Criticality, shocks and avalanches in spin glasses

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Outline

- Crackling, avalanches, and “shocks” in disordered, non-linear systems; Self-organized criticality

- Avalanches in the magnetizing process (“Barkhausen noise”)

- The criticality of spin glasses at equilibrium – why to expect scale free avalanches

- Magnetization avalanches in the Sherrington-Kirkpatrick spin glass – an analytical study.

- Outlook: finite dimensions, electron glasses,…

Crackling

*Review: Sethna, Dahmen, Myers, Nature **410**, 242 (2001).*

Crackling = Response to a slow driving which occurs in a discrete set of avalanches, spanning a wide range of sizes.

Occurs often but not necessarily out of equilibrium.

Examples:

- Earthquakes
- Crumpling paper
- Vortices and vortex lattices in disordered media etc.
- Disordered magnet in a changing external field magnetizes in a series of jumps

But: Not everything crackles!

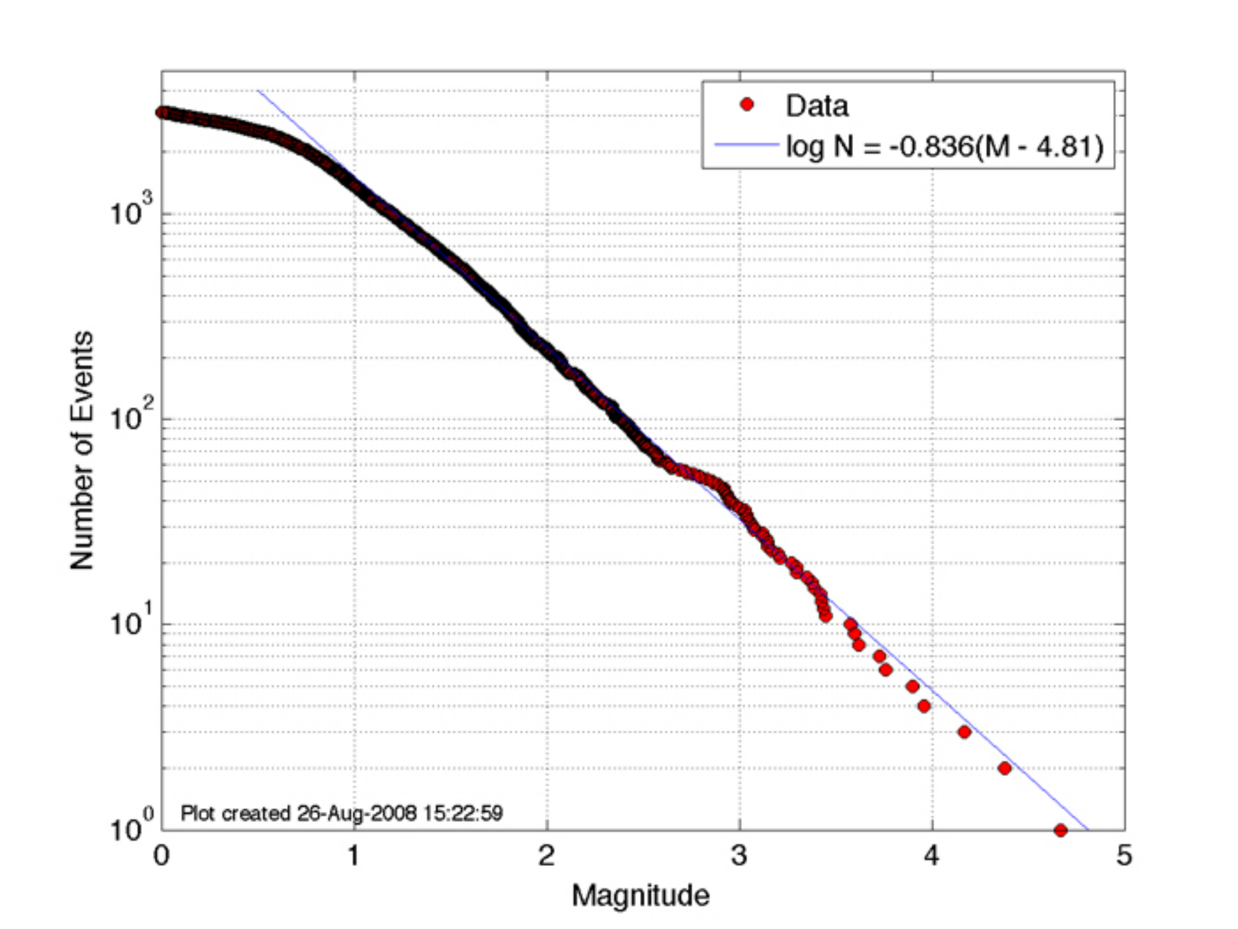
It is intermediate between snapping (e.g., twigs, chalk, weakly disordered ferromagnets, nucleation in clean systems) and popping (e.g., popcorn, strongly disordered ferromagnets)

Crackling on all scales is generally a signature of a critical state in driven, non-linear systems. It can thus be an interesting *diagnostic tool*.

Examples of crackling I

- Gutenberg-Richter law for strength of earthquakes

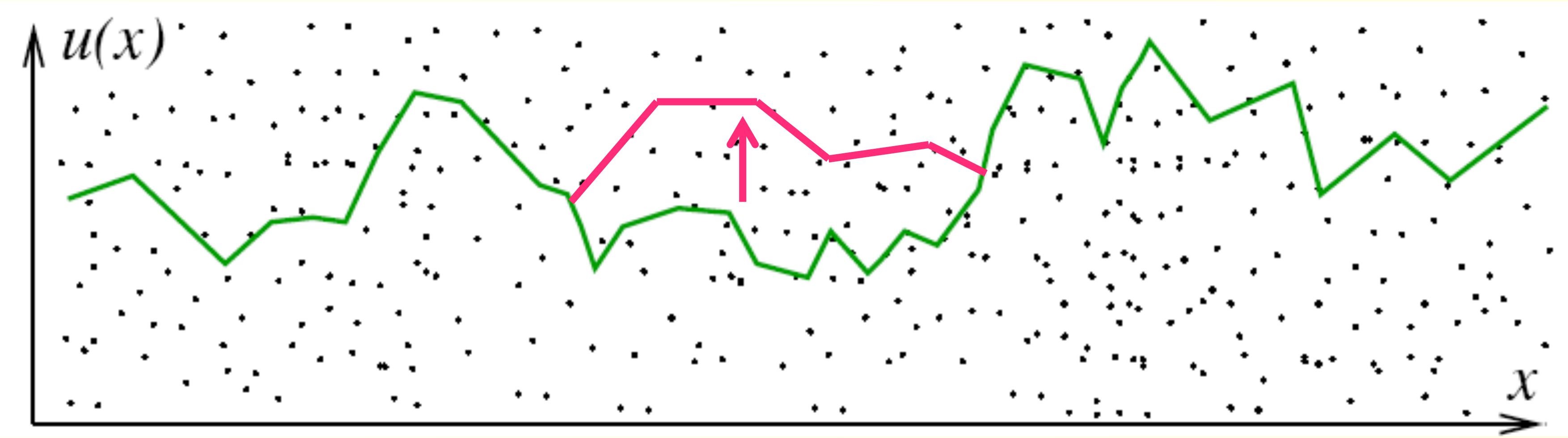
(jumps of driven tectonic plates)



Examples of crackling II

- Depinning of elastic interfaces

Liquid fronts, domain walls, charge density waves, vortex lattices:



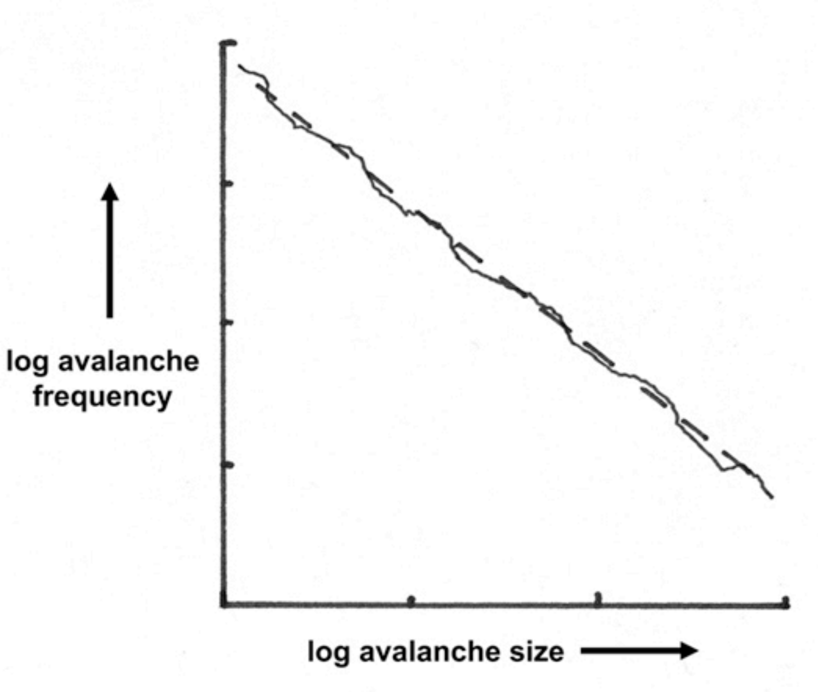
Depinning as a dynamical critical phenomenon in disordered glassy systems
Sophisticated theoretical approach: functional RG [D. Fisher, LeDoussal, etc]

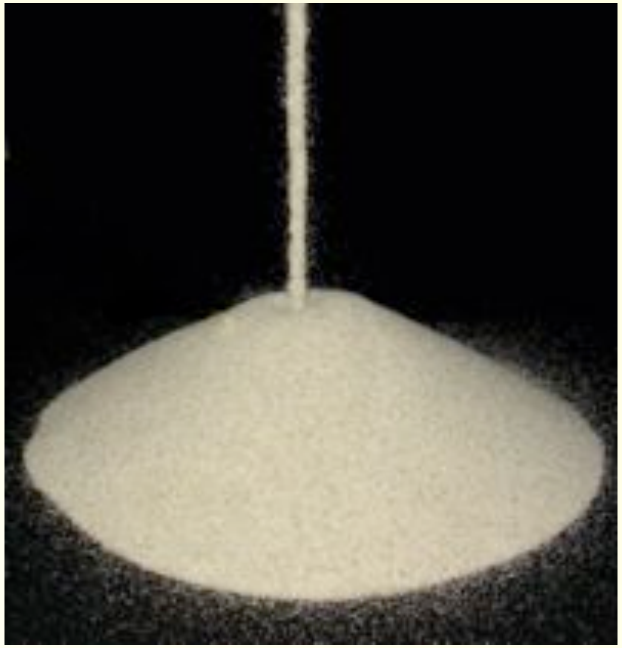
Statistics of avalanches: - mean field theory
- recent first steps and successes with FRG
find non-trivial critical power laws (without scale)

Examples of crackling III

• Power laws due to self-organized criticality:
Dynamics is attracted to a critical state, without fine-tuning of parameters

Example: sandpile model by Bak, Tang, and Wiesenfeld





Magnetic systems

- Crackling noise in the hysteresis loop: "Barkhausen noise"
- When does crackling occur in random magnets, and why?
- What happens in frustrated spin glasses
  (as opposed to just dirty ferromagnets)?

?

Equilibrium avalanches in the hysteresis reflect criticality of the glass phase! Noise as a diagnostic of a critical glass state?

Avalanches in ferromagnetic films

Direct Observation of Barkhausen Avalanche in (ferro) Co Thin Films

Kim, Choe, and Shin (PRL 2003)

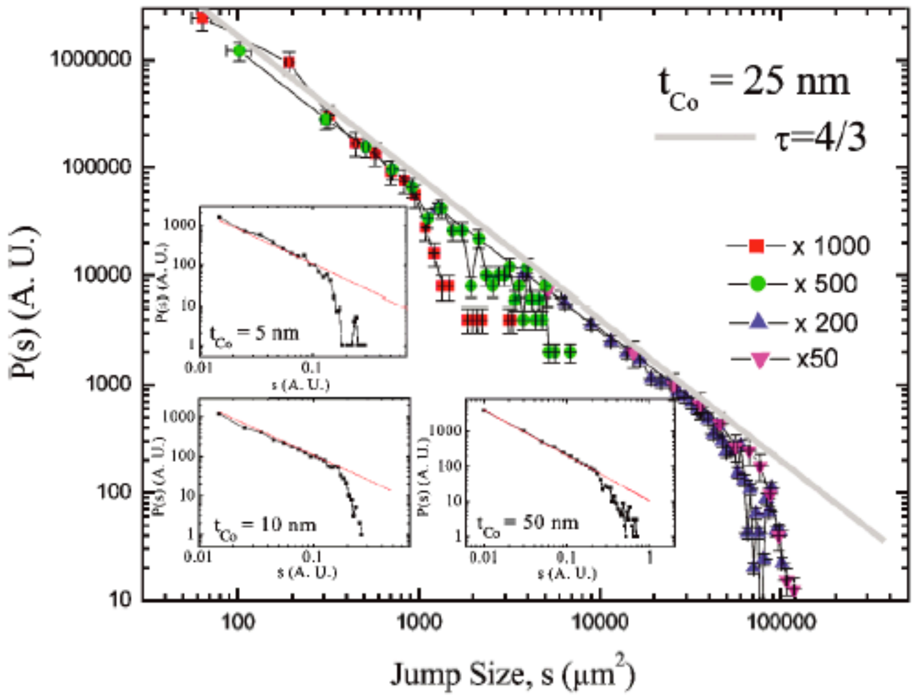


FIG. 3 (color). Distributions of the Barkhausen jump size in 25 and 50-nm Co samples. Distributions in 5, 10, and 50-nm Co samples are shown in the insets. Fitting curve with $\tau = 1.33$ is denoted at each graph.

Distribution of magnetization jumps

$$\boxed{P(s) = \frac{A}{s^{\tau}} \qquad \tau = \frac{4}{3}}$$

Cizeau et al.:
Theoretical model with **dipolar long range interactions** (believed to be crucial to get criticality)

Model ferromagnets

Dahmen, Sethna
Vives, Planes

Random field Ising model (short range):

$$H = -J \sum_{<ij>} s_i s_j - \sum_i h_i s_i - h_{ext} \sum_i s_i$$

- Generically non-critical
- Scale free avalanches require fine tuning of disorder $\Delta = \langle h_i^2 \rangle$ *and* field $h_{ext,crit}$

Experiment:

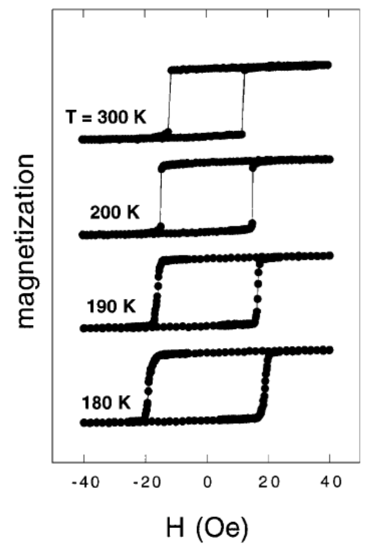


FIG. 1. $M(H)$ loops measured on a Co/CoO-bilayer structure for the temperatures indicated. The thin lines are guides to the eye.

(T tunes effective disorder)

Experiment (disordered ferro)
Berger et al. (2000)

Why is the random Ising model generally non-critical?

Pazmandi, Zarand, Zimanyi (PRL 1999):

Elastic manifolds and
ferromagnets with dipolar interactions:

They have strong frustration:
- long range interactions with varying signs and/or
- strong configurational constraints

→ Glassy systems with arbitrarily high barriers, metastable states

In contrast: RFIM is known not to have a spin glass phase
(Krzakala, Ricci-Tersenghi, Zdeborova)

→ Look at spin glasses!
(Frustration + disorder = glass and criticality!?)

SK criticality

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Self-Organized Criticality in the Hysteresis of the Sherrington-Kirkpatrick Model

Ferenc Pázmándi,[1,2,3] Gergely Zaránd,[1,3] and Gergely T. Zimányi[1]

Canonical spin glass model: Sherrington-Kirkpatrick (SK) model - fully connected

$$\boxed{H = \frac{1}{2}\sum_{i,j} J_{ij} s_i s_j - h_{ext}\sum_i s_i, \quad J_{ij}: \text{ random Gaussian } \overline{J_{ij}^2} = J^2/N}$$

• Extremely intricate mean field version of the Edwards-Anderson model in finite dimensions (but $d_{uc} = 6$)

• Known facts:

- There is a thermodynamic transition at T_c to a glass phase:
- no global magnetization, but broken Ising symmetry: $\langle s_i \rangle \neq 0$,
- measured by Edwards Anderson order parameter $Q_{EA} = \frac{1}{N}\sum_i \langle s_i \rangle^2$
- Multitude of metastable states, separated by barriers
- Correct equilibrium solution by G. Parisi : Replica symmetry breaking
- Glass phase is always critical! *(Kondor, DeDominicis)*
- This is also true in the droplet picture! (power law correlations persist)

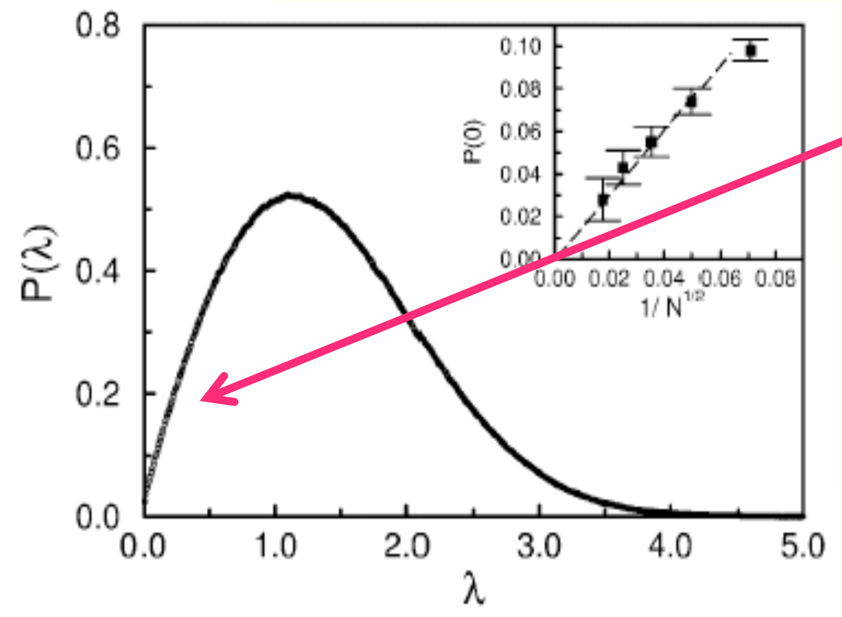
SK criticality – local fields

$$\boxed{H = \frac{1}{2}\sum_{i,j} J_{ij} s_i s_j - h_{ext}\sum_i s_i}$$

Thouless, Anderson and Palmer, (1977); Palmer and Pond (1979) Parisi (1979), Bray, Moore (1980) Sommers and Dupont (1984) Dobrosavljevic, Pastor (1999) Pazmandi, Zarand, Zimanyi (1999) MM, Pankov (2007)

Local field on spin *i*:

$$\lambda_i \equiv -\frac{\partial H}{\partial s_i} = -\sum_{j\neq i} J_{ij} s_j + h_{ext}$$



Linear "Coulomb" gap in the distribution of local fields

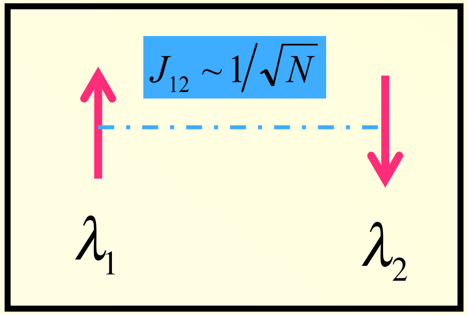
A first indication of criticality!

The linear pseudogap in SK

Thouless (1977)

Stability of ground state with respect to flipping of a pair:

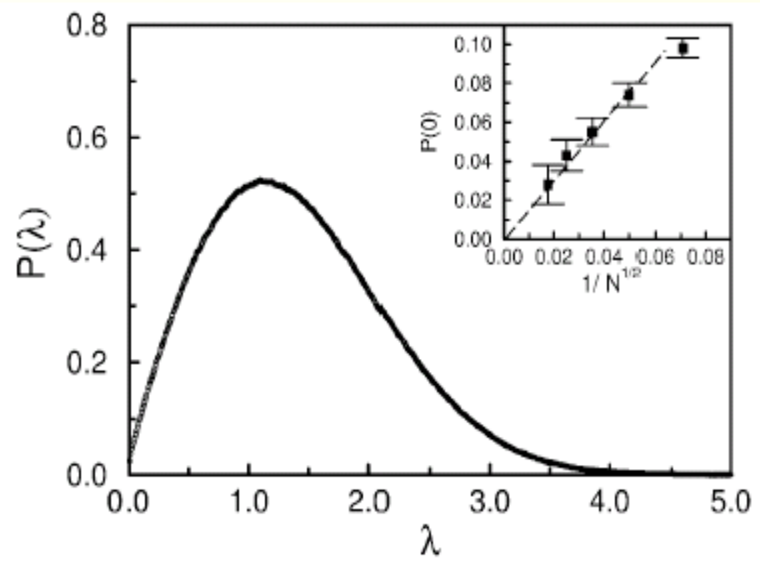
The distribution of local fields must vanish at λ=0 at T = 0!



- Suppose pseudogap $P(\lambda) \propto \lambda^{\gamma}$

→ Smallest local fields $\lambda_{\min} \propto N^{-1/1+\gamma}$

- 2-spin flip cost $E_{\cos t} \propto |\lambda_1| + |\lambda_2| - N^{-1/2} \sim N^{-1/1+\gamma} - N^{-1/2} \overset{!}{>} 0$



$\gamma \geq 1$ → At least linear pseudogap!

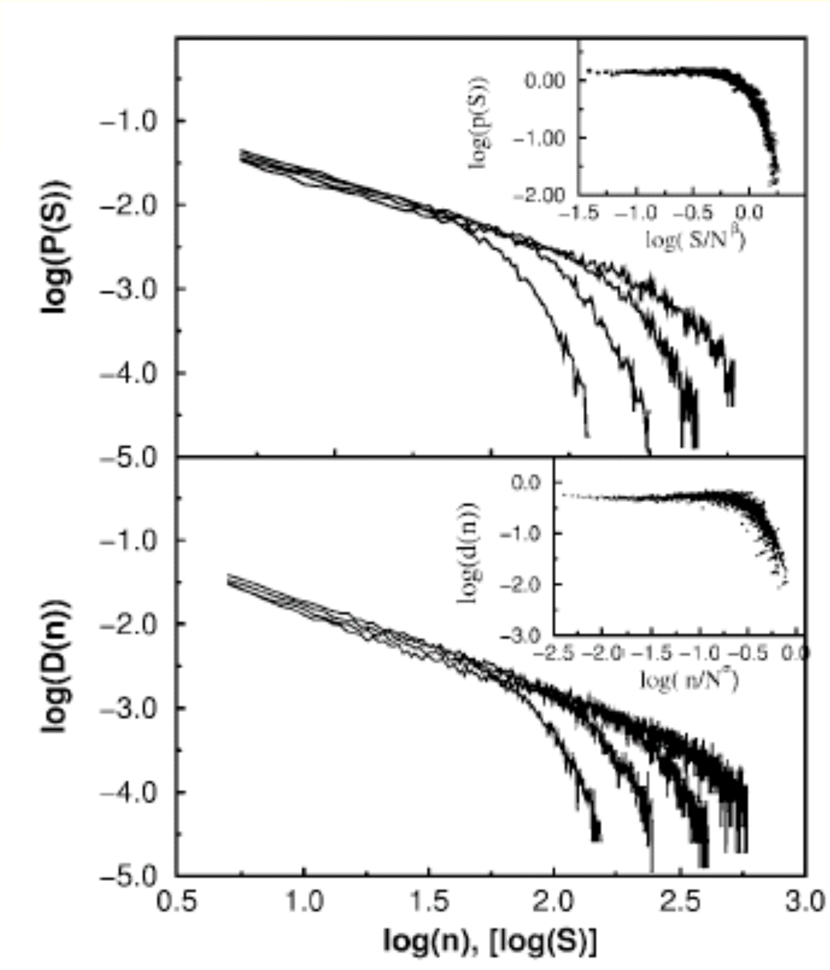
- But: $\gamma = 1$!

Largest possible density of soft spins!

Distribution is critical so that flipping the first spin by an increase of $\Delta h_{ext} = \lambda_{\min}$ can trigger a large avalanche!

“Living on the edge”

Pazmandi, Zarand, Zimanyi (1999)



Size distribution of avalanches:

- Avalanches are large
- Only cutoff: system size ($N^{1/2}$)
- Power law:
  Sign of Self-Organized Criticality

Review: Criticality and RSB

SK-model $$H = \sum_{i<j} J_{ij} s_i s_j$$

Replica trick $$-\beta F = \overline{\ln Z} = \lim_{n\to 0} \frac{\overline{Z^n} - 1}{n}$$ *Edwards, Anderson (1974)*

$$\overline{Z^n} = \overline{\exp\left(-\beta \sum_{a=1}^{n} H\left[s^a\right]\right)} = \exp\left[\frac{\beta^2 N n}{4} + \frac{\beta^2 N}{2} \sum_{1\le a<b\le n} \left(\sum_i s_i^a s_i^b / N\right)^2\right]$$

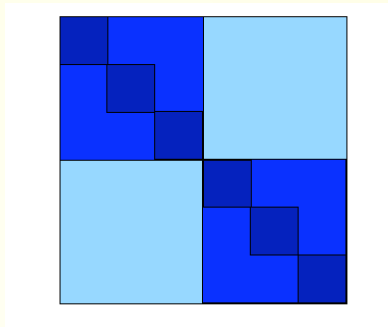
$$= \int \prod_{a<b} \frac{dQ_{ab}}{\sqrt{2\pi / \beta^2 N}} \exp(-NA[Q])$$

Free energy functional $$A[Q] = -n\beta^2/4 + \beta^2/2 \sum_{1\le a<b\le n} Q_{ab}^2 - \log\left[\sum_{\{S^a\}} \exp\left(\beta^2 Q_{ab} S^a S^b\right)\right]$$

Parisi ansatz for the saddle point:
Hierarchical replica symmetry breaking

Parisi (1979)

$$Q_{ab} =$$



SK criticality

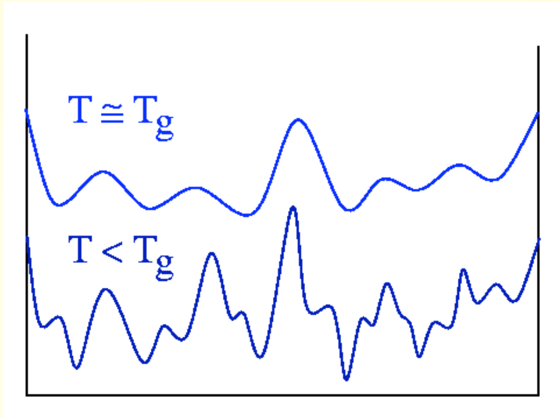
Important features of the solution in the glass phase:

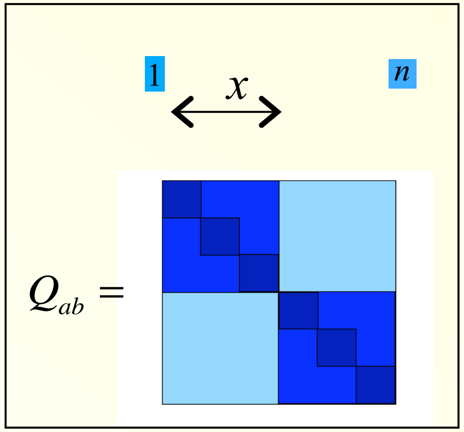
- The free energy functional is only marginally stable!
  → Full family of zero modes of the Hessian $\dfrac{\partial^2 A}{\partial Q_{ab}\partial Q_{cd}}$
- ↔ Critical spin-spin correlations in the whole glass phase! Found numerically also in finite dimensions!

$$\boxed{\overline{\langle s_i s_j \rangle^2} \sim \frac{1}{r_{ij}^{\alpha}}}$$

- Hierarchical structure of phase space and time scales
- Replica symmetry is broken continuously (at all scales)
  A continuous function Q(x), n<x<1, parametrizes Q_{ab}
- Marginality is directly related to the linear pseudogap
  The pseudogap can be calculated analytically at low T
  (Pankov)

Free energy landscape





After so much critical preparation:

- Understand shocks in spin glasses

- Calculate avalanche distribution analytically
  (at equilibrium, however!)

- Find the direct connection of scale-free
  equilibrium shocks and thermodynamic criticality

Stepwise response and shocks in spin glass models

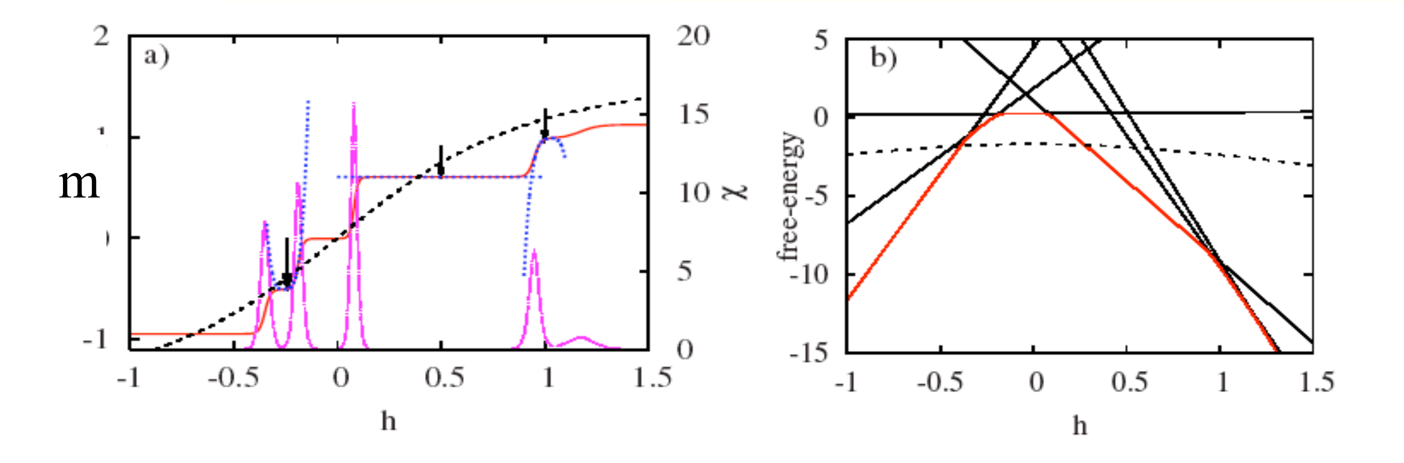
Yoshino, Rizzo (2008)

p-spin models [physics similar as in supercooled liquids]
- no continuous, but only 1-step Replica Symmetry Breaking

→ Glassy, but much simpler and non-critical

Free energy of metastable state α: $F_\alpha(h) = F_\alpha(h=0) - hM_\alpha$

Equilibrium jump/shock when two states cross: $F_\alpha(h_{shock}) = F_\beta(h_{shock})$



Mesoscopic effect: Susceptibility has spikes and does **not** self-average!

How to obtain shocks and their distribution for the SK model?

Expect: For $\Delta h=O(1)$ expect full chaos (minimal overlap) between initial and final configuration!

Question: What happens on the mesoscopic scale, and what happens to the magnetization?

Strategy of calculation

k^th cumulant of magnetization difference

$$\overline{[M(h) - M(h+\delta h)]^k} = \mathrm{Prob}(\mathrm{shock} \in [h, h+\delta h])\, \overline{\Delta M^k_{\mathrm{shock}}}^h + O\left(\delta h^2\right)$$

Shock density

$$\mathrm{Prob}(\mathrm{shock} \in [h, h+\delta h]) = \boxed{\rho_0} |\delta h|$$

Avalanche size cumulants

$$\overline{\Delta M^k_{\mathrm{shock}}}^h = \int_0^\infty d\,\Delta M \boxed{P(\Delta M; h)} \Delta M^k$$

→ Calculate $\overline{[M(h) - M(h+\delta h)]^k} \quad \to \boxed{\rho_0}, \boxed{P(\Delta M; h)}$

Natural scales:

$\boxed{\delta h \sim \lambda_{\min} \sim N^{-1/2}}$ Distance between shocks

$\boxed{\Delta M \sim \chi N \Delta h \sim N^{1/2}}$ Magnetization jumps

Strategy of calculation

→ Calculate $\overline{[M(h)-M(h+\delta h)]^k}$ $\rightarrow \boxed{\rho_0}, \boxed{P(\Delta M; h)}$

$$\overline{M(h_1)\dots M(h_k)} = (-1)^k \partial_{h_1}\dots\partial_{h_k}\overline{F(h_1)\dots F(h_k)}$$

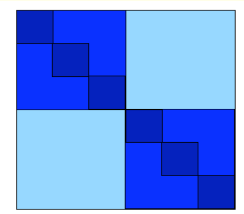
→ Calculate effective potential of n replicas:

Easy to extract in the replica limit $n \rightarrow 0$

$$\begin{aligned}
\exp\Big[W[\{h_a\}]\Big] &:= \overline{\exp\Big[-\beta\sum_{a=1}^{n} F(h_a)\Big]}^{J} \\
&= \exp\Big[-\beta\sum_{a=1}^{n}\overline{F(h_a)}^{J} + \frac{\beta^2}{2}\sum_{a,b=1}^{n}\overline{F(h_a)F(h_b)}^{J,c} - \frac{\beta^3}{3!}\sum_{a,b,c=1}^{n}\overline{F(h_a)F(h_b)F(h_c)}^{J,c} + \dots\Big] \\
&= \sum_{\{S_a^i\}}\int\prod_{a\neq b}\mathrm{d}Q_{ab}\prod_i \exp\Big[nN\frac{\beta^2 J^2}{2} + \beta^2 J^2\sum_{a\neq b}\Big(-\frac{N}{2}Q_{ab}^2 + Q_{ab}S_a^i S_b^i\Big) + \sum_a \beta h_a S_a^i\Big].
\end{aligned}$$

$N \rightarrow \infty$ limit: i) Rescale $h_a = \tilde{h}_a/\sqrt{N}$

ii) Saddle point Q_{ab}, sum over replica permutations!

$Q_{ab} =$ 

$$\exp[W[h]-W[0]] = \sum_\pi \exp\Big[\frac{\beta^2}{2}\sum_{ab} Q_{ab}\tilde{h}_{\pi(a)}\tilde{h}_{\pi(b)} + \frac{\beta^2}{2}\sum_a \tilde{h}_a^2(1-Q_{aa})\Big] \Big/ \sum_\pi 1$$

Calculation

Sum over replica permutations π in *S(n)* [a real challenge!]

$$\exp[W[h] - W[0]] = \sum_{\pi} \exp\left[\frac{\beta^2}{2}\sum_{ab} Q_{ab}\tilde{h}_{\pi(a)}\tilde{h}_{\pi(b)} + \frac{\beta^2}{2}\sum_{a}\tilde{h}_a^2(1 - Q_{aa})\right] / \sum_{\pi} 1$$

- k'th cumulant: *k* groups of *n* → *0* replicas with the same h_a
- integral representation of the magnetization cumulants

$$\overline{m_{h_1}..m_{h_p}}^{J,c} = -p(-T)^p \int d^p y \delta(\sum_i y_i)\partial_{\tilde{h}_1}..\partial_{\tilde{h}_p}\phi(0,y)$$

Generalized Parisi equations:

$$\phi(x=1;\{y_i\}) = \log\left[\sum_{i=1}^{p}\exp(y_i)\right],$$

$$\frac{\partial\phi}{\partial x} = -\frac{1}{2}\sum_{i,j=1}^{p}\frac{dq_{ij}}{dx}\left(\frac{\partial^2\phi}{\partial y_i\partial y_j} + x\frac{\partial\phi}{\partial y_i}\frac{\partial\phi}{\partial y_j}\right),$$

$$q_{ij}(x) \;=\; \beta^2 Q(x)\delta\tilde{h}^i\delta\tilde{h}^j.$$

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- k'th cumulant: k groups of $n \to 0$ replicas with the same h_a
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- limit $T \to 0$: expand in nonlinear diffusion term

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- integral representation of the magnetization cumulants
- limit $T \to 0$: expand in nonlinear diffusion term
- extract non-analytic contribution from shocks

Final result: (completely for general for any RSB pattern)

$$\rho(\Delta m)\, d(\Delta m) d\tilde{h} = \Delta m \int_{q(0)}^{q(u_c)} dq \frac{d\hat{u}(q)}{dq} \frac{\exp[-\frac{(\Delta m)^2}{4(q(u_c)-q)}]}{\sqrt{4\pi(q(u_c)-q)}}\, d(\Delta m) d\tilde{h}.$$

In 1-step RSB cases:

$$= \Delta m\, \hat{u}_c \frac{\exp[-\frac{(\Delta m)^2}{4\Delta q}]}{\sqrt{4\pi\Delta q}}\, d(\Delta m) d\tilde{h}$$

- Full shock distribution for p-spins (Rizzo-Yoshino)
- Full agreement with results in Burgers turbulence (Bouchaud-Mézard-Parisi)

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SK model: (full RSB)

$$= \Delta m \int_{C\overline{h}^{2/3}}^{1} dq \frac{\sqrt{c^*}}{2(1-q)^{3/2}} \frac{\exp[-\frac{(\Delta m)^2}{4(1-q)}]}{\sqrt{4\pi(1-q)}}\, d(\Delta m) d\tilde{h}$$

$$= \frac{\sqrt{c^*/\pi}}{\Delta m} \exp[-(\Delta m)^2/4(1-C\overline{h}^{2/3})]$$

Calculation

Sum over replica permutations π in $S(n)$ [a real challenge!]

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- limit $T \to 0$: expand in nonlinear diffusion term
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Final result:

- picture of mesoscopic avalanches ~ $N^{1/2}$ fully confirmed
- obtained critical probability distribution of avalanche sizes

$$\rho_{\mathrm{SK}}(\Delta m; \overline{h})\, d(\Delta m) d\tilde{h} \;=\; \frac{\sqrt{c^*/\pi}}{\Delta m} \exp[-(\Delta m)^2/4(1 - C\overline{h}^{2/3})]$$

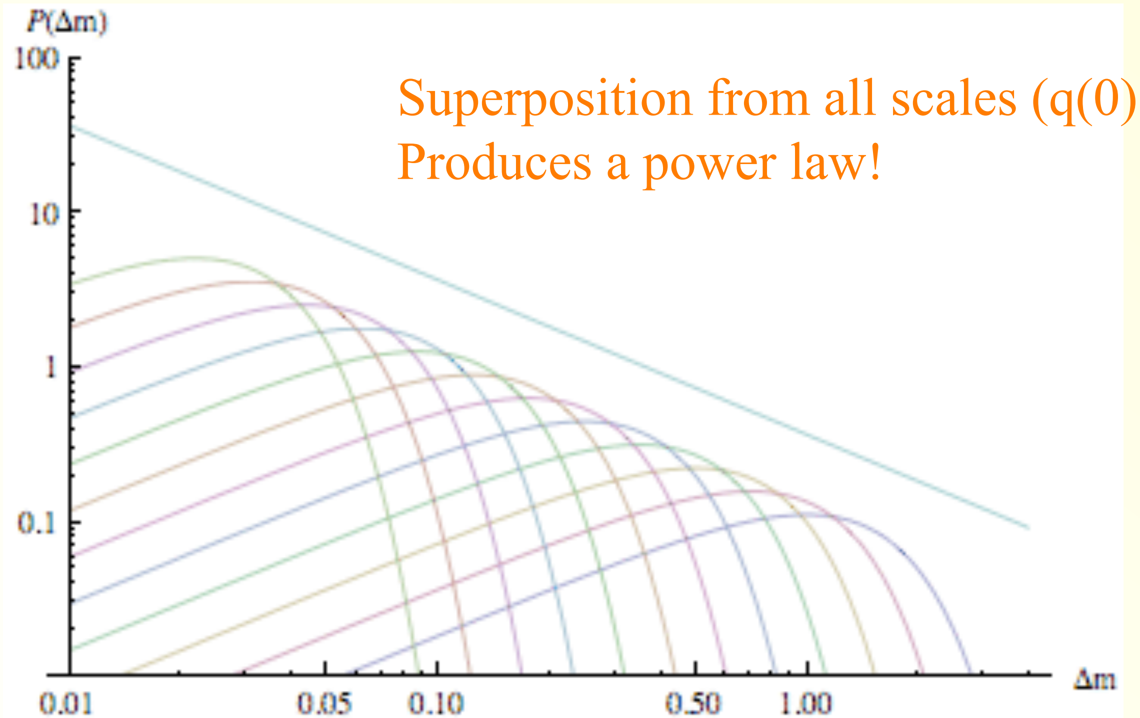
Avalanche exponent

$\tau = 1$

Comparison with numerics

Analytical result (shocks in equilibrium)

$$\rho_{\mathrm{SK}}(\Delta m; \overline{h})\, d(\Delta m) d\tilde{h} \;=\; \frac{\sqrt{c^*/\pi}}{\Delta m} \exp[-(\Delta m)^2/4(1 - C\overline{h}^{2/3})]$$



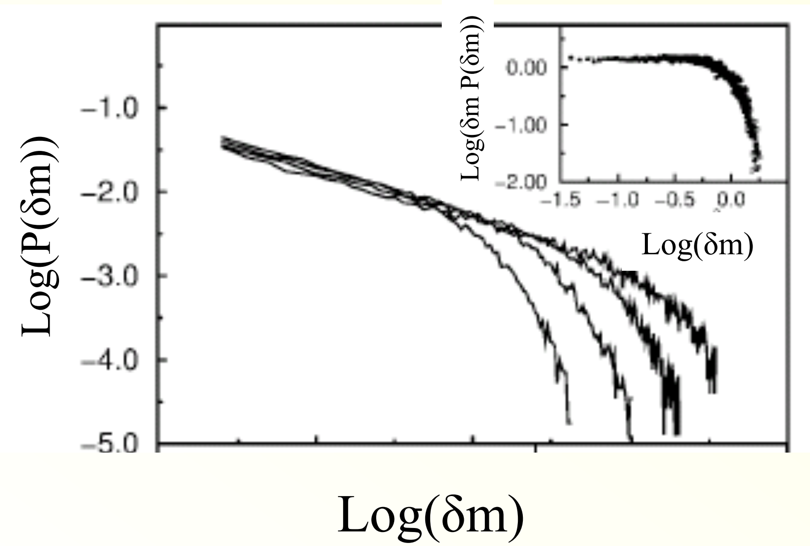
Superposition from all scales (q(0) < q <1)
Produces a power law!

Comparison with numerics

Analytical result (shocks in equilibrium)

$$\boxed{\rho_{\mathrm{SK}}(\Delta m;\overline{h})\, d(\Delta m) d\tilde{h} \;=\; \frac{\sqrt{c^*/\pi}}{\Delta m} \exp[-(\Delta m)^2/4(1 - C\overline{h}^{2/3})]}$$

Avalanches in the hysteresis loop (slowly driven, out-of-equilibrium)



Many qualitative features agree between analytics (equilibrium) and numerics (out-of equilibrium)

Pazmandi, Zarand, Zimanyi (1999)

But this may be a treacherous agreement!

A posteriori – a simple derivation !?

$$\boxed{\rho(\Delta m)\, d(\Delta m) d\tilde{h} = \Delta m \int_{q(0)}^{q(u_c)} dq \frac{d\hat{u}(q)}{dq} \frac{\exp[-\frac{(\Delta m)^2}{4(q(u_c)-q)}]}{\sqrt{4\pi(q(u_c)-q)}}\, d(\Delta m) d\tilde{h}.}$$

A heuristic derivation/interpretation – a posteriori (cf. *Franz-Parisi 1999*)

Density of states at distance q $\quad \rho(E=0,q) = \frac{1}{T} P(q) = \frac{1}{T}\frac{du}{dq} \equiv \frac{d\hat{u}}{dq}$

Relation between jump in q and M $\quad N_{\text{spinflip}} = N(1-q)/2$

$$\overline{\Delta m^2} = \overline{\Delta M^2}/N = 4N_{\text{spinflip}}/N = 2(1-q)$$

Shock location: $\quad \tilde{h} = \sqrt{N} h = E/\Delta M$

$$\rho(\Delta m)\, d(\Delta m) d\tilde{h} = \int_{q_m}^{q_c} dq \int_0^\infty dE\, \rho(E,q) \mathcal{N}\left(\frac{\Delta m}{\sqrt{2(1-q)}}\right) \delta(\tilde{h} - E/\Delta m)\, d(\Delta m) d\tilde{h}$$

Note: This interpretation suggests that full chaos in h [Δq=O(1)], while ΔM~1/√N
Equilibrium: may describe very different microscopic process from dynamics!

Conclusion

Spin glass criticality (in the SK model) is prominently reflected in scale free **equilibrium** response to a slow magnetic field change.

Connection between various manifestations of criticality:

Soft gap – avalanches, shocks– spin-spin correlations

Avalanches in Barkhausen noise:
An interesting experimental diagnostic for spin glass criticality?!

Outlook

• Calculation for dynamics/driven hysteresis loop?

• Finite d spin glasses:
- droplets vs. RSB?
- FRG for spin glasses?

• Electron glasses: (1/r interactions + disorder)
Analogies with SK model:

- Critical soft gap (Efros-Shklovskii)
- Infinite gate-induced avalanches (~ L) at T = 0
- Mean field: full RSB, critical correlations predicted

• Avalanches in other complex systems
- Optimization problems with full RSB (vertex cover, K-SAT coloring)
- economy, etc)