tunes effective disorder)

Experiment (disordered ferro)
Berger et al. (2000)

Why is the random Ising model generally non-critical?

Pazmandi, Zarand, Zimanyi (PRL 1999):

Elastic manifolds and
ferros with dipolar interactions:

They have strong frustration:
- long range interactions with varying signs and/or
- strong configurational constraints

→ Glassy systems with arbitrarily high barriers, metastable states

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→ Look at spin glasses! (Frustration + disorder = glass and criticality? yes!)

SK criticality

VOLUME 83, NUMBER 5 PHYSICAL REVIEW LETTERS 2 AUGUST 1999

Self-Organized Criticality in the Hysteresis of the Sherrington-Kirkpatrick Model

Ferenc Pázmándi,[1,2,3] Gergely Zaránd,[1,3] and Gergely T. Zimányi[1]

Canonical spin glass model: Sherrington-Kirkpatrick (SK) model - fully connected

$$\boxed{H = \frac{1}{2}\sum_{i,j} J_{ij} s_i s_j - h_{ext} \sum_i s_i, \quad J_{ij} : \text{ random Gaussian } \overline{J_{ij}^2} = J^2/N}$$

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• Known facts:

- There is a thermodynamic transition at T_c to a glass phase:
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- Multitude of metastable states, separated by barriers
- Correct equilibrium solution by G. Parisi : Replica symmetry breaking
- Glass phase is always critical! *(Kondor, DeDominicis)*

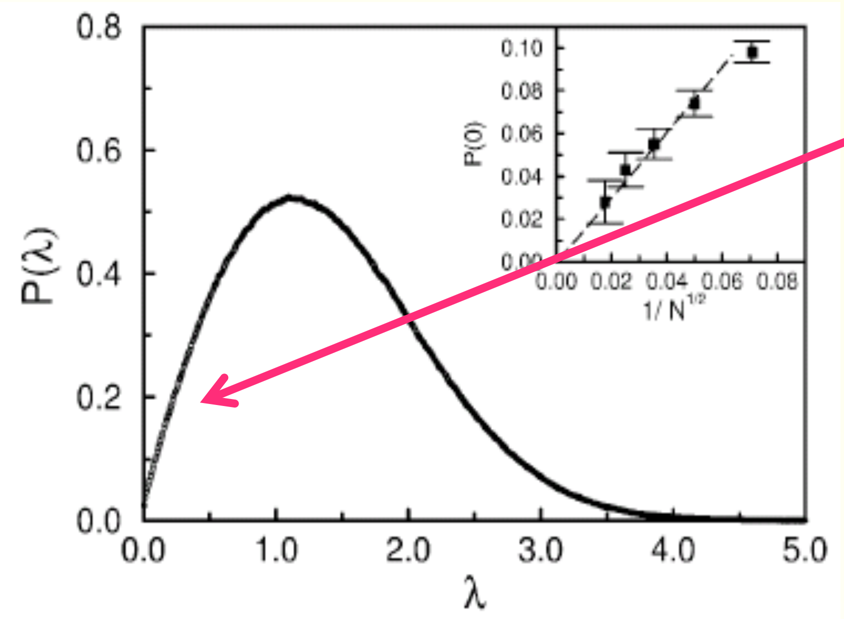
SK criticality – local fields

$$\boxed{H = \frac{1}{2}\sum_{i,j} J_{ij} s_i s_j - h_{ext}\sum_i s_i}$$

Thouless, Anderson and Palmer, (1977); Palmer and Pond (1979)
Parisi (1979), Bray, Moore (1980)
Sommers and Dupont (1984)
Dobrosavljevic, Pastor (1999)
Pazmandi, Zarand, Zimanyi (1999)
MM, Pankov (2007)

Local field on spin *i*:

$$\lambda_i \equiv -\frac{\partial H}{\partial s_i} = -\sum_{j\neq i} J_{ij} s_j + h_{ext}$$



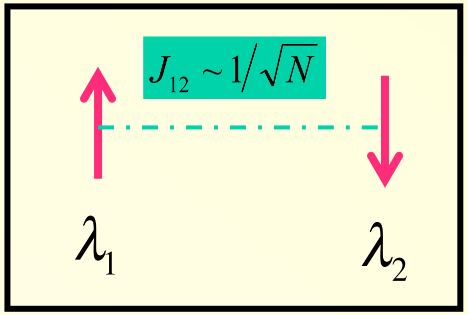
Linear "Coulomb" gap in the distribution of local fields

A first indication of criticality!

The linear pseudogap in SK

Thouless (1977)

Stability of ground state with respect to flipping of a pair:



> The distribution of local fields must vanish at λ=0 at T = 0!

- Suppose pseudogap $\quad P(\lambda) \propto \lambda^{\gamma}$

$\rightarrow$ Smallest local fields $\quad \lambda_{\min} \propto N^{-1/1+\gamma}$

- 2-spin flip cost $\quad E_{\cos t} \propto |\lambda_1| + |\lambda_2| - N^{-1/2} \quad \sim \quad N^{-1/1+\gamma} - N^{-1/2} \overset{!}{>} 0$

> $\gamma \geq 1 \rightarrow$ At least linear pseudogap!

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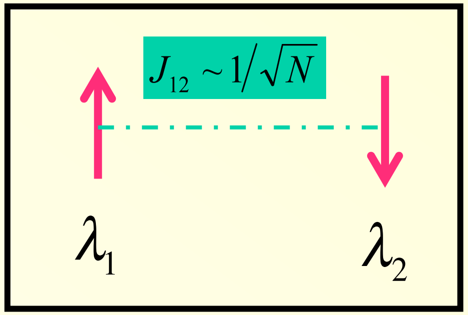

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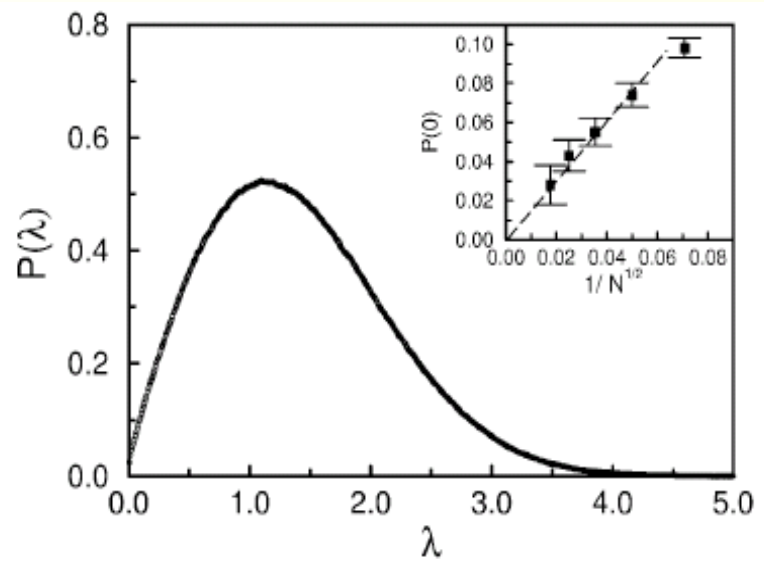


γ ≥ 1 → At least linear pseudogap!

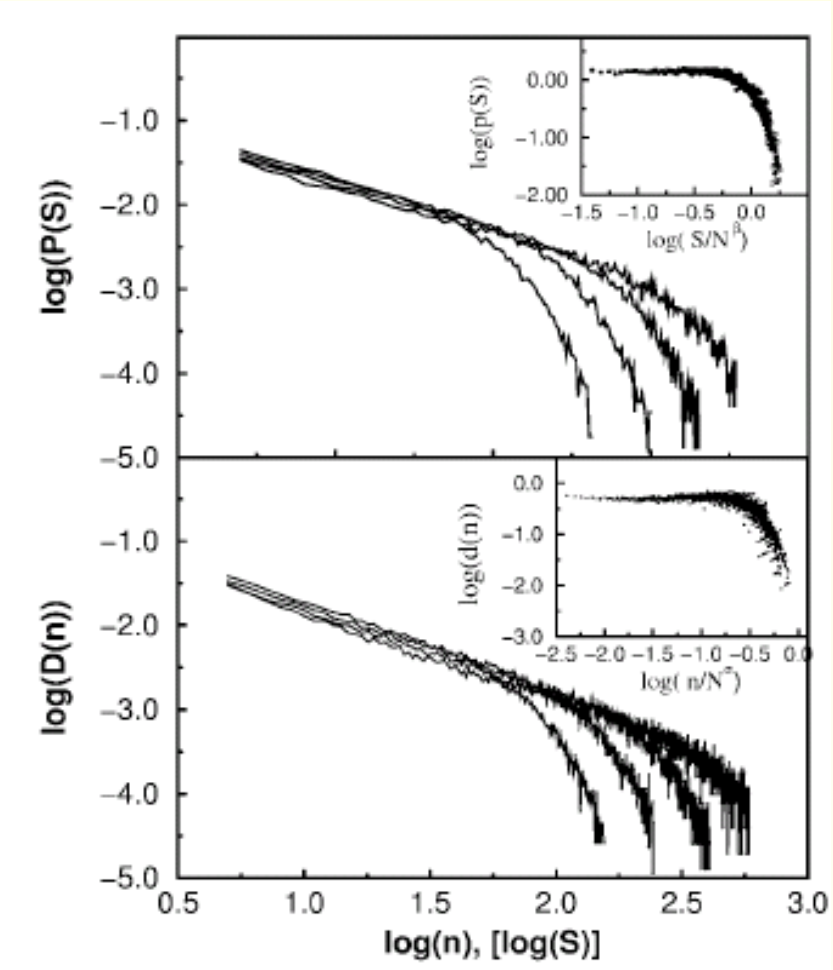
- But: γ = 1!

Largest possible density of soft spins!

Distribution is so critical that flipping the first spin by an increase of $\Delta h_{ext} = \lambda_{\min}$ can trigger a large avalanche!

“Living on the edge”

Pazmandi, Zarand, Zimanyi (1999)



Size distribution of avalanches:

- Avalanches are large
- Only cutoff: system size ($N^{1/2}$)
- Power law:
  Sign of Self-Organized Criticality

Review: Criticality and RSB

SK-model

$$H = \sum_{i<j} J_{ij} s_i s_j$$

Replica trick

$$-\beta F = \overline{\ln Z} = \lim_{n\to 0} \frac{\overline{Z^n} - 1}{n}$$

Edwards, Anderson (1974)

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$$\overline{Z^n} = \overline{\exp\left(-\beta \sum_{a=1}^{n} H\left[s^a\right]\right)} = \exp\left[\frac{\beta^2 N n}{4} + \frac{\beta^2 N}{2} \sum_{1\le a<b\le n} \left(\sum_i s_i^a s_i^b / N\right)^2\right]$$

$$= \int \prod_{a<b} \frac{dQ_{ab}}{\sqrt{2\pi / \beta^2 N}} \exp\left(-NA[Q]\right)$$

Free energy functional $$A[Q] = -n\beta^2/4 + \beta^2/2 \sum_{1\le a<b\le n} Q_{ab}^2 - \log\left[\sum_{\{S^a\}} \exp\left(\beta^2 Q_{ab} S^a S^b\right)\right]$$

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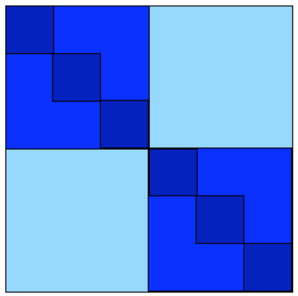
Parisi ansatz for the saddle point:
Hierarchical replica symmetry breaking

Parisi (1979)

$$Q_{ab} =$$



SK criticality

Important features of the solution in the glass phase:

- The free energy functional is only marginally stable!
  → Collection of zero modes of the Hessian $\dfrac{\partial^2 A}{\partial Q_{ab} \partial Q_{cd}}$
- ←→ Critical spin-spin correlations in the whole glass phase! Found numerically also in finite dimensions!

$$\boxed{\overline{\langle s_i s_j \rangle}^2 \sim \frac{1}{r_{ij}^{\alpha}}}$$

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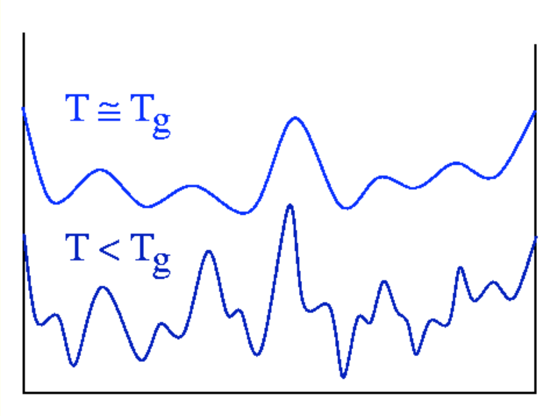
Free energy landscape



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  A continuous function Q(x), n<x<1, parametrizes Q_{ab}

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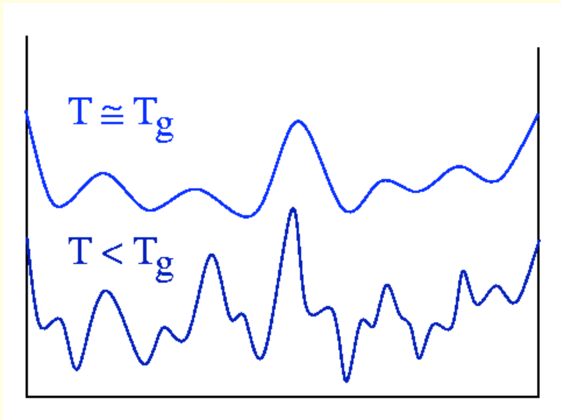

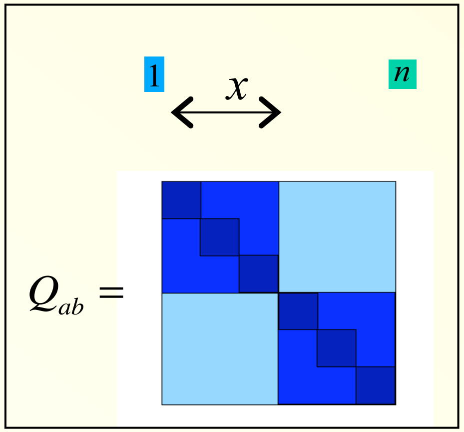

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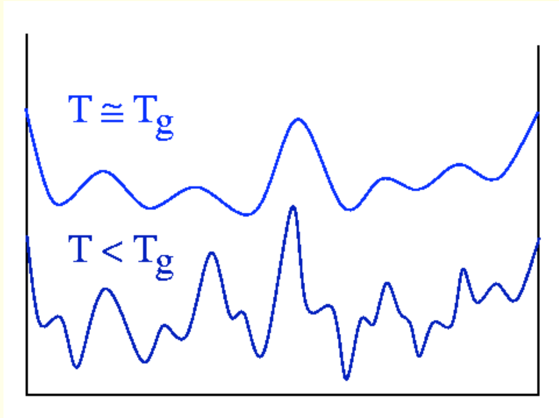
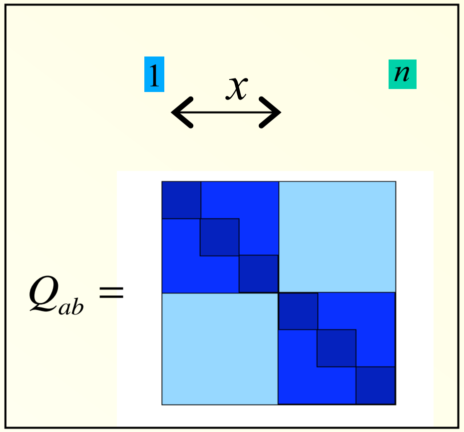
- Hierarchical structure of phase space and time scales
- Replica symmetry is broken continuously (at all scales)
  A continuous function Q(x), n<x<1, parametrizes Q_{ab}
- Marginality is directly related to the linear pseudogap
  The pseudogap can be calculated analytically at low T
  (Pankov)

Free energy landscape





After so much critical preparation:

- Understand shocks in spin glasses
- Calculate avalanche distribution analytically!
- Confirm the direct connection of scale free avalanches and thermodynamic criticality!

→ Barkhausen noise as a diagnostic tool for a glass phase?

Stepwise response and shocks in spin glass models

Yoshino, Rizzo (2008)

p-spin models [akin to supercooled liquids]
- no continuous, but 1-step RSB -
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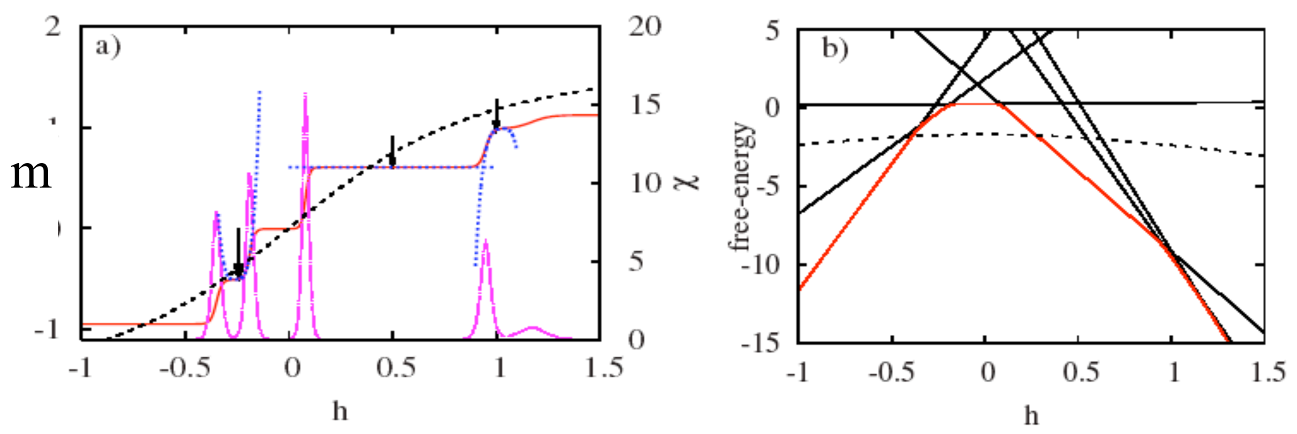
Equilibrium jump/shock when two states cross: $F_\alpha(h_{shock}) = F_\beta(h_{shock})$



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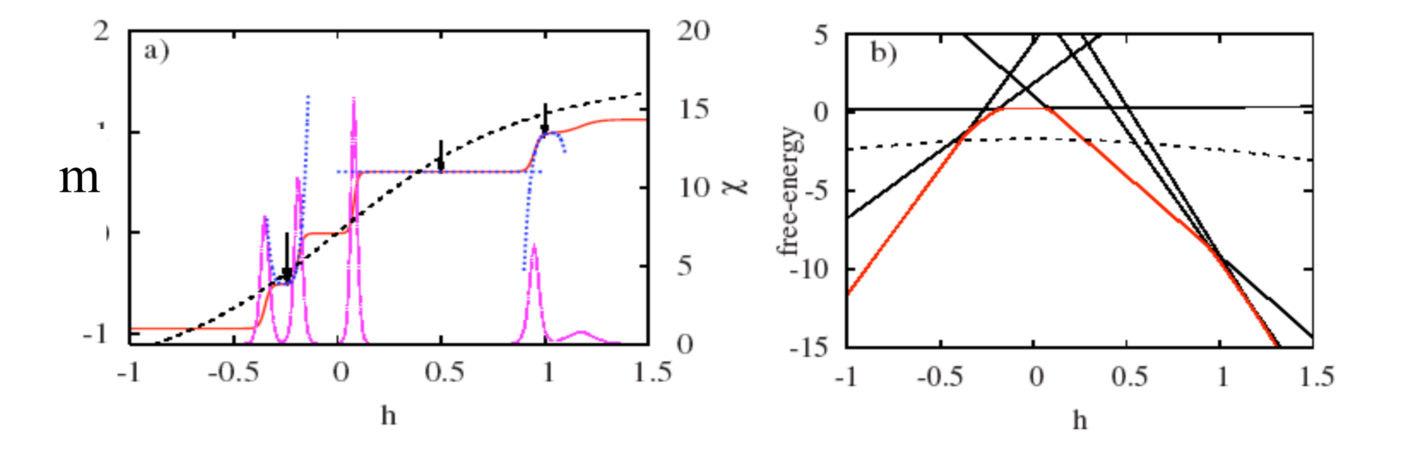


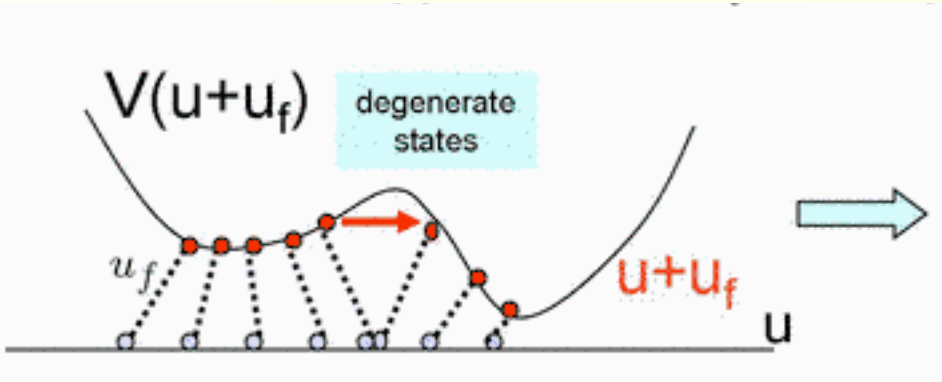
Mesoscopic effect: Susceptibility has spikes and does **not** self-average!

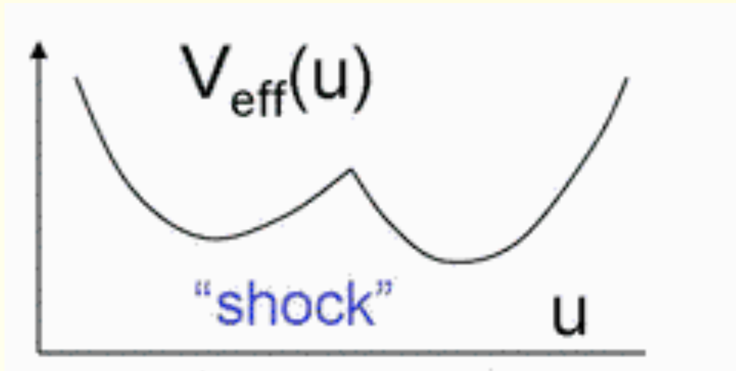
A simple picture of shocks

Balents, Bouchaud, Mezard

$$H = \frac{q^2}{2}u^2 + \frac{\Lambda^2}{2}|u_f|^2 + V(u+u_f)$$

$$H_{eff}[u] = \mathrm{Min}_{u_f} H[u, u_f] \qquad T = 0$$





Elastic system — Magnetic system

Displacement — $u \leftrightarrow h$ — Magnetic field

Effective potential — $V(u) \leftrightarrow F(h)$ — Free energy

Force — $f(u) = -V'(u) \leftrightarrow m(h) = -F'(h)$ — magnetization

Detecting shocks

2^{nd} cumulant of the magnetization ($T = 0$) *Yoshino, Rizzo (2008)*

$$\overline{M(h+\delta h)M(h-\delta h) - M(h)^2} \propto |\delta h|$$

Non-analytic cusp!

- Reflects the probability of shocks.
- The cusp is rounded at finite T.

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- The cusp is rounded at finite T.

Closely related effects:

- Functional renormalization group for collectively pinned elastic manifolds (e.g. vortex lattices):

Cusp in cumulants of effective potential $V_{eff}(u_{cm})$ at T = 0 and beyond collective pinning scale L> L_{larkin}

D. Fisher (1986)
LeDoussal, Wiese
Balents, Bouchaud, Mézard
LeDoussal, MM, Wiese

Detecting shocks

2^{nd} cumulant of the magnetization ($T = 0$) *Yoshino, Rizzo (2008)*

$$\overline{M(h+\delta h)M(h-\delta h) - M(h)^2} \propto |\delta h|$$

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- Turbulence:
Shocks in the velocity field v(x), rounded only by finite viscosity η (akin to T above)

Bouchaud, Mézard, Parisi

Strategy of calculation

k^{th} cumulant of magnetization difference

$$\overline{[M(h) - M(h+\delta h)]^k} = \text{Prob}(\text{shock} \in [h, h+\delta h])\, \overline{\Delta M^k_{\text{shock}}}^h + O\left(\delta h^2\right)$$

Shock density

$$\text{Prob}(\text{shock} \in [h, h+\delta h]) = \boxed{\rho_0} |\delta h|$$

Avalanche size cumulants

$$\overline{\Delta M^k_{\text{shock}}}^h = \int_0^\infty d\,\Delta M \boxed{P(\Delta M; h)} \Delta M^k$$

Calculate $\overline{[M(h) - M(h+\delta h)]^k} \quad \rightarrow \boxed{\rho_0}, \boxed{P(\Delta M; h)}$

Natural scales: $\boxed{\delta h \sim \lambda_{\text{min}} \sim N^{-1/2}}$ $\boxed{\Delta M \sim \chi N \Delta h \sim N^{1/2}}$

Strategy of calculation

Calculate $\overline{[M(h) - M(h+\delta h)]^k} \quad \rightarrow \boxed{\rho_0}, \boxed{P(\Delta M; h)}$

$$\overline{M(h_1)\dots M(h_k)} = (-1)^k \partial_{h_1} \dots \partial_{h_k} \overline{F(h_1)\dots F(h_k)}$$

Calculate effective potential of n replicas:

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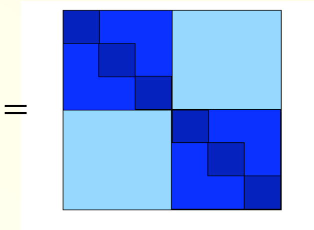
Calculate effective potential of n replicas:

Easy to extract in the replica limit $n \rightarrow 0$

$$\begin{aligned}
\exp\left[W[\{h_a\}]\right] &:= \overline{\exp\left[-\beta \sum_{a=1}^{n} F(h_a)\right]}^J \\
&= \exp\left[-\beta \sum_{a=1}^{n} \overline{F(h_a)}^J + \frac{\beta^2}{2} \sum_{a,b=1}^{n} \overline{F(h_a)F(h_b)}^{J,c} - \frac{\beta^3}{3!} \sum_{a,b,c=1}^{n} \overline{F(h_a)F(h_b)F(h_c)}^{J,c} + \dots\right] \\
&= \overline{\sum_{\{S_a^i\}} \exp\left[\beta \sum_{ij} S_a^i J^{ij} S_a^j + \beta \sum_i h_a S_a^i\right]}^J = \\
&= \sum_{\{S_a^i\}} \int \prod_{a\neq b} \mathrm{d}Q_{ab} \prod_i \exp\left[nN\frac{\beta^2 J^2}{2} + \beta^2 J^2 \sum_{a\neq b}\left(-\frac{N}{2} Q_{ab}^2 + Q_{ab} S_a^i S_b^i\right) + \sum_a \beta h_a S_a^i\right].
\end{aligned}$$

$Q_{ab} =$ 

Strategy of calculation

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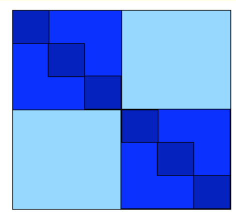
$N \rightarrow \infty$ limit: $h_a = \tilde{h}_a / \sqrt{N}$

Saddle point Q_{ab}, sum over replica permutations!

$Q_{ab} =$ 

$$\exp[W[h]-W[0]] = \sum_\pi \exp\Big[\frac{\beta^2}{2}\sum_{ab}Q_{ab}\tilde{h}_{\pi(a)}\tilde{h}_{\pi(b)} + \frac{\beta^2}{2}\sum_a \tilde{h}_a^2(1-Q_{aa})\Big]\Big/\sum_\pi 1$$

Calculation

Sum over replica permutations π in $S(n)$ [quite a challenge!]

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- k'th cumulant: k groups of $n \to 0$ replicas with the same h_a
- integral representation of W[h] and the magnetization cumulants
- limit $T \to 0$: extract non-analytic contribution from shocks

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Final result:

- picture of mesoscopic avalanches ~ $N^{1/2}$ fully confirmed
- obtain critical probability distribution of avalanche sizes

$$P(\delta m \equiv \frac{\Delta M}{\sqrt{N}})\, d(\delta m)\frac{dh}{\sqrt{N}} = \sqrt{\frac{c^*}{\pi}}\,\frac{e^{-\delta m^2}}{\delta m}\, d(\delta m)\frac{dh}{\sqrt{N}}$$

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$\tau = 1$

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Avalanche exponent

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$c^* = 0.410802$ Obtained from low T solution of the SK model

Comparison with numerics

Analytical result (shocks in equilibrium)

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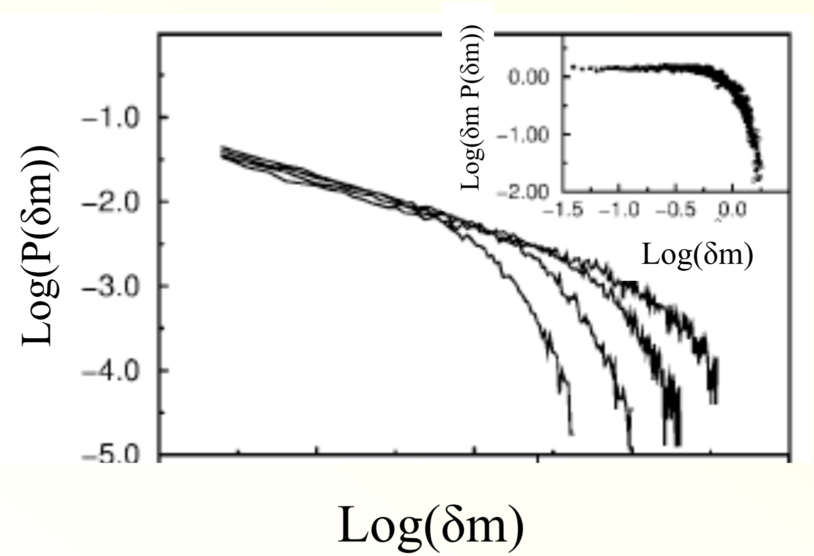
Avalanches in the hysteresis loop (slowly driven, out-of-equilibrium)



Many qualitative features agree between analytics (equilibrium) and numerics (out-of equilibrium)

Pazmandi, Zarand, Zimanyi (1999)

Remarks

Analytical result (shocks in equilibrium)

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Important remarks

- The power law arises because of the criticality of the glass

- It receives contributions from all scales and distances within the hierarchical organization of states

- Nearly no dependence on the external field, except in the cutoff scale:

The spin glass is critical even in finite field.

Conclusion

Spin glass criticality (in the SK model) is prominently reflected in scale free response to a slow magnetic field change.

There is a deep connection between various manifestations of this criticality:

Soft gap – avalanches – spin-spin correlations – abundant collective low energy excitations

Outlook

- Finite d spin glasses:
  - Is criticality revealed in avalanches?
  - Experimental probe for criticality via Barkhausen noise?
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- Avalanches in other complex systems (computer science, optimization, economy, etc)