Visualization of quantized vortex dynamics

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We discuss an experimental technique developed to visualize quantized vortices in helium II. We illuminate micron-sized solid particles of hydrogen suspended in the fluid, and mark quantized vortex cores within the volume observed by a camera. While, under some circumstances, the particles modify vortex dynamics by the action of viscous drag and by pinning vortex intersections, we do capture basic vortex dynamics such as vortex ring decay, vortex reconnection, and superfluid turbulence.

1 Introduction

At the Symposium, the senior author of this article presented a talk describing the gains made by working with cryogenic helium in the experimental studies of fluid dynamics. Among these advantages is the ability to generate flows at very high Reynolds and Rayleigh numbers and to span many decades of these parameters in a single apparatus. These aspects have been described in adequate detail in a recent review article [1], and need not be restated here. We describe briefly the second part of the talk dealing with the fluid dynamics of helium II. Even though certain properties of helium II such as superfluidity have no counterparts in classical fluid dynamics, we shall be concerned here with the dynamics of vortices associated with the superfluid part of helium II. These vortices are of the order of an angstrom in diameter and the circulation around them is quantized. However, their properties beyond a few diameters from the core of these vortices are quite classical in nature and can be understood largely in those terms.

Above $T_\lambda \approx 2.17K$, liquid helium behaves like a classical fluid and the Navier-Stokes equations describe its dynamics. As the fluid temperature is lowered, a phase transition at T_λ introduces more complicated behavior, and the fluid acquires superfluid properties. At these lower temperatures, we must appeal to quantum mechanics to describe even the macroscopic motions of the fluid [2]. One consequence of this is that vortices can have quantized circulation [3], rather than arbitrary circulation as in a classical fluid. We discuss an experimental technique presented by Bewley et al. [4] for marking the cores of these quantized vortices in the superfluid phase of liquid helium. We show that the vortices trap small particles of solid hydrogen, and observe the behavior of filaments formed by the collection of many such particles along the vortex core. For the first time, it is possible to observe directly in this way the interactions of quantized vortices and to locally probe superfluid turbulence.

There is a long tradition of studying fluid behavior by observing the motions of tracer particles (for examples, see [5]), and several groups have used small particles to study the motions of liquid helium (e.g., [6] [7] [8]). In liquid helium, the challenge is to generate particles that are small enough to trace accurately the flow [9], and to understand the response of the particles to the motions of the superfluid [10].

Other researchers have detected quantized vortices indirectly by a variety of means, including using the vortex attenuation of second sound in the superfluid [11]. Until now, individual vortices have been resolved only in two-dimensional projections, where the vortices are straight and parallel to each other. This technique was perfected by Yarmchuk et al. [12], who observed vortices at their intersection with the top boundary of the fluid, and determined that in solid body rotation, the superfluid formed a triangular lattice of vortices. Our uniqueness of our method is that it allows the study of the dynamics of quantized vortices in the bulk of the fluid.

2 Apparatus

We generate small solid hydrogen particles in liquid helium contained in a cryostat with optical access provided by four windows [13]. We inject a room-temperature mixture of hydrogen gas diluted with helium gas into normal liquid helium, at a temperature above T_λ. The injector tube has a $3mm$ inner diameter, and opens below the free surface of the liquid helium. The procedure yields particles with a wide size distribution, though most are probably about $2\mu m$ in diameter or smaller. We estimate the characteristic particle size by dividing the known mass of hydrogen injected among the number of particles in the volume of liquid, which we find by counting particle images. Our experience has been that the observations described below are possible only if the particles form in the warmer phase of liquid helium. The suspension of particles is subsequently cooled through the transition temperature to a desired temperature below T_λ. We cool the liquid helium by evaporation using a mechanical vacuum pump. The pumping rate is controlled manually using a diaphragm valve, and we measure temperature by determining the resistance of a calibrated semiconductor probe.

Some of our images are acquired when the cryostat is rotating, others are not. In the absence of system rotation, we acquire images of the particle suspension during the cooling process, and record the time and temperature for each image. We use a digital movie camera with $16\mu m/pixel$ resolution that is focused on a laser-illuminated sheet approximately $100\mu m$ thick. We do not disturb the fluid, except by the thermally driven flows caused by the cooling process.

The cryostat and camera are mounted on an air bearing that allows them to rotate freely about the vertical axis. The laser beam is passed to the rotating apparatus in such a way that they rotate in unison. Rotation complicates the method for acquiring images because the cryostat is initially tethered to a vacuum pump in order to cool the liquid helium. In the absence of cooling, the temperature of the fluid rises due to heat leaks into the cryostat. We therefore cool the fluid to below the phase transition temperature before spin-up, and disconnect the vacuum pump in order to spin the cryostat independently of the pump. We cool the fluid by an amount sufficient to keep the fluid below the transition temperature throughout the spin-up of the cryostat and fluid, and for the period thereafter during which measurements are made. The steady state behavior of superfluid helium at a fixed rotation rate does not depend on temperature [14].

3 Results

3.1 Evidence

Images acquired above T_λ show that particles are randomly distributed (see Fig. 1a). As we cool the fluid without rotation, the images show that a fraction of the suspended particles collect onto slender filaments, often several millimetres in length (see Fig. 1b). These appear when the temperature is from a few millikelvin below T_λ down to $1.9K$, which is the lowest temperature we have explored. The filaments evolve slowly as they drift upward through the observation volume at a rate of roughly $1mm/s$. The remaining particles are randomly distributed, as is the case above T_λ (see Fig. 1a). In Bewley et al. [4], we presented briefly the evidence that suggests the observed filaments are formed by particles trapped on quantized vortex cores. Here, we consider this evidence and our assumptions in greater detail.

The first piece of evidence that the particles mark the cores of the quantized vortices is that the filaments appear only in images taken at temperatures when quantized vortices exist—that is, at temperatures below T_λ. Parks and Donnelly [15] describe a mechanism by which quantized vortices trap ions. Ions are drawn to the cores of quantized vortices by the steep pressure gradient supporting the circulating superfluid. Hydrogen particles are hydrodynamically comparable to ions, though the ions are much smaller. Since both the ions and the hydrogen particles are substantially larger than a quantized vortex core, the mechanism is equally applicable to both entities.

A second piece of evidence is that when the liquid helium cell is set in steady rotation, the particles arrange themselves along uniformly spaced lines parallel to the rotation axis, as shown in Fig. 1c. This observation agrees with the expectation that quantized vortices form a rectilinear lattice aligned with the axis [2], since the vorticity of rotation resides in the quantized vortices.

Feynman [2] showed that the population of quantized vortices aligned with the axis of rotation in steadily rotating liquid helium should have at equilibrium a number density equal to

$$n_o = 2\Omega/\kappa = 2000\Omega vortices/cm^2 \tag{1}$$

where Ω is the angular velocity of the system in $radians/s$ and $\kappa = h/m$ is the quantum of circulation, h being the Planck's constant and m the mass of the helium atom. Additional evidence that the columns of particles described above are vortices is that the number density of lines per unit area normal to the axis of rotation is consistent with Feynman's rule (1) for a series of rotation rates. For system rotation rates from $0.15Hz$ to $0.46Hz$, we calculate the number density of vortex lines observed in each state, according to assumptions described below. The value we measure is typically 25% larger than is predicted by (1).

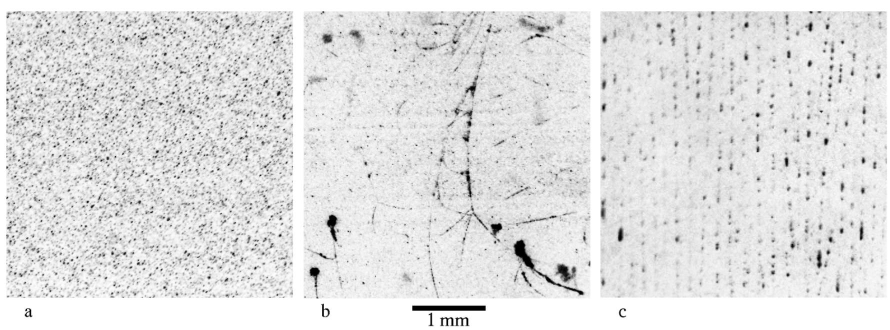

Fig. 1. (a) shows hydrogen particles in liquid helium at a temperature above T_λ. (b) shows similar particles after the fluid is cooled to a temperature below T_λ. Some of the particles have collected along branching filaments. (c) shows particles arrayed along vertical lines in liquid helium that is steadily rotating at $\Omega = 0.3Hz$ at a temperature slightly less than T_λ.

3.2 Discussion of assumptions

From the images of particles in rotating superfluids helium, we calculate the number density of lines under several assumptions. We argue that the measurement of line density is not sensitive to these assumptions, and that the conclusion is favorable even when the conditions hold only approximately.

The array of vortices appears rectilinear in the plane of observation, but we cannot know in what way the vortices are arranged in the normal plane. In (1), we have assumed that the intersections of the vortex cores with the light sheet are at the corners of the theoretical triangular array (see [16]) and that the laser light sheet illuminates a section of such a triangular array. If, for example, the lines in the array were on the corners of squares instead of triangles, we err by about 14% in computing the vortex line density, which nudges the corresponding theoretical line towards the data.

We assume that the light sheet is aligned with the lattice in such a way that the vortices appear equally spaced in the image, and that the separation between them is the minimum spacing of vortices in the lattice. Deviations from these assumptions lead to a calculation of line density that is in excess by about 33%. We also assume that the thickness of the illuminating sheet is roughly equal to the lattice spacing, so that it illuminates only one row of lines in the lattice. As mentioned above, the thickness of the illuminating sheet is about $100\mu m$. For the range of rotation rates explored we expect from Feynman's rule (1) that the minimum separation between vortices is between 140 and $250\mu m$, and so our assumption is reasonable.

Finally, we assume that every vortex in the fluid is decorated with particles, and will be observed if illuminated. In making our estimates, we have assumed that all vortices are singly quantized, as favored by energy considerations. Our vortex count provides only a lower bound for the total circulation they produce, if there exist multiply quantized vortices. We take our results to be consistent with the view that each of the lines revealed by particles is singly quantized.

3.3 Vortex dynamics

Using the technique for marking quantized vortex cores, we are able to study fundamental vortex dynamics. We observe, for example, the decay of vortex rings and the reconnection of pairs of vortices (see Fig. 2). In addition, we have observed phenomena which indicate that under certain conditions, particles are not passive tracers of vortex dynamics. For example, continuously decorated filaments form branches rather than reconnecting, and a vortex ring decays more slowly than predicted by the theory when its core is completely covered with particles.

It is evident in Fig. 1b that the continuously decorated filaments form networks with stable forks, whereas simulations show vortex filaments as smooth curves with brief dynamic intersections through reconnections [19]. We are unaware of any discussion in the literature of the possibility that particles in superfluid helium transform the topology of a vortex tangle. This phenomenon may be related to ability of a particle to trap more than one quantized vortex, analogous to a condition examined by Tsubota in computer simulations [17].

We attribute the retarded decay time of a vortex ring (see Fig. 2, top row) to viscous drag on the particles, and conjecture that this drag is important

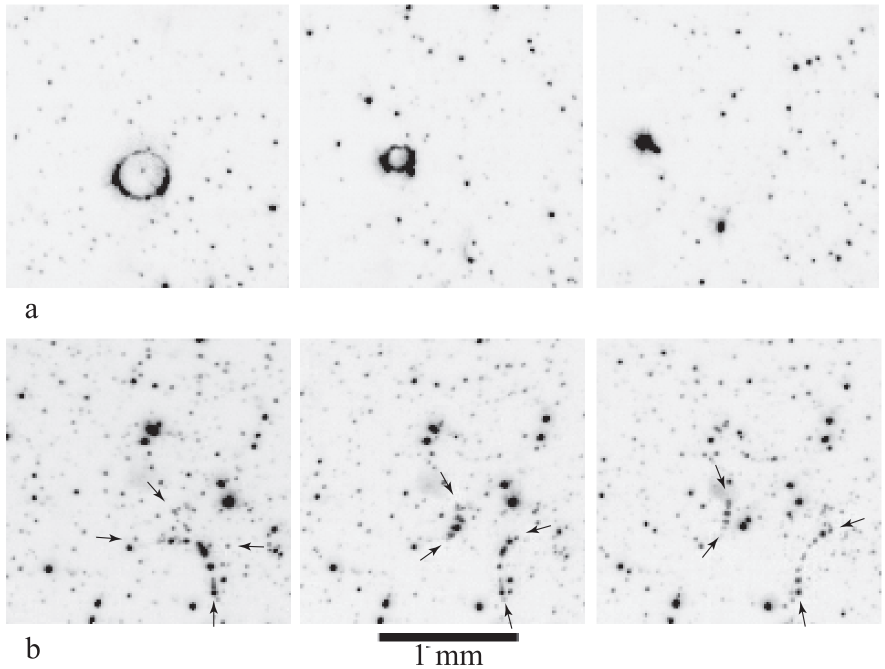

Fig. 2. Row (a) shows images taken 1.5s apart of a ring whose diameter decays until the particles form an amorphous aggregate. The theory [14] predicts that the ring should decay more than 3 times faster than it does. Row (b) shows what we believe is the reconnection of two vortices that have crossed. The vortices are marked by particles that have space between them, rather than by a continuous coating of particles. The left-most frame shows the two vortices just before the moment of reconnection. We have indicated where we believe the vortices leave the thin plane of illumination, and become invisible to the camera, with arrows. The subsequent two frames are taken 0.25 and 0.50s after the first, and show that the vortices have exchanged ends and are recoiling.

when the particles are spaced by less than about 10 particle diameters according to the following argument. We compare the drag per unit length on a vortex line due to friction with the normal fluid [14], which is a normal part of quantized vortex dynamics, to the Stokes drag on a spherical particle on the line per sphere diameter, which could modify the behavior of the quantized vortex. To a rough approximation, this ratio is $\rho_s \kappa / 3\pi\mu$, and its value is always less than one. This result suggests that the drag of particles trapped on the line dominates, as long as the line is continuously decorated with particles. However, for a vortex line with particles every 10 particle diameters, the drag contributions are comparable for a range of temperatures between 1.5 and 2.1K.

Under the condition that quantized vortices are decorated with particles spaced by about ten diameters, we observe vortex reconnections (see Fig. 2, bottom row). The evolution of the vortices after reconnection is in qualitative agreement with simulations [18] [19]. Details regarding these observations will appear in forthcoming publications.

4 Summary

We find that particles collect along filaments in superfluid helium and present evidence that these filaments mark the cores of quantized vortices. In this way, we have found a method for tracking the vortices, so that their cores can be viewed in the bulk for the first time. We observe that the filaments behave in some respects as is expected of quantized vortices. They align with the axis when the fluid is rotating, and the diameter of a ring decays with time due to interaction with the normal fluid. Because of this, we identify the filaments with quantized vortices. However, we propose that the hydrogen particles modify the behavior of the vortices in two ways. First, the viscous drag on the hydrogen particles is more important than mutual friction when the particles completely cover the line, and second, the hydrogen particles stabilize vortex intersections, which might not otherwise exist. Our observation of what appears to be a decaying vortex ring further validates an explanation of dissipation in quantized vortex turbulence, and confirms that the hydrogen spines do not immobilize the quantized vortices, or render them rigid. We predict that if particles are spaced along a vortex line by more than ten diameters, the vortex may behave as if bare of particles, but still be observable. In fact, we have recorded images of what we believe are vortices with widely and evenly spaced particles, and captured such dotted lines reconnecting with each other.

References

1. Niemela JJ, Sreenivasan KR (2006) The use of cryogenic helium for classical turbulence: Promises and hurdles. J. Low Temp. Phys. 143:163–212.
2. Feynman RP (1955) Application of quantum mechanics to liquid helium. In: Gorter CJ (ed) Progress in low temperature physics vol 1. North Holland, Amsterdam
3. Onsager L (1953) Proc Intern Conf Theor Phys, Kyoto and Tokyo, Science Council of Japan, Tokyo:877–880
4. Bewley GP, Lathrop DP, Sreenivasan KR (2006) Nature 441:588.
5. Batchelor GK (1967) An introduction to fluid dynamics. Cambridge University Press, Cambridge
6. Chopra KL, Brown JB (1957) Phys Rev 108:157
7. Murakami M, Ichikawa N (1989) Cryogenics 29:438–443
8. Zhang T, Van Sciver, SW (2005) Nature Physics 1:36–38

9. White CM, Karpetis AN, Sreenivasan KR (2002) J Fluid Mech 452:189–197
10. Poole DR, Barenghi CF, Sergeev YA, Vinen WF (2005) Phys Rev B 71:064514-1–16
11. Vinen WF (1957) Proc Rot Soc A 240:114–127
12. Yarmchuk EJ, Gordon MJV, Packard RE (1979) Phys Rev Lett 43:214–218
13. Bewley GP, Sreenivasan KR, Lathrop DP (2007) submitted to Expts in Fluids
14. Donnelly RJ (1991) Quantized Vortices in Helium II. Cambridge University Press, Cambridge
15. Parks PE, Donnelly RJ (1966) Phys Rev Lett 16:45–48
16. Campbell LJ, Ziff RM (1979) Phys Rev B 20:1886–1902
17. Tsubota M (1994) Phys Rev B 50:579–581
18. Koplik J, Levine H (1993) Phys Rev Lett 71:1375–1378
19. Schwartz KW (1985) Phys Rev B 31:5782–5804