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Cryogenic Ultra-High Rayleigh Number Turbulence

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Turbulent thermal convection can be described by three dimensionless parameters: the Rayleigh number, $Ra = \alpha g \Delta T L^3/\nu\kappa$, Prandtl number $Pr = \nu/\kappa$, and aspect ratio, $\Gamma = D/L$, where g is the acceleration of gravity, ΔT is the vertical temperature difference across the fluid layer of height L, and α, ν and κ are respectively the thermal expansion coefficient, kinematic viscosity and thermal diffusivity of the fluid. D is a characteristic horizontal length scale; for a cylindrical cell it is the diameter. We report heat transfer measurements in turbulent thermal convection using helium gas in the temperature and pressure ranges $4.3K < T < 6K$ and 0.1 mbar$<$ P $<$3 bar, contained in a large Rayleigh-Benard cell of height $L = 1m$, covering 11 decades in Ra, $10^6 \leq Ra \leq 10^{17}$ [1]. Our data overlap previous measurements [2, 3] without relying on the divergence of fluid properties near the critical point, resulting in nearly constant Pr and negligible non-Boussinesq effects in the overlap region. To facilitate comparison with [2, 3], we retain the aspect ratio $\Gamma = 1/2$.

It is of considerable importance to predict the efficiency of heat transport at large Ra, represented by the Nusselt number, Nu, defined as the total heat flux normalized by its conductive contribution. Simple scaling arguments and experiments have suggested $Nu \sim Ra^{\beta}$. As summarized in [4], marginal stability arguments and dimensional reasoning produced the "classical" result: $\beta = 1/3$, while the so-called asymptotic "Kraichnan regime" suggests $\beta = 1/2$. Helium gas experiments of Wu [2] up to 10^{14} yielded exponents closer to 2/7 and somewhat higher for aspect ratio 0.5 (see below) stimulating other scaling theories, also reviewed in [4].

In contrast to [2], cryogenic helium experiments by Chavanne et al.[3] displayed a continual increase in β with increasing Ra, attaining a value close to 0.4 at $Ra \approx 10^{14}$. This was attributed to a transition to the asymptotic regime; however, it has not been observed in recent Hg experiments up to $Ra \approx 10^{11}$ [5], nor in recent SF_6 experiments [6], where (for constant Pr) a power law with $\beta = 0.3 \pm 0.03$ is reported for all Ra up to 10^{14}.

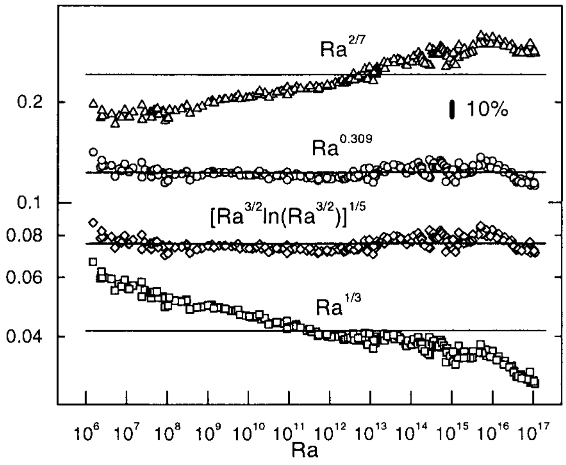

Figure 1: The log-log plot of Nu normalized by different scaling functions.

Our measurements in a $\Gamma = 1/2$ cell [1] also do not show any transition to an asymptotic regime, and yield an overall exponent for developed thermal turbulence $\beta \approx 0.31$ (preliminary data in a $\Gamma = 1$ cell are consistent with this scaling) over the whole range $10^6 \leq Ra \leq 10^{17}$. Our results are shown normalized to various expressions in figure 1. It includes the least-squares fit $Nu = 0.124 Ra^{0.309 \pm 0.004}$, where the three significant figures result from the fitting procedure alone. Our data can be equally well described by a 'mean-field' result [7] $Nu \sim (Ra^{3/2} ln Ra^{3/2})^{1/5}$, with only one adjustable parameter (the prefactor) instead of two for the simple power law. Exponents 1/3 and 2/7 are ruled out: the latter perhaps could be applied to a narrow region up to $Ra \approx 10^8$.

We remark on the newer theory of Grossman and Lohse [8]. They propose a superposition of power laws $Nu = 0.27 Ra^{1/4} Pr^{1/8} + 0.038 Ra^{1/3}$ that can mimic a single power law over many decades of Ra. It can be satisfactorily fitted to our data with slightly different prefactors (approximately 0.2 and 0.04).

To first order, β does not change significantly if one or more decades of Ra in the developed turbulence range are removed from the fitting procedure. It can depend, however, on the fluid properties used to evaluate Nu and Ra. The currently accepted property values [9] have changed significantly from the older ones [10], used to evaluate Ra and Nu in Wu's original experiments[2]. By far the most significant change is a decrease by $(5-10)\%$ in the thermal conductivity. We have recalculated Wu's original $\Gamma = 0.5$ data to take into account all changes in property values as well as a small correction due to the adiabatic gradient.

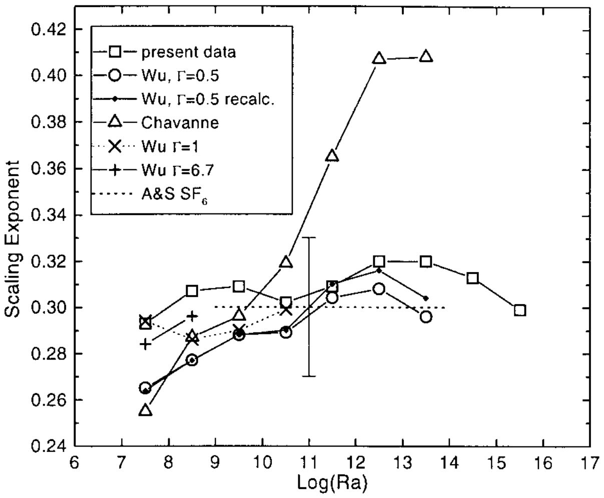

Figure 2: Figure 2. Local $Nu - Ra$ scaling exponents for successive 3-decade intervals, plotted against Ra at the mid-point of each interval.

The scaling for Wu's $\Gamma = 0.5$ data is affected primarily for $Ra > 10^{10}$, where the fractional change in thermal conductivity between [9] and [10] begins to vary significantly - otherwise Nu is simply shifted up with no change in slope.

Although the $Nu - Ra$ scaling for our data can be described to a good approximation by a single exponent 0.31 over the entire range of Ra, we examine it "microscopically" in figure 2, along with similar analysis for helium data of Wu [2] and Chavanne [3]. The local scaling exponents are calculated from least-squares fits over 3-decade intervals in Ra and are plotted versus Ra that corresponds to the middle of each interval. The first interval covers the range $10^6 \le Ra \le 10^9$. From the tabulated values of Nu and Ra in Chavanne's Thesis [3], only those points determined to be Boussinesq by the author's prescription were used. Included in the plot are data points derived from the recalculated $Nu - Ra$ data of Wu [2] for the $\Gamma = 0.5$ cell, as well as for the $\Gamma = 1$ and $\Gamma = 6.7$ cells. We note that the local exponent corresponding to the highest Ra for each data set was calculated from an interval slightly less than 3 decades. The recent SF_6 data [6] are represented as a horizontal dotted line with an associated error bar. Although we hesistate to extract too much information from this kind of detailed analysis, we can nonetheless make a few general observations.

First, the use of revised fluid properties [9] to recalculate Wu's $\Gamma = 0.5$ data results in a clear increase in the scaling exponent above $Ra \approx 10^{10}$, as expected from considerations above, and closely matches our results. Indeed, the scaling relation derived from Wu's re-analyzed data yields $Nu = 0.146Ra^{0.299}$ for all $Ra > 5x10^7$, where the turbulence is presumably fully developed. This scaling exponent is significantly greater than the previously reported value 0.290±0.005.

Second, it appears that the 2/7 law can be applied over narrow regions for most data sets. It is indeed not inconsistent with our data over the limited range $10^6 < Ra < 10^9$ where the calculated exponent is 0.29.

Third, the scaling exponents derived from Wu's and our helium experiments display a spread of values that are more or less consistent with the size of the error bar for the SF_6 experiments [6]. In accord with the discussion above, those from Chavanne's data show significant differences from the others for $Ra > 10^{11}$.

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