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# Some comments on nonlinear dynamics

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**Abstract.** I summarize here the remarks made at the closing of the Conference and Research Workshop: Perspectives on Nonlinear Dynamics, held in Trieste in July 2007.

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The late Joseph Ford of the Georgia Institute of Technology often declared that nonlinear dynamics and chaos formed the third scientific revolution of the twentieth century, on par with quantum mechanics and general relativity [1]. Several seminal papers that laid the foundation for modern nonlinear dynamics appeared in the 1960s and continued through the 1980s. By the middle of this period, a reasonably articulate community of nonlinear dynamics and chaos had emerged. Though the members of this community were initiated in disparate disciplines, they seemed to be able to communicate with each other sensibly. Some in this audience will no doubt remember the excitement from the late seventies and early eighties. The excitement was that one could understand our world to be both random and deterministic at the same time, and that the juxtaposition between randomness and determinism was not the property of some special circumstances but of nearly every dynamical behavior. This broad concept and the detailed work that followed somehow brought disparate fields under one umbrella.

I myself became actively involved in the community in the early 1980s, and remember being thrilled when asked to lecture in nonlinear dynamics meetings. It was indeed then that my interest in multidisciplinary science was truly aroused. Even if we ignore the pop-culture exaggerations that surrounded the subject on a few occasions [2], it was clear that something fresh was in the air. The freshness was not especially that the physical problems that were being addressed were new, although that too was true, but had to do with the new way of doing science.

So what exactly was new? I think that it was the realization that: (1) simple dynamical equations can display rich structure which could be understood mathematically; (2) simple dynamical equations apply to complex problems, at least to some aspects thereof; (3) there exists a great degree of universality, which had some profound mathematical underpinnings, thus connecting several branches of nonlinear science, and fostering interdisciplinarity; (4) careful and controlled experiments do indeed support this universality, at least in some circumstances (which, however, are not clear *a priori*).

We are now so used to these ideas, or expectations, that it is useful to recall how nonlinear science was being done in the immediately preceding era. The subject consisted in part of special solutions which were extracted ingeniously from dynamical equations without actually solving them [3]. There was the notion of scaling that came from Kolmogorov's brilliant work in hydrodynamic turbulence [4], culminating in critical phenomena and renormalization [5]. Then came a set of new developments in ODE's such as Smale's horseshoe [6], strange attractors [7], algorithmic complexity [8], scenarios of chaos [9], multifractals [10], and so forth. These and other pieces were pulled together in this new field, and the effort has been advancing ever since. We no doubt also remember that several new journals came up in nonlinear science all at once [11].

What is happening presently? Needless to say, my view is at best partial. I fear that a little bit of disappointment has set in, and the promise of the heady days does not seem to have been realized. We know how much mathematical work is involved in proving theorems about the simplest of dynamical systems [12]. While it is true that simple dynamics could lead to complex behaviors, it is not the case that all complex systems are governed by simple dynamics. It appears not to be true that universality is the overriding principle that organizes nonlinear systems; if one digs deeper, many of the crucial effects in important problems are produced by non-universal effects coming from boundary and initial conditions, memory effects, and so on. And, as in biology, sometimes one has to know nearly everything about a system before one understands it well enough for functional purposes. Perhaps chemistry offers a better comparison: there are general principles just as there are particular aspects of a complex compound. One cannot easily transfer the experience of one field to another simply because some results in the two fields are superficially similar; a great deal of effort is needed to make such a transition, bringing back memories of earlier days I just mentioned. Altogether, nonlinear dynamics seems to be subject to the same splintering and centrifuging effects of the past.

Does it mean that a viable nonlinear dynamics community does not exist anymore? Or, more practically, does it mean that the community such as now exists is simply working in the backwaters of a short-lived era of glory? I think not, but do believe that the community needs some reorientation, as I shall discuss below. It is partly because of this belief that I was strongly in favor of this conference in the first place and why I am writing this short piece.

I think that the spirit of the early mesmerizing days of nonlinear dynamics is valuable to preserve. The view that somehow one can understand anything nonlinear, even if it is only up to a point, by following common threads of analysis, is a valuable lesson that one cannot and should not let go. The confidence that one can understand any nonlinear problem by bringing to bear the cumulative experience with other nonlinear problems is important for the growth of this science. This attitude is also important because of the cross fertilization that will invariably emerge, eventually fulfilling the expectations – even if the progress might be slow. For instance, that Levy flights and random walks underlie the spread of infectious diseases is both exciting and practical; that one can generate a beautiful mathematical problem such as topological frustrations from the messy problems of glassy systems is not a trifle.

Nevertheless, the field is becoming somewhat unglued and the standards for its evolution are becoming uneven. Great papers that are torch-bearers for the field are becoming increasingly rare. This situation may well be a reflection of the inherent complexity of nonlinear problems. It shows that we have only scratched the surface of our understanding of nonlinear problems, and that much more lies ahead. While we continue to attack new problems, however, I fear that much of the work in the field is becoming mundane. This situation could be detrimental for the future of nonlinear science and I worry that this is already the perception of people from the outside. A case in point is that very few people in nonlinear science have actually become famous in a broader setting, received highly prestigious prizes, and so forth. More important perhaps is that the penetration of the ideas of nonlinear dynamics in other fields has not been all that spectacular.

For the field to thrive and optimism to build, nothing will serve the cause better than new and solid results even if in specialized areas. There needs to be an honest sense of self-examination which can separate the wheat from chaff, discouraging the mediocre and supporting the substantial. This condition is necessary in any field but is especially true in something inherently interdisciplinary such as nonlinear dynamics, where the tendency sometimes is to scratch the surface but claim the bulk as finished, or simulate a problem under certain specific conditions but believe that the outcome is general. We have to keep supporting the best work possible by recognizing it to be as such and encouraging youngsters to acquire a sense of what it means to do good work without worrying about fame, prizes, and such external recognitions.

One problem I see is that the education in nonlinear dynamics is not solid. Students are exposed to a few tools and guided in other aspects by shallow exposures. I argue in favor of evolving something like a minimum education for a new recruit into nonlinear dynamics (see Appendix), and would like to see at least one graduate program of this sort to evolve. Unless we demand of students a certain set of minimum standards of knowledge in different sciences in which nonlinear dynamics plays a crucial role, the common threads that hold our field together will weaken. We should not let this happen.

Overall, my feeling is one of optimism and pessimism both at the same time. Perhaps one calls it realism that comes with age.

## Appendix

To contribute something lasting and yet retain interdisciplinarity, one has to know the substance of several fields and cannot move across them with only the knowledge of a set of tools. A good student in nonlinear dynamics should learn about the important dynamical problems in a few fields just as she should master essential techniques. One can imagine a number of courses – typical of the preparatory requirements of a good graduate program in many US universities – which might more or less fill these needs. I have attempted to list my view of what those courses may contain, being fully aware that it might need modifications, and that some students may want to skip one or two of these courses. The list should be supplemented by basic courses in mathematical, numerical and experimental methods, and I offer it as a target for criticism.

### *1,2. Mathematics*

Nonlinear oscillations, dynamical systems, bifurcations, critical points, singularities, two and three body problems, Lagrangian methods, KAM theory, ergodic theory, invariant measures, dynamical chaos, strange attractors, routes to chaos, quantum chaos, multifractals, elementary methods for spatially extended systems, etc.

Time series analysis, wavelets, stochastic differential equations, Levy flights and stable distributions, etc.

### *3,4. Physics*

Brownian motion, equilibrium statistical mechanics, phase transitions and critical phenomena, Landau theory, fluctuations and anomalous dimensions, scaling (static, dynamic and non-equilibrium), self-similarity, renormalization group, etc.

Far-from equilibrium systems, self-organization, pattern formation, defects, wave propagation, instabilities and hydrodynamic turbulence, complexity, etc.

### *5. Biology*

Biological clocks and rhythms, neurons and networks, DNA sequencing and genomics, protein dynamics, collective motion, competition models and extinction, population models and evolutionary biology, ecological models, biological complexity, animal gait, visual systems, biological information and complexity, etc.

### *6,7. Special topics*

Plasticity and thermomechanics, granular flows, polymers, liquid crystals, DLA, plasma dynamics and instabilities, magnetic reconnections, astrophysical problems, nonlinear optics, chemically reacting systems, Bose–Einstein condensation, percolation, disordered materials, glassy systems, etc.

### *8. Earth sciences*

Nonlinear dynamics of the lithosphere and earthquake prediction, volcanoes, sand dunes, transport in partially saturated heterogeneous soils and fractured rocks and ground water systems, ocean–atmosphere coupling, large-scale weather systems including monsoons, climate change, paloeclimate, etc.

*9. Social sciences*

Financial and monetary policies, theories of risk, business cycles, emergence of social, cultural and economical order, epidemiology, etc.

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[2] J Gleick, *Chaos: Making a new science* (Viking Penguin, 1987)
[3] Perhaps a good example of this sort of work is fluid dynamics, especially from the boundary layer theory; see, for example, H Schlichting, *Boundary-layer theory* (McGraw Hill, 1956)
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[10] See, for example, T C Halsey, M H Jensen, L P Kadanoff, I Procaccia and B I Shraiman, *Phys. Rev.* **A33**, 1141 (1986)
B B Mandelbrot, *Proc. R. Soc.* **A423**, 4 (1989)
[11] At about the same time, several journals such as Journal of Nonlinear Science, International Journal of Bifurcation and Chaos, Chaos, Nonlinearity, came into existence
[12] See, for example, M Benedicks and L Carleson, *Ann. Math.* **133**, 73 (1991)