

Velocity-force characteristics of a driven interface in a disordered medium

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Using a dynamic functional renormalization group treatment of driven elastic interfaces in a disordered medium, we investigate several aspects of the creep-type motion induced by external forces below the depinning threshold f_c . (i) We show that in the experimentally important regime of forces slightly below f_c the velocity obeys an Arrhenius-type law $v \sim \exp[-U(f)/T]$ with an effective energy barrier $U(f) \propto (f_c - f)$ vanishing linearly when f approaches the threshold f_c . (ii) Thermal fluctuations soften the pinning landscape at high temperatures. Determining the corresponding velocity-force characteristics at low driving forces for internal dimensions $d=1,2$ (strings and interfaces) we find a particular non-Arrhenius-type creep $v \sim \exp[-(f_c(T)/f)^\mu]$ involving the reduced threshold force $f_c(T)$ alone. For $d=3$ we obtain a similar v - f characteristic, which is, however, nonuniversal and depends explicitly on the microscopic cutoff.

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I. INTRODUCTION

The influence of disorder on the static and dynamic properties of elastic systems has been intensely studied in recent years.^{1,2} Various physical systems including flux lattices in superconductors,³ domain walls in magnets,^{4,5} and charge density waves in solids⁶ significantly change their properties upon introduction of even a small amount of disorder. Subject to a disorder landscape, these systems transform to a glassy state characterized by a nontrivial scaling of the displacement correlation functions⁷ and a vanishing linear response to external driving forces,⁸⁻¹¹ e.g., the current induced Lorentz force acting on vortices or the magnetic field driving the domain walls in magnets. The determination of the velocity-force characteristics of a driven elastic manifold subject to a disorder landscape is a challenging problem: while the behavior at small distances and large drives is amenable to perturbation theory, the most interesting long distance/weak drive regime can be attacked only via nonperturbative methods. In this paper, we consider some aspects of the creep-type dissipative motion of a driven elastic interface with d internal dimensions, moving along one transverse direction in a disorder landscape [($d+1$)-dimensional random manifold problem].

Depending on the value of the temperature T and the external force f several regimes can be distinguished (see Fig. 1). At $T=0$, the velocity v is zero as long as f does not exceed the critical force f_c , whereas for $f > f_c$ the system starts moving, $v(f) \neq 0$. In particular, one finds $v(f) \propto (f - f_c)^\beta$ near the threshold (the depinning transition), with a nontrivial critical exponent β .¹²⁻¹⁵ For large drives $f \gg f_c$ the disorder becomes irrelevant and the velocity-force characteristic turns linear, $v \sim f/\eta$, with η the friction coefficient characteristic of the dissipative dynamics.

At finite temperatures $T > 0$, thermal fluctuations induce a creep-type motion resulting in an exponentially small but finite velocity even below threshold $f < f_c$ (see Fig. 1). At small drives $f \rightarrow 0$ an Arrhenius-type law $v(f) \propto \exp[-U(f)/T]$ holds, with a diverging activation barrier $U(f \rightarrow 0) \rightarrow \infty$ (glassy response).

Close to threshold $f \sim f_c$ one may distinguish two interesting regimes. (i) Fixing the force f at its critical value $f = f_c$, thermal fluctuations smooth the depinning transition and the velocity $v_{f_c}(T) \propto T^{1/\delta}$ is expected to scale as a power of temperature.^{16,17} (ii) Fixing the temperature T and increasing the force toward threshold $f \rightarrow f_c$, a creep-type response is expected with a vanishing activation barrier $U(f \rightarrow f_c) \propto (f_c - f)^\alpha \rightarrow 0$; here we are interested in the second situation.

The scaling theory of creep⁸⁻¹¹ predicts that $U(f \rightarrow 0) \propto U_c (f_c/f)^\mu$, with a characteristic energy scale U_c set by the disorder landscape. On the other hand, when the force f approaches f_c from below one expects that the barrier behaves like

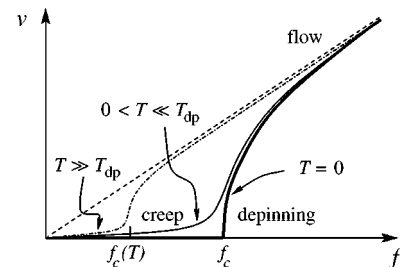


FIG. 1. Velocity-force characteristic of a driven interface at various temperatures (T_{dp} denotes the depinning temperature above which thermal fluctuations smear the disorder landscape). The thick solid line is the zero-temperature result with a threshold force f_c below which the velocity vanishes. Beyond f_c the velocity first rises following the scaling law $v \propto (f - f_c)^\beta$ and then crosses over to the linear dissipative regime with $v \propto f$. The thin line shows the behavior at finite but low temperatures $T < T_{dp}$ with a creep regime at low forces $f \ll f_c$, $v \propto \exp[-(U_c/T)(f_c/f)^\mu]$. Close to threshold we find that the creep barriers scale linearly in f , $v \propto \exp[-(U_c/T)(1 - f/f_c)]$. At high temperatures ($T > T_{dp}$, dash-dotted line) thermal fluctuations become particularly important in dimensions $d=1,2$; the threshold force $f_c(T)$ is strongly reduced by thermal fluctuations and we find a non-Arrhenius glassy response at small drive with $v \propto \exp[-(f_c(T)/f)^\mu]$ determined by the renormalized critical force alone.

$$U(f) \propto (f_c - f)^\alpha, \quad (1)$$

with α an exponent depending on the dimensionality of the space and the elastic manifold. The parameter α determines the relaxation of magnetization in superconductors at currents close to the critical one. In his original description of magnetic relaxation, Anderson¹⁸ assumed that $\alpha=1$ and explained the famous logarithmic decay of the magnetic field trapped inside a superconductor.¹⁹ Note that the regime $f \approx f_c$ is an experimentally important one: the observation of the system response at small driving forces $f \ll f_c$ involves long relaxation times $\propto \exp[(U_c/T)(f_c/f)^\mu]$ and hence this regime is more difficult to access experimentally. More quantitatively, the maximal creep barrier U that can be observed after a waiting time t is given by $U(t) \approx T \ln(1+t/t_0)$ with t_0 a characteristic time scale involving details of the critical state.^{20,3} For the vortex creep problem this time scale typically is of order 10^{-6} s and thus the experimentally attainable value of U/T is limited to factors $\sim 30-40$.

From a theoretical point of view the calculation of the barrier exponent α in Eq. (1) near criticality still remains a problem. In fact, one may expect that critical fluctuations of the manifold near the threshold will affect the creep motion. In this paper we study the behavior of $U(f)$ near the threshold using dynamical renormalization group theory¹⁵ and show that if the pinning of the manifold is due to a short-range correlated random potential (e.g., due to pointlike impurities) the effective barrier behaves as

$$U(f) \approx U_c(1 - f/f_c), \quad f \rightarrow f_c, \quad (2)$$

with U_c a characteristic energy scale set by the disorder landscape. This result is independent of the dimensionality of the manifold and confirms the original assumption of Anderson.¹⁸

In addition, we investigate creep at high temperatures, again using the dynamical functional renormalization group technique. In this case, the dimensionality of the manifold is particularly important: It is well known that the mean thermal displacement $\langle u^2 \rangle_{\text{th}}$ of a manifold with internal dimension $d \geq 3$ is bounded, the maximum displacement depending on the microscopic short-scale cutoff of the elastic system. Strings and surfaces ($d=1,2$), however, exhibit thermal fluctuations $\langle [\mathbf{u}(\mathbf{z}) - \mathbf{u}(\mathbf{0})]^2 \rangle$ that grow unboundedly with separation z . At high temperatures, the manifold probes an effective disorder landscape averaged over thermal displacements that are bounded only through the disorder-induced pinning at large scales, resulting in a strongly reduced disorder strength. In particular, the critical force $f_c(T)$ is found to decrease as a power law with increasing temperature, $f_c(T) \sim f_c(T_{\text{dp}}/T)^\kappa$ with $\kappa=7$ (2) in $d=1$ (2) dimensions (see Fig. 1). The characteristic temperature T_{dp} determining the cross-over from the low to the high temperature regime is given by the bare disorder energy scale U_c , $T_{\text{dp}} = U_c$. For the effective barrier in $d=1,2$ we find

$$U(f) \approx T \{ [f_c(T)/f]^\mu - 1 \}, \quad (3)$$

depending only on the renormalized pinning force $f_c(T)$, confirming the results obtained previously via scaling esti-

mates (see Ref. 3). As for the low T creep, the exponent $\mu = (d+2\zeta-2)/(2-\zeta)$ again is determined through the static roughness exponent ζ . The large thermal fluctuations modify the characteristic energy scale U_c of the problem to $U_c \rightarrow T$, leading to a peculiar non-Arrhenius form for the v - f characteristic in the high T creep regime. For $d=3$ the velocity-force characteristic also takes a non-Arrhenius form but with an exponent additionally modified through the temperature dependence of the creep barrier $U_c(T) \gg T$. Note, however, that Eqs. (2) and (3) make sense only if the temperature T is still small enough to produce an exponent $U(f)/T \gg 1$.

In Sec. II below we will first analyze creep near the critical force and derive Eq. (2) while Sec. III is devoted to the study of creep at high temperatures with a derivation of the result Eq. (3) for the creep barrier.

II. CREEP NEAR THRESHOLD

In the following we will concentrate on the case of $(d+1)$ -dimensional elastic media, with d internal dimensions and one single transverse direction. Typical realizations are strings confined to a plane [a $(1+1)$ -dimensional manifold] or two-dimensional membranes embedded in three-dimensional space [a $(2+1)$ -dimensional manifold]. These models describe domain walls in thin film and bulk random magnets, for example. The motion of the elastic manifold is governed by the equation

$$\eta \partial_t u = c \nabla_{\mathbf{z}}^2 u + f_{\text{pin}}(u, \mathbf{z}) + \zeta(\mathbf{z}, t) + f, \quad (4)$$

where the friction and external driving forces are given by $\eta \partial_t u$ and f , respectively, and the additional forces acting on the manifold are those due to elasticity $c \nabla_{\mathbf{z}}^2 u$, pinning $f_{\text{pin}}(u, \mathbf{z})$, and thermal fluctuations $\zeta(\mathbf{z}, t)$; η and c denote the viscosity and the elasticity per unit volume. We assume that the pinning force is a Gaussian random variable with zero mean and a correlator $\langle f(u, \mathbf{z}) f(u', \mathbf{z}') \rangle = \Delta(u - u') \delta^d(\mathbf{z} - \mathbf{z}')$ of width ξ , the typical length scale of the disorder landscape. The statistics of the stochastic force $\zeta(\mathbf{z}, t)$ is Gaussian as well and the correlator is related to the viscosity η and the temperature T via $\langle \zeta(\mathbf{z}, t) \zeta(\mathbf{z}', t') \rangle = 2\eta T \delta^d(\mathbf{z} - \mathbf{z}') \delta(t - t')$.

The calculation of the average velocity $v = \langle \partial_t u \rangle$ as a function of f and T is a difficult problem in the creep regime $f < f_c$ since most of the time the manifold is pinned by the random potential and only rarely will a strong thermal fluctuation drive it into a neighboring metastable state. Obviously, this type of motion cannot be described perturbatively. However, it can be rigorously proved that the velocity-force characteristic is unique.²¹

A powerful method to study random elastic manifolds is the functional renormalization group²²⁻²⁴ (FRG) with various extensions dealing with finite temperature²⁵ and velocity.¹²⁻¹⁵ For dimensionalities d of the manifold larger than 4, the effect of disorder can be taken into account perturbatively, whereas in fewer than four dimensions an ϵ expansion allows one to study the properties of the system at small $\epsilon = 4 - d$. The FRG has provided numerous results in the investigation of static and dynamic properties of elastic

manifolds. The static wandering exponent²² ζ as well as the dynamic exponent¹⁴ z have been determined for different types of disorder. Furthermore, the depinning transition at $T=0$ has been analyzed and the critical exponent β in the depinning law $v \propto (f-f_c)^\beta$ has been calculated.¹²⁻¹⁵

The dynamical extension of the FRG by Chauve *et al.*¹⁵ allows investigation of the creep regime and confirms the creep law $U(f) \propto U_c (f_c/f)^\mu$ derived earlier via scaling arguments. In addition, it turns out that the method allows for the determination of different characteristics of the manifold's dynamics without additional physical assumptions (cf. Refs. 12–14). The dynamical FRG starts from a Martin-Siggia-Rose action²⁶ obtained from the equation of motion Eq. (4) and proceeds with the elimination of large momentum fluctuations. Thereby the parameters entering Eq. (4) are renormalized and characterize a system for which disorder is less and less relevant. Finally, the flow is cut off at a scale where the effect of disorder can be taken into account perturbatively.

Our starting point is the system of equations derived in Ref. 15 describing the renormalization of the parameters entering Eq. (4), to lowest nontrivial order,

$$\begin{aligned} \partial_l \tilde{\Delta}_l(u) = & (\epsilon - 2\zeta) \tilde{\Delta}_l(u) + \zeta u \tilde{\Delta}'_l(u) + \tilde{T}_l \tilde{\Delta}''_l(u) \\ & + \int_{s>0, s'>0} e^{-s-s'} \{ \tilde{\Delta}''_l(u) [\tilde{\Delta}_l(\lambda_l(s'-s)) \\ & - \tilde{\Delta}_l(u+\lambda_l(s'-s))] - \tilde{\Delta}'_l(u-\lambda_l s') \tilde{\Delta}'_l(u+\lambda_l s) \\ & + \tilde{\Delta}'_l(\lambda_l(s'+s)) [\tilde{\Delta}'_l(u-\lambda_l s') - \tilde{\Delta}'_l(u+\lambda_l s)] \}, \end{aligned} \quad (5)$$

$$\partial_l \ln \lambda_l = 2 - \zeta - \int_{s>0} e^{-s} s \tilde{\Delta}''_l(\lambda_l s), \quad (6)$$

$$\partial_l \tilde{f}_l = (2 - \zeta) \tilde{f}_l + c \Lambda^2 \int_{s>0} e^{-s} \tilde{\Delta}'_l(\lambda_l s), \quad (7)$$

$$\partial_l \ln \tilde{T}_l = -\theta + \int_{s>0} e^{-s} \lambda_l s \tilde{\Delta}'''_l(\lambda_l s), \quad (8)$$

with $\tilde{\Delta}_l(u) = (A_d \Lambda^{d-4}/c^2) \Delta_l(u)$, $\lambda_l = (\eta v)_l / c \Lambda^2$, $\tilde{T}_l = A_d \Lambda^{d-2} T_l / c$, and $\tilde{f}_l = f_l - (\eta v)_l$. The exponents ζ and $\theta = d - 2 + 2\zeta$ describe the scaling of the roughness and the energy, respectively. A_d is the surface of the unit sphere in d dimensions and Λ denotes the short-scale cutoff of the theory. Note the important effect of the dynamics in rendering the equations nonlocal on the scale λ_l proportional to the center of mass velocity v of the manifold [note that the latter involves the dynamical exponent z in its scaling relation $\partial_l v_l = (\zeta - z) v_l$; on the other hand, the dynamical exponent disappears from the combination ηv]. The main goal of this section is to investigate these equations in the limit when the external force acting on the manifold is slightly below the threshold force, $f < f_c$.

We first analyze the system of equations (5)–(8) for the case of an infinitesimal velocity $v=0+$ and concentrate on low temperatures. Equations (5) and (8) then reduce to the static FRG equations²²

$$\begin{aligned} \partial_l \tilde{\Delta}_l(u) = & (\epsilon - 2\zeta) \tilde{\Delta}_l(u) + \zeta u \tilde{\Delta}'_l(u) + \tilde{T}_l \tilde{\Delta}''_l(u) + \tilde{\Delta}''_l(u) \\ & \times [\tilde{\Delta}_l(0) - \tilde{\Delta}_l(u)] - \tilde{\Delta}'_l(u)^2, \end{aligned} \quad (9)$$

$$\partial_l \tilde{T}_l = -\theta \tilde{T}_l. \quad (10)$$

The flow takes the correlator $\tilde{\Delta}_l$ through a special point $l_c \approx (1/\epsilon) \ln \{ \epsilon / [3 \tilde{\Delta}''_0(0)] \}$, $L_c \approx \Lambda^{-1} e^{l_c} \approx \{ \epsilon c^2 \xi^2 / [A_d \Delta_0(0)] \}^{1/(4-d)}$, where it becomes singular at the origin in the limit $T \rightarrow 0$; this is easily seen from the equation

$$\partial_l \tilde{\Delta}''_l(0) \approx \epsilon \tilde{\Delta}''_l(0) - 3 \tilde{\Delta}'_l(0)^2, \quad (11)$$

satisfied by the second derivative of the correlator at low temperatures; exploiting the fact that $\tilde{\Delta}''_0(0) < 0$ we have $|\tilde{\Delta}''_l(0)| \approx |\tilde{\Delta}''_0(0)| e^{\epsilon l} / \{ 1 - [3 \tilde{\Delta}'_0(0) / \epsilon] (e^{\epsilon l} - 1) \}$. The curvature $\tilde{\Delta}''_l(0)$ diverging at l_c marks the occurrence of a nonanalyticity at the origin which is reflected in the appearance of a cusp in $\tilde{\Delta}_{l>l_c}$ at $u=0$. Although the initial correlator $\tilde{\Delta}_0(u)$ is usually an analytic and even function of the coordinate u with vanishing odd derivatives at the origin, the function $\tilde{\Delta}_{l>l_c}$ has a cusp with a nonzero slope $\tilde{\Delta}'_l(0+) < 0$ at the origin when $T=0$. Asymptotically, $\tilde{\Delta}_l(u)$ approaches a zero-temperature “cuspy” fixed point $\tilde{\Delta}^*(u)$ describing the disordered phase with a nontrivial roughness exponent ζ . If the rough shape of the fixed point function $\tilde{\Delta}^*(u)$ is assumed at the Larkin scale l_c we can easily find its characteristics. The width

$$\xi^* \approx \xi \exp(-\zeta l_c) \quad (12)$$

of $\tilde{\Delta}^*(u)$ follows from integrating the second term in Eq. (9). Comparing terms in Eq. (9) at the origin $u=0$ we find $\tilde{\Delta}^{*2}(0+) \sim \epsilon \tilde{\Delta}^*(0)$, and combining this with the relation $|\tilde{\Delta}^{*'}(0+)| \xi^* \sim \tilde{\Delta}^*(0)$ we find the estimates $\tilde{\Delta}^*(0) \sim \epsilon \xi^2 \exp(-2\zeta l_c)$ and

$$|\tilde{\Delta}^{*'}(0+)| \approx \epsilon \xi e^{-\zeta l_c}. \quad (13)$$

Let us then analyze the force flow (7) in the light of these results. The scale l_c divides the flow into two distinct regimes, the Larkin regime at small scales $l < l_c$ and the random manifold regime ($l > l_c$). For $l < l_c$ we have $\tilde{\Delta}'_l(0) = \tilde{\Delta}'_l(0+) = 0$ and the force \tilde{f}_l obeys the equation $\partial_l \tilde{f}_l = (2 - \zeta) \tilde{f}_l$, i.e., $\tilde{f}_l = e^{(2-\zeta)l} c \tilde{f}$ grows exponentially. At the point $l = l_c$ the integral term on the right-hand side of Eq. (7) jumps from zero to a finite value, since the slope of the correlator at the origin does not vanish any longer. If this contribution overcompensates the scaling term, i.e., $c \Lambda^2 |\tilde{\Delta}'_l(0+)| > (2 - \zeta) e^{(2-\zeta)l_c} c \tilde{f}$, the force will start renormalizing to zero while in the opposite case it will continue to

increase. This can be interpreted in the following way. If the initial value f of the external force is smaller than a critical value f_c the force will eventually renormalize downward and cannot move the manifold. For $f > f_c$ the manifold starts moving and one should take into account Eq. (6) since the problem is not static any more. We therefore come to the conclusion that for a system with a dynamics described by Eq. (4) there exists a finite threshold force f_c at $T=0$. Using the above condition one easily finds $f_c \approx [c\Lambda^2 |\tilde{\Delta}^{*'}(0+)| / (2-\zeta)] e^{-(2-\zeta)l_c} \approx [\epsilon / (2-\zeta)] c \xi / L_c^2$ with $L_c = \Lambda^{-1} e^{l_c}$. Note that the expression for f_c coincides with the result obtained from simple scaling estimates.³

At finite temperatures $T > 0$ it is no longer possible to define the critical force density f_c as the threshold below which there is no center of mass motion of the manifold, as thermally activated jumps lead to an average velocity $v > 0$ at any finite force f . Of course, at low temperatures the velocity is exceedingly small, given that it obeys an Arrhenius-type law. Therefore the time needed to observe this velocity might well exceed the time scale of the experiment, i.e., from an experimental point of view the critical force density still exists with the threshold f_c separating creep-type motion from viscous flow. On the mathematical level the nonexistence of the critical force density can be explained as follows. At $T > 0$, the slope $\tilde{\Delta}'(0+)$ remains zero beyond l_c , $\tilde{\Delta}'_{>l_c}(0+) = 0$, and the renormalized force density will continue to grow beyond the length scale l_c even if the initial force density f is smaller than f_c . The flow of λ_l then has to be included in our consideration and the renormalization of \tilde{f}_l will be found to stop at a larger scale.

Let us then analyze the flow of the correlator $\tilde{\Delta}_l$, Eq. (5), at finite temperatures in more detail. The nonlocalities introduced by the finite value of λ_l in Eq. (5) can be neglected as long as λ_l is smaller than the length scale introduced by the finite temperature, and we can therefore continue to use the quasistatic equation (9). Below, we will make use of the flow equations only in the regime where this condition holds. We also neglect the disorder contribution to the temperature renormalization in Eq. (8) since it does not influence the main result to the accuracy desired here.

At finite but small $T > 0$ the correlator flow below l_c does not differ much from the zero-temperature case. However, at l_c no cusp occurs at the origin—rather, the correlator remains rounded on a characteristic scale u_l^T . Assuming that outside the thermally dominated region close to the origin the correlator has approached its zero-temperature fixed point shape $\tilde{\Delta}^*(u)$, we may estimate u_l^T from Eq. (9) by equating the third and fourth terms on the right-hand side (RHS),

$$u_l^T \approx \frac{\tilde{T}_l}{|\tilde{\Delta}^{*'}(0+)|} \approx \frac{A_d \Lambda^{d-2} T e^{-\theta l}}{\epsilon c \xi e^{-\zeta l_c}} \approx \xi^* \frac{T}{U_c} e^{-\theta(l-l_c)}, \quad (14)$$

with U_c the typical elastic energy on the Larkin scale L_c , $U_c \approx (\epsilon/A_d) c \xi^2 L_c^{d-2}$. Obviously, for low temperatures

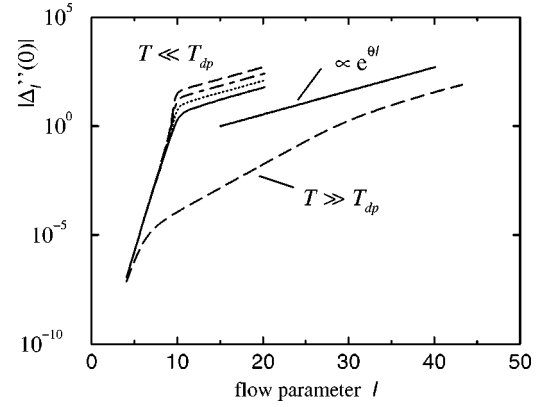


FIG. 2. Numerical integration of the curvature $|\tilde{\Delta}_l''(0)|$ in the force correlator for $d=1$, [see Eq. (9)]. Four initial temperatures $T_0 < T_{dp}$ below thermal depinning at $T_{dp} = U_c = c \xi^2 / L_c$ with values $T_0 = T_{dp} / \alpha$, $\alpha = 40, 20, 10, 5$ have been chosen, while the high temperature curve starts with $T_0 = 20 T_{dp}$. The initial growth through the Larkin regime involves the exponents $\epsilon = 3$ and $\epsilon - 5 \zeta_{th} = 1/2$ at low and high temperatures, respectively. Beyond the Larkin regime the flow of $\tilde{\Delta}_l''(0) \propto -1/\tilde{T}_l$ is characterized by the temperature exponent $\theta = 2\zeta - 1 = 0.2495$ [cf. Eq. (15) with a wandering exponent ζ given by the one-loop fixed point value (Ref. 22) $\zeta \approx 0.2083 \epsilon$]. The crossover at the Larkin scale is sharp and independent of temperature below T_{dp} ; the uniform vertical spacing of the asymptotic curves reflects the temperature independence of the energy scale U_c . For high temperatures above T_{dp} the crossover is shifted to $l_c(T) \gg l_c$ and the pinning energy $U_c(T)$ depends on temperature, $U_c \approx T$ [note that $|\tilde{\Delta}_{l_c(T)}''(0)| \sim 1$].

$T \ll U_c$, the thermal rounding of the cusp involves a scale u_l^T much smaller than the width ξ^* of the correlator.

The finite temperature curvature of the correlator follows from a comparison of the term $\tilde{T}_l \tilde{\Delta}_l''$ with either the fourth or the last term in Eq. (9),

$$\tilde{\Delta}_{l \gg l_c}''(0) \approx - \frac{\tilde{\Delta}^{*'}(0+)^2}{\tilde{T}_l} \approx - \frac{U_c}{T} e^{\theta(l-l_c)}, \quad (15)$$

the finite temperature fluctuations thus regularize the “cuspy” divergence occurring at $T=0$. The above estimates are correct up to numerical prefactors only; a more rigorous derivation can be found in Ref. 15.

For the following it is crucial to establish that the behavior described by Eq. (15) is valid already soon after l_c , as the flow equation (6) for λ_l is quite sensitive to the curvature of the correlator at $u=0$. Indeed, as we have checked numerically (see Fig. 2), after a rapid growth within the Larkin regime, the curvature $\tilde{\Delta}_l''(0)$ saturates at a value $\sim U_c/T$ with a slow further growth due to temperature rescaling, $\tilde{\Delta}_l''(0) \propto \tilde{T}_l^{-1} \propto e^{\theta l}$. As long as we are interested in the threshold behavior of the barrier close to f_c it is sufficient to establish a rapid crossover of the curvature from a steep growth below l_c to a gentle increase above l_c (which we will neglect for small $l-l_c > 0$). Also note that the crossover occurs essentially at the same value of l_c independent of the tempera-

ture T , while the uniform vertical spacing of the asymptotic curves reflects the temperature independence of the energy scale U_c .

Let us now analyze the system of equations (5)–(8) for $T > 0$, $v > 0$ close to criticality $f < f_c$. We assume a fixed deviation $f_c - f$ from the threshold and a small temperature $T \ll U_c$. The velocity then is (exponentially) small as well and represents the smallest parameter in the problem. In the opposite case (T small and fixed while $f_c - f \rightarrow 0+$) the velocity $v \propto T^{1/\delta}$ has been argued to scale as a power of temperature,^{17,16} the numerical value of the exponent δ still being a matter of controversy.^{17,27,28}

Again, we consider separately the two regimes below and above l_c . Throughout the Larkin regime the parameter λ_l remains small and can be set to zero in Eqs. (5) and (7). Furthermore, the temperature dependent term in Eq. (9) may be neglected initially—it will become relevant when $\tilde{\Delta}_l''(0)$ becomes of the order of U_c/T . The flow of λ_l through the Larkin regime then follows from expanding the flow equations for $\tilde{\Delta}_l''(0)$ and λ_l , Eqs. (5) and (6), to second order in the small parameter λ_l and setting $\tilde{T}_l = 0$,

$$\partial_l [\ln |\tilde{\Delta}_l''(0)| - 3 \ln \lambda_l] = \epsilon - 3(2 - \zeta) + O(\lambda_l^4). \quad (16)$$

Integrating from 0 to l_c and using $\tilde{\Delta}_{l_c}''(0) \approx U_c/T$ we obtain (up to numerical factors)

$$\lambda_{l_c} \approx \frac{\eta v L_c^2}{c} e^{-\zeta l_c} \left(\frac{U_c}{T} \right)^{1/3}. \quad (17)$$

Note that λ_{l_c} grows as a *power* of U_c/T as the temperature approaches zero, whereas in the following depinning regime λ_l will be *exponentially* sensitive to T .

Going beyond the Larkin regime $l > l_c$ the function $\tilde{\Delta}_l(u)$ quickly approaches its fixed point form except for a small thermally smoothed region of size u_l^T around the origin with the second derivative given by Eq. (15). As long as $\lambda_l < u_l^T$ one can keep $\lambda_l = 0$ in the integral on the RHS of Eq. (6) and a simple integration from l_c to l provides the result

$$\begin{aligned} \lambda_l &\approx \lambda_{l_c} \exp \left[(2 - \zeta)(l - l_c) + \frac{U_c}{\theta T} (e^{\theta(l - l_c)} - 1) \right] \\ &\sim \lambda_{l_c} \exp \left[\frac{U_c}{T} (l - l_c) \right], \end{aligned} \quad (18)$$

where in the last step we have assumed that $T/U_c \ll l - l_c \ll 1$.

Turning next to the force equation (7) we note that at finite temperature the disorder contribution adds in only at a larger scale $l_d > l_c$ where λ_l becomes of the order of the thermal rounding u_l^T , in contrast to the zero-temperature case where disorder jumps in at l_c . The condition

$$\lambda_{l_d} \approx u_{l_d}^T \approx \tilde{T}_{l_d} / |\tilde{\Delta}_{l_d}^{*'}(0+)| \quad (19)$$

then determines a relation connecting the crossover scale l_d with the initial velocity v ,

$$v \sim \frac{f_c}{\eta} \left(\frac{T}{U_c} \right)^{4/3} \exp \left[- \frac{U_c}{T} (l_d - l_c) \right]. \quad (20)$$

Below l_d the flow of \tilde{f}_l is determined by the scaling term alone, $\partial_l \tilde{f}_l = (2 - \zeta) \tilde{f}_l$, and a simple integration gives $\tilde{f}_l = f \exp[(2 - \zeta)l]$, where we have dropped the small correction ηv in the definition of \tilde{f} , $\tilde{f} = f - \eta v$. At l_d the disorder correction turns on rapidly and we enter the depinning regime¹⁵ at scales $l > l_d$. In this regime we can substitute the fixed point correlator $\tilde{\Delta}^*$ for $\tilde{\Delta}_l$ since now $\lambda_l \gg u_l^T$. Furthermore, since still $\lambda_l \ll \xi^*$ we can set λ_l equal to $0+$ in Eq. (7) and we obtain a disorder correction $c \Lambda^2 \tilde{\Delta}^{*'}(0+)$. As argued before when determining the threshold force f_c , we have to prevent the force f_l from running away to $\pm \infty$ and thus the disorder term has to match the scaling term; we then arrive at a second relation expressing l_d in terms of the applied driving force f ,

$$f e^{(2 - \zeta)l_d} \approx \frac{c \Lambda^2 |\tilde{\Delta}^{*'}(0+)|}{2 - \zeta} \equiv f_c e^{(2 - \zeta)l_c}. \quad (21)$$

With l_d close to l_c we can expand, $l_d - l_c \approx (2 - \zeta)^{-1} (1 - f/f_c) \ll 1$, and combining with Eq. (20) we arrive at the final result for the average velocity v ,

$$v \propto \exp \left\{ - \frac{U_c}{T} \frac{f_c - f}{f_c} \right\}, \quad (22)$$

where we have dropped an inessential numerical factor in a redefinition of U_c . Also, our analysis is not sufficiently precise to specify the prefactor.

Summarizing, we find that close to threshold with $T/U_c \ll 1 - f/f_c < 2/(1 + \mu)$ the velocity obeys an Arrhenius-type law with an energy barrier decreasing linearly on approaching f_c . On the other hand, the usual glassy behavior $U(f) \sim U_c (f_c/f)^\mu$ is valid at small forces $f/f_c < 2^{-1/\mu}$. In typical experiments the measured barriers are related to the waiting time t in the experiment, $U(f) \sim T \ln(t/t_0)$, and only a limited regime of forces with barriers $5 < U(f)/T < 30$ is available. This regime is, by making use of an extended temperature interval, still sufficient to probe both the linear and glassy regimes close to threshold and at low drives, respectively (see Fig. 3).

III. CREEP AT HIGH TEMPERATURES

In this section we consider thermal creep of $d = 1, 2$ elastic interfaces (strings and interfaces moving in one transverse direction) at high temperatures and for small driving forces. We show that the velocity-force characteristic exhibits a non-Arrhenius-type behavior $v \propto \exp\{-[f_c(T)/f]^\mu\}$. This dependence derives from a creep-type motion with a renormalized activation barrier of the order of temperature, $U_c(T) \sim T$, and involves only the renormalized critical force $f_c(T)$. For $d = 3$ the parameters $f_c(T)$ and $U_c(T) \gg T$ depend on temperature as well, leading to a non-Arrhenius-type creep that is nonuniversal, however, with a result depending explicitly on the chosen cutoff. In Sec. III A we show how to calculate the renormalized energy barrier $U_c(T)$ and the threshold force

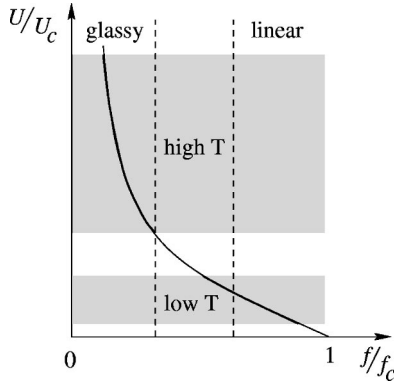


FIG. 3. Effective creep barrier U at low temperatures as a function of external force f . The thin line follows the interpolation formula $U(f) \approx U_c [(f_c/f)^\mu - 1]$, properly interpolating between the glassy and linear regimes at small drives and close to threshold, respectively. The slow relaxation governed by the logarithmic decay law $U(f) \approx -T \ln(1+t/t_0)$ limits the experimental window to the interval $5 < U/T < 30$ depending on temperature. Typically, a low (high) temperature measurement then probes the linear (glassy or nonlinear) regime.

density $f_c(T)$ using scaling arguments and then present a more rigorous analysis using (dynamical) FRG in Secs. III B and III C.

A. Scaling analysis

While the mean squared displacement $\langle u^2 \rangle_{\text{th}} \approx T\Lambda/\pi c$ is bounded in $d=3$ (with Λ^{-1} the intrinsic cutoff of the manifold), the thermal displacement (or wandering) of strings and interfaces ($d=1,2$) grows unboundedly, with either distance z or time t ,

$$\begin{aligned} \langle u^2(\mathbf{z}, t) \rangle_{\text{th}} &\equiv \langle [u(\mathbf{z}, t) - u(\mathbf{0}, 0)]^2 \rangle_{\text{th}} \\ &= \frac{2T}{c} \int \frac{d^d q}{(2\pi)^2} q^{-2} \text{Re}[1 - e^{i\mathbf{q} \cdot \mathbf{z}} e^{-(c/\eta)q^2 t}] \\ &= \begin{cases} \frac{T}{\pi c} [z^2 + (c/\eta)t]^{1/2}, & d=1 \\ \frac{T}{2\pi c} \ln \left[\frac{z^2 + (c/\eta)t + \Lambda^{-2}}{\Lambda^{-2}} \right], & d=2; \end{cases} \end{aligned} \quad (23)$$

the pinning length $L_c(T)$ set by the disorder landscape then has to provide the necessary cutoff, which in $d=3$ is given by the intrinsic cutoff Λ^{-1} [note that the q integral in Eq. (23) is dominated by small (large) q for low (high) dimensions; hence, depending on the dimensionality of the system the amplitude of thermal fluctuations is determined by short ($d=3$) or long ($d=1,2$) scales]. This implies the existence or absence of a separation of scales for thermal and disorder effects: While for $d=3$ these scales are separated, $\Lambda^{-1} \ll L_c(T)$, no such separation is effective in dimensions $d=1,2$; thermal effects smearing the disorder landscape are active on scales $L < L_c(T)$ while disorder takes over for L

$> L_c(T)$, i.e., at the same scale. The temperature-induced smoothing of the disorder potential then follows different rules in low and high dimensions, as we are going to discuss now.

In order to discuss pinning and creep we have to determine the renormalized disorder landscape. Assuming pinning to involve longer time scales than thermal fluctuations we average the pinning potential over thermal excursions,³

$$\begin{aligned} \langle \langle E_{\text{pin}}^2(L) \rangle \rangle_t &= \int_0^{t_0} \frac{dt}{t_0} \int_0^{t_0} \frac{dt'}{t_0} \int d^d z \int d^d z' \\ &\quad \times \langle V(u(\mathbf{z}, t), \mathbf{z}) V(u(\mathbf{z}', t'), \mathbf{z}') \rangle \\ &= L^d \int_0^{t_0} \frac{dt}{t_0} \int_0^{t_0} \frac{dt'}{t_0} K[|u(t) - u(t')|] \\ &\approx L^d K(0) \left(\frac{\xi^2}{\langle u^2(\Lambda^{-1}, L) \rangle_{\text{th}}} \right)^{1/2}, \end{aligned} \quad (24)$$

where the mean squared thermal fluctuations $\langle u^2(\Lambda^{-1}, L) \rangle_{\text{th}}$ are cut off by Λ^{-1} or $L = L_c(T)$ in high and low dimensions, respectively. Here, $K(u)$ denotes the potential correlator, which is related to the force correlator $\Delta(u)$ used above via $-K''(u) = \Delta(u)$. The result (24) tells us that at high temperatures thermal fluctuations replace the basic length scale ξ of the disorder landscape by the scale $\langle u^2(\Lambda^{-1}, L) \rangle_{\text{th}}^{1/2} > \xi$ (note that the energy scale of the disorder potential remains unchanged). Comparing this smoothed pinning energy with the elastic energy $c \langle u^2(\Lambda^{-1}, L) \rangle_{\text{th}} L^{d-2} \sim T$ we obtain the new pinning scale replacing the $T=0$ Larkin length $L_c = (c^2 \xi^4 / K(0))^{1/(4-d)}$,

$$L_c(T) \sim L_c \left(\frac{c^2 \langle u^2(\Lambda^{-1}, L) \rangle_{\text{th}}^{5/2}}{K(0) \xi} \right)^{1/(4-d)}. \quad (25)$$

For high dimensions we find $L_c(T) \sim L_c (T/T_{\text{dp}})^{1/2 \xi_F}$ with the Flory exponent $\xi_F = (4-d)/5$ and the depinning temperature $T_{\text{dp}} = c \xi^2 / \Lambda$ (this result follows from simple scaling $u \propto L^{\xi_F}$ and $u \propto T^{1/2}$). In dimensions $d=1, 2$ the corresponding result takes the form (here we concentrate on the case $n=1$; see Ref. 29 for a discussion of the marginal situation in $d+n=1+2$)

$$L_c(T) \sim L_c \left(\frac{T}{T_{\text{dp}}} \right)^\lambda, \quad (26)$$

with the temperature exponent $\lambda = 5(5/4)$ and the depinning temperature $T_{\text{dp}} = [cK(0)\xi^2]^{1/3} (c\xi^2)$. Comparing with the above Flory exponent we see that thermal fluctuations indeed are much more important in $d=1$, while for $d=2$ the corrections are only logarithmic [not shown in Eq. (26)].

The energy barrier and the threshold force are renormalized correspondingly; for $d=3$ we have $U_c(T) \sim U_c (T/T_{\text{dp}})^{7/2} \gg T$, with $U_c = c \xi^2 L_c$ and $T_{\text{dp}} = c \xi^2 / \Lambda$, while for $d=1,2$ the barrier ‘‘saturates’’ above T_{dp} , $U_c(T) \sim T$. The critical force density is renormalized according to $f_c(T) \sim f_c (T_{\text{dp}}/T)^{9/2}$ in $d=3$ and takes the form

$$f_c(T) \sim c \frac{\langle u^2 [L_c(T)] \rangle_{\text{th}}^{1/2}}{L_c(T)^2} \sim f_c \left(\frac{T_{\text{dp}}}{T} \right)^\kappa, \quad (27)$$

with the temperature exponent $\kappa=7(2)$ in dimensions $d=1(2)$, restricting ourselves again to the case $n=1$. Using the usual creep formula $U(f)/T=(U_c/T)(f_c/f)^\mu$ and inserting the temperature dependent values for $U_c \rightarrow T$ and $f_c \rightarrow f_c(T)$, we obtain the creep exponent $(f_c/f)^\mu (T_{\text{dp}}/T)^{\kappa\mu}$ producing a non-Arrhenius type creep in $d=1,2$ involving only the renormalized critical force density $f_c(T)$. For higher dimensions $d>2$ the renormalized barrier $U_c(T) \gg T$ remains large, a consequence of the separation of scales mentioned above. In the following we rederive these scaling results via the more rigorous analysis provided by the (dynamical) FRG scheme.

B. Functional renormalization group

In a first step we rederive the crossover scale $L_c(T)$ via the functional renormalization group. The nonlinear terms in the flow equations are still small during the initial stage of the RG flow—neglecting them, we first solve the linear equation. The length $L_c(T)$ then appears as the characteristic length where the nonlinear corrections become of the order of the linear terms. The analysis is conveniently carried out for the potential correlator $\tilde{K}_l(u)$ which follows the flow equation

$$\begin{aligned} \partial_l \tilde{K}_l(u) &= (\epsilon - 4\zeta) \tilde{K}_l(u) + \zeta u \tilde{K}'_l(u) + \tilde{T}_l \tilde{K}''_l(u) \\ &+ \frac{1}{2} \tilde{K}''_l(u)^2 - \tilde{K}''_l(u) \tilde{K}''_l(0), \end{aligned} \quad (28)$$

while the temperature flow is given by

$$\partial_l \tilde{T}_l = -\theta \tilde{T}_l. \quad (29)$$

Using the ansatz

$$\tilde{K}_l(u) = \exp\{[\epsilon - 4\zeta - (2-d)/2]l\} \tilde{P}_l(u e^{\theta l/2}), \quad (30)$$

the linear part of the flow transforms into a Fokker-Planck equation describing the probability distribution $\tilde{P}_l(u)$ for an overdamped particle moving in a parabolic potential at constant temperature \tilde{T}_0 ,²⁹

$$\partial_l \tilde{P}_l(u) = \frac{2-d}{2} \partial_u [u \tilde{P}_l(u)] + \tilde{T}_0 \partial_u^2 \tilde{P}_l(u), \quad (31)$$

for which the fundamental solution is well known. Solving the initial value problem for \tilde{P}_l and inserting in Eq. (30), we obtain

$$\begin{aligned} \tilde{K}_l(u) &= \exp[(4-d-5\zeta)l] \left[\frac{2-d}{4\pi\tilde{T}_0 e^{-\theta l} (1-e^{-(2-d)l})} \right]^{1/2} \\ &\times \int du' \tilde{K}_0(u') \exp \left[-\frac{(2-d)(u-u' e^{-\zeta l})^2}{4\tilde{T}_0 e^{-\theta l} (1-e^{-(2-d)l})} \right]. \end{aligned}$$

For high temperatures such that $\tilde{T}_0 e^{-\theta l} (1-e^{-(2-d)l}) / (2-d) > \xi^2 e^{-2\zeta l}$ the factor $\tilde{K}_0(u')$ acts as a δ function and can be extracted from the integral

$$\int \tilde{K}_0(u) du \approx \begin{cases} e^{(3-5\zeta)l} \frac{\exp[-u^2/4\tilde{T}_0 e^{-\theta l}]}{(4\pi\tilde{T}_0 e^{-\theta l})^{1/2}}, & d=1 \\ e^{(2-5\zeta)l} \frac{\exp[-u^2/4\tilde{T}_0 l e^{-\theta l}]}{(4\pi\tilde{T}_0 l e^{-\theta l})^{1/2}}, & d=2 \\ e^{(1-5\zeta)l} \frac{\exp[-u^2/4\tilde{T}_0 e^{-2\zeta l}]}{(4\pi\tilde{T}_0 e^{-2\zeta l})^{1/2}}, & d=3. \end{cases} \quad (32)$$

The above constraint on the temperature simplifies to $\tilde{T}_0 > e^{-1}\xi^2$, $\tilde{T}_0 > \xi^2/l$, and $\tilde{T}_0 > \xi^2$ for $d=1,2,3$, respectively, where ξ is the initial width of the correlator \tilde{K}_0 ; expressing \tilde{T}_0 and l through the physical quantities T and L we recover the condition $\langle u^2(\Lambda^{-1}, L) \rangle_{\text{th}} > \xi^2$.

The solutions (32) still involve the wandering exponent ζ . Although the final physical results do not depend on the particular choice, it is a matter of convenience to adopt the thermal values $\zeta_{\text{th}}=1/2$, $\theta_{\text{th}}=0$ for $d=1$, $\zeta_{\text{th}}=0$, $\theta_{\text{th}}=0$ for $d=2$, and $\zeta_{\text{th}}=0$, $\theta_{\text{th}}=1$ for $d=3$ and have the correlator flow toward a thermal fixed point. The renormalized correlator (32) then behaves very differently for large and small dimensions. In $d=3$ the transverse scale u does not change (as we chose $\zeta=0$) and the initial correlator of width ξ is replaced with a new correlator of width $\langle u^2 \rangle_{\text{th}} \sim \tilde{T}_0 = T\Lambda/c$. This contrasts with the situation in $d=1$ where u does rescale (as we chose $\zeta=1/2$) and the physical width of the correlator increases with l to follow the mean thermal displacement amplitude $\langle u^2(L) \rangle_{\text{th}} \sim TL/c$. For $d=2$ the physical width grows only logarithmically.

The flow (32) indicates that the thermal fixed point is unstable as the amplitude of the disorder grows exponentially under the FRG transformation. As the nonlinear terms in Eq. (28) become large beyond the scale $L_c(T) = \Lambda^{-1} e^{l_c(T)}$ we cannot neglect them any longer and the flow crosses over to approach the disorder dominated fixed point (the wandering exponent ζ then has to be modified accordingly, $\zeta=0.2083\epsilon$ for random bond disorder²²). The pinning length $L_c(T)$ replaces the $T=0$ Larkin length L_c and can be found from a comparison of linear and quadratic terms in the flow equation (28),

$$\tilde{K}_{l_c(T)}(0) \approx \tilde{K}''_{l_c(T)}(0)^2, \quad (33)$$

making use of the result (32). It is easily verified that the crossover condition (33) together with the explicit solution (32) of the linearized flow equations then yields the results (25) for the crossover length $L_c(T)$ obtained above with the help of scaling arguments. Note that for $d=1,2$ the linear term $\tilde{T}_l \tilde{K}''_l$ in Eq. (28) gains in importance as we integrate through the Larkin regime; hence thermal rounding persists on all scales $l < l_c(T)$. On the contrary, for $d=3$ the thermal rounding term is most important at small scales $l \sim 1$ where it

quickly replaces the width ξ of the correlator by the mean thermal displacement amplitude $\langle u^2 \rangle_{\text{th}}^{1/2}$; upon further scaling the effective temperature decreases and at the Larkin scale the thermal term is down by a factor of $e^{-l_c(T)}$. This again reflects the different role the temperature plays for different internal dimensions of the elastic manifold.

C. Dynamic functional renormalization group

Finally, let us see how the high temperature creep [as characterized by the temperature dependent critical force density $f_c(T)$ for $d=1,2$] appears directly from the dynamic FRG treatment. The analysis parallels the treatment at low temperatures; however, we have to be more careful in distinguishing between the cases $d=1,2$ and $d=3$.

Let us analyze the flow of the force correlator $\tilde{\Delta}_l(u) = -\tilde{K}_l''(u)$. Following the full flow up to $l_c(T)$ the force correlator assumes a shape with a height and width as given by Eq. (32). At $l_c(T)$ the nonlinear terms in the flow equation (28) have become important; beyond $l_c(T)$ the correlator quickly flows toward the disorder dominated fixed point function $\tilde{\Delta}^*(u)$, the linear temperature term $\tilde{T}_l \tilde{\Delta}_l''(u)$ smoothing the flow in a region of size u_l^T around the origin. Assuming again that the fixed point function $\tilde{\Delta}^*(u)$ derives its rough shape from the correlator $\tilde{\Delta}_{l_c(T)}(u)$ at crossover, we can use the result (32) in combination with the flow equation (9) to find the characteristic features $\tilde{\Delta}^*(0)$, ξ^* , and $\tilde{\Delta}^{*'}(0+)$ of the fixed point function and the rounding parameters $\tilde{\Delta}_l''(0)$ and u_l^T of the cusp.

Using Eq. (32) and the crossover condition (33) we find the height $\tilde{\Delta}^*(0) = -\tilde{K}_{l_c(T)}''(0) \approx \tilde{T}_0$. The slope of the fixed point function at $u=0+$ again follows from comparing terms in the flow equation $|\tilde{\Delta}^{*'}(0+)| \approx \tilde{\Delta}^*(0)^{1/2} \approx (\tilde{T}_0)^{1/2}$ and we find the width $\xi^* \approx (\tilde{T}_0)^{1/2} \approx \langle u^2 [L_c(T)] \rangle_{\text{th}}^{1/2} e^{-\zeta_{\text{th}} l_c(T)}$ [see also the result (32)]. The width u_l^T of thermal rounding derives from $u_l^T \approx \tilde{T}_l / |\tilde{\Delta}^{*'}(0+)|$ and we find the result $u_l^T / \xi^* \sim e^{-\theta(l-l_c(T))}$ in $d=1,2$, with $\theta = d-2+2\zeta$ and ζ the random manifold exponent; thus at $l_c(T)$ the width of thermal rounding equals the width of the correlator, $u_l^T \approx \xi^*$. The curvature $\tilde{\Delta}_l''(0) \sim \tilde{\Delta}^*(0) / \tilde{T}_l$ is correspondingly small, $\tilde{\Delta}_l''(0) \approx -e^{(l-l_c(T))}$, (see Fig. 2); comparing this result with Eq. (15) we conclude that the barriers “saturate” to follow the temperature, $U_c(T) \approx T$. This is quite different from the case of $d=3$: Here, the thermal rounding affects only the narrow regime $u_l^T \approx \xi^* e^{-l_c(T)} e^{-\theta(l-l_c(T))} \approx \xi^*(T/U_c(T)) e^{\theta(l-l_c(T))}$ around the origin and the curvature is already large at $l_c(T)$, $\tilde{\Delta}_l''(0) \approx -e^{l_c(T)} e^{\theta(l-l_c(T))} = -(U_c(T)/T) e^{\theta(l-l_c(T))}$, where $U_c(T) = c \langle u^2 \rangle_{\text{th}} L_c(T) \gg T$. Thus, as anticipated above, the curvature of the correlator at $l_c(T)$ is thermally reduced to the order of 1 in $d=1,2$ while it is large in $d=3$. Hence for $d=3$ the situation at high temperatures $T > T_{\text{dp}}$ is not different from that at low temperatures.

Next we integrate the flow for the velocity and force parameters λ_l and f_l . We determine the creep scale $l_d(T)$

twice, using the condition $\lambda_{l_d(T)} = u_{l_d(T)}^T$ to relate the velocity v and the scale $l_d(T)$ and a second time from the onset of the disorder term in the force equation (21), providing a relation between f and $l_d(T)$; combining these results we obtain the desired velocity-force characteristics. In doing so, we have to be careful to use the above high temperature estimates for $\tilde{\Delta}^{*'}(0+)$ in Eqs. (19) and (21).

Integrating the flow equation (6) for λ_l through the Larkin regime and then up to $l_d(T)$ we find the first relation

$$v \propto \exp[-e^{\theta(l_d(T)-l_c(T))}]. \quad (34)$$

Integrating next the force equation (8) we obtain

$$f_{l_d(T)} \sim f e^{(2-\zeta_{\text{th}})l_c(T) + (2-\zeta)[l_d(T)-l_c(T)]} \quad (35)$$

and equating this to the disorder-induced term $c\Lambda^2 |\tilde{\Delta}^{*'}(0+)|$ in the flow equation (8) we arrive at the second relation,

$$f \sim f_c(T) e^{-(2-\zeta)[l_d(T)-l_c(T)]} \quad (36)$$

with the critical force density $f_c(T) = c\tilde{T}_0^{1/2} \Lambda^2 e^{-(2-\zeta_{\text{th}})l_c(T)} \sim c \langle u^2 [L_c(T)] \rangle_{\text{th}}^{1/2} / L_c^2(T)$, in agreement with Eq. (27). Combining the results (34) and (36) we find the velocity-force characteristic describing the non-Arrhenius-type creep at high temperature,

$$v \propto \begin{cases} \exp\left[-\left(\frac{f_c(T)}{f}\right)^{\theta/(2-\zeta)}\right], & d=1,2 \\ \exp\left[-\frac{U_c(T)}{T} \left(\frac{f_c(T)}{f}\right)^{\theta/(2-\zeta)}\right], & d=3; \end{cases} \quad (37)$$

In conclusion, using dynamical functional renormalization group theory we have derived the linear scaling of the creep barriers close to f_c and have put the non-Arrhenius-type high temperature creep of low-dimensional manifolds on a firm basis. The simple behavior of the creep barrier close to threshold appears surprising—considering the nontrivial threshold exponents due to a diverging nucleus obtained for elastic manifolds trapped in a washboard potential (see Ref. 3) one is tempted to expect a nontrivial exponent for the random case as well. However, from our analysis we conclude that there is no new diverging scale associated with creep near threshold. The linear decay of the creep barrier then follows from a regular expansion and no effects of critical fluctuations are picked up.

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