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THERMAL CONVECTION AT HIGH RAYLEIGH NUMBERS

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ABSTRACT

Some general comments on turbulent thermal convection are made. Most remarks will be restricted to experiments. The references cited towards the end should be consulted for specific results.

1. GENERAL REMARKS

Turbulent thermal convection is ubiquitous in engineering practice as well as geophysical and astrophysical flows; it is important in such varied contexts as energy transport within stars, weather patterns, ocean currents, and the generation of the earth's magnetic field within the fluid core. Convection controls global heat-transport through the outer regions of stars *(Spiegel 1971)*, the reversals of the Earth's magnetic field *(Glatzmeier et al 1999)*, and other such large-scale issues. Accurate and direct predictions of such features are out of reach for the state-of-the-art numerical simulations and modern theory. This is so because turbulent convection involves many spatial and temporal scales which interact nonlinearly; the influence of boundaries and geometry can often be important as well.

Since it is not realistic, nor necessarily desirable, to address the problem in its full glory, a suitable system must be selected for study: this must be sufficiently simplified to allow for theoretical and experimental progress, yet contain the essential properties of the "real" problem. Such an idealization of the thermal convection problem is the so-called Rayleigh-Benard convection (RBC), which considers a fluid vertically confined between two horizontal surfaces separated by a vertical distance H, heated from below and cooled from above, resulting in a vertical temperature difference DT. The temperature difference DT is maintained small enough, so that the fluid properties are nominally constant all through the fluid, with the obvious exception of the density (whose variation is necessary for buoyancy forces to be felt); this is the essence of the so-called Boussinesq approximation *(e.g., Tritton 1988)* which is almost always incorporated in the theory.

There are various time scales that apply to RBC: for heat and momentum diffusion and buoyancy these are $\tau q = H^2/k$, $\tau_v = H^2/n$, and $\tau_B = (H/agDT)^{1/2}$, where n, k, a and g are, respectively, the kinematic viscosity, thermal diffusivity, isobaric coefficient of thermal expansion, and the gravitational acceleration. Two dynamical similarity parameters, the Rayleigh number, Ra, and the Prandtl number, Pr, which specify the dynamical state of the flow for a given set of boundary conditions, can be obtained from ratios of time scales: $Ra \equiv (agDTH^3)/(nk) = \tau_?\tau_v/\tau_B^2$ and $Pr \equiv n/k = \tau_?/\tau_v$. Most geophysical and astrophysical situations correspond to very large values of Ra, and Pr can vary from being very small (of the order of $10^{??}$ for the Sun) to very large (as high as 10^6 inside Earth's mantle).

Experimental understanding of turbulent RBC is being acquired, using two related approaches. In one, experiments have pursued detailed knowledge of the flow for some fixed values of Ra, in the hope that the state of the system is representative of the fully developed turbulent state with universal properties (see section 3). In the other, one has tended to vary the range of parameters as widely as possible and measure a small number of chosen parameters with great accuracy, so that the scaling patterns and possible transitions from one state to another, if they exist at all, can be discerned. The second approach has been possible by working with particularly chosen fluids and exploiting the fact that one can work near their critical point with relative ease. In particular, the large variation of the thermal expansion coefficient of these fluids leads directly to a large variation in Ra; further, very low values of the kinematic viscosity and thermal diffusivity of the fluid (also a by-product of the nearness to the critical region) result in large values of Ra *(see Frost 1975, Ashkenazi & Steinberg 1999)*. The most effective fluid in this regard is cryogenic helium (Threlfall 1975). Using helium at low temperatures, very large values of Ra as well as very large ranges of Ra have been obtained *(Castaing et al. 1989, Niemela et al. 2000, Chavanne et al. 2001, Niemela & Sreenivasan 2003)*, exceeding by several orders of magnitude anything that is likely with conventional fluids. The record today for one apparatus is 11 decades of Ra, with the maximum of 10^{17} *(Niemela et al. 2000)*. This is only a few orders of magnitude smaller than is relevant to convection in the Sun. (The use of a fluid near the critical point introduces its own complications, which we shall not discuss here).

2. NON-UNIVERSAL ASPECTS

Let us concentrate first on the variation of global heat transport as a function of Ra. The heat transport is measured in ratio to what would be possible when the fluid is at rest, this ratio being known as the Nusselt number, Nu. Progress in our understanding of the Nu vs Ra relation has not always been straightforward: the topic has undergone "swings" from being declared as essentially "solved" to one in which too many details are seen to matter. In no small measure, this swing of views has been due to a constant influx of new data, from both laboratory and numerical work, and to the recognition that issues that are often regarded as unimportant theoretically could be important in practice. For instance, one must scrutinize the correspondence between the assumptions underlying a theory and experimental realities, such as whether the fluid motion is fully turbulent, whether the sidewalls of an experimental or numerical container can ever be disregarded fully to correspond to theoretical idealizations, whether the Boussinesq approximation might be violated in an experiment in some subtle manner even if it holds globally, and so forth.

Taking the example of finite boundaries, it is enough, in many circumstances, to understand the physics when the lateral boundaries are far enough away and, if needed, to correct for the small influences that may be present under such circumstances. However, for some purposes, it is likely that the walls alter conditions in a subtle way that may lead to unforeseen changes, at least for some range of Rayleigh numbers. The simplest nondimensional parameter that measures sidewall effects is the aspect ratio G=D/H, where D and H are characteristic horizontal and vertical length scales of the confinement. However, G is not a sufficient indication of the smallness of the sidewall effects. For instance, in detail, a square crosssection may be different from a circular crosssection *(see, e.g., Daya & Ecke 2001, Verzicco & Camussi 2003)*. This situation has been encountered, and well appreciated, near the onset of convection. Since it is possible experimentally to have very large values of G near the onset, and the nonlinearity is weak, the problem near the onset is amenable to rigorous theoretical treatment. In turbulence experiments, one requires very large Ra. For this purpose, the height of the apparatus must be made as large as possible (taking advantage of its cubed contribution to Ra), and then it is practically difficult to scale up the other dimension sufficiently as well, in order to keep G large. Thus, turbulence measurements have occurred in apparatus with relatively small aspect ratios whose relevance to RBC is worrisome. For modern experiments with very high values of Ra, G=_ has typically been used *(see, e.g., Castaing et al. 1989, Niemela et al. 2000, Chavanne et al. 2001, Niemela & Sreenivasan 2003a)* to reach very high Ra. In *Niemela & Sreenivasan (2003a, b)*, an aspect ratio unity was used, but this is still small by convention. One desires an aspect ratio greater than about 4.

Even the aspect ratio of might have been adequately representative of turbulent RBC with no confining boundaries—indeed it was thought to be so not too long ago—if it were not for the role played by the large scale circulation, or "mean wind", which appears to link the boundaries directly to the bulk flow *(Daya & Ecke 2001, Verzicco & Camussi 2003)*. Numerical simulations of Verzicco & Camussi show that this link is very complex for aspect ratio ½ and that the interaction occurs also between thermal boundary conditions on the sidewall and the mean flow, further complicating the picture. In addition, the mean flow can assume the form of vertically stacked roll-like convection cells, more akin to convection in a vertical pipe *(Tritton 1988)* than RBC. Finally, optical measurements of velocity *(see, e.g., Qui, Yao & Tong 2000, Qui & Tong 2001)* have shown that the mean flow for aspect ratio ½ is less robust at a given Ra than for aspect ratio unity *(see Niemela et al. 2001, Sreenivasan, Bershadskii & Niemela 2002, Niemela & Sreenivasan 2003b)*.

Finally, the finite top-to-bottom conductivity of the lateral boundaries can lead to measurable effects on the mean wind *(Ahlers 2001, Roche et al. 2001, Verzicco 2002, Niemela & Sreenivasan 2003a)*, but these effects become negligibly small at high Ra *(Niemela et al. 2001, Sreenivasan et al. 2002, Niemela et al. 2003a)*. For low Ra, it may be possible to avoid some of these complications through, for example, the choice of the construction material for the apparatus, but ultimately the different types of sensitivity of the system shows signs of being non-universal in detail. This remains a significant motivation for more measurements and simulations at very high Ra.

3. "UNIVERSAL" ASPECTS

While the global properties are affected, in various degrees of detail, by the myriad details of the flow, the general belief is that the local or small scales properties of turbulence are universal—that is, they do not depend on flow details. There is a large history behind this thinking in the turbulence literature, starting with the seminal paper of *Kolmogorov (1941)*, which concerned itself with velocity fluctuations. His arguments have been extended to admixtures such as temperature when the heating is low enough that it does not affect turbulence dynamics *(see Monin & Yaglom 1975)*. The temperature fluctuations that do not affect the dynamics, but whose evolution depends on the flow, are called "passive". In turbulent convection, temperature fluctuations are clearly "active", because the turbulent motion is entirely the result of density differences induced by the temperature differential. The question is whether there are any attributes at all of temperature fluctuations that are shared by another class of turbulent fluctuations. In particular, is there some commonality with passive scalar fluctuations, despite the fact that the situation here is far from being passive? The question, while admittedly a bit vague, is within the spirit of a search for "universality".

This and similar questions concerning universality have turned out to be somewhat tricky to answer definitively. In the past, in large measure because there is a good collapse of the energy spectrum at small scales of velocity fluctuations, as suggested by *Kolmogorov (1941)*, there has been the hope that small scales are universal. This has turned out to be overly optimistic because high-order moments display many features of non-universality, and one has to worry about the effects of shear and other sources of anisotropy and forcing. Fortunately, there are ways by which one can separate the universal and non-universal features with respect to the so-called structure functions (i.e., moments of increments of the fluctuating variable such as velocity and temperature), at least in principle *(Arad et al. 1998)*. For velocity structure functions, this scheme of separating the two parts has been implemented for both experimental and numerical data *(Kurien & Sreenivasan 2000, 2001, Biferale & Toschi 2001, Biferale et al. 2002, Shen & Warhaft 2002)*. Its adaptation to scalars is substantially easier in technical details, and its implementation for the third-order structure function can be found in *Kurien, Aivalis and Sreenivasan (2001)*.

A less formal way of separating the effects of shear has been described in *Sreenivasan & Dhruva (1988)*.

Even with all these efforts, the question of universality has been difficult to address because the scaling exponents of structure functions cannot be calculated theoretically, nor determined experimentally or numerically, with the accuracy needed for the purpose —although much progress has been made in recent years in the understanding of the issues involved. Progress in one particular direction has been very interesting and needs some emphasis. By proposing a modified form of scaling, the so-called Extended Self-Similarity (ESS), into whose details we do not enter here, *Benzi et al. (1996)* have enabled a number of scaling issues to be addressed with a better sense of certainty. In this sense, it appears reasonably clear that three-dimensional homogeneous turbulence, velocity fluctuations in thermal convection and magneto-hydrodynamic turbulence all belong to the same class of universality. From a number of tests made in our group, it appears that the temperature fluctuations in passive and active situations, based on ESS exponents, belong to the same universality class (different, of course, from that just cited above). This conclusion appears to be consistent with the theoretical work of *Ching et al. (2003)*. In this paper, it is shown that the statistics of active scalars are also dominated by the same types of structures (in some well-defined sense) as those of the passive scalars advected by the same velocity field. While this is somewhat of an unexpected result, it lends credibility to experimental findings that the scaling exponents, determined through ESS, are the same between active and passive fields. In this sense, there is some hope for universality in turbulence, and the area continues to be very active at present.

4. CONCLUDING REMARKS

Turbulent thermal convection is a grand problem because of its rich physics and wide applicability. Research in this area has been extremely active in the last twenty five years or so. Several important results have been obtained (though we have not mentioned most of them here). It is, however, clear that many basic elements have not been understood, and that the problem will remain active for the foreseeable future.

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REFERENCES

- Ahlers, G. 2001 *Effect of sidewall conductance on heat-transport measurements for turbulent Rayleigh-Benard convection.* Phys. Rev. E 63, art. no. 015303.

- Arad, I., Dhruva, B., Kurien, S., L'vov, V.S., Procaccia, I. & Sreenivasan, K.R. 1998 *The extraction of anisotropic contributions in turbulent flows.* Phys. Rev. Lett. 81, 5330-5333.
- Ashkenazi, S. & Steinberg, V. 1999 *High Rayleigh number turbulent convection in a gas near the gas-liquid critical point.* Phys. Rev. Lett. 83, 3641-3644.
- Benzi, R., Biferale, L., Ciliberto, S., Struglia, M.V. & Tripiccione, R. 1996 *Generalized scaling in fully developed turbulence.* Physica D 96, 162-181.
- Biferale, L. and Toschi, F. 2001 *Anisotropic homogeneous turbulence: Hierarchy and intermittency of scaling exponents in the anisotropic sectors.* Phys. Rev. Lett. 86, 4831-4834.
- Biferale, L., Daumont I., Lanotte, A. & Toschi, F. 2002 *Anomalous and dimensional scaling in anisotropic turbulence.* Phys. Rev. E 66, art. no. 056306.
- Castaing, B., Gunaratne, G., Heslot, F., Kadanoff, L., Libchaber, A., Thomae, S., Wu, X.Z., Zaleski, A. & Zanetti, G. 1989 *Scaling of hard thermal turbulence in Rayleigh-Benard convection.* J. Fluid Mech. 204, 1-30.
- Chavanne, X., Chilla, F., Chabaud, B., Castaing, B. & Hebral, B. 2001 *Turbulent Rayleigh-Benard convection in gaseous and liquid He.* Phys. Fluids 13, 1300-1320.
- Ching, E.S.C., Cohen, Y., Gilbert, T. & Procaccia, I. 2003 *Active and passive fields in turbulent transport: The role of statistically preserved structures.* Phys. Rev E 67, art. no. 016304.
- Daya, Z.A. & Ecke, R.E. 2001 *Does Turbulent convection feel the shape of the container?* Phys. Rev. Lett. 87, 184501-184504.
- Frost, W. 1975 *Heat transfer at low temperatures.* Plenum Press, New York.
- Glatzmaier, G.A., Coe, R.C., Hongre, L. & Roberts, P.H. 1999 *The role of the Earth's mantle in controlling the frequency of geomagnetic reversals.* Nature 401, 885-890.
- Kolmogorov, A.N. 1941 *The local structure of turbulence in incompressible viscous fluid for very large Reynolds number.* Dokl. Akad. Nauk. SSSR 30, 9-13 (reprinted in English in Proc. Roy. Soc. Lond. A 434, 9-13, 1991).
- Kurien, S. & Sreenivasan, K.R. 2000 *Anisotropic scaling contributions to high-order structure functions in high-Reynolds-number turbulence.* Phys. Rev. E 62, 2206-2212.
- Kurien, S. & Sreenivasan, K.R. 2001 *Measures of anisotropy and the universal properties of turbulence. In New Trends on Turbulence,* NATO Advanced Study Institute, Les Houches, Springer and EDP-Sciences, 2001, pp. 53-111.

- Kuiren, S., Aivalis, K. & Sreenivasan, K.R. 2001 *Anisotropy of small-scale scalar turbulence*. J. Fluid Mech. 448, 279-288, 2001

- A.S. Monin & Yaglom, A.M. 1975 *Statistical Fluid Mechanics, vol. 2* (MIT Press, Cambridge, MA)

- Niemela, J.J. & Sreenivasan, K.R. 2003a *Confined turbulent convection*. J. Fluid Mech. 481, 355-384.

- Niemela, J.J. & Sreenivasan, K.R. 2003b *Rayleigh number evolution of large scale coherent motion in turbulent convection*. Europhys. Lett. 62, 829-833.

- Niemela, J.J., Skrbek, L., Sreenivasan, K.R. & Donnelly, R.J. 2001 *The wind in confined thermal convection*. J. Fluid Mech. 449, 169-178.

- Niemela, J.J., Skrbek, L., Sreenivasan, K.R. & Donnelly, R.J. 2000 *Turbulent convection at very high Rayleigh numbers*. Nature 404, 837-840.

- Qiu, X.-L., Yao, S.-H. & Tong, P 2000 *Large-scale coherent rotation and oscillation in turbulent thermal convection*. Phys. Rev. E 61, R6075-R6079.

- Qiu, X.-L. & Tong, P 2001 *Onset of coherent oscillations in turbulent RayleighBenard convection*. Phys. Rev. Lett. 87, 094501-04.

- Roche, P., Castaing, B., Chabaud, B., Hebral, B. & Sommeria, J. 2001 *Sidewall effects in Rayleigh-Benard convection*. Euro. Phys. J. 24, 405-408.

- Spiegel, E.D. 1971 *Convection in stars*. Annu. Rev. Astron. Astrophys. 9, 324-367.Sreenivasan, K.R. & Dhruva, B. 1988 *Is there scaling in high-Reynolds-number turbulence?* Prog. Theo. Phys. 130, 103-120.

- Sreenivasan, K.R., Bershadskii, A. & Niemela, J.J. 2002 *Mean wind and its reversal in thermal convection*. Phys. Rev. E 65, article no.056306.Threlfall, D.C. 1975 *Free convection in low-temperature gaseous helium*. J. Fluid Mech. 67, 17-28.

- Tritton, D.J. 1988 *Physical Fluid Dynamics*. Clarendon Press, Oxford.

- Verzicco, R. 2002 *Side wall finite conductivity effects in confined turbulent thermal convection*. J. Fluid Mech. 473, 201-210.

- Verzicco, R. & Camussi, R. 2003 *Numerical experiments on strongly turbulent thermal convection in a slender cylindrical cell*. J. Fluid Mech. 477, 19-49.

- Warhaft, Z. & Shen, X. *On the higher order mixed structure functions in laboratory shear flow*. Phys. Fluids 14, 2432-2438.