Formation of the "Superconducting" Core in Turbulent Thermal Convection

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A sinusoidal temperature perturbation is superimposed on the bottom plate of a cylindrical convection cell, and its decay is measured at the cell midheight. Rayleigh numbers up to $\mathrm{Ra} = 10^{13}$ and aspect ratios 1 and 4 are considered. The technique allows a dynamic measurement of the height of the layer interposed between the superconducting core and the boundary. This deduced height is in good agreement with results from recent numerical simulations.

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Turbulence in thermal convection enhances heat transport manyfold. The enhancement factor could be a few thousand in geophysical convection and larger by another order of magnitude in stellar convection [1]. A simplified paradigm of convection is the so-called Rayleigh-Bénard problem, which consists of a layer of fluid between two horizontal plates, with the bottom plate heated and the top one cooled; the side walls are nonconducting. For analytical understanding, it is conventional to divide the convection layer into the thermal boundary layer and the bulk [2,3]. One occasionally assumes a three-layer structure [4] to account for anomalous heat transport. In this Letter, we experimentally assess the structure of the flow by sinusoidally modulating the heating of the lower boundary about a mean value and observing the decay of the thermal wave at the midheight of the cell.

For reference, one defines the characteristic measure of the vertical temperature difference across the fluid layer as the Rayleigh number $\mathrm{Ra} \equiv \alpha \Delta T g H^3/\nu\kappa$, where α is the isobaric thermal expansion coefficient of the fluid, ΔT the temperature difference between the bottom and top plates separated by the vertical distance H, and g the acceleration due to gravity; ν and κ are, respectively, the kinematic viscosity and thermal diffusivity of the fluid ($\kappa \equiv k/\rho C_P$, where k is the thermal conductivity, ρ is the density, and C_P is the isobaric specific heat of the fluid). The ratio of the total heat flux to the pure conduction value (corresponding to the same temperature difference) is the Nusselt number Nu, which is a function of the Rayleigh number when the fluid Prandtl number is fixed (as is true for the present measurements). The dynamical characteristics of the flow originate from plumes and thermals emanating from unsteady boundary layers, leading to a large-scale mean wind circulating along the periphery of the convection cell [5–8]. These are all coupled phenomena.

The notion that there is a central "superconducting" core is fundamental to all arguments seeking an asymptotic heat transfer law. The measurements presented here are designed to shed light on this feature of turbulent convection. At moderately low Ra, we demonstrate the absence of the turbulent core, whereas, for very high Ra, the existence of the core region can indeed be deduced; the results agree well with profiles of the root-mean square (rms) of the vertical temperature presented by Amati *et al.* [9]. The intervening fluid layer is substantially larger than the traditional thermal boundary layer and appears to scale differently.

The apparatus is the same as that described earlier [10–12]. Briefly, it consists of a cylinder of 50 cm diameter with stainless steel sidewalls, 0.267 cm thick, containing helium gas near 5 K, and surrounded by several radiation shields within a common vacuum space. The height could be adjusted to give different aspect ratios Γ, defined as the ratio of the diameter to the height. The 3.8 cm thick top and bottom plates were fabricated from copper annealed under oxygen-free conditions, with thermal conductivity of the order of 1 $\mathrm{kW\,m^{-1}\,K^{-1}}$ at the temperature of these measurements. Measurements were initiated only after the steady state was attained.

For all experiments, the top plate of the cell was operated at a regulated constant temperature maintained by means of a resistance bridge and servo. The bottom plate was subject to a time-varying heat flux about a nonzero mean, such that its temperature is $T_B(t) = \langle T_B \rangle + T_{B0}\cos(\omega t)$, with the modulation frequency $\omega = 2\pi f_M$, where f_M has units of Hertz and $\langle \ldots \rangle$ refers to averaging over integral periods of the modulation. The negligible heat capacity of the copper plates at the low temperatures greatly facilitates the temperature modulation. We can define a dimensionless amplitude as $\Delta_M = (T_{B0})_{\mathrm{rms}}/\langle \Delta T \rangle$, where $(T_{B0})_{\mathrm{rms}}$ is the rms value of the measured amplitude of the temperature modulation at the bottom plate. In response to the modulation, a thermal wave propagates into the fluid at the forcing frequency ω and is damped out exponentially with the rms value at a position z from the bottom wall given by

$$T_M^{\mathrm{CALC}}(z, f_M) = (T_{B0})_{\mathrm{rms}} \exp(-z/\delta_S), \qquad (1)$$

where δ_S is the so-called Stokes layer thickness, or the penetration depth, given by

$$\delta_S = \sqrt{\frac{\kappa^{\text{eff}}}{\pi f_M}}. \tag{2}$$

Here κ^{eff} is an effective thermal diffusivity characteristic of the turbulent fluid. By the definition of the Nusselt number as the ratio of the effective thermal conductivity to its molecular value, it follows that

$$\kappa^{\text{eff}} = \text{Nu}(\text{Ra})\kappa. \tag{3}$$

Indeed, for moderate Ra, it is possible to estimate Nu from the spectral density of fluctuations measured at the cell midheight, as will be demonstrated below. The physics is more interesting when this estimate does not work. Specifically, at very high Ra we may expect that a nearly superconducting core region develops, within which the thermal wave propagates without attenuation. From the measured temperature amplitude of the thermal wave at the cell midheight, $T_M^{\text{MEAS}}(H/2, f_M)$, and, by using (1)–(3), we can deduce the height of the layer $\tilde{z}$ near the boundary within which all of the attenuation occurs:

$$\tilde{z} = \ln\left(\frac{(T_{B0})_{\text{rms}}}{T_M^{\text{MEAS}}(H/2, f_M)}\right)\sqrt{\frac{\text{Nu}(\text{Ra})\kappa}{\pi f_M}}. \tag{4}$$

Clearly, $\tilde{z} \leq H/2$, with the equality corresponding to the absence of a core region.

Figure 1 shows a segment of the bottom plate temperature measured by an imbedded resistance thermometer, together with the cosine fit. The standard deviation of the resulting temperature oscillation is $(T_{B0})_{\text{rms}} = 0.007\,06$ K and the dimensionless modulation amplitude $\Delta_M \equiv (T_{B0})_{\text{rms}}/\langle\Delta T\rangle \simeq 0.05$. Here the time-averaged $\langle\text{Ra}\rangle = 3.43 \times 10^9$, $\kappa = 4.03 \times 10^{-2}$ cm^2 s^{-1}, and $f_M = 0.01$ Hz, which is about a fifth of the frequency associated with the mean wind f_P—which is given roughly by the wind speed divided by the effective circumferential path (see, e.g., Ref. [6]). The resulting time-averaged Nusselt number $\langle\text{Nu}\rangle = 98.6$, while for the same Ra the compilation of Ref. [12] for the case of steady heating gives Nu = 98.8, demonstrating that the modulation has a negligible effect on the time-averaged heat transport. As a check on repeatability, the present steady-state measurement of Nu = 99.8 at Ra = 3.54×10^9 agrees with Nu = 99.7 for the same Ra [12] well within measurement resolution. For purposes of these comparisons, we have used Nusselt numbers uncorrected for the effects of sidewall conductance [12,13] and finite plate conductivity [14], but these corrections do not affect the conclusions.

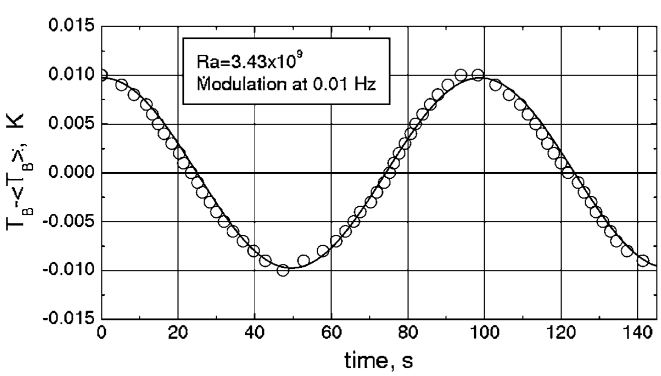

FIG. 1. A segment of the measured bottom plate temperature: $f_M = 0.01$ Hz, $\Delta_M = 0.05$, and Ra = 3.43×10^9. The mean temperature has been subtracted. The solid line is the cosine fit to the data.

Figure 2 shows temperature spectra obtained at the cell midheight using a 250 μm cubic germanium temperature sensor placed about 4 cm radially inboard from the sidewall. Temperature fluctuations were recorded at a rate of 50 Hz by using the off-balance signal from an audio frequency bridge circuit with lock-in detection.

Figure 2(a) shows the spectrum for steady heating and the location of the mean-wind peak for Ra = 3.5×10^9 and $\kappa = 4.03 \times 10^{-2}$ cm^2/s. The peak at f_P is not sharp, reflecting the wobbliness of the phenomenon. In Fig. 2(b),

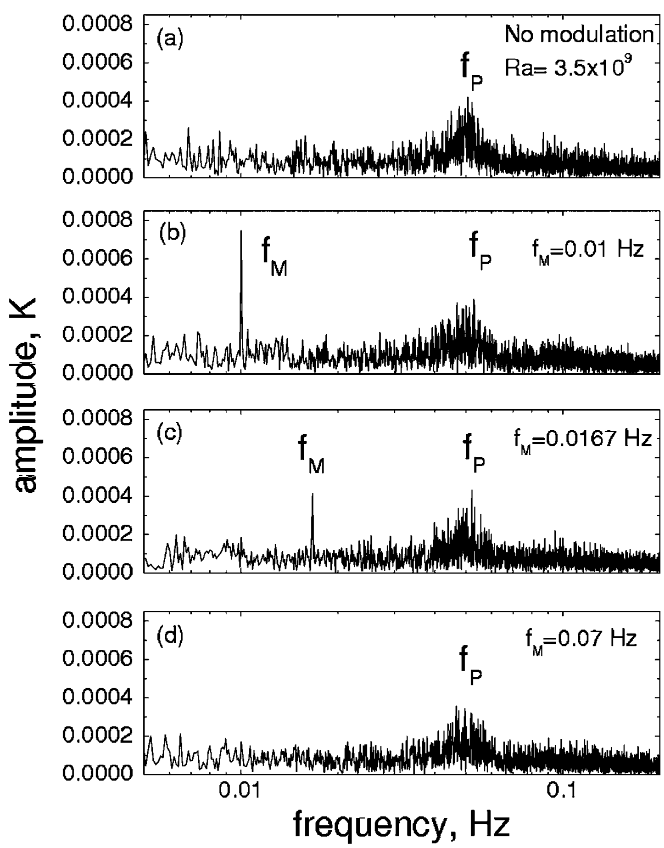

FIG. 2. Amplitude spectra for both steady and modulated heating. (a) Steady heating, Ra = 3.5×10^9; (b) modulated at $f_M = 0.01$ Hz, $\langle\text{Ra}\rangle = 3.4 \times 10^9$, $\Delta_M = 0.05$; (c) $f_M = 0.0167$ Hz, $\langle\text{Ra}\rangle = 3.5 \times 10^9$, $\Delta_M = 0.047$; (d) $f_M = 0.07$ Hz, $\langle\text{Ra}\rangle = 3.5 \times 10^9$, $\Delta_M = 0.022$. The position and amplitude of the spectral peak at f_P due to the mean wind remains essentially unchanged by the modulation. In (d), the modulation is too weak to be detected (see text).

we show spectra obtained for slightly smaller Ra $= 3.43 \times 10^9$, with $f_M = 0.01$ Hz, $(T_{B0})_{\rm rms} = 7.06 \times 10^{-3}$ K, and $\Delta_M = 0.05$. The measured spectral amplitude at the cell midheight 7.5×10^{-4} K is in excellent agreement with that calculated from (1), namely, $T_M^{\rm CALC}(H/2, f_M) = 7.7 \times 10^{-4}$ K. Evaluating (4) gives $\tilde{z}/H = 0.51$, demonstrating the absence of a turbulent core for these moderate Ra. Figure 2(c) shows the spectrum corresponding to Ra $= 3.5 \times 10^9$, with $f_M = 0.0167$ Hz, $(T_{B0})_{\rm rms} = 6.76 \times 10^{-3}$ K, and $\Delta_M = 0.047$. Here the spectral amplitude is 4.1×10^{-4} K, while (1) and (4) yield $T_M^{\rm CALC}(H/2, f_M) = 3.9 \times 10^{-4}$ K and $\tilde{z}/H = 0.49$, respectively, again showing that the predicted spectral amplitude is consistent with the absence of the turbulent core. Finally, in Fig. 2(d), we show the spectrum for Ra $= 3.5 \times 10^9$, $f_M = 0.07$ Hz, $(T_{B0})_{\rm rms} = 3.16 \times 10^{-3}$ K, and $\Delta_M = 0.022$. In this case, (2) yields $T_M^{\rm CALC}(H/2, f_M) = 9 \times 10^{-6}$ K, which is well below the background level of the temperature fluctuations, explaining the absence of the spectral peak corresponding to the modulation. There is no observable effect on the mean-wind peak for any modulations studied at these moderate values of Ra.

To check these results, we have performed similar experiments also in a cell of aspect ratio $\Gamma = 4$, for which the distance between the plate and the midplane sensors was 6.25 cm instead of 25 cm. Heat transport measurements in this cell are reported in Ref. [10]. For these experiments, Ra $= 1.9 \times 10^9$ with $\kappa = 7.04 \times 10^{-3}$ cm^2 s^{-1} and modulation parameters $f_M = 0.032$ Hz $\simeq f_P$, $(T_{B0})_{\rm rms} = 0.0229$ K, and relatively large $\Delta_M = 0.15$. In this case, simultaneous measurements were made near the sidewall (about 4 cm inboard as before) and at the cell center, both at the midplane height. The results of these measurements are shown in Figs. 3(b) and 3(c). The center and sidewall spectra have nearly identical values of the peak amplitude corresponding to the modulation frequency, namely, 1.78×10^{-3} and 1.71×10^{-3} K, respectively. These are in excellent agreement with the predicted values according to (1), using $T_M^{\rm CALC}(H/2, f_M) = 1.69 \times 10^{-3}$ K. Correspondingly, (4) yields $\tilde{z}/H = 0.49$ and 0.50, respectively, again showing the absence of a core region, as for aspect ratio unity.

Let us now consider higher Ra. The highest Ra investigated for $\Gamma = 1$ was Ra $= 10^{13}$ [Fig. 3(a)] with $\kappa = 4.79 \times 10^{-4}$ cm^2 s^{-1}, $(T_{B0})_{\rm rms} = 0.009$ K, $f_M = f_P = 0.025$ Hz, and $\Delta_M = 0.22$. From (1) we obtain $T_M^{\rm CALC}(H/2, f_M) = 1.41 \times 10^{-6}$ K, which should be well below the background fluctuation level. However, Fig. 3(a) shows a peak of 1.91×10^{-4} K, which is higher by more than 2 orders of magnitude. An additional run (not shown) at similarly high Ra $= 4 \times 10^{12}$ [with $\kappa = 7.93 \times 10^{-4}$ cm^2 s^{-1}, higher $f_M = 0.04$ Hz $> f_P = 0.029$ Hz, $(T_{B0})_{\rm rms} = 0.016$ K, and $\Delta_M = 0.028$] yielded a spectral amplitude of 1.508×10^{-4} K, while (1) gave $T_M^{\rm CALC}(H/2, f_M) = 7.9 \times 10^{-7}$ K, which again is smaller

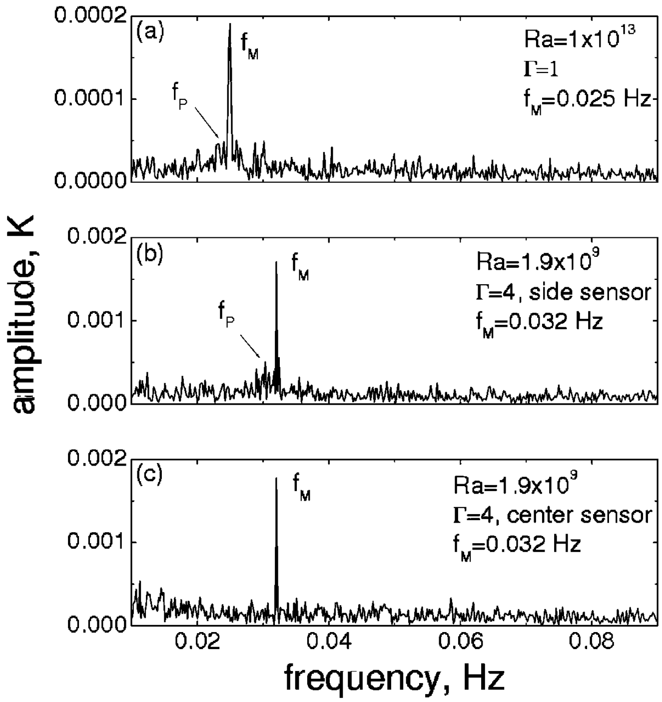

FIG. 3. Spectral density for (a) $\Gamma = 1$, high Ra $= 1 \times 10^{13}$, $f_M = 0.025$ Hz, $\Delta_M = 0.22$. (b) $\Gamma = 4$, Ra $= 1.9 \times 10^9$, $f_M = 0.032$ Hz, $\Delta_M = 0.153$, side sensor; (c) the same as (b) but for the sensor at cell center. The peak corresponding to the weak mean wind is absent in the center spectrum for $\Gamma = 4$, as expected [10]. Note, in addition, the order of magnitude difference in the ordinate scales between (a), corresponding to $\Gamma = 1$, and (b) and (c) corresponding to $\Gamma = 4$.

by 2 orders of magnitude or so than the measured peak and so should have been below the background noise.

If we compute $\tilde{z}$ by using the measured Nu in (4), we find $\tilde{z}/H = 0.21$ for Ra $= 10^{13}$ and $= 0.22$ for Ra $= 4.5 \times 10^{12}$. Thus, both high Ra measurements reveal the presence of a substantial core region, which can be correlated only with the Rayleigh number. This latter fact has been demonstrated by the large variation in the range of parameters used for the low Ra experiments discussed above, as well as their variation at high Ra (differing modulation frequency commensurate and noncommensurate with f_P). Furthermore, the highest Ra experiment was performed with the same general conditions as for the lowest Ra experiment, namely, a large amplitude and a modulation frequency set to f_P. The conclusion appears inescapable that the amplitude decay at high Ra occurs entirely in a region close to the bottom boundary, whose thickness $\tilde{z}$ is smaller than the cell half-height but much larger than the thermal boundary layer.

To assess if these values of $\tilde{z}$ are reasonable, we show in Fig. 4 the rms temperature profiles in turbulent convection for $\Gamma = 1/2$, taken from the simulation of Amati *et al.* [9]. While these simulations used $\Gamma = 1/2$ to achieve the necessary high Ra, other simulations using the same code reveal no significant differences in the profiles between

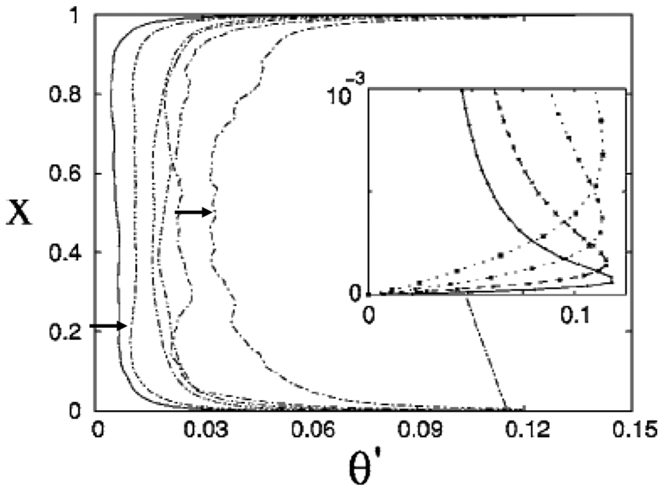

FIG. 4. Profiles of the rms temperature from Amati *et al.* [9]. The curves from left to right vary by a factor of 10 from 2×10^{14} to 2×10^9. The arrows indicate the estimate of $\tilde{z}/H$, which, for high Ra, coincides with the vanishing of the gradient in the profile.

$\Gamma = 1/2$ and $\Gamma = 1$ for overlapping Ra; indeed, there is no reason to suspect that significant differences will arise at higher Ra [15]. The curves in Fig. 4 correspond to Ra values that increase successively by an order of magnitude starting at $\mathrm{Ra} = 2 \times 10^9$. We have marked by a horizontal arrow the $\tilde{z}/H$ for 1.9×10^9 on the curve corresponding to $\mathrm{Ra} = 2 \times 10^9$; another horizontal arrow marks $\tilde{z}/H$ for 10^{13} on the curve corresponding to the closely similar value $\mathrm{Ra} = 2 \times 10^{13}$. From the rms profiles in Fig. 4, it is clear that there is no turbulent core region for $\mathrm{Ra} = 2 \times 10^9$, consistent with our measured $\tilde{z}/H \sim 0.5$. At $\mathrm{Ra} = 2 \times 10^{13}$, on the other hand, there is an obvious core region starting at about 20% of the cell height, in good agreement with our estimate of $\tilde{z}/H \approx 0.2$.

Finally, we observe a weak redistribution of energy in the temperature spectra when the applied modulation coincides with the frequency of the mean wind, and the variance associated with f_P becomes slightly lower. This coincides with a similarly weak sharpening of the distribution of the mean wind speed. Other than these modest effects, perhaps surprisingly, there are no major effects on the fluctuations even for the strongest modulation considered here. We stress that the modulation did not have any particular phase relation to the mean wind [8].

We have shown for a variety of modulation parameters in two cells of different aspect ratios, at both the sidewall and the cell center, that the measured damping of the oscillatory heating applied to the lower boundary is consistent, at moderate Ra, with the absence of a superconducting turbulent core region. At high Ra, however, the measured spectral amplitudes at the cell midheight show the development of such a core region, the remainder being about 20% of the cell height. Both of these observations are in good agreement with the rms temperature profiles recently obtained by Amati *et al.* by numerically solving Boussinesq equations. This is the first time that a dynamic measurement has been used to explore the highly conducting core in turbulent convection at very high Ra. The conclusion is consistent with our earlier observation [12] that, for high values of Ra exceeding about 10^{13}, fluctuations dominate the mean wind, which therefore does not appear to survive; this indicates the emergence of strong mixing in the core region, which takes place on a time scale much smaller than that of the mean wind.

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