Appendix to: Atmospheric simulations using a GCM with simplified physical parametrizations. I: Model climatology and variability in multi-decadal experiments

by Franco Molteni

Abdus Salam International Centre for Theoretical Physics, Trieste, Italy

A Formulation of physical parametrizations.

A.1 General definitions.

Physical parametrization tendencies at time step t are computed from (time-filtered) model variables at step *t-1*, and integrated forward to step *t+1*. All parametrization schemes described below are run as independent modules, except for computation of surface fluxes, which uses the downward radiative flux at the surface as an input to define a (diagnostic) skin temperature over land, and returns an upward flux of longwave radiation to the radiation code.

The parametrization driver first computes grid-point values of the 'primary' variables (U and V wind components, temperature T, specific humidity Q, geopotential Φ, surface pressure p_s) from their spectral representations, then defines a number of additional diagnostic variables:

- saturation specific humidity: Q^{sat} ;

- relative humidity: RH ;

- dry static energy: $SE = c_p T + \Phi$;

- moist static energy: $MSE = SE + L_c Q$;

- saturation moist static energy: $MSS = SE + L_c Q^{sat}$;

where c_p and L_c are the specific heat at constant pressure and the latent heat of condensation respectively.

All variables are defined at full levels; however, values of some variables are also needed at half-levels (layer boundaries) to define fluxes. Let A_k be the value of a generic variable A at full level k (k=*1,...,N*, with k increasing with σ, i.e from top to bottom). Defining h=1/2 layer, let A_{k+h} be the value at the half-level representing the lower boundary of layer k, and A_{k-h} the value at its upper boundary. Unless stated otherwise, half-level values used by parametrization schemes are obtained by linear interpolation in $\log(\sigma)$:

$$A_{k+h} = A_k + (A_{k+1} - A_k) \frac{log(\sigma_{k+h}) - log(\sigma_k)}{log(\sigma_{k+1}) - log(\sigma_k)} \tag{1}$$

Most parametrization schemes work by defining upward and downward fluxes of momentum, energy or moisture at layer boundaries. For a generic variable A, upward and downward fluxes at half-level $k \pm h$ will be indicated by ${}^{u}F^{A}_{k\pm h}$ and ${}^{d}F^{A}_{k\pm h}$ respectively. The flux *absorbed* in layer k is:

$$\Delta F^{A}_{k} = (\ {}^{u}F^{A}_{k+h} -\ {}^{u}F^{A}_{k-h}\ ) + (\ {}^{d}F^{A}_{k-h} -\ {}^{d}F^{A}_{k+h}\ ) \tag{2}$$

Fluxes of momentum and moisture are converted into wind and moisture tendencies by:

$$\frac{\partial A_k}{\partial t} = \frac{g\ \Delta F^{A}_{k}}{\Delta p_k} \tag{3}$$

where $A\ =\ U,\ V$, or Q, and:

$$\Delta p_k = p_s\ (\ \sigma_{k+h} - \sigma_{k-h}\ ) \tag{4}$$

while energy fluxes are converted into temperature tendencies by:

$$\frac{\partial T_k}{\partial t} = \frac{g\ \Delta F^{EN}_{k}}{c_p\ \Delta p_k} \tag{5}$$

where $EN = SE,\ SR$ (shortwave radiation), or LR (longwave radiation).

A.2 Convection.

The convection scheme is a simplified version of the mass-flux scheme developed by Tiedke (1993). It represents the updraft of saturated air from the PBL to a suitably defined "top-of-convection" (TCN) level in the middle or upper troposphere, and the compensating large-scale descending motion. Entrainment into the updraft occurs in the lower troposphere above the PBL, while detrainment is only allowed in the TCN layer (which greatly simplifies the computation of latent heat release). The scheme also represents a "secondary" exchange of moisture between the PBL and the layers below the TCN level.

The convection scheme is activated in conditionally unstable regions where MSS decreases with height, and where (either relative or specific) humidity in the PBL (level N) exceeds a prescribed threshold. Specifically, convection occurs at a given grid-point if:

$$Q_N > Q_{thr} = min\ (\ Q_{cnv}\ ,\ RH_{cnv}\ Q^{sat}_{N}) \tag{6}$$

and if a tropospheric level exists (with $1 < k < N - 1$), such that:

$$MSS_N > MSS_{k+h} \tag{7}$$

The TCN level is defined as the highest tropospheric level for which Eq. 7 is satisfied.

Over the selected grid points, the scheme first defines the fluxes of mass *(m)*, humidity and dry static energy at the top of the PBL:

$$^{u}F^{m}_{N-h} = \, ^{d}F^{m}_{N-h} = F^{*} \tag{8}$$

$$^{u}F^{Q}_{N-h} = F^{*} \cdot Q^{sat}_{N} \ \ ; \ \ ^{d}F^{Q}_{N-h} = F^{*} \cdot Q_{N-h} \tag{9}$$

$$^{u}F^{SE}_{N-h} = F^{*} \cdot SE_{N} \ \ ; \ \ ^{d}F^{SE}_{N-h} = F^{*} \cdot SE_{N-h} \tag{10}$$

The closure of the scheme (i.e. the determination of F^{*}) is obtained by requiring the convective moisture tendency in the PBL to be equivalent to a relaxation of humidity towards the threshold specified by Q_{thr} with relaxation time τ_{cnv}:

$$\left(\frac{\partial Q_N}{\partial t}\right)_{cnv} = -\frac{g\ F^{*}\ (\ Q^{sat}_{N} - Q_{N-h}\ )}{\Delta p_N} = -\frac{Q_N - Q_{thr}}{\tau_{cnv}} \tag{11}$$

Solving Eq. 11 for F^{*} gives:

$$F^{*} = \frac{\Delta p_N}{g\ \tau_{cnv}}\ \frac{Q_N - Q_{thr}}{Q^{sat}_{N} - Q_{N-h}} \tag{12}$$

In the 'intermediate' layers (between the PBL and the TCN layer), the upward fluxes are increased by the contribution due to entrainment (E^{m}), while the downward fluxes are computed from the modified mass flux and the half-level values of humidity and dry static energy (since the upward and downward mass fluxes have the same magnitude, just one value needs to be defined):

$$E^{m}_{k} = \epsilon(\sigma_k)\ F^{m}_{k+h} \tag{13}$$

$$F^{m}_{k-h} = F^{m}_{k+h}\ +\ E^{m}_{k} \tag{14}$$

$$^{u}F^{Q}_{k-h} = \, ^{u}F^{Q}_{k+h}\ +\ E^{m}_{k}\ Q_k \ \ ; \ \ ^{d}F^{Q}_{k-h} = F^{m}_{k-h}\ Q_{k-h} \tag{15}$$

$$^{u}F^{SE}_{k-h} = \, ^{u}F^{SE}_{k+h}\ +\ E^{m}_{k}\ SE_k \ \ ; \ \ ^{d}F^{SE}_{k-h} = F^{m}_{k-h}\ SE_{k-h} \tag{16}$$

where the entrainment coefficient $\epsilon(\sigma_k)$ varies linearly with σ, from 0 at $\sigma = 0.5$ to a maximum value at σ_{N-1}.

It should be noted that, since no detrainment occurs in 'intermediate' layers, there is no need to compute the condensation in the updraft within these layers, since the energy released by such a process is simply transported upwards and does not modify the local temperature tendency. Therefore, condensation is only computed in the TCN layer, where the fluxes at the upper boundary are set to zero. A part of the upward moisture flux at the TCN lower boundary in converted into convective precipitation:

$$P_{cnv} = {}^{u}F^{Q}_{k0+h} - F^{m}_{k0+h}\, Q^{sat}_{k0+h} \tag{17}$$

where $k0 = k(TCN)$. Then, the net fluxes of moisture and energy into the TCN layer are modified as follows:

$$\Delta F^{Q}_{k0} = {}^{u}F^{Q}_{k0+h} - {}^{d}F^{Q}_{k0+h} - P_{cnv} \tag{18}$$

$$\Delta F^{SE}_{k0} = {}^{u}F^{SE}_{k0+h} - {}^{d}F^{SE}_{k0+h} + L_c\, P_{cnv} \tag{19}$$

From Eqs. 8 to 19, net fluxes of moisture and dry static energy can be computed for all tropospheric layers. These represent the effects of deep convective systems extending up to the TCN layer. The effects of shallower, non-precipitating convective systems are crudely represented by an additional diffusion of moisture between the PBL and the 'intermediate' layers, which acts wherever the relative humidity in the the latter layers falls below a threshold $RH'_{cnv} < RH_{cnv}$. In such a case, the net moisture flux in the *k-th* intermediate layer is increased by a quantity

$$\Delta' F^{Q}_{k} = \epsilon'\, F^{*}\, (\, RH'_{cnv}\, Q^{sat}_{k} - Q_k\,) \tag{20}$$

while the net moisture flux in the PBL is decreased by a corresponding amount.

A.3 Large-scale condensation.

Large-scale condensation is modelled as a relaxation of humidity towards a reference value, which occurs in tropospheric layers whenever relative humidity exceeds a σ-dependent threshold RH_{lsc}. In such a case, a humidity tendency is defined as:

$$\left(\frac{\partial Q_k}{\partial t}\right)_{lsc} = -\,\frac{Q_k - RH(\sigma_k)_{lsc}\, Q^{sat}_{k}}{\tau_{lsc}} \tag{21}$$

and the corresponding temperature tendency as:

$$\left(\frac{\partial T_k}{\partial t}\right)_{lsc} = -\,\frac{L_c}{c_p}\left(\frac{\partial Q_k}{\partial t}\right)_{lsc} \tag{22}$$

where the dependence of RH_{lsc} on σ is as follows:

$$RH(\sigma_k)_{lsc} = RH^1_{lsc} \; + \; \Delta RH_{lsc} \; (\sigma_k^2 - 1) \tag{23}$$

The precipitation flux due to large-scale condensation is given by (the opposite of) the vertical integral of humidity tendencies in the troposphere:

$$P_{\,lsc} = - \, \frac{1}{g} \sum_{k=2}^{N} \Delta p_k \, \left(\frac{\partial Q_k}{\partial t} \right)_{lsc} \tag{24}$$

A.4 Clouds and shortwave radiation.

Although the cloud and radiation schemes used in SPEEDY are much simpler than the schemes used in state-of-the art GCMs, still they represent the most complex part of the parametrization package, and that which depends on the largest number of parameters. For reasons of brevity, only the basic principles of the schemes will be described here. A more detailed description can be found in Corbetta (1999).

Clouds properties are defined diagnostically from values of relative and specific humidity in the tropospheric air column. Clouds are assumed to have their base at the interface between the lowest two model layers, and their top at the upper boundary of highest layer in which the following conditions are both satisfied:

$$RH_k \quad > \quad RH_{cl} \tag{25}$$

$$Q_k \quad > \quad Q_{cl}(\sigma_k) \tag{26}$$

where RH_{cl} is a constant and Q_{cl} a function of σ. The cloud-top layer will be denoted by k_{cltop}.

The total cloud cover CLC is defined as a piecewise linear function of

$$RH_{max} = max \; [\; RH_k \;\, , k_{cltop} \leq k \leq N-1 \;] \tag{27}$$

Namely, CLC is 0 when $RH_{max} < RH_{cl}$, 1 when $RH_{max} > RH'_{cl}$, and varies linearly with RH_{max} between these two thresholds.

Once cloud properties are defined, the shortwave radiation (SR) starts by computing the incoming flux of solar radiation ${}^{d}F_0^{sol}$ at the top of the atmosphere from astronomical formulae. Since the model has just one layer above the troposphere, which is intended as a "dynamical boundary" for the tropospheric motion rather than as a representation of the vertically-averaged upper atmosphere, the code is designed to reproduce lower-stratosperic temperatures in the first (highest) model layer. The absorption by ozone in the lower stratosphere is defined by an idealised function of latitude; the corresponding flux (ΔF_{lst}^{ozone}) is considered as absorbed by the first model layer. In addition, as a crude representation of the radiative effects of the *unresolved* upper stratosphere, a small latitudinally-dependent fraction (ΔF_{ust}^{ozone}) of the incoming solar radiation is turned directly into outgoing longwave radiation.

After subtracting the ozone absorption, the residual downward flux into the first model layer:

$$ {}^{d}F_{h}^{SR} = {}^{d}F_{0}^{sol} - \Delta F_{ust}^{ozone} - \Delta F_{lst}^{ozone} \quad (28) $$

is partitioned into two bands, one corresponding to the visible part of the spectrum, the second to the near-infrared. For each band and for each tropospheric layer, a transmissivity τ_k^{SR} is defined as a function of the daily-averaged zenith angle (again a prescribed function of latitude), layer depth, specific humidity and cloud properties, and the downward propagation of SW radiation is simply modelled (for each band) by:

$$ {}^{d}F_{k+h}^{SR} = {}^{d}F_{k-h}^{SR} \;\; \tau_k^{SR} \quad (29) $$

In the cloud-top layer, the radiation entering the upper boundary is first modified by subtracting the flux reflected by clouds, so that for $k = k_{cltop}$ Eq. (29) becomes:

$$ {}^{d}F_{k+h}^{SR} = {}^{d}F_{k-h}^{SR} \; (1 - A_{cl} \; CLC) \;\; \tau_k^{SR} \quad (30) $$

where A_{cl} is the cloud albedo. (For simplicity, cloud effects are neglected in the near-infrared band).

At the surface (i.e. level $N{+}h$), a climatological albedo A_s is defined as a function of seasonally varying fields of sea ice and snow depth. The upward flux at the surface is defined as:

$$ {}^{u}F_{s}^{SR} = {}^{d}F_{s}^{SR} \; A_s \quad (31) $$

and the upward propagation of shortwave radiation is modelled by:

$$ {}^{u}F_{k-h}^{SR} = {}^{u}F_{k+h}^{SR} \;\; \tau_k^{SR} \quad (32) $$

with the flux reflected by clouds added at the upper boundary of the cloud-top layer. Since practically the whole flux in the near-infrared band is absorbed in the downward propagation, the upward part is only modelled for the visible band.

A.5 Longwave radiation.

In the parametrization scheme for longwave radiation (LR), the infrared spectrum ($5\mu m \leq \lambda \leq 50\mu m$) is partitioned into four regions (again referred to as "bands", although two of them include different spectral intervals with similar optical properties):

- the so-called "infrared window" between 8.5 and 11 μm (band 1);
- the band of strong absorption by CO_2 around 15 μm (band 2);
- the aggregation of regions with weak or moderate absorption by water vapour (band 3);
- the aggregation of regions with strong absorption by water vapour (band 4).

As in the SR code, a transmissivity is computed for each band and model layer as a function of layer depth, humidity and cloud properties. The effect of clouds is modelled as a (strong) decrease in the transmissimity in the "window" band. So, if

$$\Delta p'_k = \frac{p_s}{p_0} (\sigma_{k+h} - \sigma_{k-h}) \tag{33}$$

is the normalised layer depth (where $p_0 = 10^5$ hPa), the transmissivity in the "window" band is given by

$$\tau^{LR}_{k,1} = exp \, [\, - \, (\, \alpha^{LR}_{win} + \alpha^{LR}_{cl} \, CLC) \, \Delta p'_k \,] \tag{34}$$

in cloudy regions/layers (i.e. where $CLC > 0$ and $k_{cltop} \leq k \leq N - 1$), by

$$\tau^{LR}_{k,1} = exp \, (\, - \, \alpha^{LR}_{win} \, \Delta p'_k \,) \tag{35}$$

otherwise. For the CO_2 and water-vapour bands, transmissivities are given respectively by:

$$\tau^{LR}_{k,2} = exp \, (\, - \, \alpha^{LR}_{CO_2} \, \Delta p'_k \,) \tag{36}$$

$$\tau^{LR}_{k,3} = exp \, (\, - \, \alpha^{LR}_{wv1} \, Q_k \, \Delta p'_k \,) \tag{37}$$

$$\tau^{LR}_{k,4} = exp \, (\, - \, \alpha^{LR}_{wv2} \, Q_k \, \Delta p'_k \,) \tag{38}$$

where the α coefficients are constant absorptivity parameters. Typically, in the middle and lower troposphere (in clear-sky conditions),

$$\tau_{k,4}^{LR} < \tau_{k,2}^{LR} < \tau_{k,3}^{LR} < \tau_{k,1}^{LR} \tag{39}$$

except in the tropics, where $\tau_{k,3}^{LR} < \tau_{k,2}^{LR}$ close to the surface. In the upper troposphere and in the stratosphere, the CO_2 band has the lowest clear-sky transimissivity.

After setting the downward flux at the upper boundary to zero, the LR code proceeds by computing the downward transmission and emission in each band:

$${}^{d}F_{k+h}^{LR} = {}^{d}F_{k-h}^{LR} \; \tau_k^{LR} + (1 - \tau_k^{LR}) \, {}^{d}B_k \tag{40}$$

In the equation above (where the band index is omitted for simplicity), ${}^{d}B_k$ represents the downward blackbody emission in a given spectral band. This is computed as a weighted function of temperature at the centre and at the lower boundary of the layer:

$${}^{d}B_k = f_b(T_k) \, \sigma_{SB} \, [\, T_k^4 + w_k^{LR} \, (\, T_{k+h}^4 - T_k^4 \,) \,] \tag{41}$$

where σ_{SB} is the Stefan-Boltzmann constant, f_b gives the fraction of energy emitted in each band as a function of temperature, and the weight w_k^{LR} depends on the layer transmissivity in a given band ($w_k^{LR} = 0$ when $\tau_k^{LR} = 1$).

At the lower boundary, the blackbody emission from the surface ${}^{u}B_s$ (multiplied by a constant surface emissivity ϵ_s) is partitioned among the four bands, and a small contribution from LR reflection is added:

$${}^{u}F_s^{LR} = \epsilon_s \;\; f_b(T_s) \, {}^{u}B_s + (1 - \epsilon_s) \, {}^{d}F_s^{LR} \tag{42}$$

Then, for each band the upward propagation is modelled by:

$${}^{u}F_{k-h}^{LR} = {}^{u}F_{k+h}^{LR} \; \tau_k^{LR} + (1 - \tau_k^{LR}) \, {}^{u}B_k \tag{43}$$

$${}^{u}B_k = f_b(T_k) \, \sigma_{SB} \, [\, T_k^4 + w_k^{LR} \, (\, T_{k-h}^4 - T_k^4 \,) \,] \tag{44}$$

where the upward emission is computed from the temperature at the centre and the upper boundary of each layer. Eqs. (41) and (44) imply that, when the layer transmissivity is close to zero, the upward and downward emissions are dependent on the temperatures at the respective boundaries, while they approach each other (as functions of mid-layer temperature) when transmissivity is high.

Finally, since in the (dry) stratospheric layer neither the water-vapour nor the ozone emission/absorption are explicitely modelled, a seasonally-varying, zonally-symmetric correction term is added to the LR flux emitted by such a layer.

A.6 Surface fluxes.

In SPEEDY, surface fluxes are modelled using rather standard aerodynamic formulas (see e.g. chapter 4 in Hartmann 1994). However, since the PBL is represented by just one layer, one cannot use variables at the lowest model level as approximations of near-surface variables. Also, vertical gradients between two model levels cannot be used to estimate PBL stability properties which may affect the definition of exchange coefficients.

Therefore, the first step in the estimation of surface fluxes is the definition of near-surface atmospheric values of wind, temperature and humidity (U_{sa}, V_{sa}, T_{sa}, Q_{sa}, RH_{sa}) through a suitable extrapolation procedure.

Near-surface wind is simply assumed to be proportional to the wind at the lowest full level:

$$U_{sa} = f_{wind}\, U_N \quad ; \quad V_{sa} = f_{wind}\, V_N \tag{45}$$

For temperature and moisture, two options are available. The simplest method, normally used in GCMs with multi-level PBL, assumes that near-surface values of potential temperature and specific humidity are the same as at the lowest model level. In the second method (used in the integrations described below), temperature is extrapolated at σ=1 using values at the two lowest levels (T_N, T_{N-1}) and assuming a linear profile in $\log(\sigma)$ (although T_{sa} is not allowed to be lower than T_N). Then, relative humidity is set equal to the lowest-level value ($RH_{sa} = RH_N$), and Q_{sa} is recomputed from T_{sa} and RH_{sa}.

From the variables defined above, surface air density (ρ_{sa}) is also computed, and an "effective" surface wind speed is defined as:

$$|V_0| = \; (\; U_{sa}^2 + V_{sa}^2 + V_{gust}^2 \;)^{1/2} \tag{46}$$

where the V_{gust} constant represents the contribution of unresolved wind variability.

With regard to temperature and moisture over land-surface, currently these are mainly derived from seasonally-varying climatological fields (see Sect. 2.3). For temperature, the climatological value T_{land} (appropriate for the first 5-10 cm of soil) is corrected using the net flux of solar radiation at the surface to define a skin temperature:

$$T_{skin} = T_{land} + \beta_{skin}\ F_s^{SR}\ cos(\phi) \tag{47}$$

where ϕ is latitude and β_{skin} a constant of the order of $10^{-2}\ {}^oK\ \ W^{-1} m^2$.

Note that this correction is always positive, so it is not meant to represent soil temperature anomalies arising from land-atmosphere interactions. Rather, it tends to compensate for the underestimation of daily-mean heat fluxes by SPEEDY's parametrizations, due to the absence of non-linear effects associated with the daily cycle. A simple scheme to define a soil temperature anomaly based on a surface energy balance model is currently being tested.

As far as soil moisture is concerned, the information regarding the water content in the top soil layer (W_{top}) and in the root layer below (W_{root}) is condensed into a climatological soil-water availability index α_{sw}. This non-dimensional ratio depends on vegetation fraction (f_{veg}), and on soil moisture values at field capacity (W_{cap}) and wilting point (W_{wil}) (see e.g. Viterbo and Beljiars 1995 for the definition of these parameters and their use in more complex schemes):

$$\alpha_{sw} = \frac{D_{top}\ W_{top}\ +\ f_{veg}\ D_{root}\ \ max\ (\ W_{root} - W_{wil},\ 0\ )}{D_{top}\ W_{cap}\ +\ D_{root}\ (\ W_{cap} - W_{wil}\ )} \tag{48}$$

where D_{top} and D_{root} are the layer depths.

Once the appropriate variables are defined, surface stresses, sensible heat flux and evaporation are computed over land surface as:

$$^uF_{ls}^U = \tau_{ls}^U = -\rho_{sa}\ C_l^D\ |V_0|\ U_{sa} \tag{49}$$

$$^uF_{ls}^V = \tau_{ls}^V = -\rho_{sa}\ C_l^D\ |V_0|\ V_{sa} \tag{50}$$

$$^uF_{ls}^{SE} = SHF_{ls} = \rho_{sa}\ C_l^H\ |V_0|\ c_p\ (T_{skin} - T_{sa}) \tag{51}$$

$$^uF_{ls}^Q = E_{ls} = \rho_{sa}\ C_l^H\ |V_0|\ \ max\ [\ \alpha_{sw} Q^{sat}(T_{skin}, p_s) - Q_{sa}\ ,\ 0\ ] \tag{52}$$

where the drag coefficient C_l^D is an increasing function of topographic height, and the heat exchange coefficient C_l^H is dependent on the difference of potential temperature between the surface and the lowest model layer.

Over sea surface, surface stresses have the same formulation as over land, but with a constant drag coefficient C_s^D, while sensible heat and moisture fluxes:

$$ {}^{u}F_{ss}^{SE} = SHF_{ss} = \rho_{sa}\ C_s^H\ |V_0|\ c_p\ (T_{sea} - T_{sa}) \tag{53} $$

$$ {}^{u}F_{ss}^{Q} = E_{ss} = \rho_{sa}\ C_s^H\ |V_0|\ \ max\ [\ Q^{sat}(T_{sea}, p_s) - Q_{sa}\ ,\ 0\ ] \tag{54} $$

use a heat exchange coefficient C_s^H with a different mean value but the same dependence on potential temperature differences as for the land fluxes.

Differently from most GCMs, SPEEDY uses a fractional land-sea mask rather than a binary one. This feature allows a more accurate interpolation of surface fluxes over sea in view of future coupling with ocean models of different resolution and domain. So, in all grid points where the land-fraction is between 0.1 and 0.9, both land and sea surface fields are defined, and grid-point-average surface fluxes are defined as weighted averages of land and sea fluxes. This approach is also used to compute the emission of longwave radiation from the surface from the Stefan-Boltzmann formula.

A.7 Vertical diffusion.

Three different processes are modelled by the vertical diffusion scheme:

- a shallow convection, which redistributes moisture and dry static energy between the two lowest layers in cases of conditional instability;
- a (slow) diffusion of moisture in stable conditions;
- a (fast) redistribution of dry static energy occuring when the lapse rate is close to the dry-adiabatic limit.

For shallow convection (acting between levels N and $N - 1$), the condition of occurrence is:

$$ MSE_N > MSS_{N-1} \tag{55} $$

where the use of MSE instead of MSS at level N prevents the process from occurring in dry regions. If the above condition is satisfied, net upward fluxes of dry static energy and moisture are defined at the interface between the two layers:

$$ {}^{u}F_{N-h}^{SE} \ = \ F_{shc}^{*}\ (MSE_N - MSS_{N-1}) \tag{56} $$

$$ {}^{u}F^{Q}_{N-h} \;=\; F^{*}_{shc}\; Q^{sat}_{N} \;\; max\;(RH_N - RH_{N-1}\;,\;0\;) \tag{57} $$

where

$$ F^{*}_{shc} \;=\; \frac{\Delta p_N}{g\;\tau_{shc}} \tag{58} $$

In stable conditions, or at higher levels, vertical diffusion is activated wherever the vertical gradient of a (scalar) variable is outside some reference bounds. Let $\mathit{\Gamma}^A$ be the reference gradient of variable A with respect to a generic vertical coordinate Z. The difference between the values of A at adjacent levels is checked against $\mathit{\Gamma}^A$, and if:

$$ A_{k+1} - A_k \;>\; \Gamma^A\;(\;Z_{k+1} - Z_k\;) \tag{59} $$

then a net upward flux of A is defined at the interface between the two layers:

$$ {}^{u}F^{A}_{k+h} \;=\; F^{*}_{vdf}\;(\;A^{*}_{k} - A_k\;) \tag{60} $$

where

$$ F^{*}_{vdf} \;=\; \frac{C_0\;p_s}{g\;\tau_{vdf}} \tag{61} $$

and A^{*}_{k} is defined below.

If the adimensional coefficient C_0 is equal to the average σ-depth of model layers, then the tendencies implied by Eq. (60) are (approximately) equivalent to a relaxation of A_k towards A^{*}_{k} with time scale τ_{vdf}. Different time scales may be used for different variables and/or stability conditions.

For moisture diffusion, a reference gradient of relative humidity ($\mathit{\Gamma}^{RH}$) with respect to σ is specified, so that Eqs. (59-60) become:

$$ RH_{k+1} - RH_k \;>\; \Gamma^{RH}\;(\;\sigma_{k+1} - \sigma_k\;) \tag{62} $$

$$ {}^{u}F^{Q}_{k+h} \;=\; F^{*}_{vdf}\;Q^{sat}_{k}\;(\;RH_{k+1} - RH_k\;) \tag{63} $$

For dry static energy, the gradient $\mathit{\Gamma}^{SE}$ with respect to geopotential is used in Eq. (59); this parameter is linearly related to the ratio between the actual and the dry-adiabatic lapse rate:

$$ \Gamma^{SE} \;=\; \frac{\partial SE}{\partial \Phi} \;=\; 1 + \frac{c_p}{g}\,\frac{\partial T}{\partial z} \tag{64} $$

Diffusion occurs when $SE_k < SE^{*}_{k}$, where:

$$ SE^{*}_{k} \;=\; SE_{k+1} \;+\; \Gamma^{SE}\;(\;\Phi_k - \Phi_{k+1}\;) \tag{65} $$

A.8 Table of constants

Symbol	Equation	Process/Definition	Value
		Convection	
RH_{cnv}	6	Relative humidity threshold in PBL	0.8
Q_{cnv}	6	Absolute humidity threshold in PBL	15 g/kg
τ_{cnv}	11, 12	Relaxation time for PBL humidity	6 hours
RH'_{cnv}	20	Relative humidity threshold in intermediate layers	0.7
ϵ'	20	Reduction factor for mass flux of non – precip. clouds	0.5
		Large – scale condensation	
τ_{lsc}	21	Relaxation time for humidity	4 hours
RH^{1}_{lsc}	23	Relative humidity threshold at $\sigma = 1$	0.9
ΔRH_{lsc}	23	Vertical range of relative humidity threshold	0.1
		Clouds and shortwave radiation	
RH_{cl}	25	Relative humidity limit corresp. to cloud cover $= 0$	0.45
RH'_{cl}	(text)	Relative humidity limit corresp. to cloud cover $= 1$	0.85
Q_{cl}	26	Absolute humidity threshold for cloud cover	0.1 g/kg $(\sigma < 0.5)$
			1.0 g/kg $(\sigma \geq 0.5)$
A_{cl}	30	Cloud albedo (for cloud cover $=$ 1)	0.4
		Longwave radiation	
α^{LR}_{win}	34	Absorptivity coef. for dry air in window band	0.7
α^{LR}_{cl}	34	Absorptivity coef. for clouds in window band	12.0
$\alpha^{LR}_{CO_2}$	36	Absorptivity coef. in CO_2 band	4.0
α^{LR}_{wv1}	37	Absorptivity coef. in weak – abs. water vapour band	0.7 $(g/kg)^{-1}$
α^{LR}_{wv2}	38	Absorptivity coef. in strong – abs. water vapour band	50.0 $(g/kg)^{-1}$
ϵ_s	42	Surface longwave emissivity	0.98
		Surface fluxes	
f_{wind}	45	Ratio of near – surf. wind to lowest – level wind	0.6
V_{gust}	46	Wind speed of sub – grid scale gusts	5 m/s
β_{skin}	47	Coef. for diurnal cycle correction of skin temperature	10^{-2} °K W^{-1} m^2
D_{top}	48	Depth of top soil layer	7 cm
D_{root}	48	Depth of root layer	21 cm
W_{cap}	48	Soil wetness at field capacity (volume fraction)	0.30
W_{wil}	48	Soil wetness at wilting point (volume fraction)	0.17
C^{D}_{l}	49, 50	Drag coefficient over land	$1.8 \cdot 10^{-3}$ (1)
C^{D}_{s}	(text)	Drag coefficient over sea	$0.8 \cdot 10^{-3}$
C^{H}_{l}	51, 52	Heat exchange coefficient over land	$1.2 \cdot 10^{-3}$ (2)
C^{H}_{s}	53, 54	Heat exchange coefficient over sea	$0.8 \cdot 10^{-3}$ (2)
		Vertical diffusion	
τ_{shc}	58	Relaxation time for shallow convection	24 hours
τ_{vdf}	61	Relaxation time for vertical diffusion	40 hours (humidity)
			5 hours (dry st. en.)
Γ_{RH}	62	Reference gradient of relative humidity (w.r.t. σ)	0.5
Γ_{SE}	64, 65	Reference gradient of dry static energy (w.r.t. geopot.)	0.1

(1) Value for topographic height = 0 . (2) Value for neutral stability conditions.

A.9 References

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