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Multiscaling of cosmic microwave background radiation

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Abstract

The multiscaling features of structure functions of cosmic microwave background (CMB) radiation have been studied using the new data available with the resolution of the order of a degree (combined QMAP and Saskatoon CMB radiation maps). It is shown that the scaling exponents of the CMB structure functions in the angular scale interval 0.9–4° are nearly identical to those of universal velocity structure functions in fluid turbulence. This indicates a strong connection between fluid turbulence and the origin of the observed CMB angular anisotropy in this interval of scales. Since the atmosphere and foreground radiation give minor contribution to these data the most likely explanation for this finding is primordial turbulence. © 2002 Elsevier Science B.V. All rights reserved.

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The cosmic microwave background (CMB) radiation in the microwave wavelengths is believed to be the remnant of the early universe, corresponding to the recombination time at which baryonic matter and radiation were decoupled. The properties of CMB are, therefore, of great interest for the understanding of the formation of early universe. The early data obtained from the Cosmic Background Explorer, or the COBE satellite, were used to obtain some insight into this question. However, low angular resolution of COBE measurements (about 7°) does not allow make a significant advance into this subject. More recently, extensive CMB maps with angular resolution of the order of a *degree* have become available (see, for instance, [1] and references therein). The new maps have been generated from both balloon-borne and ground-based observations, and have been tested thoroughly, compared with each other and with the previously obtained satellite data. While the CMB temperature fluctuations are themselves Gaussian, we show that CMB structure functions, which are moments of various order of the CMB angular *increments*, exhibit multiscaling with respect to angular increments. This multiscaling is found in the interval between 0.9 and 4°. Though the range is small, it is robust and the scaling exponents are remarkably close to those of velocity structure functions

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in fluid turbulence. This effect is not marginal in the sense that the entire angular increments in this interval appear to be generated by a fluid turbulence.

At present, we can suppose three possible origins of this phenomenon: atmospheric turbulence, foreground radiation from galactic and unresolved point sources, and primordial turbulence. We will discuss these sources in some detail, but it is worth noting in advance that, on balance, primordial turbulence appears as the most plausible candidate.

We will use maps made from the QMAP (balloon) and Saskatoon (ground-based) experiments [1–4]. The observations were made in Ka and Q bands (26–36 and 36–46 GHz, respectively) for different polarization channels. QMAP was designed to measure the CMB anisotropy by direct mapping. The Saskatoon data set is different in the sense that it does not contain simple sky temperature measurements. Instead, it contains different linear combinations of sky temperatures with complex set of weighting functions. The Saskatoon measurements probe a circular sky patch within about 16° diameter, centered on the North Celestial Pole. Then the QMAP and Saskatoon data were combined to produce a CMB map named QMASK. The map was generated by subdividing the sky into square pixels of side $\Theta \simeq 0.3°$ and consists of 6495 pixels. The data are represented using three coordinates x, y, z of a unit vector $\mathbf{R}$ in the direction of the pixel in the map (in equatorial coordinates). Since some pixels are much noisier than others and there are noise correlations between pixels, Wiener filtered maps are more useful than the raw ones. Wiener filtering suppresses the noisiest modes in a map and shows the signal that is statistically significant.

The main tool for extracting cosmological information from the CMB maps is the angular power spectra. However, such spectra use only a fraction of the information on hand. Taking advantage of the good angular resolution of the QMASK map, we can study statistical properties of angular *increments* of CMB temperature for different values of angular separation. The increments are defined as

$$\Delta T_r = \big(T(\mathbf{R}+\mathbf{r}) - T(\mathbf{R})\big), \tag{1}$$

where $\mathbf{r}$ is dimensionless vector connecting two pixels of the map separated by a distance r, and the structure functions of order p as $\langle |\Delta T_r|^p \rangle$ where $\langle\cdot\rangle$ means a statistical average over the map.

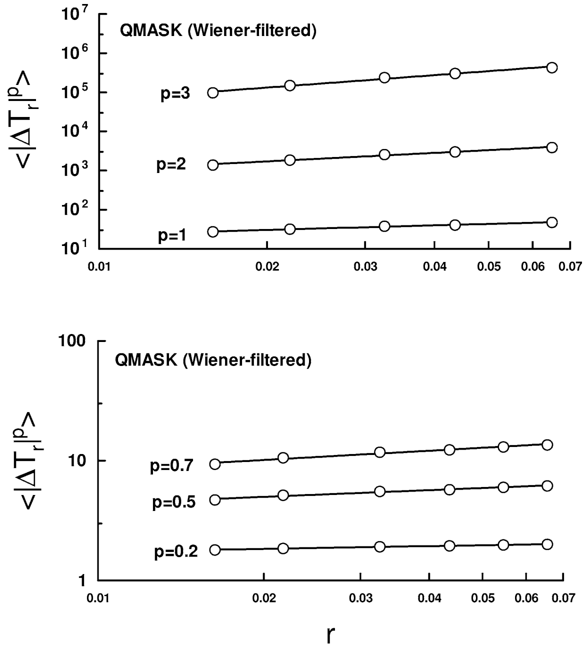

Fig. 1. Logarithm of moments of different orders $\langle |\Delta T_r|^p \rangle$ against $\log_{10} r$ for the Wiener filtered QMASK map. The straight lines (the best fit) are drawn to indicate the scaling (2).

Fig. 1 shows, in log–log scales, the dependence of the moments of different orders, $p = 0.2$, 0.5, 0.7, 1, 2, 3 of $\langle |\Delta T_r|^p \rangle$ against r for the Winer filtered QMASK map. The best linear fits are drawn to indicate the scaling

$$\big\langle |\Delta T_r|^p \big\rangle \sim r^{\zeta_p}. \tag{2}$$

Fig. 2 shows the scaling exponents ζ_p extracted from the QMASK data shown in Fig. 1 (circles). We also show in this figure (as crosses) the scaling exponents corresponding to velocity increments in fluid turbulence ([5,6]). The correspondence is rather remarkable (especially for low moments for which the calculations are also the most reliable). Let us recall that the values of ζ_p for $p = 1$ and $p = 2$ in fluid turbulence are direct consequence of Kolmogorov's cascade hypothesis, whereas the exponent $\zeta_3 = 1$ is a rigorous consequence of the dynamic equations of fluid motion; see, for instance [5].

It appears that for the angular interval between 0.9 and 4°—this being the most reliable range for the data [4]—the temperature increments in QMASK

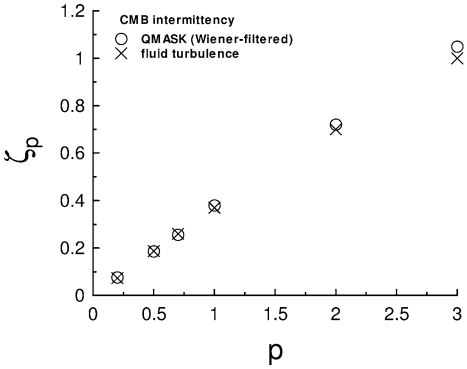

Fig. 2. The scaling exponents ζ_p, corresponding to Eq. (2), for the QMASK map (circles). The crosses (the data are taken from Table 1 of [5] and from [6]) represent the scaling exponents corresponding to the Kolmogorov fluid turbulence.

map, scale essentially as velocity increments in fluid turbulence.

It should be noted that under certain circumstances scaling of the moments (2) for $p = 2$ can be related to scaling of corresponding spectrum. One of the necessary conditions for such relation is sufficiently fast decrease of the spectrum with increasing of wave numbers $k > \eta^{-1}$, where η is the lower edge of the scaling interval of r. For fluid turbulence such fast decay of the spectral asymptotic indeed takes place (η is so-called Kolmogorov or viscous scale there). However, it seems that for the CMB data this is not the case and the strong relation between fluid turbulence and the CMB space fluctuations takes place in a restricted interval of scales only (and does not determine the spectral asymptotic).

Background signal can be distinguished from the atmospheric and foreground contamination signals by their frequency dependence, their frequency coherence and their spatial power spectra. Employing these methods, it has been shown in a series of papers [1–4,7–9] that all known atmospheric and foreground sources give a minor contribution to the QMASK map.

It is always possible that extragalactic sources with parameters different from the known foreground radiation could alter these maps but the commonly accepted opinion (see, for instance [8]) is that the effect would be small in comparison with background radiation.

We are thus led to conclude that the background (primordial) turbulence is the most plausible reason why the degree-scale CMB maps resemble fluid turbulence in their intermittence properties. Primordial turbulence was proposed in Refs. [10] and [11] to explain the formation of galaxies and clusters of galaxies. The theory was criticized on the basis that turbulent eddies would completely decay before galaxies were formed because of intense acoustic radiation. After the discovery of CMB this criticism was found to be irrelevant (cf. also [12]). The reason is that a sound speed in plasma, coupled strongly with radiation, is likely to have been very high, so that the turbulent velocity would be very low in comparison, thus rendering acoustic radiation essentially insignificant. However, other mechanisms for the formation of galaxy and galaxy clusters, such as gravitational instability [13], turned out to be more popular, and, due to lack of experimental support, the idea of primordial turbulence was poorly developed (though not completely neglected, see [14,15] and references therein).

In the last few years, however, the interest in primordial turbulence has been renewed in relation to another problem: the origin of magnetic fields observed in galaxies. It has been suggested that primordial magnetic fields might arise during the early cosmic phase transitions. Such a field could then contribute to the seed needed to understand the presently observed galactic magnetic fields [16], which have been measured in both the Milky Way and other spiral galaxies, including their halos.

At earliest times, the magnetic fields are generated by particle physics processes, with length scales typical of particle physics. If the inflation hypothesis is correct, long correlation lengths can be expected following the inflation. In general, the primordial turbulence is an essential feature of such phenomena [17]. It is shown in [18] that rotational velocity perturbations characteristic to the primordial magnetohydrodynamic (MHD) turbulence can produce significant angular scale anisotropies in CMB through the Doppler effect. On the other hand, it is well-known (see [19] for a recent review) that the scaling laws of fluid turbulence are rather surprisingly persistent in astrophysical plasma (MHD-) turbulence. It is conceivable that the primordial turbulence might be significantly different from the interplanetary and interstellar plasma turbulence, but it seems from our observations that the general features are the same.

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