

## Preface

One of the most important conceptual advances of the modern era has been the realization that the temporal evolution of many nonlinear systems, independent of their physical origin, is profoundly similar. The similarity is dynamical and quantitative, not merely statistical and descriptive. The subject that cultivates and explores—to some degree or another—this universality of nonlinear problems is called nonlinear science, or nonlinear dynamics, or chaos, depending to some extent on the orientation of the researcher. Initial progress in the area was rapid and simultaneously occurred in several disciplines in the natural and mathematical sciences. As we have begun to appreciate better the richness of nonlinear systems, the challenges have proliferated as well.

From the time of inception of this multidisciplinary subject, several groups of scientists in India have contributed to it. These groups were initially scattered with no serious contact among them. In 1987, a summer school on the subject was organized in Bangalore, under the auspices of the Department of Science and Technology, with the intention of providing an impetus for further research and for developing a sense of community within the country. Many bright and young scientists participated with excitement and eagerness. Some of the seeds sown at the time have now matured, and the nonlinear science community seems to have established an identity of its own.

It was felt, ten years later, that a collection—in a single place—of the current work in the field would spur further synergism and collaborations. Thus came the inspiration for this special volume. The scope was restricted, by and large, to the physical sciences. Various researchers active in the field within the country (and some living abroad with strong ties to local activities) were invited to contribute to the volume. Given the special nature of the volume, they were encouraged to include in their contributions both review elements and new results. The enthusiasm with which the proposal was greeted by them is a testimony to both the interest that the field commands and the way in which the idea resonated with their own thinking. The articles were all subjected to peer review. While the papers are representative of the authors' specific interests and individually provide a glimpse of their own area of inquiry, it is our belief that the collection provides a comprehensive account of the work occurring in the field as a whole.

Given the vast scope of the work that this volume represents, it is difficult to pigeonhole it into a few neat categories but an attempt—even if imperfect—seems worthwhile. The forty two contributions have been grouped in six sections as follows.

A mathematical tool that is much used in nonlinear science, especially in quantum chaos and increasingly in other sub-fields of dynamical systems as well, is the theory of random matrices. Mehta's article, which begins this volume as well as Section I on mathematical aspects, is a comprehensive survey and summary of the most important results in this area. Other articles in the section by Balakrishnan, Nicolis and Nicolis, by Nandakumaran, and by Sinha and Das deal with iterated unimodal maps which have been important to the mathematical understanding of chaotic dynamics. Kolwankar and Gangal discuss the connection between fractional differentiability and the fractal dimension of curves; this topic is of considerable interest because of the ubiquitous

presence of fractals in chaotic systems. The meaning of the power spectrum and its generic features form the focus of the article by Valsakumar, Satyanarayana and Sridhar, who are concerned with signal processing and data analysis. Finally, Rangarajan deals with a new technique for integrating the equations of motion of a Hamiltonian system while explicitly respecting its symplectic property.

Dynamical systems that are exactly integrable—and therefore nonchaotic—have been studied in great detail well before the spurt of activity in chaos theory. Integrable systems are constrained by independent conserved quantities, and support coherent solutions such as solitons. The properties of solitons and their many applications form the focus of Section II which contains articles by Porsezian on the Painlevé property, by Lakshmanan and Radha on solitons in higher dimensions, and by Balakrishnan on the geometric aspects of nonlinear evolution equations. An area where the concepts of solitons has seen much application is that of particle physics and field theory, and the article by Sriram and Segar discusses some of these issues in the context of nonlinear chiral models.

An aspect of chaotic systems that has been at the forefront of recent research is their control. Several groups in the country are involved in devising, implementing and studying control methods. Articles in this area are grouped in Section 3. The papers by Gadre and Varma, Sharma and Gupte, and Rajasekhar are representative of the work in this area. Related concerns deal with extended dynamical systems, namely coupled lattice systems, where spatiotemporal chaos is of interest. Articles by Sinha, by Parekh, Ravi Kumar and Kulkarni, and by Roy and Amritkar are on the latter topic.

While temporal chaos has unearthed exceedingly interesting behaviours of nonlinear systems, their connections to fluid turbulence are still tenuous. In the background however, there exists the hope that the knowledge one acquires from simpler spatiotemporal systems and the methods developed to probe them will enhance our ability to understand the richness of fully developed turbulence. Section IV contains two articles that deal with aspects of turbulence from a relatively modern perspective: that by Bhattacharjee deals with statistical mechanical aspects of the randomly stirred fluid, while the paper by Dhar, Sain, Pande and Pandit deals with developments in the scaling phenomenology, and emphasizes the lessons learnt from the GOY model.

The field of quantum chaos attempts to understand the difference in behaviour between systems that are (in some appropriate limit) classically chaotic and those that are classically integrable. There are, in contrast to classical chaos, a multitude of definitions of what constitutes quantum chaotic behaviour. One very successful classification of systems is based on the different universality classes of random matrices. Papers in Section V have analysed problems in quantum chaos from a variety of points of view. Ganesan and Gebarowski look at chaos in the hydrogen atom in different external fields, while Santhanam, Sheorey and Lakshminarayan study the morphology and characteristics of eigenstates of quartic oscillators; the paper by Mahapatra, Sathyamurthy and Ramaswamy analyzes chaos and resonances in a molecular collision system, while Kudrolli and Sridhar report on results of an experiment to study quantum chaos in a microwave cavity. Pal examines chaos in nuclear motion and its role in the collective dynamics, using results from a quantum chaotic analysis in a classical model for dissipation in nuclear fission. Periodic orbits have played a major role in semiclassical theories, and the papers by Biswas, by Lakshminarayan, by Gaspard and Jain and by Khare deal with different aspects of trace formulae. The similarity between the

localization problem in disordered systems and a class of quantum chaos problems, especially in periodically driven systems, forms the focus of articles by Lahiri, Roy and Bhowal, Satija and Ketoja, and Bala Sundaram.

The final section, broadly classified as **Applications**, shows why chaos has enjoyed universal appeal: rarely, it seemed, had the methods developed in *modern* mathematics found such an immediate relevance in real-life, although one often had to look for the connections rather meticulously: Chaos is in the details!

Mehra, Nayak and Ramaswamy apply the methods of nonlinear dynamics to understand the mesoscopic behavior of clusters of a small number of atoms. Lakshmibala, Bambah, Sriram and Mukku review studies of dynamical chaos in classical gauge theories using both analytic as well as numerical techniques. Ambika discusses the complex behaviour of Josephson junctions, such as its phase locked and chaotic states, scaling near basin boundaries, and so forth. The classical chaos elements of collective thermalization of the quark-gluon problem are discussed by Sengupta, Bhatt, Kaw and Parikh. The paper by Kaw and Sen, which deals with the nonlinear propagation of electromagnetic waves in plasmas, is concerned with applications to a variety of current experiments in laser fusion and plasma based accelerators. Olinger, Chhabra and Sreenivasan describe experimental results in the wake of a transversely oscillating cylinder in a fluid flow, which, they demonstrate, belongs to the same universality class as circle maps with cubic nonlinearity. The low-dimensional chaotic behavior of the stick-slip phenomenon common in plastic instability of flowing materials is discussed by Noronha, Ananthakrishna, Quaouire and Fressengeas. Goswami dwells on the limits to predictability of Indian monsoons due to the phase and amplitude irregularities of interseasonal oscillations in a dynamic model. Puri discusses phase ordering of random magnets and phase separation in binary fluids, while Das treats the nonlinear dynamics of fluctuations and relaxation times in supercooled liquids. Finally, Bhattacharjee, Mehta and Luck discuss the role of infrared divergences in a perturbation theory of noisy coupled maps.

As this summary suggests, the richness and variety of the subject and its practice in the country, which this collection reflects, is immense. The subject brings together, in a unique way, a unification of mathematics, fundamental physics in several of its branches, as well as applications. We have greatly enjoyed putting the volume together, and hope that you, the reader, will share our sense of discovery and delight.

However, none of this could have happened without the support and cooperation of the nonlinear science community. As authors and referees, they have played the vital and crucial role in giving this volume its present shape and they deserve our warmest thanks.

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