# CONCLUDING REMARKS ON LUDWIG PRANDTL AND GÖTTINGEN

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Abstract: I shall offer some personal remarks on Ludwig Prandtl and his memory in Göttingen, and on circumstances that led to the IUTAM meeting celebrating 100 years on the boundary layer theory.

Key words: Ludwig Prandtl, Boundary Layer Theory, IUTAM meeting, Göttingen.

## 1. INTRODUCTION

Ludwig Prandtl occupies a unique place in fluid dynamics. To him alone we owe the concept of the boundary layer [1] that integrated the two diverging areas of hydraulics and hydrodynamics, and shaped them into the rich subject of modern fluid dynamics. James Lighthill, the great twentieth century fluid dynamicist, has said [2] that the boundary layer theory holds a place comparable to Einstein's theory of relativity. G.I. Taylor, another all-time giant in fluid dynamics, called Prandtl "our chief" [3] and held his work in the highest regard.

Prandtl's short paper, presented in 1904 at the Third International Mathematical Congress at Heidelberg, is now well known to us all: in few short pages that are dense with ideas, Prandtl laid out an agenda for a good part of the twentieth century fluid dynamics. It is difficult to imagine modern fluid dynamics without these ideas; equally difficult it is to appreciate that Prandtl's paper invited little attention for quite some time after it was published. Perhaps the paper's brevity, perhaps the mismatch of the audience at the meeting, contributed to this delay [4].

The person who saw the importance of the paper immediately after it was read was Felix Klein, who promptly brought Prandtl from Hanover to Göttingen. That Prandtl came there to occupy a lower position than the full professorship he held in Hanover says something about his priorities: he must have liked the idea of being in the middle of the thriving intellectual atmosphere of Göttingen, and the position itself seemed less important. For some thirty-odd years until his death, Göttingen was Prandtl's home. It was here that he created and led an exceptionally successful institute [5] devoted to basic and applied work on fluid dynamics, with particular attention paid to aerodynamics. Göttingen owed its fame in fluid dynamics entirely to Prandtl.

Yet, the post-war Göttingen establishment of fluid dynamics, namely the DLR and the Max Plank Institute (both of which are Prandtl's legacies), had refrained in unspoken ways from highlighting their continuity with Prandtl. I myself was educated in the Aeronautical Engineering Department of the Indian Institute of Science, Bangalore. That department was started by one of Prandtl's Indian students, V.M. Ghatge; one of the later heads of that department was O.J. Tietjens of the Prandtl-Tietjens fame [6]. Even this somewhat distant link nurtured my abstract kinship to Prandtl. Thus, when I first visited Göttingen several years later, and then spent parts of 1983-84 as Humboldt Fellow, I was struck by the absence of visible ties to Prandtl: there was no Prandtlstrasse in the town that has had the broadmindedness to name several of its streets after scientists, or a Prandtl building in the complex of DLR and the Max Planck Institute; while in many institutions local celebrities of minor distinction are honored with annual lectures named after them, there was nothing like a Prandtl lecture, nor a Prandtl visiting Professorship. At around the same time, the Prandtl Ring, an honor that had been conferred upon some selected fluid dynamicists, had also been discontinued. At different times, I had discussions about these issues with various people in Göttingen and soon discovered that this benign neglect was in part the desire to start afresh after the war—somewhat in consonance with the spirit that swept through the country as a whole—but in part also reflecting the ambivalence of von Karman's unflattering description [7] of Prandtl for his apparent "lack of sensitivity to the enormity of crime that was taking place in Germany".

During one of my later visits to Göttingen, Professor Ernst Müller, who was then the director of the Max Planck Institute, presented me with a copy of Prandtl's biography written by his daughter [8]. The appendix of that German biography highlighted one of Prandtl's letters to Hermann Goering in which he strenuously argued, at much risk to personal safety, that "German Science"—one of the rallying cries of the Nazis—was an abhorrent concept. I read that letter with astonishment and fascination; it confirmed to me that Prandtl's values were in the right direction—even if he felt that his obligations were mostly to his science. Thus arose my conviction that Göttingen ought to reclaim its continuity with Prandtl more willingly.

I witnessed a gradual change in Göttingen during my later visits. A few photographs of Prandtl were already in display in the Theoretical Institute headed at the time by Professor Herbert Oertel. In the Max Planck Institute, there was a designated Prandtl room that stored additional photographs and housed his desk and chair. When, a few years later, there was a possibility of my heading the Max Planck Institute, Professor Vladik Fisdon of Poland, a visitor at the time, took me around to Prandtl's room and sat me in Prandtl's chair while we went over several charming photographs from the past. This event only enhanced my awareness that the Göttingen fluid mechanics community was yet to own up much of its continuity with Prandtl. My subsequent connection with Göttingen became sporadic because of other preoccupations, but I continued to feel that the disconnect between Prandtl and Göttingen was still a long way from being mended.

Thus, for me, 2004 was to be the year not only to celebrate the 100th anniversary of the advent of the boundary layer but also to reinvigorate Göttingen's links to Prandtl. While it was clear that an appropriate meeting was to be held in Göttingen celebrating the 100th anniversary of the boundary layer theory, the details had assumed a certain level of complexity.

Around 2000 when I started thinking about these matters concretely, the situation was this: Prandtl's formal successor, Professor Theo Giesel, a distinguished atomic physicist, was not entrenched in the fluid mechanics community enough to assume the major responsibility for the event (though he was most supportive of the idea); and Prandtl's intellectual heirs in DLR, such as Professor Gerd Meyer, were on throes of retirement by then. At some point, Professor Roddam Narasimha and I made an informal inquiry with Professor Keith Moffatt, the President of IUTAM at the time, asking if IUTAM would sponsor a meeting in Prandtl's honor. Moffatt replied with enthusiasm but also noted, in response to our query, that Göttingen itself had not generated such a request (which all of us thought was the best possible scenario): It was less meaningful to hold the meeting at places other than Göttingen, and a strong willingness on the part of the Göttingen locals seemed hard to come by. The idea seemed to have hit an impasse.

It was thus a great relief to have received a note sometime later from Meyer suggesting that we jointly propose an IUTAM meeting in Göttingen. I now know that various other discussions took place before that event—and I am thankful to all for their contributions. The proposal was finally made and approved, the organizing committee established, speakers chosen, and so forth. I should here record my particular pleasure in having worked with Meyer and other members of the organizing committee. On behalf of all of us, I would like to express my grateful thanks to Dr. Hajo Hienemann who lent his invaluable administrative help in arranging the meeting and preparing its proceedings, which you have in front of you now.

I have already referred to the Max Planck Report on Prandtl's biography and his letter in which he decried the notion of German Science as an exclusionary entity. Professor Ernst Müller was very keen to have the report translated into English, but the various efforts I made at the time, with AIP and Springer [9], did not yield fruit for different reasons. My goal was to have the biography translated into English by the 2004 meeting. Some two years ago, I approached Dr. V. Vasanta Ram of Bochum to prepare the translation. He has completed the task with devotion in spite of his physical handicap. Alas, the book could not be released in time for the meeting for a number of invidious reasons, but I have every expectation that it will appear soon.

Prandtl's boundary layer has secured a matchless place in the development of mechanics as a whole, as well as applied mathematics, and other branches of physical sciences where singular perturbations play a key role. It will remain to be taught as a fundamental concept even in these days of computer hegemony. Prandtl's many students and colleagues [10], themselves of no mean accomplishment, were living testimonials to his greatness as a mentor. That Prandtl was a man of personal integrity seemed clear to those who knew him well. It is impossible to put oneself precisely in the shoes of another person—least of all those of someone who in personal science was at the peak but saw his country decimated twice in his lifetime because of the vicious policies pursued by his country—but I, for one, would like to see Göttingen build greater ties with the best of Prandtl's memory. I hope that there will be an annual Prandtl lecture, an occasional visiting Professorship inn his name, and, of course, the resumption of the Prandtl Ring. And I hope that the world of fluid dynamics will view these developments as desirable.

References

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8. Johanna Prandtl-Vogel. "Fully Developed Two-Dimensional Channel Flow", *Int. J. Heat and Fluid Flow,* **23**, 678-, 2002 (Data Code Number CH12_PG.WL6, University of Tokyo).
9. AIP and Springer. AIP at the time was willing to publish the book if I got it translated and offered to buy a certain number of copies. Springer was willing to consider its publication though it did not regard the book to be a scientific biography. Springer also did not foresee selling many copies of the book, thus implying that the price would have to be quite high.

# Divide, Conquer and Unify!

*A hundred years ago Ludwig Prandtl proposed an idea that brought to end the 19th century wars between hydraulics and hydrodynamics, and created modern fluid dynamics.*

**RODDAM NARASIMHA**

In August 1904 a 29-year old engineer named Ludwig Prandtl read a remarkable paper before the Third International Mathematical Congress at Heidelberg. (He had already been for three years professor of mechanics at the Technical University of Hanover.) The paper was a kind of scientific time bomb: it made no great impact at the Congress (except on that perceptive mathematician Felix Klein, who promptly got Prandtl to migrate to Göttingen), and was not translated to English till 1928. But by the 1920s and 30s the powerful idea in that paper, and the reputation of its author, had spread across the world, even as they helped create modern fluid dynamics out of ancient hydraulics and 19th century hydrodynamics.

In retrospect, the paper can be seen as full of understated daring. In the first place it talks to its distinguished mathematical audience through 12 photographs of visualized water flow past bodies, ten other professionally-drafted diagrams, and only eight equations. The heart of the paper is in one paragraph (see figure) sandwiched between a mathematical preliminary which leads to it, and a terse but wide-ranging exposition of the explanatory power of the idea that follows it. That exposition covers how vortices emerge in mixing layers, behind cylinders, at the edge of a plate moving normal to its plane etc., and of when and why flows separate from the solid surface they are supposed to follow.

The tone of that key paragraph is deceptively casual. The formal problem it tackled is the flow past an aligned flat plate – a problem that is trivial in inviscid hydrodynamics but was crucial to the new fluid dynamics that was being created. First, as the title of the paper indicated (Über Flüssigkeitsbewegung bei sehr kleiner Reibung, = On the motion of fluids of small viscosity), a small parameter $\varepsilon$ is identified in the problem – in this case viscosity, or more precisely a reciprocal of the Reynolds number (say Re, = product of flow velocity and length scales divided by kinematic viscosity). Classical perturbation methods would not work here, for the limit $\varepsilon \to 0$ is singular: the limit of the full solution (e.g. at the surface) is not the solution of the limiting equation. Prandtl's method was to divide the flow into different regions in which the dynamics are different – an outer inviscid flow, and a thin inner viscous ('boundary') layer next to the surface. In each of these regions only the dominant terms in the Navier-Stokes equations were collected. The different equations so found were separately solved, but their boundary conditions were cleverly selected so that the solutions blended into each other to yield a 'unified' approximation.

This paper will certainly prove to be one of the most exciting papers of this century, and
[illegible]

And in that same paragraph Prandtl also showed how the partial differential equations governing the boundary layer could be reduced to an ordinary differential equation using a combination of the independent variables (utilizing a group property of the equation), solved numerically the resulting nonlinear o.d.e. (without writing it down explicitly!), and ended by giving a rough value for the drag of the plate and a sketch of the solution – all in some 25 lines.

But such a potentially precise calculation of laminar-flow flat-plate drag was clearly much more than a future entry in an engineering handbook, for it opened a door between the previously sealed chambers of hydraulics and hydrodynamics. Their practitioners had for long poured scorn on each other: hydraulics was often called a science of variable constants, and hydrodynamics the mathematics of dry water. Reality, Prandtl demonstrated, was not only *within* the Navier-Stokes equations, but even accessible to mathematics.

In succeeding decades the idea was developed and extended, chiefly through Prandtl's many able pupils, who flocked to Göttingen to work with the master. (Felix Klein had meanwhile helped him to establish what became a renowned centre of fluid dynamics research, and was called by a grateful Prandtl his 'destiny'). And the theory showed excellent agreement with experiment. In spite of this, however, for nearly five decades it astonishingly retained the image of no more than a clever and successful approximation or 'model'. It was eventually the work of Paco Lagerstrom and his colleagues at Caltech that distilled a systematic 'method of matched asymptotic expansions' (MAX for short) as the mathematical idea inherent in Prandtl's work. In the simplest case MAX involves studying the limiting solutions as $\varepsilon \to 0$ separately in inner and outer regions, making in each of them appropriate asymptotic expansions in so-called inner and outer variables, matching the two expansions by equating (to appropriate order) the outer limit of the inner expansion with the inner limit of the outer expansion, and finally constructing a unified ('composite') solution that is everywhere asymptotic to the exact solution.

That formal mathematization of solving a whole class of singular perturbation problems had several important sequels. First it was discovered that many puzzles – from d'Alembert's famous paradox to others even in low Re Stokes-type hydrodynamics (e.g. Stokes, Filon) – yielded to MAX. (Garrett Birkhoff wrote a whole book about these and other fluid-dynamical paradoxes just a little too early – even as the machinery of MAX was being perfected.) Then it turned out that the many other problems that Prandtl had brilliantly solved – e.g. finite wings in aerodynamics – had essentially been based on use of MAX too. Eventually when Milton van Dyke wrote the first book on the subject he crowned MAX with the adjective 'rational' – distinguishing it from the numerous non-rational, more or less ad hoc, approximations and theories that then abounded in fluid dynamics.

Prandtl was too much of an engineering physicist to dwell on his mathematics. When he was made an Honorary Fellow of the German Physical Society in 1948, its president Werner Heisenberg (an old warrior himself in pre-quantum days on hydrodynamic stability, which Prandtl and his students pursued more relentlessly and successfully) said Prandtl had 'the ability to see the solution of equations without going through the calculations'. Prandtl demurred: 'No, I strive to form in my mind a thorough picture . . . The equations come only later when I believe I have understood . . . [and are] good means of proving my conclusions in a way that others can accept'. Replying to an invitation by the Indian Mathematical Society in 1949 he demurred again, 'I can make no claim for the description of myself [as an eminent mathematician – as IMS had called him]. I am an engineer . . . or a theoretician in engineering subjects'. After all, his boundary layer idea had been triggered by frustration with a lossy diffuser he was asked to tackle during a year he spent in industry before becoming an academic.

People have often called him modest, but this I believe is misjudgement. He himself said 'I am proud of what I have achieved, but I never made use of it'. His papers have a simplicity and directness born of supreme self-confidence, but they have no trumpet parts, no strong criticism of others. Instead they get on with solving the central problems using all the tools available – observation (plenty of it!), mathematics, calculation, modelling. His methodological eclecticism set the style of fluid dynamics research in the 20th century. No wonder G.I. Taylor called him 'our chief', and helped nominate Prandtl for a Nobel prize he never won.

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Further reading:

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Figures:

1. A portrait of Prandtl, painted by his Indian student V M Ghatage (Ph.D. 1936).
2. Was modern fluid dynamics born here? Extract from Prandtl's paper at the 3rd International Mathematical Congress.