

Remanent Magnetization: Signature of Many-Body Localization in Quantum Antiferromagnets

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(Received 8 September 2016; published 8 June 2017)

We study the remanent magnetization in antiferromagnetic, many-body localized quantum spin chains, initialized in a fully magnetized state. Its long time limit is an order parameter for the localization transition, which is readily accessible by standard experimental probes in magnets. We analytically calculate its value in the strong-disorder regime exploiting the explicit construction of quasilocal conserved quantities of the localized phase. We discuss analogies in cold atomic systems.

DOI: 10.1103/PhysRevLett.118.237202

Introduction.—The nonequilibrium dynamics in disordered, isolated quantum systems have been subject to theoretical investigations ever since the notion of localization was introduced in Ref. [1]. Spin systems in random fields are prototypical models to analyze the disorder-induced breakdown of thermalization: a large number of numerical studies on disordered spin chains [2–13] has provided evidence for a dynamical phase transition between a weak-disorder phase which thermalizes, and a many-body localized phase in which excitations do not diffuse, ergodicity is broken, and local memory of the initial conditions persists for infinite time [14–16].

Signature of many-body localization (MBL) are found in the properties of individual many-body eigenstates: even highly excited eigenstates exhibit area-law scaling of the bipartite entanglement entropy [4,7,8,17] and Poissonian level statistics [2,18,19], both being incompatible with thermalization [20–22]. Novel dynamical properties such as the logarithmic spreading of entanglement have been observed in direct simulations of the time evolution [3,23]. This phenomenology arises due to an extensive set of conserved operators whose local structure prevents both transport and thermalization [24–28]. While mathematically rigorous results are available for certain localized spin chains [28], the existence of such conserved quantities, and thus of MBL, in dimensions $d \geq 2$ is debated [29].

The nonequilibrium physics of many-body localized systems has been probed experimentally in artificial quantum systems made of cold atomic gases [30,31] and trapped ion systems [32], while an indirect signature via a strong suppression of microwave absorption was found in electron glasses [33]. However, direct observations of MBL in the solid state are still lacking.

In this work, we propose a readily observable consequence of MBL in quantum magnets: the out-of-equilibrium remanent magnetization that persists after ferromagnetically

polarizing an antiferromagnet whose total magnetization is not conserved. As an example, we consider an antiferromagnetic, anisotropic Heisenberg spin-1/2 chain,

$$H = \sum_k \left(h_k \sigma_k^z - \sum_{\alpha=x,y,z} J_\alpha \sigma_k^\alpha \sigma_{k+1}^\alpha \right), \quad (1)$$

subject to random fields h_k along the easy axis. We assume $J_z < 0$, as well as $J_x \neq J_y$ to ensure the nonconservation of the total magnetization. Such Hamiltonians can be realized, e.g., in Ising compounds with both exchange and dipolar interactions. However, essentially any sufficiently strongly disordered quantum antiferromagnet with nonconserved magnetization should exhibit the same phenomenology as we describe below.

Consider the following protocol: First, the chain is fully polarized by a strong magnetic field [equilibrated by a (very weak) coupling to a bath]. Then the field is switched off at time $t = 0$ (cf. Fig. 1). For magnets with spin-phonon coupling much weaker than the spin-spin interactions (see, e.g., Ref. [34]), the dynamics is governed by Eq. (1) alone over a long intermediate time window, and thus behaves like a closed system. Interchain couplings in quasi-1D magnets are weak and will not modify the short-time phenomenology. On the other hand, the conjectured breakdown of MBL at long times due to very rare regions in higher dimensions [29] is expected to be a much less efficient channel of delocalization than that provided by realistic couplings to phonons.

Ergodic spin dynamics would relax the initial magnetization completely. A finite remanent magnetization thus implies nonergodicity (see also Refs. [37,38] for disorder-free examples), and can serve as an order parameter in the many-body localized phase of the isolated system. This is a magnetic analogue of the remanent density modulation

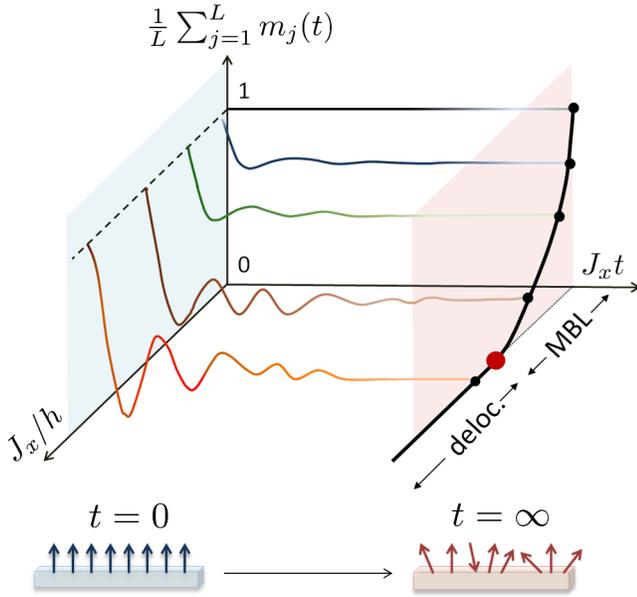


FIG. 1. Relaxation of the total magnetization from a fully polarized initial state. The black curve is the stationary value $L^{-1} \sum_j \hat{m}_j$; it vanishes (possibly discontinuously [35,36]) at the critical point separating the many-body localized and delocalized phases (red point). It is nonanalytic for $J_x/h \ll 1$, cf. Eq. (13).

considered in Ref. [39] and measured in recent cold-atom experiments [30,31]. However, it is experimentally much simpler to access, since it focuses on the total magnetization which can be readily picked up, e.g., by a squid, without requiring scattering measurements or microscopy to resolve spatial patterns.

Conserved dressed spins.—We consider random fields h_k uniformly distributed in $[-h, h]$, and assume strong anisotropy of the couplings, $|J_y| \ll |J_x| \ll |J_z|, h$. For simplicity we set $J_y = 0$. For $J_x = 0$, the spin chain is classical and trivially localizes dynamically, as the σ_k^z form a complete set of commuting, strictly local conserved operators, and the eigenstates are product states in this basis. Perturbative arguments as in Refs. [15,16] predict that localization remains intact for small quantum fluctuations $|J_x| \ll h$. This comes along with a complete set of mutually commuting conserved operators I_k . Those are predicted to be *quasilocal* [25–27], their action decaying exponentially with the distance from a localization center, with a finite correlation length ξ . The I_k are dressed (rotated) versions of the spin degrees of freedom. They can be constructed explicitly by applying perturbation theory to the σ_k^z and resumming the divergences associated with sparse local “resonant” regions in the chain where the effect of quantum fluctuations is nonperturbative [27]. Their existence follows as a corollary of Ref. [28], where MBL was shown to occur due to the quasilocality of the unitary operator U that diagonalizes the disordered Hamiltonian: U is proved to be a sequence of small unitary transformations almost everywhere in the chain, up to rare

resonant regions. The inverse U^{-1} deforms σ_k^z into the conserved quasilocal operator $I_k = U \sigma_k^z U^{-1}$ which admixes only degrees of freedom in the vicinity of k .

Below, we explicitly construct the dressed spin operators following the recipe of Ref. [27]. We then use them to calculate analytically the remanent magnetization to low orders in the quantum fluctuations $|J_x|/h$. This method is very efficient [40] to describe the asymptotic magnetization in nonresonant regions where the transverse couplings act perturbatively. However, we will see that the dominant effect of weak quantum fluctuations arises from rare resonant regions, where the perturbative expansion in J_x has to be resummed to infinite order.

Computation of remanent magnetization.—We consider the dynamical evolution of the fully magnetized initial state $|\psi_0\rangle$ with density matrix $|\psi_0\rangle\langle\psi_0| = \prod_i (1 + \sigma_i^z)/2$, governed by Eq. (1). [41] We are interested in the long time remanence of the magnetization, and thus consider the time averaged magnetization at site j :

$$\hat{m}_j = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt m_j(t); \quad m_j(t) = \langle \psi_0 | \sigma_j^z(t) | \psi_0 \rangle. \quad (2)$$

For $J_x = 0$, the local magnetization is trivially conserved, $m_j(t) = 1$. For finite J_x , the nontrivial dynamics of $\sigma^z(t)$ reduces \hat{m}_j . In the many-body localized regime, the time evolution is strongly constrained by the conservation of the dressed spins I_k with $|k - j| \lesssim \xi$. As a consequence, partial memory of the initial order $\langle \sigma_j^z \rangle = 1$ is retained for an arbitrarily long time, resulting in a finite remanence of the site-averaged magnetization $\hat{m} = L^{-1} \sum_j \hat{m}_j$.

In the absence of spectral degeneracies, Eq. (2) can be expressed via a Lehmann representation as

$$\hat{m}_j = \sum_{\alpha} \langle \psi_0 | P_{\alpha} \sigma_j^z P_{\alpha} | \psi_0 \rangle, \quad (3)$$

where $P_{\alpha} = |\psi_{\alpha}\rangle\langle\psi_{\alpha}| = \prod_{k=1}^L (1 + i_k^{(\alpha)} I_k)/2$ projects onto the eigenstate labeled by the quantum numbers $i_k^{(\alpha)} \in \{\pm 1\}$ of the dressed spins I_k . Using the operator identity

$$\sum_{\alpha} P_{\alpha} \sigma_j^z P_{\alpha} = \sigma_j^z + \sum_{n=1}^L \sum_{k_n > k_{n-1} > \dots > k_1} \prod_{l=1}^n \left(\frac{I_{k_l}}{2} \right) \times [[\sigma_j^z, I_{k_1}], I_{k_2}], \dots, I_{k_n}], \quad (4)$$

we obtain (cf. Supplemental Material [42] for details)

$$\hat{m}_j = 1 + \sum_{n=1}^L \sum_{k_n > k_{n-1} > \dots > k_1} \times \text{Tr} \left\{ \prod_{i=1}^n \left(\frac{I_{k_i}}{2} \right) [[\sigma_j^z, I_{k_1}], I_{k_2}], \dots, I_{k_n}] \prod_{\ell=1}^L \left(\frac{1 + \sigma_{\ell}^z}{2} \right) \right\}, \quad (5)$$

where $\text{Tr}\{\dots\}$ denotes the trace, and some fixed ordering among the labels of the operators I_k is assumed [44].

For the dressed spins I_k we write the formal expansion

$$I_k = \sigma_k^z + \delta I_k^{(1)} + \delta I_k^{(2)} + \dots \quad (6)$$

with $\delta I_k^{(n)} = O(J_x^n)$. At any order n , $\delta I_k^{(n)}$ is uniquely determined by the constraints $[I_k, H] = 0$ and $I_k^2 = 1$ [27], see also the Supplemental Material [42]. For the Hamiltonian Eq. (1) with $J_y = 0$, the first order terms read

$$\begin{aligned} \delta I_k^{(1)} = & \sum_{\rho, \tau = \pm 1} (A_{\rho\tau}^{(k)} O_{\rho\tau}^{(k)} - A_{\rho\tau}^{(k-1)} O_{\rho\tau}^{(k-1)}) \\ & + \sum_{\rho, \tau = \pm 1} (B_{\rho\tau}^{(k)} \Delta_{\rho\tau}^{(k)} + B_{\rho\tau}^{(k-1)} \Delta_{\rho\tau}^{(k-1)}), \end{aligned} \quad (7)$$

where we define the local operators

$$\begin{aligned} O_{\rho\tau}^{(k)} &= \frac{1 + \rho\sigma_{k-1}^z}{2} [\sigma_k^+ \sigma_{k+1}^- + \text{H.c.}] \frac{1 + \tau\sigma_{k+2}^z}{2}, \\ \Delta_{\rho\tau}^{(k)} &= \frac{1 + \rho\sigma_{k-1}^z}{2} [\sigma_k^+ \sigma_{k+1}^+ + \text{H.c.}] \frac{1 + \tau\sigma_{k+2}^z}{2}, \end{aligned} \quad (8)$$

and the coefficients

$$\begin{aligned} A_{\rho\tau}^{(k)} &= -\frac{J_x}{h_k - h_{k+1} + J_z(\tau - \rho)}, \\ B_{\rho\tau}^{(k)} &= -\frac{J_x}{h_k + h_{k+1} - J_z(\tau + \rho)}. \end{aligned} \quad (9)$$

At low orders, the sum over multi-indices in Eq. (5) reduces to the few terms involving indices sufficiently close to k , since other commutators vanish. The lowest order corrections to \hat{m}_j are given by the terms with $n = 1, 2$ in Eq. (5). Inserting Eq. (7) into Eqs. (4) and (3) we find [45]

$$\hat{m}_j = 1 - (B_{1,1}^{(j)})^2 - (B_{1,1}^{(j-1)})^2 + O(J_x^3). \quad (10)$$

The above method is easily extended to higher orders on the majority of sites around which the quantum fluctuations act perturbatively. However, we see that the site average of $\langle \hat{m}_j \rangle_{\text{dis}}$ is ill defined for $|J_z| < h$. The apparent divergence is due to rare resonances, i.e., realizations of neighboring local fields that give rise to small denominators in Eq. (9). Resonances may also arise at higher order n in perturbation theory, if two nearly degenerate classical configurations hybridize strongly due to a coupling $\sim J_x^n$. The corresponding small denominators render the naive perturbative expansion Eq. (6) nonconvergent, as a small denominator generated at order n reappears repeatedly in higher order terms, giving rise to norm-divergent operator subsequences. However, a defining feature of MBL is that resonances are sparse in space, each of them involving spins only within the typical range of the correlation length ξ . [27,28].

Thus, *local* resummations (or equivalently, exact diagonalizations of resonant subsystems of typical size ξ) suffice to cure the divergences of the perturbation theory (in analogy with the single particle case [1]) and yield a “regularized,” norm-convergent operator expansion for the I_k .

Let us illustrate this resummation for resonances involving spins on neighboring sites, as those dominate in the strong-disorder limit. Resumming all terms in Eq. (6) containing higher powers of the resonant J_x coupling is equivalent to determining the operators \tilde{I}_i (satisfying $\tilde{I}_i^2 = 1$) that are conserved by the reduced Hamiltonian

$$H^{(k)} \equiv \sum_{i=1}^L (h_i \sigma_i^z - J_z \sigma_i^z \sigma_{i+1}^z) - J_x \sigma_k^x \sigma_{k+1}^x, \quad (11)$$

where only the resonant J_x coupling is retained. These operators serve as a new basis for the perturbation theory in the remaining, nonresonant J_x couplings. They can be constructed explicitly by determining a local rotation that acts nontrivially in the vicinity of the sites $k, k+1$, deforming the pair $\sigma_k^z, \sigma_{k+1}^z$ into two modified, exactly conserved operators $\tilde{I}_k, \tilde{I}_{k+1}$ (with $\tilde{I}_i = \sigma_i^z$ for $i \neq k, k+1$), similarly to Ref. [28]. For the particular model Eq. (1) the resulting $\tilde{I}_k, \tilde{I}_{k+1}$ are again linear combinations of Eq. (8), albeit with modified coefficients Eq. (9) given in the Supplemental Material [42]. Inserting the conserved integrals into Eq. (3), we find an expression like Eq. (10), but with the substitution [46]

$$B_{\rho\tau}^{(j)} \rightarrow -\frac{J_x}{([h_j + h_{j+1} - J_z(\tau + \rho)]^2 + J_x^2)^{1/2}}, \quad (12)$$

a result one can also derive from exact diagonalization of two neighboring spins in the field of their polarized neighbors, and computing their diminished magnetization. From this we obtain the remanent magnetization

$$\langle \hat{m}_j \rangle_{\text{dis}} = 1 - \frac{\pi |J_x|}{h} \left(1 + \frac{J_z}{h} \right) + O(J_x^2), \quad (13)$$

which for $|J_z| < h$ is nonanalytic in J_x . This feature arises due to resonances. The nonanalytic cusp at $J_x = 0$ has the largest magnitude in the limit of vanishing Ising interactions, $J_z \rightarrow 0$ (recall that $J_z < 0$), as it creates the lowest effective disorder.

Atomic analogues.—The gauge transformation $\mathcal{U} = \prod_{j=1}^L \exp[i(\pi/2)j\sigma_j^x]$ maps the antiferromagnetic chain Eq. (1) into its ferromagnetic counterpart with $J_x \rightarrow J_x$, $J_{y,z} \rightarrow -J_{y,z}$, and the initial state $|\psi_0\rangle$ into a Néel state. The order parameter is mapped into the staggered magnetization. Such a quantity has been studied numerically in Ref. [47] for disordered, long-range transverse field Ising chains, modeling the ion-trap quantum simulators explored

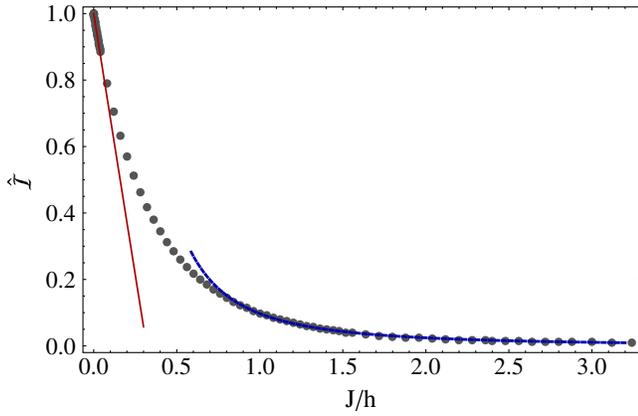


FIG. 2. Dependence of the remanent density imbalance on the hopping strength J for a chain of noninteracting fermions ($L = 100$, 5×10^3 realizations). The continuous red line is the analytical estimate, Eq. (13), with $J_x = J$, $J_z = 0$. The blue dashed line is a power law fit $a + c(J/h)^{-2}$, with $a = 0.003$, $c = 0.101$.

experimentally in Ref. [32]. The staggered magnetization is a close analogue of the particle imbalance studied as an experimental probe of MBL in cold atoms [30]:

$$\mathcal{I}(t) = \frac{2}{L} \sum_{j=1}^L (-1)^j \langle n_j(t) \rangle. \quad (14)$$

Here n_j is the occupation number of site j , after preparation in an initial density wave $n_j(t=0) = [1 + (-1)^j]/2$.

A ferromagnetic spin chain with $J_x = J_y$ is equivalent, via the Jordan-Wigner transformation, to a one-dimensional model of interacting spin-less fermions in a disordered potential. For $J_z = 0$ it reduces to the noninteracting Anderson model

$$H = -J \sum_{i=1}^{L-1} (c_i^\dagger c_{i+1} + \text{H.c.}) + 2 \sum_{i=1}^L h_i n_i \quad (15)$$

for which the imbalance is a sum over single particle contributions, weighted with the occupation probability of eigenstates in the initial state. A standard calculation leads to the remanent imbalance Eq. (14) in the form

$$\hat{\mathcal{I}} = \frac{1}{L} \sum_{\alpha=1}^L \left(\sum_{k=1}^L (-1)^k \phi_\alpha^2(k) \right)^2, \quad (16)$$

where $\phi_\alpha(i)$, with $1 \leq \alpha, i \leq L$ are the localized single particle eigenstates of the quadratic Hamiltonian Eq. (15). This solvable case is interesting as it can be analyzed deeper into the weak disorder limit.

Figure 2 shows the imbalance as a function of J/h , as obtained by exact diagonalization. At small J/h a linear cusp with the slope derived in Eq. (13) (using $J_z = 0$, $J_x = J$) is seen. For large J/h , $\hat{\mathcal{I}}$ decays algebraically as $(J/h)^{-2}$, as derived in the Supplemental Material [42] and verified numerically in Fig. 2. [48].

Let us now discuss the qualitative effects of fermionic interactions. The addition of a term $U \sum_{i=1}^L n_i n_{i+1}$ (the equivalent of Ising interactions) to the Hamiltonian Eq. (15) may have opposite effects, depending on the value of J/h . For $J/h \ll 1$, the interaction broadens the distribution of the energy denominators, and thus acts as an additional source of disorder, which reduces the deviation of $\langle \hat{\mathcal{I}} \rangle_{\text{dis}}$ from the classical limit. The same holds in the magnetic analogue as confirmed by Eq. (13). For larger $J/h > 1$, the single particle localization length becomes substantial. The dominant effect of interactions is then to mediate (virtual) scattering between single particle states, as discussed in Refs. [15,16]. One expects that this suppresses the remanent imbalance, as was indeed observed in the experiments of Ref. [30]. At large enough interactions (and finite temperature), delocalization is induced by a proliferation of resonant regions, which leads to the vanishing of the order parameter $\langle \hat{\mathcal{I}} \rangle_{\text{dis}}$ at a U -dependent critical hopping $J^*(U)/h$. The perturbative arguments in Refs. [15,16,27] predict that the localized phase is stable for $U < U^* \propto \delta_\xi / \log(\mathcal{W}/\delta_\xi)$. Here \mathcal{W} is the total bandwidth of the noninteracting Hamiltonian Eq. (15) and $\delta_\xi \sim \mathcal{W}/\xi$ the average energy gap between single-particle states localized within the same region of size ξ , which is assumed to be much larger than the lattice constant, $\xi \gg a$. This corresponds to $J/h \gg 1$, implying $\xi \sim (J/h)^2$ and $\mathcal{W} \approx J$. However, as was pointed out recently [49], previous studies neglected the phenomenon of spectral diffusion [50,51], which is expected to reduce the critical interaction strength in the weak disorder limit to $U^* \propto \delta_\xi (\delta_\xi / \mathcal{W})$ (up to logarithmic corrections).

Discussion and conclusion.—We have proposed and analyzed the presumably simplest possible protocol for quantum disordered magnets to exhibit the absence of ergodic dynamics, and thus of MBL in the form of remanent magnetization in initially ferromagnetically polarized antiferromagnets. The present calculation illustrates how our explicit recipe for constructing the conserved quantities allows for analytic predictions for quantities of experimental relevance. It is an analytical alternative to several numerical schemes based on DMRG [52–55] or quantum Monte Carlo calculations [56] that allow one to study properties of specific many-body localized eigenstates. The simple formula, Eq. (5), can be applied to the conserved pseudospins constructed numerically in Refs. [57–59] for nonperturbative interactions by means of renormalization procedures or diagonalizing flows. It would be interesting to extend this calculation beyond the lowest orders, possibly exploiting the expansion for the conserved quantities in the forward approximation [27,60], to discuss the behavior of the typical value of the remanent magnetization when approaching the delocalization threshold.

An interesting question concerns the vanishing of the order parameter, Eq. (16), at criticality. If it is continuous, it

might exhibit nontrivial scaling with the system size, potentially reflecting multifractality of critical wave functions. However, numerical studies of the scaling of the entanglement entropy of small subsystems [36,61] have suggested that at the delocalization transition long-time averages of typical observables jump discontinuously, implying a discontinuity of the order parameter (remanent magnetization or imbalance). Given the difficulty in accessing numerically the relevant regimes, the prospect of probing these scenarios experimentally is appealing.

Part of this work was done at the Kavli Institute for Theoretical Physics at the University of California, Santa Barbara and supported in part by the National Science Foundation under Grant No. NSF PHY11-25915. V. Ros thanks the Paul Scherrer Institute in Villigen for hospitality and support.

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 [41] One may also view this protocol as a quantum quench: First a high energy eigenstate of the Hamiltonian with $J_x = 0$ is prepared, and then the quantum fluctuations $J_x \sigma_k^x \sigma_{k+1}^x$ are switched on abruptly at time $t = 0$.
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