Local dissipation scales in turbulence

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1 Motivation

The Kolmogorov theory of turbulence assumes the existence of a single mean dissipation length at the small-scale end of the turbulent cascade where the viscous dissipation term of the underlying Navier-Stokes equations becomes comparable to the nonlinear advection term. This scale is known as the Kolmogorov length η_K and is defined as $\eta_K = \nu^{3/4}/\langle\epsilon\rangle^{1/4}$ where ν is the kinematic viscosity and $\langle\epsilon\rangle$ the mean energy dissipation rate of the flow. The classical definition of the dissipation scale does not, however, capture the strongly intermittent nature of the energy dissipation field which is now a well-accepted fundamental building block of our understanding on turbulence. It therefore seems natural to include these fluctuations and to extend the notion of a single mean dissipation scale to that of a whole continuum of local dissipation scales, i.e. to a fluctuating random field η. The finest local dissipation scales will then be associated with the steepest velocity gradients, or alternatively, with the most intensive energy dissipation events in turbulence. These ideas were put forward within the multifractal formalism (see [1] for references).

In a recent theoretical approach which stayed close to the underlying Navier-Stokes equations, the distribution of local dissipation scales was directly calculated by one of the authors [2]. Here, we want to present high-resolution direct numerical simulations (DNS) of homogeneous isotropic turbulence, that confirm the theoretically predicted shape of the local dissipation scale distribution. A standard pseudospectral method is used to simulate homogeneous isotropic turbulence in a fully periodic box. However, grid resolutions used were significantly finer than the ones applied in standard cases. We enforce the spectral resolution criterion to values of $k_{max}\eta_K \geq 10$. Four different runs are conducted at Taylor microscale Reynolds numbers $R_\lambda = 10, 24, 42$ and 64 with grid resolutions of $N^3 = 512^3, 1024^3, 1024^3$ and 2048^3 points, respectively (for more details, see [1]).

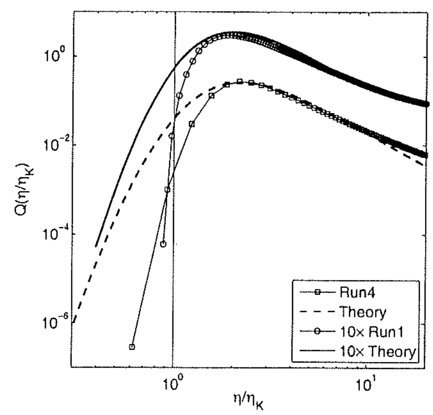

Fig. 1. Comparison of numerical and theoretical results for $Q(\eta, Re)$ for Runs 1 ($R_\lambda = 10$) and 4 ($R_\lambda = 64$). The data for Run 1 are shifted upwards in the diagram for a better visibility.

2 Results

Figure 1 shows the distribution $Q(\eta)$ of the local dissipation scales as determined from DNS for two Reynolds numbers. The distribution agrees qualitatively with the theoretical predictions [2]. First, one can observe that the range of excited sub-Kolomogorov scales increases with growing Reynolds number, which underlines that intermittent fluctuations from the inertial range sweep deeper into the viscous scale range. While the tail of the distribution for scales $\eta > \eta_K$ agrees well with the theory, deviations for the tail $\eta < \eta_K$ are detected.

An interesting connection of these results to the decay of the energy spectrum $E(k)$ for wavenumbers $k > \eta_K^{-1}$ should follow. The yet unanswered question is how the intermittency, which is detected in physical space, manifests itself in the decay of the energy spectrum in the viscous range. In order quantify the exponential decay of the energy spectra in the dissipation range we fit in Figure 2 the following dimensionless local slope to the data:

$$\frac{\mathrm{d}\log(\tilde{E}(k))}{\mathrm{d}\log(\tilde{k})} = \alpha - \beta\tilde{k} \quad \text{for } \tilde{k} \geq 1\,, \tag{1}$$

which transforms the exponential decay law of the spectrum into a linear function. Here, $\tilde{E}(k) = E(k)/(\nu^5\langle\epsilon\rangle)^{1/4}$ and $\tilde{k} = k\eta_K$. Our findings extend former results for the spectral tails [3, 4] to larger spectral resolutions and support the validity of (1). The lower panels of Figure 2 show both coefficients as functions of the Reynolds number. They are in the same range as in [3, 4].

Unfortunately, their asymptotic values for high Reynolds numbers cannot be deduced from data in this limited range.

This behavior at the crossover between inertial and viscous ranges of turbulence might may have consequences for the turbulent mixing of scalar concentration fields at large Schmidt numbers, i.e. when the scalar diffusivity is significantly smaller than the kinematic viscosity of the fluid. They will be discussed elsewhere.

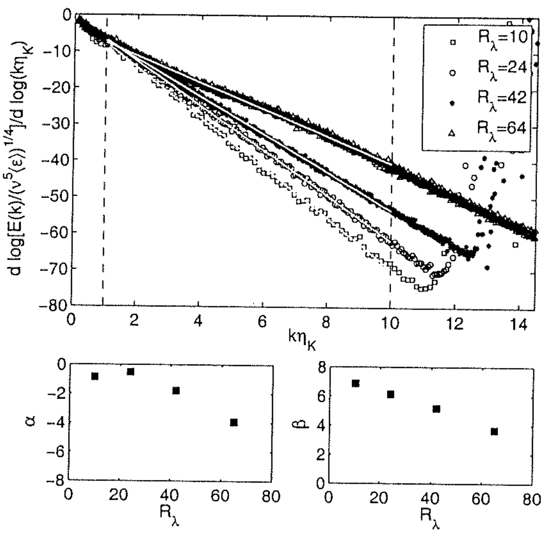

Fig. 2. Decay of the energy spectra $E(k)$ in the viscous scale range for different Reynolds numbers. Upper picture: Local slope of the spectrum as a function of the wavenumber. Least square fits for all data were performed between $1 \leq k\eta_K \leq 10$ (dashed lines). The fit results are indicated as gray lines for each data set. Lower pictures: Constants α (left) and β (right) as a function of the Reynolds number.

References

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