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Self-Sustained Large-Scale Flow in Turbulent Cryogenic Convection

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Turbulent convection is studied in a cell filled with cryogenic helium gas. At high Rayleigh numbers a large-scale circulation persists and has a size comparable to the container. Over a wide range of time scales greater than its characteristic turnover time, this mean flow exhibits occasional and irregular reversals of direction without a change in magnitude. We study this feature in an apparatus of aspect ratio unity, in which the highest attainable Rayleigh number is about 10^{16}.

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1. INTRODUCTION

The turbulence generated by thermal convection in a finite container has several features of organized motion that are known to exist. Diffusive boundary layers form on the upper and lower heated surfaces and periodically eject "plumes" of relatively warmer (lower surface) or colder (upper surface) fluid into the bulk. Under ideal conditions both boundary layers are dynamically identical, and the emitted plumes from each can synchronize to drive a large scale flow about the periphery of the cell, often called the "wind."[1,2] This self-sustained flow exists up to very high values of the control parameter, and the existence of such organized and persistent motion amidst a sea of strong fluctuations is a fascinating feature of fully developed turbulence.

In the present experiments, we have observed that the wind, while persistent, does not have a stable direction, exhibiting occasional and irregular

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reversals. To study the long-time dynamics of this phenomena, we have measured the wind speed and direction for continuous periods up to one week in a cryogenic Rayleigh-Benard cell filled with helium gas. The relevant control parameters are the Rayleigh number, defined as $Ra \equiv \alpha \Delta g L^3/\nu\kappa$ and the Prandtl number, $Pr = \nu/\kappa$. Here α is the isobaric thermal expansion coefficient of the fluid in the container, Δ is the temperature difference between the bottom and top walls, g is the acceleration due to gravity, L is the height of the convection cell, ν is the kinematic viscosity of the fluid and κ its thermal diffusivity. The Rayleigh number measures the ratio between the rate of release of gravitational potential energy to the rate at which it is dissipated by viscosity and thermal diffusion. The Prandtl number represents the ratio of characteristic diffusion times for heat and momentum in the fluid.

2. APPARATUS

The basic apparatus has been described in detail elsewhere[3] and here we briefly outline its salient features. The experimental cell is cylindrical and has a diameter-to-height aspect ratio $\Gamma \equiv D/L = 1$, for D,L= 0.5 m. We use cryogenic helium gas as the working fluid, and by varying the mean pressure and temperature of the gas, in addition to the application of heating, we are able to achieve Ra between 10^6 and 10^{16} in the same apparatus.

The top and bottom plates of our cell were made of annealed OFHC copper with a thermal conductivity of about 2 kW m^{-1} K^{-1}. The side wall was made of thin stainless steel. Three radiation shields surrounded the cell and resided in a common vacuum space. The upper copper plate was connected to a helium reservoir through a distributed and adjustable thermal link containing helium gas, and its temperature was held constant via a resistance bridge and servo. The lower plate was uniformly heated using a metal film heater distributed over the entire area of its outer horizontal surface.

Temperature fluctuations in the fluid were measured by small sensors made of neutron-transmutation-doped germanium crystal cubes, 250 μm on a side. To measure the large scale flow, two vertically aligned sensors were placed on the center plane of the cell at a radial distance $w = 4.4$ cm from the side wall and separated by a vertical distance $d = 1.27$ cm. Temperature fluctuations $\theta_1(t)$ and $\theta_2(t)$ were read by the two sensors, which were each part of a separate bridge circuit and sampled at a frequency of 50 Hz over long times (up to one week). We define a short-time correlation between θ_1 and θ_2 as

$$G(\tau) = \Sigma_{i=1}^{i=N} \theta_1(t_i)\theta_2(t_{i+n}), \tag{1}$$

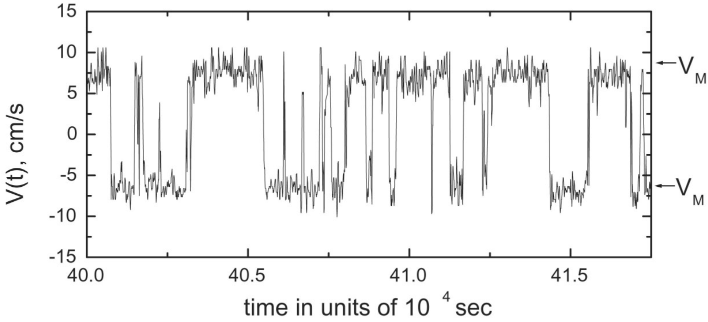

Fig. 1. Large scale velocity determined by correlation of side-wall temperature sensors for $Ra = 1.5 \times 10^{11}$.

where t_i is the i-th sample of a digitized signal, $\tau = t_{i+n} - t_i$. The minimum window width N for calculating the correlation is typically of the order of one convective turn-over time for the large scale flow. The delay time τ_M for which G has a maximum defines the average phase lag between the two signals θ_1 and θ_2. The speed of the mean flow V is then given by $V = d/\tau_M$, where τ_M can be of either sign. Because V is averaged over the window N, the method cannot resolve fluctuations that oscillate more rapidly than its width and thus is sensitive only to the large scale flow, which is our primary objective.

Here we note that the large scale flow has the form of a single roll, which is verified by observing that sensors placed at 180 degrees about the circumference indicate the opposite sense of flow and show a strong correlation consistent with the measured speed. Presumably, experimental imperfections (e.g., welding seams, sensor placement, apparatus tilt, etc.) break the cylindrical symmetry enough to prevent azimuthal wandering of the wind.

3. RESULTS

Figure 1 shows a small segment of the velocity record determined as described above for $Ra = 1.5 \times 10^{11}$. It is immediately clear that the flow switches direction "suddenly," in the sense that it occurs over a time comparable to the turnover time for the flow. In addition, there is no obvious regularity to the directional switching. The distribution of flow direction is

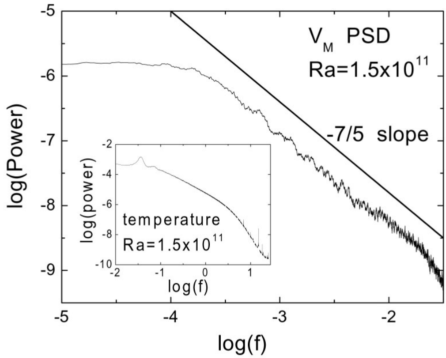

Fig. 2. PSD for V_M at $Ra = 1.5 \times 10^{11}$. The solid line has a slope -7/5, similar to the scaling of the temperture PSD shown in the inset. The temperture PSD shows a strong periodic component associated with plume emission.

bimodal and highly symmetric; i.e., in the longest single run (one week) the amount of time spent in either direction is the same to within 1.5%. The mean speed V_M, indicated on the right of Figure 1, is the same for both orientations of the flow.

In Figure 2, we show the power spectral density (PSD) for the velocity. The inset shows the corresponding PSD for the temperature fluctuations from one of the sensors. While both spectra possess a nearly identical power-law scaling, the temperature PSD has a well-defined peak and a weak harmonic, corresponding to the emission of plumes from the heated surfaces, while the velocity PSD does not exhibit any obvious periodic features. In addition, there is an orders-of-magnitude difference in scales between the temperature fluctuations and the directional switching of the large scale flow.

It appears that for small time scales the flow reversal has a fractal dimension, which we determine as follows. We divide the total velocity time series into boxes or bins of size r and count the number of boxes for which there is at least one zero-crossing as a function of box size. For a fractal, the number of non-empty boxes of size r, N_r, should exhibit a power-law relation with r; i.e., $N_r \sim r^{-\gamma}$ where γ is the fractal dimension. A typical plot of log N_r vs log r is shown in Figure 3. The small scales possess a dimension of 0.4, represented by the solid line, while the larger scales become space-filling with a slope of 1 as shown by the dashed line.

A Reynolds number, or dimensionless velocity, defined as $Re \equiv V_M L/\nu$ can be calculated from the mean value of the wind speed V_M. The relation between Re and the driving parameter Ra is shown in Figure 4. A

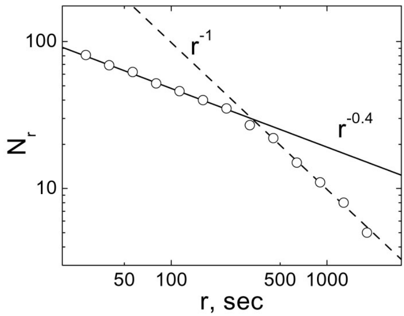

Fig. 3. The log-log plot of N_r vs r for $Ra = 1.1 \times 10^{10}$. The existence of a power-law behavior for small r suggests that the zero-set has a fractal dimension γ given by the magnitude of the power-law exponent. The dimension here is about 0.4 corresponding to the solid line. Larger scales are space-filling and follow the dashed line representing r^{-1}.

dependence on Pr of the form $Re \sim Pr^{-\alpha}$ is assumed in order to account for its variation. Chavanne, et al [4] adopted a value $\alpha \approx 0.72$ for which they found $Re \propto Ra^{0.49} Pr^{-0.72}$. To facilitate comparison we have chosen the same exponent, and a subsequent least squares fit to our data gives $Re = 0.17 Ra^{0.49} Pr^{-0.72}$ over seven decades of Ra in good agreement with these authors. Here, Pr varies between 0.7 to 1.2.

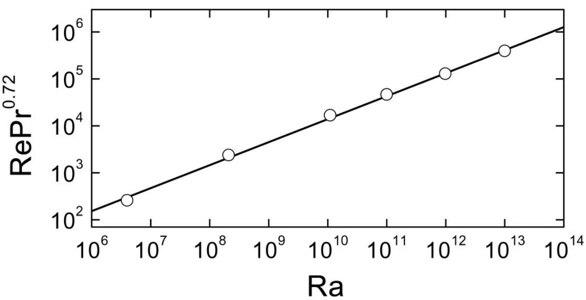

Fig. 4. Scaling of $RePr^{0.72}$ vs Ra. The solid line is a least squares fit to the data $Re = 0.17 Ra^{0.49} Pr^{-0.72}$.

4. DISCUSSION

The switching of the mean flow shows interesting generic qualities; e.g., there are striking similarites to the paleomagnetic record of non-periodic and "sudden" reversals in the polarity of the dipolar part of the Earth's magnetic field, which is generated by turbulent convection currents in the outer core. This irregular reversal process has also been observed in simulations [5]. There is a similar orders-of-magnitude spread in time scales: geomagnetic reversals also take place "suddenly" (of order 5000 years!), while the polarity may remain constant for periods that range up to order 500,000 years. This can be compared to the range of time scales observed here, which in order of magnitude vary between approximately 10^1 sec and 10^3 sec.

We can speculate on the origin of the wind reversals by considering the following: the wind will carry plumes away from the horizontal surfaces and this will act locally to reduce buoyancy. If, by chance, a relatively larger plume is emitted at one position along the wall the mean flow will become more susceptible to small perturbations which, unknowable *a priori*, may serve to either reinforce or reverse the flow. This could then lead to the chaotic process observed for the wind reversals.

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