Comments on high Rayleigh number convection

J.J. Niemela, L. Skrbek, K.R. Sreenivasan[†], and R.J. Donnelly
University of Oregon, Eugene, OR 97403
[†] *Yale University, New Haven, CT 06520*

Abstract.
We have recently conducted a series of experiments on turbulent convection in the range of Rayleigh numbers between 10^6 and 10^{17} (Niemela *et al.* 1999). The working fluid is cryogenic helium gas. The eleven decades of dynamic range enable us to make a few conclusive observations. Among them, the following aspects are noteworthy.

1. Scaling of the heat transport

The Nusselt number, Nu, scales with Ra according to $Nu = 0.124 Ra^{0.309 \pm 0.0043}$, essentially over all eleven decades of Ra (Fig. 1)[1]. The data can also be fitted equally well by a $3/10^{\text{ths}}$ power of Ra with logarithmic corrections, $Nu \sim (Ra^{3/2} \ln Ra^{3/2})^{1/5}$. The form of this latter expression is derived from a weakly nonlinear theory for conditions just past the onset of convection. This theory, due in various stages to the efforts of Howard, Roberts, Stewartson, and Herring (see also Toomre *et al.* 1977) consists of calculating the steepest variation of the Nu by a single- mode solution of weakly nonlinear convection. The functional form fits the data very well, but the coefficient in front is measured to be smaller by a factor of about 4. Regardless of its perceived applicability, we note that the number of adjustable parameters in this expression is only one (namely the prefactor); even the simple power-law fit has two unknown coefficients. The Rayleigh number span of the data is large enough to rule out, for the present convection cell, the classical $1/3^{\text{rd}}$ power (Malkus 1954, Priestly 1959) and the more recent $2/7^{\text{th}}$ power (Castaing *et al.* 1989, Shraiman and Siggia 1990). In particular, we do not observe a transition to Kraichnan's (1962) asymptotic regime (see also Howard 1963, Doering and Constantin 1996). This observation is consistent with the recent finding of Glazier et al. 1999. Over the Rayleigh number range covered in the experiment, it is possible to discern the differences between the formula culled from the weakly nonlinear theory and the upperbound result of Constantin

[1] The precise value of the exponent depends on how well one knows the physical properties of helium gas. We have used the most recently available standard data. See Niemela et al. (2000) for some details.

T. Kambe et al. (eds.), IUTAM Symposium on Geometry and Statistics of Turbulence, 269–277.

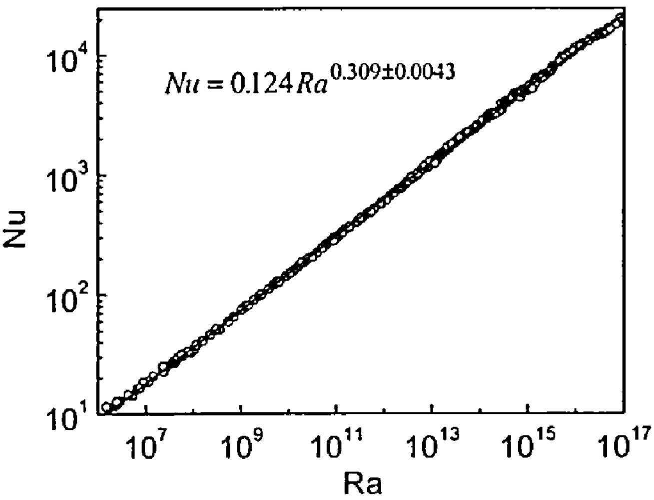

Figure 1. Log-log plot of the Nusselt number versus Rayleigh number. The line through the data is the least squares fit over the entire range of Ra.

and Doering (1999) for the case of very large Prandtl numbers, namely $Nu \leq Ra^{1/3}(1 + \log Ra)^{2/3}$

2. The mean wind

The so-called mean wind is the strong recirculating motion in the convection cell. At low Rayleigh numbers it does seem to exist in the form of unidirectional circulation, but the situation is complex at high Ra. If averaged over a suitable intervals of time, a semblance of the mean wind can be observed in this latter regime as well, but it is small compared to the free-fall velocity by a factor of 10 to 30 (depending on details of how the mean wind is estimated). In particular, this mean wind seems to alternate its direction quite frequently (Fig. 2a). This conclusion is not based on a direct measurement of velocity, but on correlating signals from two neighboring temperature probes. By necessity, the measurement technique does not discern velocity fluctuations whose time scales are smaller than the averaging time scale. If the averaging time becomes smaller, the distinctly bimodal nature of the distribution disappears (see Figs. 2b and 2c). Thus, a realistic picture may be one of weak large-scale circulation, upon which strong small-scale velocity fluctuations are superimposed. This does not necessarily mean that the theories invoking mean wind are incorrect: for their purposes, the

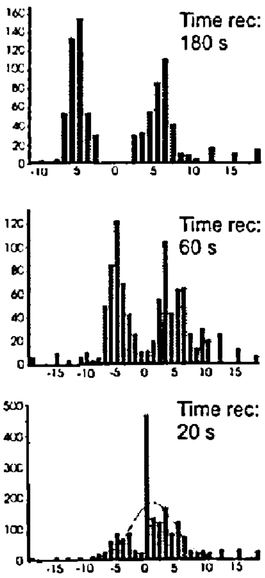

Figure 2. A rough measure of the large-scale velocity in the cell. The data are obtained by correlating the temperature signals from two neighboring probes, allowing for a time delay from one of them so as to maximize the correlation. The correlation is obtained by averaging the two signals over a certain amount of time. These time scales are different for the three cases shown here (180s, 60s, and 20s for (a), (b) and (c) respectively). By construction, velocities corresponding to time scales smaller than the averaging time cannot be discerned from these measurements. The distinctly bi-modal nature seen in (a) becomes less clear as the averaging time becomes smaller. The conclusion appears to be that the so-called mean wind is a manifestation of the large scale when small-scales are suitably averaged out.

shearing motion established by the somewhat random large-scales is perhaps adequate.

3. Prandtl number variation

In general, the Nusselt number depends not only on the Rayleigh number (the dynamical parameter), but also on the Prandtl number (which is a fluid property) and the aspect ratio (a geometric property). In our measurements, the Prandtl number (Pr) was constant up to an Ra of 10^{13} . Beyond this, Pr eventually increased to about 30 (Fig. 3), staying less than or of order unity for Ra up to 10^{15} and increasing and increasing to a maximum value of about 30 at the highest Rayleigh number. Using the first seven decades of the Rayleigh number available in the regime of strictly constant Prandtl number, we obtained

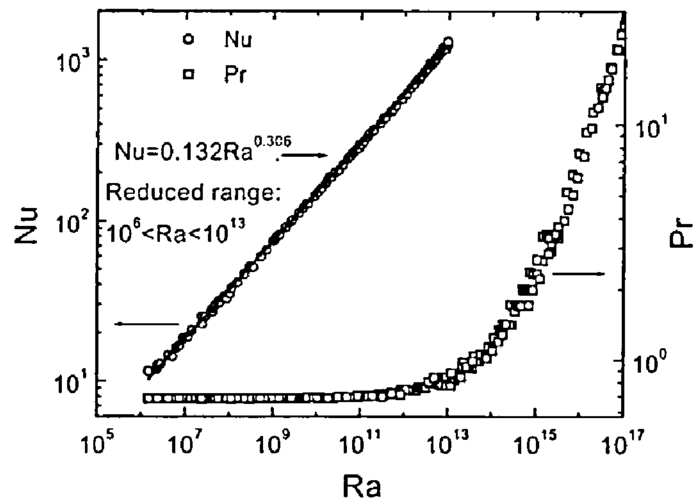

Figure 3. The Prandtl number corresponding to different Rayleigh numbers measured. The Nusselt number data for the first seven decades of Rayleigh number, over which the Prandtl number is strictly constant, are fitted by a power law in this figure. This power-law is given by $Nu = 0.132Ra^{0.306}$

the Rayleigh number scaling at fixed Prandtl number;this power-law fit is given by $Nu = 0.132Ra^{0.306}$. This does not differ significantly from the fit obtained earlier (Fig. 1) for the entire range of Ra. This means that the Prandtl number effects are relatively weak for Pr in the range considered here. These small effects can be estimated, assuming that no other transition occurs, by plotting the variation of the ratio $Nu/0.132Ra^{0.306}$ against Pr. This is done in Fig. 4. The Nusselt number ratio decreases weakly for increasing Prandtl numbers. If this decrease is fitted by a power law, even if less than convincingly, we obtain the Prandtl number effect to $Pr^{-0.09}$. Taking these results in conjunction with an earlier study of Verzicco et al. (1998) at low Prandtl numbers, we summarize the Prandtl number variation as follows:

$$\begin{array}{lll} Nu \sim Pr^{0.14} & \text{for} & Pr < 0.1 \\ Nu \sim Pr & \text{for} & 0.1 < Pr < 5 \\ Nu \sim Pr^{-0.09} & \text{for} & Pr > 5. \end{array}$$

This last formula is not inconsistent with the theory of Shraiman & Siggia (1990).

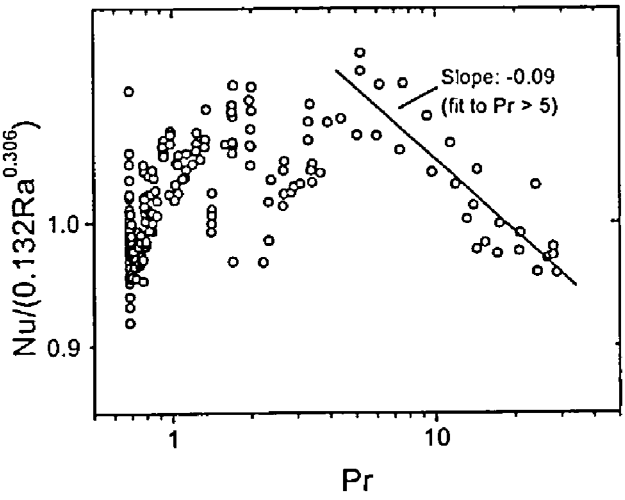

Figure 4. An estimate of the effects of variable Prandtl number. The ordinate is the measured Nusselt number divided by $0.132Ra^{0.306}$, the latter being the fit to the data in the region of constant Prandtl number. If the variability seen in the data is attributed to Prandtl number changes, we obtain, roughly, something like $Pr^{-0.09}$

4. Aspect ratio variation

The present experiments pertain to a fixed aspect ratio of 1/2. In order to get a sense of the effects of the aspect ratio, we collected various data on how the Nusselt number varies with aspect ratio, keeping both Rayleigh and Prandtl numbers fixed. This issue has been discussed by others (e.g., Castaing *et al.* 1989) before, but we carry it one modest step further. Figure 5 shows the results. To the lowest approximation, the Nusselt number decreases with increasing aspect ratio; the aspect ratio ceases to be important probably when it is as high as 5 to 10. Perhaps the increase for smaller aspect ratio is related to the increased importance of side-wall boundary layer. In detail, however, the Nusselt number dependence appears to be non-monotonic (see the dashed line in Fig. 5). That particular behavior is not understood, but is perhaps related to the accommodation of the most efficient heat transfer modes (largest scales) by the finite geometry. If so, it must vanish in the limit of very large aspect ratio, as it indeed seems to do.

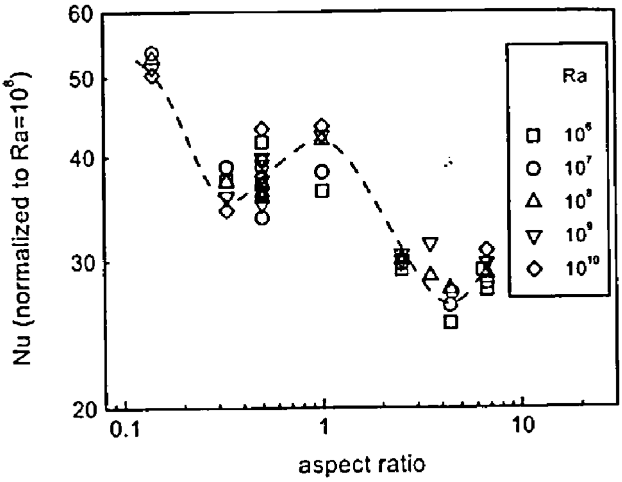

Figure 5. Effects of aspect ratio on the Nusselt number by fixing the Rayleigh number and the Prandtl number. (The latter is not strictly constant but does not vary over a large range, and the Pr variation over that range is small.) The data are normalized to those at Ra = 108, and are extracted from the following references: Rossby (1969), Garon & Goldstein (1973), Chu & Goldstein (1973), Threlfall (1975), Tanaka & Miyata (1980), Wu (1991), Chavanne (199?), and Niemela et al. (1999).

5. The dissipation rate

A few comments can be made on the dissipation rate in the convection cell. By integrating the equations of motion satisfying the Boussinesq approximation,it is possible to derive an exact equation for the turbulent energy dissipation averaged over the entire cell. The relevant expression for the non-dimensional dissipation rate is $(Nu-1)Ra$ (Howard 1963). This quantity is plotted in Fig. 6. There seems to be a unique power law for all Rayleigh numbers, as could have been guessed from Fig. 1. This power-law exponent is measurably distinct from 3/2 the latter being the expectation from Kolmogorov-type dimensional arguments.

Grossmann & Lohse (1999) have split the energy dissipation into a bulk contribution and the boundary layer contribution. Another parameter to contend with is the ratio of the thermal boundary layer to that of the momentum boundary layer. Depending on which effect dominates, these authors propose different power laws for the Nusselt/Rayleigh number relation; they note that, in a phase plane of $Ra - Pr$, different areas can be expected to have different scaling exponents. This is depicted in Fig. 7. The present experimental data are overlaid on

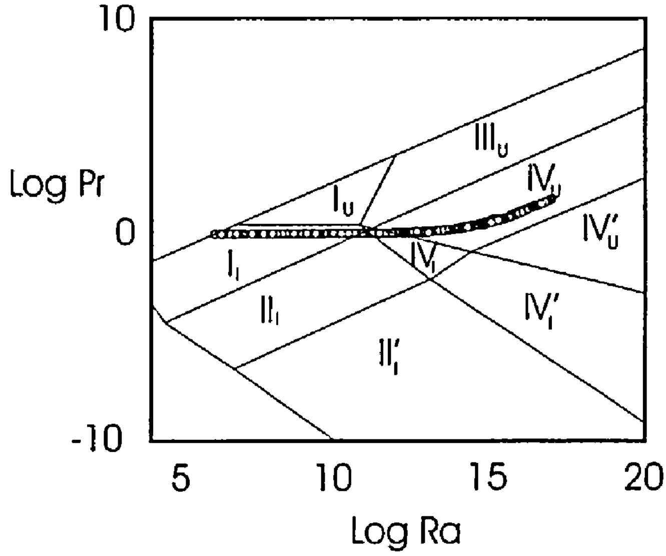

Figure 6. The product (Nu-1)Ra, which is the average energy dissipation in the whole convection cell, as a function of Ra. It is fitted essentially by a single power law, this being different from the 3/2 power expected by dimensional argument of the type employed by Kolmogorov in his 1941 theory.

that diagram. Although linear combinations of different power laws can mimic a single power law relationship over many decades (as noted by Grossman & Lohse 1999), no statistical advantage is gained over fitting the data with just one value of the power-law exponent, and the latter's simplicity makes its use somewhat more compelling. One possible explanation for why a single power law distinct from 3/2 can correctly describe the data is that the energy dissipation contained in the boundary layer never becomes unimportant (as would have to be the case beyond some Rayleigh number if the Kolmogorov scaling were to be valid). If, on the other hand, the boundary layer contribution remains the same fraction of the overall dissipation at all Rayleigh numbers, the power-law cannot be estimated by dimensional arguments. This may well be the case because, although the boundary layer becomes progressively thinner and occupies smaller volume at increasingly large Ra, the relevant velocity gradients within the boundary layer become correspondingly larger. It is thus conceivable that the boundary layer contribution to the dissipation never ceases to be important.

In order to test this idea, it is necessary to measure the boundary layer in detail. This has not yet been done. For a partial explanation, one can turn to the case of turbulent boundary layer flows, and inquire if the so-called wall region carries increasingly smaller or larger fraction as the Reynolds number is increased. Relegating details to another place,

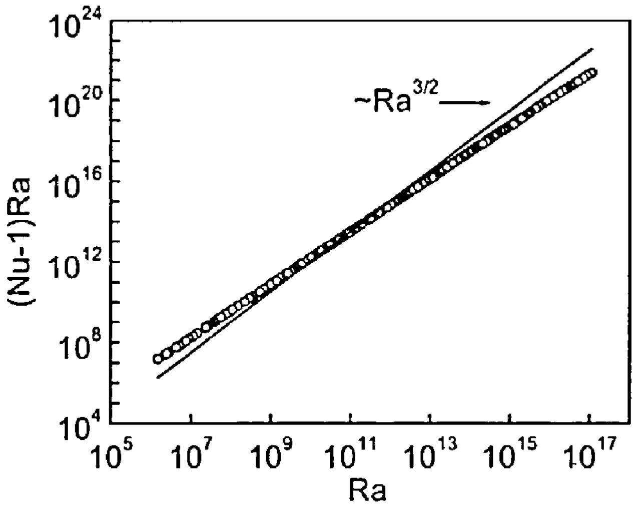

Figure 7. The different scaling regimes proposed by Grossmann & Lohse (1999) in the Pr-Ra plane. The darker regions are prohibited. The theory assigns a separate scaling exponent to each of the other regions. The interpretation is that the power law observed in an experiment is a superposition of more than one of those basic power laws. The present data are superimposed on this phase plane. They show that the data span more than one of the scaling regions of Grossmann & Lohse (1999), implying that the present power law is possibly a superposition of more than one of the basic power-laws. This exercise yields a reasonable fit to the data, but does not necessarily verify the basic tenets of the theory.

we simply note here that the basic idea presented here appears to be borne out roughly.

6. Concluding remarks

We have considered some aspects of high-Rayleigh-number convection. These aspects include the scaling of heat transport, the so-called mean wind, the effects of variable Prandtl number and aspect ratio, and the scaling of the energy dissipation rate. The problem of convection is extremely rich, and we have merely added to the existing knowledge on a few of its facets. Not all data acquired in our apparatus have been analyzed at this date. In particular, temperature fluctuation data from multiple probes have not yet been analyzed. We hope to be able to do this soon.

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