Comment on "Isotropic Turbulence: Important Differences between True Dissipation Rate and Its One-Dimensional Surrogate"

In a previous Letter, Hosokawa, Oide, and Yamamoto [1] examined direct numerical simulation (DNS) of isotropic turbulence to document differences between the real dissipation rate of turbulent kinetic energy, $\epsilon = \nu \partial_i \times u_j(\partial_i u_j + \partial_j u_i)$, and its one-dimensional surrogate, $\epsilon' = 15\nu(\partial_1 u_1)^2$. The box-averaged dissipation rates (using 3D or 1D boxes of linear size r) are denoted by ϵ_r and ϵ'_r, respectively. Hosokawa *et al.* [1] found significant differences in the probability densities of velocity increments conditioned on ϵ'_r instead of ϵ_r. They also found differences for the scaling exponents of ϵ_r and ϵ'_r, and concluded that the inferences drawn from the 1D surrogate must be reconsidered. While we welcome the results documented in Ref. [1], and certainly do not dispute that the differences between ϵ_r and ϵ'_r need to be better understood, we disagree with some of the authors' conclusions.

On a conceptual level, it needs to be emphasized that experimental circumstances at high Reynolds numbers continue to dictate the choice of ϵ'_r instead of ϵ_r. For this reason, Kolmogorov's refined similarity hypotheses were reformulated explicitly [2] for ϵ'_r instead of ϵ_r. This reformulation has a theoretical counterpart [3] and has successfully passed several tests (see [2] and references therein). Therefore, the established results for ϵ'_r at high Reynolds numbers can be regarded as fundamental.

The discrepancy found in [1] regarding the trend of the support of the probability density function of Kolmogorov's variable with increasing ϵ'_r in the dissipation range (a decreasing trend in [1] and an increasing one in [2,3]) is an artifact: In [1] Kolmogorov's variable was normalized with its root mean square; in [2,3] it was not. While we agree with Hosokawa *et al.* about the crucial dynamic role of the skewness in turbulence—see, e.g., [2]—their statement that it is difficult to incorporate the effects of skewness in the scheme of Refs. [2,3] is unfounded (see Ref. [4]).

It is similarly useful to study the multifractal properties of the surrogate. Figure 4 of Ref. [1] compares exponents of ϵ'_r obtained from single lines cutting across the computational domain and of ϵ_r from volumes over the 3D space. The latter give significantly lower generalized dimensions D_q, at high q. As argued below, this result can be attributed to a difference in statistical sample as opposed to inherent differences between the scaling of ϵ_r and ϵ'_r. The 1D segments used in Ref. [1] to compute moments of ϵ'_r consisted of only 128 or 512 points, which miss the intense but sparse events most of the time (and thus give higher D_q). The volumetric samples used for ϵ_r, on the other hand, consisted of 128^3 or 512^3 points, which can capture the strong events that dominate high-order moments. Independently of whether the dissipation is defined as the real or surrogate value, infrequent events can be captured by averaging over very long data records, or from DNS by averaging the moments over all 1D lines *before evaluating the exponents.* In Ref. [5], measurements of ϵ'_r scaling exponents (which, for high q, superseded the original short data segment measurements of Ref. [6]) were made based on long data records. The results, reproduced in Fig. 1, show that long 1D records of ϵ'_r are in good agreement (for positive q) with the 3D DNS results of Ref. [1]; the existing differences are within the accuracy to which the exponents can be determined, experimentally or numerically.

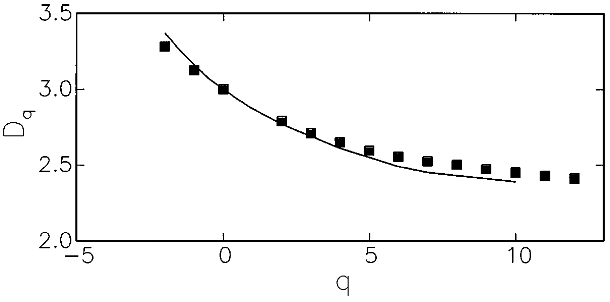

FIG. 1. Solid line: Generalized dimensions obtained for ϵ'_r [5] ($D_q = D_q^{(1)} + 2$). Squares: DNS results for ϵ_r [1].

In summary, we believe that studies of the surrogate dissipation, when properly interpreted, continue to yield useful and basic results, especially considering that it is not yet possible to measure the full dissipation at high Reynolds numbers (Re), and DNS is limited to low Re.

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