Numerical experiments of turbulent thermal convection at high Rayleigh numbers

K. Koal[1,2], G. Amati[2], F. Massaioli[2], K.R. Sreenivasan[3] and R. Verzicco[4]

[1] TU, Dresden, Germany k.koal@caspur.it
[2] CASPUR, Via dei Tizii 6b, Roma, Italy g.amati/f.massaioli@caspur.it
[3] ICTP, Trieste, Italy krs@ictp.it
[4] DIMeG & CEMeC Politecnico di Bari, Bari, Italy verzicco@poliba.it

Understanding turbulent thermal convection is crucial for industrial, geophysical, and astrophysical problems but experiments at high Rayleigh numbers (Ra) sometimes show contradictory results. This is due to unavoidable technical limitations of the experimental set–ups that produce flows slightly different from the ideal Rayleigh-Bénard problem. Numerical simulations intended as ideal experiments can help understanding these issues since all the extraneous factors can be cleanly sorted out. In this paper Direct Numerical Simulations (DNS) of a turbulent Boussinesq convection in a cylindrical cell of aspect-ratio $\Gamma = 1/2$ up to $Ra = 2 \cdot 10^{14}$ are presented and discussed.

1 The problem

Consider a Boussinesq fluid with α, ν, k and λ, respectively indicating the thermal expansion coefficient, kinematic viscosity, thermal diffusivity and thermal conductivity, contained in a cylindrical cell of diameter d and height h. The fluid is heated from below and cooled from above while the lateral wall is adiabatic. There is no slip at any of the walls. Main flow parameters are:

$$Ra = g\alpha\Delta h^3/(\nu k) \qquad Pr = \nu/k \qquad \Gamma = d/h \qquad Nu = Hh/\lambda\Delta, \tag{1}$$

where the Rayleigh number (Ra) is the forcing parameter, the Prandtl number (Pr) characterizes the fluid, the aspect ratio (Γ) accounts for the cell geometry and the response of the flow, the Nusselt number (Nu), is the total heat flux normalized by the purely convective value. In the above expressions g is the acceleration of gravity, Δ the temperature difference between the horizontal plates and H the specific heat flux transferred within the cell. An important issue is to verify how much the turbulence enhances the heat transfer, namely how Nu increases with Ra. Some experiments give contradictory results, presumably owing to finite-conductivity effects (i.e. on the lateral wall and on the upper/lower plates), various degrees of deviation from the Boussinesq approximation, Prandtl number variation and control of the temperature boundary conditions on the heated and cooled plates [1, 2, 3]. For these reasons numerical simulations could be seen as ideal experiments, where all flow parameters are under strict control.

Table 1. Variation of the Kolmogorov scale (η) and thermal boundary layer thickness (δ_θ) with Ra; these values are computed from present simulations.

Ra	η/h	δ_θ/h	$N_\theta \times N_r \times N_x$
2×10^{12}	8.2×10^{-4}	3.76×10^{-4}	$257 \times 193 \times 769$
2×10^{13}	4.3×10^{-4}	1.73×10^{-4}	$385 \times 301 \times 1381$
2×10^{14}	2.4×10^{-4}	7.00×10^{-5}	$513 \times 401 \times 1801$

2 Governing equations and set–up

Under the Boussinseq approximation, thermal convection is governed by the three dimensional unsteady Navier-Stokes equations as follows:

$$\frac{Du}{Dt} = -\nabla p + \theta \hat{x} + \left(\frac{Pr}{Ra}\right)^{\frac{1}{2}} \nabla^2 u, \qquad \nabla \cdot u = 0, \tag{2}$$

$$\frac{D\theta}{Dt} = \left(\frac{1}{RaPr}\right)^{\frac{1}{2}} \nabla^2 \theta. \tag{3}$$

Here $\hat{x}$ is the vector pointing in the opposite direction with respect the gravity and θ is the non-dimensional temperature ($0 < \theta < 1$). A second-order accurate finite-difference scheme in cylindrical coordinates discretized on a staggered mesh as in [4] has been used for the integration of the equations. In a direct numerical simulation it is mandatory to adequately resolve both velocity and thermal boundary layers near the walls and the smallest between the Kolmogorov and Corrsin scales (η) in the bulk, hence ask for a *very fine* spatial resolution. For this reason an over-resolved simulation at $Ra = 2 \cdot 10^{11}$ was performed to check the effect of grid resolution. With the mesh in the bulk equal to 1.2η and 10 points in the thermal boundary layer, a Nusselt number of $Nu = 440.3 \pm 10$ was obtained, to be compared with the previous result of $Nu = 447.2 \pm 11.7$ obtained with a grid in the bulk 4 times larger than η and only 5 points in the thermal boundary layer [4]. *A posteriori* checks of the thickness of both boundary layers and additional integral quantities confirmed the adequacy of the discretization used. Three simulations at $Pr = 0.7$ and $Ra = 2 \cdot 10^{12}, 2 \cdot 10^{13}, 2 \cdot 10^{14}$ have been performed. In table 1 the discretization for the three simulations performed are presented. They were performed using a IBM p690 Power4 32-CPU node (for $Ra = 2 \times 10^{12}$ and $Ra = 2 \times 10^{13}$) and using a NEC SX-6+ 8-CPU node (for $Ra = 2 \times 10^{14}$) asking for about 150'000 CPU hours and producing about 1 TB of raw data. To our knowledge these are the biggest simulations of fully confined turbulence ever performed.

3 Results, comments and conclusion

In Fig. 1 we report Nu as function of Ra for the present results and those by [4] in comparison with some experiments performed in the same geometry and under apparently identical conditions [1, 2, 3]. Although the agreement might seem satisfactory for a first look, a different representation of the same data (Fig. 2a), clearly shows that differences up to 20% are present. Some of the differences between the results can be explained in terms of different Pr numbers. In the experiments, in fact, Pr is constant only for $Ra < 10^{12}$ while for higher Ra it attains higher values. In Fig. 2b the Prandtl numbers for experiments and simulation are reported in a $Ra - Pr$ plane divided into different regions, according to the model by Stringano & Verzicco [5], which indicate the most likely mean flow structure. The $Pr - Ra$ plane is divided in a region where only a single roll can be found (1R region), where the mean flow is characterized by two rolls (2R region) and region where no mean flow is found (NMF region); in the latter the upper and lower thermal boundary layers do not 'communicate', and, following Malkus [6], we have $Nu \sim Ra^{1/3}$. These findings are confirmed by the flow visualizations of Fig. 3) where snapshots of the temperature field for points of the $Pr - Ra$ plane belonging to regions 1R, 2R and NMF confirm the predictions of Fig. 2b. Further details on the flow dynamics can be found in [7]. The main result of this paper is to show how numerical simulations can help the understanding of laboratory experiments thus allowing for a deeper comprehension of the problem. In this particular case the possibility of keeping Pr strictly constant for every value of Ra allowed us to find a no-mean-flow (NMF) region where, according to [6], we might expect $Nu \sim Ra^{1/3}$. In the experiments, probably owing to difficulties in keeping Pr constant, the above power-law was not observed.

References

1. Niemela J, Sreenivasan KR (2003) J. Fluid Mech. 481:355–384
2. Roche P, Castaing B, Chabaud B, Hebral B (2002) Europhys. Lett. 58:693–698
3. Chavanne X, Chillà F, Chabaud B, Castaing B, Hebral B (2001) Phys. Fluids 13:1300–1320
4. Verzicco R, Camussi R, (2003) J. Fluid Mech. 477:19–49
5. Stringano G, Verzicco R, (2005) J. Fluid Mech (in press).
6. Malkus W.V.R. (1954) Proc. Roy. Soc. Lond. A225:196–212
7. Amati G, Koal K, Massaioli F, Sreenivasan KR, Verzicco R (2005) Phys. of Fluids (in press).

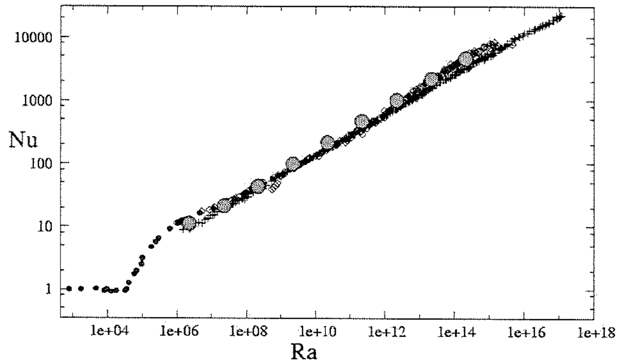

Fig. 1. Nu vs. Ra for numerical simulation (big circles) and experiments by [1] (+), [2] ($\diamond$) and [3] (little circles).

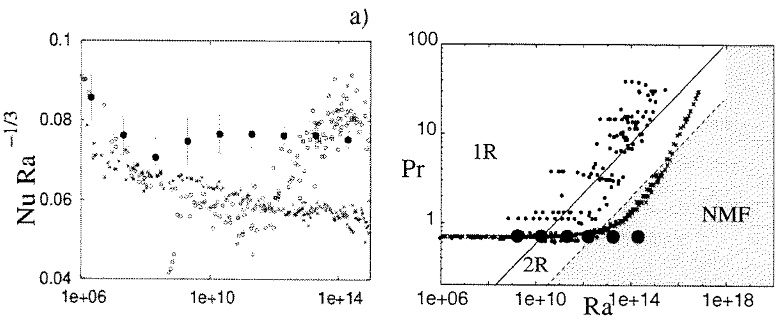

Fig. 2. Figure a): Compensated $Nu \cdot Ra^{-1/3}$ for numerical simulation (circle) and experiments by [1] (x) and [2] ($\diamond$). Figure b): Pr vs. Ra for numerical simulation (circles) and experiments by [1] (x) and [2] ($\diamond$).

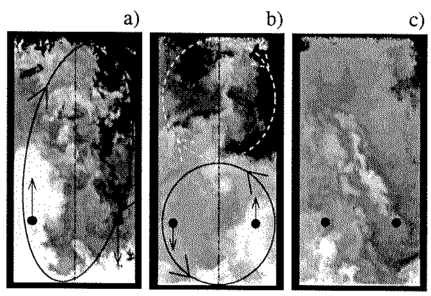

Fig. 3. Snapshots of temperature showing the possible mean flow configurations: *a)* $Ra = 2 \times 10^{10}$, *b)* $Ra = 2 \times 10^{12}$, *c)* $Ra = 2 \times 10^{14}$, Only the temperature range $0.475 \leq \theta \leq 0.525$ is represented with $\Delta\theta = 0.005$.