# The passive scalar spectrum and the Obukhov–Corrsin constant

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It is pointed out that, for microscale Reynolds numbers less than about 1000, the passive scalar spectrum in turbulent *shear flows* is less steep than anticipated and that the Obukhov–Corrsin constant can be defined only if the microscale Reynolds number exceeds this value. In flows where the large-scale velocity field is essentially isotropic (as in grid turbulence), the expected 5/3 scaling is observed even at modest Reynolds numbers. All known data on the Obukhov–Corrsin constant are collected. The support for the notion of a "universal" constant is shown to be reasonable. Its value is about 0.4. © *1996 American Institute of Physics.* [S1070-6631(96)01801-4]

## I. INTRODUCTION

It is well known that the heuristic ideas of Kolmogorov[1] lead to a simple formula for the inertial-range spectral density of turbulent velocity fluctuations. In particular, for the so-called longitudinal spectrum, one has

$$\phi(k_x)=C_\kappa\langle\epsilon\rangle^{2/3}k_x^{-5/3}, \tag{1}$$

where the mean-square velocity fluctuation in the longitudinal direction $x$ is given by

$$\langle u^2\rangle=\int_0^\infty \phi(k_x)\,dk_x, \tag{2}$$

$k_x$ is the wavenumber component in the direction $x$, and $\langle\epsilon\rangle$ is the mean value of the energy dissipation rate. The prefactor $C_\kappa$ is the Kolmogorov constant. In a previous paper,[2] the experimental support for the notion of "universality" of $C_\kappa$—by which is meant that its numerical value is independent of the flow configuration and the Reynolds number—was examined. The conclusion was that $C_\kappa$ was *roughly* universal if the microscale Reynolds number exceeded about 50: Increasing the Reynolds number increases the scaling range but does not appear to alter the constant itself—at least to the extent discernible from the accuracy of the data. While this conclusion should be tempered because the constant in the transverse spectra attains universality only at much higher Reynolds numbers, the notion of a universal Kolmogorov constant seems plausible at "high enough" Reynolds numbers.

Our concern in this paper is the passive scalar mixed by turbulence—for example, small amount of heat injected into a turbulent flow—and the experimental support for the universality of the so-called Obukhov–Corrsin constant, named after the two people who independently extended Kolmogorov's arguments to the passive scalar.[3,4] For the passive scalar, one obtains various scaling regions depending on the ratio of the scalar diffusivity, $D$, to fluid viscosity, $\nu$ (see, for example, Tennekes and Lumley[5]). We shall restrict attention to the so-called inertial–convective range in which the local Reynolds and Peclet numbers are large and the direct effects of fluid viscosity and scalar diffusivity are unimportant. Let $\theta$ be the concentration of the passive scalar and let $\psi(k_x)$, defined *via*

$$\langle\theta^2\rangle=\int_0^\infty \psi(k_x)\,dk_x \tag{3}$$

be the one-dimensional spectral density of the scalar. Let $\langle\chi\rangle$ be the rate of "dissipation" of its variance, given by

$$\chi=2D\langle(\partial\theta/\partial x)^2+(\partial\theta/\partial y)^2+(\partial\theta/\partial z)^2\rangle. \tag{4}$$

In the inertial–convective range, dimensional analysis shows that

$$\psi(k_x)=C_\theta\langle\epsilon\rangle^{-1/3}\langle\chi\rangle k_x^{-5/3}, \tag{5}$$

where $C_\theta$ is the Obukhov–Corrsin constant. The most serious past attempt in assessing the universality of $C_\theta$ is that of Monin and Yaglom[6] and Yaglom,[7] although the issue has been raised periodically by others as well (see, for example, Refs. 8–15).

We have examined all the spectra of which we are aware. The following few remarks concern the procedure used for determining $C_\theta$. First, the data examined here are from single-point measurements in which Taylor's hypothesis has been invoked to relate frequency spectrum to wavenumber spectrum. The effect of this plausible approximation is not known precisely—despite a few laudable efforts[16,17] to quantify them—and no further comments will be made on this matter. Second, all dissipation measurements in shear flows have been made by assuming local isotropy for both the velocity and scalar fields. The question of local isotropy of the scalar is a matter of intense discussion (see, for example, Ref. 18 for a recent summary), and the effect of this approximation will be remarked upon briefly. Third, whenever the authors did not quote the value of $C_\theta$ themselves, we have plotted the compensated spectral quantity $\Psi(k_x)=k_x^{5/3}\psi(k_x)\langle\epsilon\rangle^{1/3}/\langle\chi\rangle$ and determined that there was indeed a wavenumber region in which $\Psi$ was *reasonably* flat. Fourth, it appears that a consideration of possible intermittency corrections is futile—as we shall see shortly—because the issue of the precise spectral exponent is infested with even greater uncertainties. Finally, we shall plot the Obukhov–Corrsin constant against the microscale Reynolds number $R_\lambda\equiv u'\lambda/\nu$, where $u'$ is the root-mean-square of the velocity fluctuation $u$ in the direction of the mean velocity $U$ and $\lambda$ is the Taylor microscale. One could plot the data against Peclet number instead, but this will not be attempted

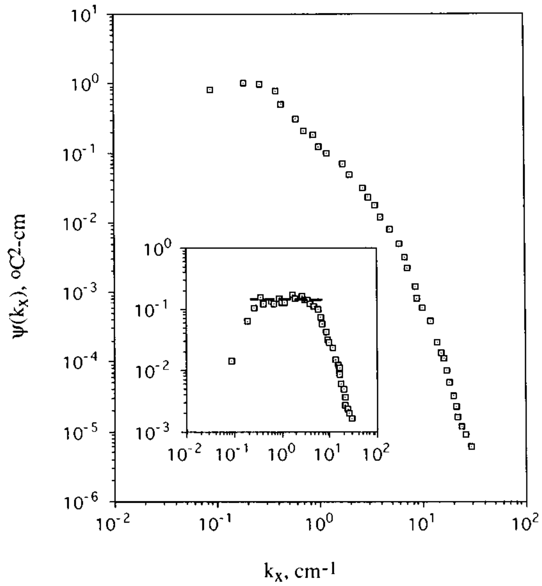


FIG. 1. The temperature spectral density, $\psi(k_x)$, and the compensated compensated quantity $\Psi(k_x)$ (see inset) plotted against the wavenumber component $k_x$ for one of the heated grid experiments of Lin and Lin,[32] $R_\lambda=150$. The flat region in the inset indicates that a reasonable, albeit small, 5/3 region exists.

here. In the literature to be examined below, $R_\lambda$ is often provided by the authors themselves, who usually obtain $\lambda$ from the relation

$$\lambda=(\langle u'^2\rangle/\langle(\partial u/\partial x)^2\rangle)^{1/2}, \tag{6}$$

with $\partial u/\partial x$ replaced by $-(1/U)(\partial u/\partial t)$ according to Taylor's hypothesis. When the authors do not report $R_\lambda$, as is the case for some atmospheric data, it is estimated from other means and the manner of estimation is indicated.

## II. THE SCALAR SPECTRUM IN THE INERTIAL–CONVECTIVE REGION

The first question to consider is whether the passive scalar spectrum possesses a 5/3 region. Here, it is necessary to consider grid turbulence and shear flow turbulence separately.

### A. Grid turbulence

Temperature fluctuations can be produced in grid turbulence either by heating the turbulence-generating grid itself (e.g., Refs. 19–27), or by heating a secondary device made of fine wires or ribbon elements situated behind the grid.[28,25,29] (This latter scheme was used first by Kellogg[30]—see also Ref. 31—for producing a spectrally local disturbance in grid turbulence.) If the wires or ribbons, with which the heating device is constructed, are operated at low enough Reynolds numbers, they shed no vortices of their own and add little turbulence to that generated by the grid. There are other unconventional possibilities of the type used by Lin and Lin.[32] In all these cases, the temperature spectrum has a modest 5/3 regime for microscale Reynolds numbers in excess of about 50. This can be seen, for example, in Fig. 1 constructed from the data of Lin and Lin, the concentration spectra of Gibson and Schwarz[33] in water and the more detailed experiments of Jayesh *et al.*[34] These last-mentioned authors emphasize that, in grid turbulence, a 5/3 scaling in temperature spectra occurs even when there is no such scaling for the velocity. Even earlier, Venkataramani and Chevray[24] had found that the temperature spectrum displays better scaling than the velocity field under the same conditions.

The situation appears to be the same even if the temperature field is not homogeneous (as long as the velocity field is nearly isotropic). For example, the spectra measured by Jayesh *et al.*[34] in grid turbulence with linear mean temperature gradient show an unmistakable 5/3 region at modest Reynolds numbers. It therefore follows that one can define, in grid turbulence, the Obukhov–Corrsin constant even at modest Reynolds numbers, and discuss the question of its universality.

### B. Shear flow turbulence

The scalar spectrum in shear flows behaves quite differently. Although this was pointed out originally in Ref. 18 and highlighted again,[34] it bears repetition here. Figures 2(a) and 2(b) show the temperature spectra in two different shear flows. While a credible power-law region exists in each of the spectra, the roll-off rates are unambiguously less steep than 5/3. As in grid turbulence, the scaling is better-defined than that found in velocity spectra for the same conditions. A plot of the spectral slope as a function of $R_\lambda$ is shown in Fig. 3. We have included the spectra for all shear flows in a single plot because a suitably defined non-dimensional shear rate is comparable in all of them. To the extent that one can draw any conclusions from these scanty data, it appears that the slope is a monotonic function of $R_\lambda$ and asymptotes to the expected value of 5/3 only for $R_\lambda>1000$ or so. (In lower Reynolds number shear flows, it is conceivable that details of generation of the flow are important to different degrees depending on the manner of generation.)

For these reasons, it was suggested in Ref. 18 that a useful expectation for the scalar spectrum is of the form

$$\psi(k_x)=C^*\langle\epsilon\rangle^{-1/3}\langle\chi\rangle k_x^{-5/3}(k_xL)^{-\gamma+\delta}, \tag{7}$$

where $L$ is an integral scale of turbulence, $C^*=C_\theta R_\lambda^{-3\delta/4}$ and $\delta$ are Reynolds-number-dependent constants and $\gamma$ is the usual intermittency exponent (which is being ignored here).

The conclusion is clearly that the *scalar spectra in shear flows cannot be used to define the Obukhov–Corrsin constant except at high Reynolds numbers ($R_\lambda\sim1000$ and larger).* This is so for yet another reason. In nearly all estimates of scalar dissipation, one uses local isotropy and replaces the full expression in Eq. (4) by $6D\langle(\partial\theta/\partial x)^2\rangle$, where the space derivative is approximated, as usual, by Taylor's hypothesis. This procedure is known to yield significant errors[35] in moderate-Reynolds-number shear flows.

We shall thus consider, among shear flows, only data obtained in geophysical flows; even though geophysical flows are not well controlled, practitioners generally pick conditions which are nearly steady—and thus provide valuable high-Reynolds-number data. One cannot, however,

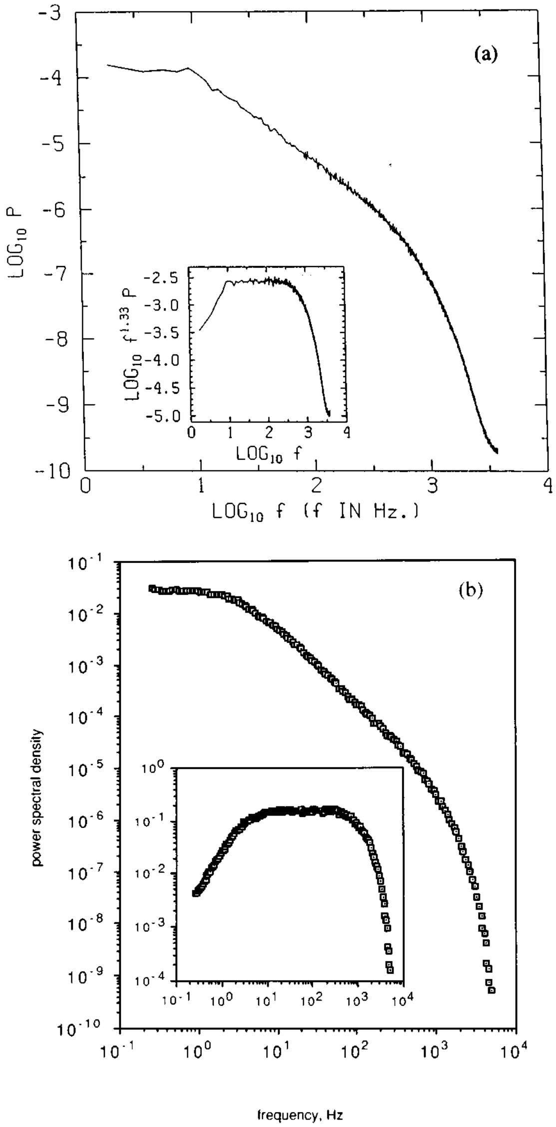


FIG. 2. The one-dimensional temperature spectrum, $\psi(k_x)$, and, in the inset, the quantity $k_x^{m_\theta}\psi(k_x)$ in two shear flows at moderate Reynolds numbers. (a) The flow is the wake of a circular cylinder, $R_\lambda = 175$, $m_\theta = 1.33$. The figure is essentially taken from Ref. 18. (b) The flow is the boundary layer in an air-sea interaction wind tunnel, $R_\lambda = 616$, $m_\theta = 1.49$. Data taken from Ref. 14.

overemphasize the need for high-Reynolds-number measurements of velocity and temperature under controlled conditions.

### C. Velocity spectra

Even though the question of velocity spectra is not central to the paper, a brief digression is worthwhile because it puts in perspective the behavior of scalar spectra. It was indicated in Refs. 2 and 18 that, while the inertial-range spectral slope for the longitudinal velocity in shear flows attains the value of 5/3 for $R_\lambda \sim 50$, the slope in the transverse spectra does not asymptote except for $R_\lambda \sim 1000$. Two examples, one in a pipe flow[36] [Fig. 4(a)] and another in a mixing layer[37] [Figs. 4(b)], will suffice to make the point. Further, the slopes in the normal and transverse spectra are

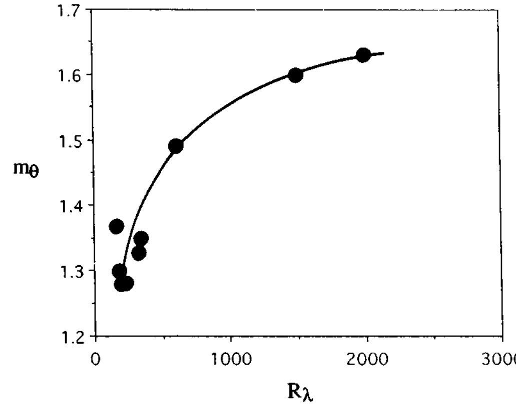


FIG. 3. The variation of the spectral slope, $m_\theta$, from scalar experiments in various shear flows. For details, see Ref. 18 from where the figure is taken. The spectral exponent approaches the expected value of 5/3 only for $R_\lambda$ of the order 1000 or so. The smooth fit through the data is drawn chiefly as a visual aid.

not the same, and we cannot discern from measurements any simple order as to which of the two is the larger. We plot in Fig. 5 the smaller of the spectral slopes of the normal and transverse velocity components as a function of $R_\lambda$ in various shear flows. Despite the scantiness of data and large scatter, it is clear that the transverse spectra do not possess the 5/3 slope except, again, at large microscale Reynolds numbers of 1000 and larger. It follows that Kolmogorov's formalism applies to the whole velocity field, not just selectively to a component, only when $R_\lambda$ is of the order 1000. Stewart and Townsend[44] as well as Corrsin,[45] had in fact estimated that comparable Reynolds numbers would be required for local isotropy—a precursor of universality—to be observed.

We do not have a full understanding of why the longitudinal velocity component possesses Kolmogorov spectrum at far lower Reynolds numbers than transverse components. The following remark might shed some light on the issue. Consider two shear flows at about the same moderately large microscale Reynolds number, say of the order 250, one of which is homogeneous (with constant mean velocity gradient) and the other is inhomogeneous, say in the logarithmic region of the boundary layer. Such comparative studies suggest that, while the latter shows a 5/3 slope in the spectral density of the longitudinal velocity component (see, for example, Fig. 6 of Ref. 38), the corresponding slope in the former case is less than 5/3 (see, for example, Figs. 2(a) and 2(b) of Ref. 46). In fact, this latter value is not inconsistent with our expectations based on Fig. 4, and it stands to reason that the spectral exponents would asymptotically attain the expected value of 5/3. The main point is that, in homogeneous shear flows where there is no strong large-scale overturning and mixing, the moderate-Reynolds-number spectra of *all* velocity components possess exponents smaller than 5/3. The principal difference between the log-region of the boundary layer and the homogeneous shear flow may well be that, in the former, the shear effect tends to be mitigated by the large-scale transport which does not seem as significant in the latter.

Be that as it may, it is easy to see that Figs. 3 and 5 are quite similar. This similarity, which we believe is not coin-

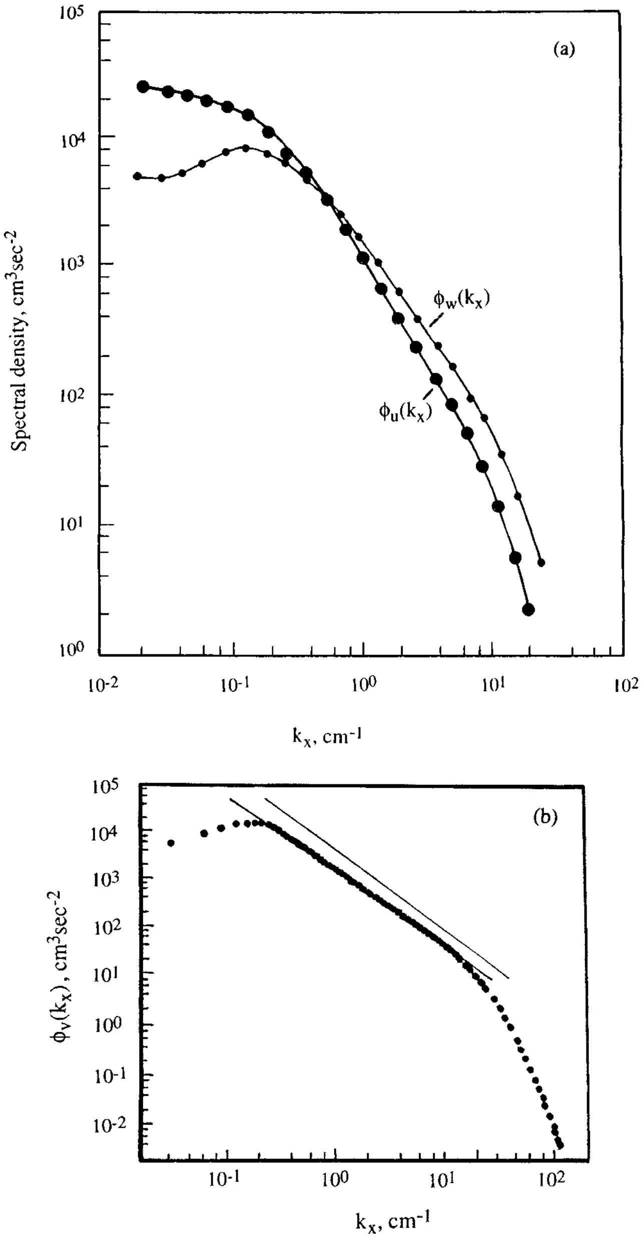


FIG. 4. Part (a) illustrates the fact that, at moderate Reynolds number, the spectral density of the azimuthal component of velocity, $w$, is less steep than 5/3 while the longitudinal component has the expected slope. The data are from Laufer's pipe flow,[36] $R_\lambda = 450$, 68% of the radius away from the wall. The normal component also has a smaller slope than 5/3, and is intermediate between the two shown here. Part (b) also illustrates the same feature in a plane mixing layer measured by Champagne *et al.*[37] The continuous line, which parallels the longitudinal spectrum (not shown here), has the 5/3 slope in a reasonable range of wavenumbers. The spectral density for the normal component, $v$, does exhibit a power law but with smaller slope. $R_\lambda = 330$, $y/x = -0.015$.

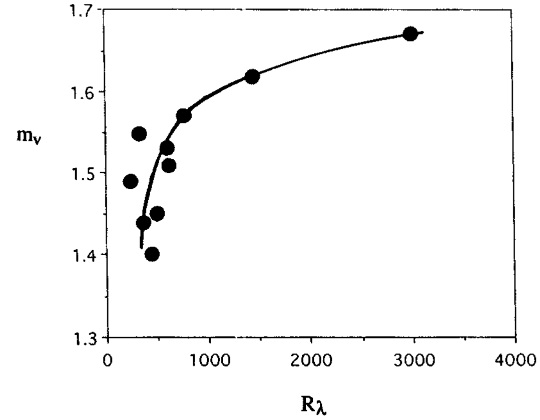


FIG. 5. The variation of the spectral slope, $m_v$, for transverse velocity components in various shear flows, obtained from figures like 4(a) and (b). The data were obtained from the following sources: Klebanoff,[38] boundary layer, normal velocity component, $R_\lambda = 230$, $m_v = 1.49$; Champagne *et al.*,[37] mixing layer, normal velocity component, $R_\lambda = 330$, $m_v = 1.55$; Comte-Bellot,[39] channel flow, normal velocity component, $R_\lambda = 370$, $m_v = 1.44$; Laufer,[36] pipe flow, azimuthal velocity component, $R_\lambda = 450$, $m_v = 1.41$; Saddoughi and Veeravalli,[40] boundary layer, normal and spanwise velocity components, $R_\lambda = 500$, $m_v = 1.45$ (not a good power law), $R_\lambda = 600$, $m_v = 1.53$ and 1.58, $R_\lambda = 1450$, $m_v = 1.62$ and 1.67. Gibson,[41] jet, normal velocity component, $R_\lambda = 780$, $m_v = 1.57$; Grant and Moilliet,[42] tidal channel, normal velocity component, $R_\lambda = 3000$, $m_v = 1.67$. Everitt and Robbins[43] observe a similar behavior in plane jets, although it is hard to determine the correct slope from their data. The smooth line through the data is drawn chiefly as a visual aid.

cidental, clearly suggests that, *in inhomogeneous shear flows*, the scalar field attains a semblance of universality only if the velocity field in its entirety is universal (not just one of its components).

It should be stressed, however, that no unique value of the Reynolds number can be specified for all aspects of the scalar field to attain approximate universality in shear flows. Indeed, it is conceivable that different aspects of the scalar field may attain different approximations to universality at different Reynolds numbers. For example, the fractal dimension of the scalar interface seems to attain (at least in some flows) an asymptotic value at far lower Reynolds numbers than does the spectral density;[47] indeed, Hunt and Vassilicos[48] have shown, using model calculations, that one can expect the fractal dimension to attain its asymptotic value at lower Reynolds numbers than the spectral exponent. On the other hand, there is some evidence[49,18] to suggest that the scalar derivative skewness does not attain its expected value of zero even at a microscale Reynolds number as high as a few tens of thousands, which makes one wonder if all aspects of the scalar field ever attain strict universality!

## III. THE OBUKHOV–CORRSIN CONSTANT

Two preliminary comments are worthwhile. First, several authors define the scalar dissipation rate by half that used in Eq. (3), and so obtain twice the numerical value for the Obukhov–Corrsin constant. We have taken some care to present data consistently. Second, one often denotes by Obukhov–Corrsin constant the prefactor in the three-dimensional spectral density in wavenumber magnitude as well as that in the second-order structure function for temperature increments. If local isotropy holds, the former is equal to 5/3 $C_\theta$ and the latter is about 4.02 $C_\theta$ (see, for example, Ref. 6).

Table I presents data collected from grid flows where the velocity field is essentially isotropic. If we ignore the data of Lin and Lin,[32] where—as already emphasized in Ref. 23—the isotropy of the velocity field may be in some doubt because of the unusual grid configuration, there seems to be remarkable consistency among the data: the Obukhov–

TABLE I. Sources and Reynolds numbers for the passive scalar spectrum in grid-generated turbulence experiments. Except for Gibson and Schwarz' data,[33] which were obtained for salinity fluctuations in water, all other data correspond to temperature fluctuations in air.[a]

| Source | $R_\lambda$ | $C_\theta$ | Remarks |
|---|---|---|---|
| Gibson and Schwarz[33] | 40–60 | 0.33–0.44 | The favored value seems to be about 0.35. |
| Jayesh *et al.*[34] | 30–130 | — | Authors do not quote $C_\theta$, but state consistency with previous atmospheric data from Ref. 50 (and the numerical data from Ref. 51). |
| Kistler *et al.*[19,20] | 26 | — | No 5/3 region was found |
| Lanza and Schwarz[52] | 53 | 0.4 | These data were available for some time at the Johns Hopkins University in the form of an unpublished report. |
| Lesieur and Rogallo[51] | — | 0.4 | Obtained from Fig. 4(b) of authors' paper. They quote a slightly larger value of 0.53, but 0.4 seems to fit their data better. Result from large-eddy simulations. |
| Lin and Lin[32] | 150 | 0.60 | The grid configuration was unusual. |
| Warharft and Lumley[28] | 45 | — | Heating introduced by a secondary screen; no good 5/3 region exists. |
| Yeh and Van Atta[22] | 35 | 0.38 | Same as Sepri.[23] |

[a]We have not listed the value of $C_\theta$ from the data of Ref. 24 because our estimate from that reference leads to an absurdly low value of the order $10^{-3}$, which clearly implies that something is amiss.

Corrsin constant is about 0.4, not too different from that obtained in the very first set of measurements by Gibson and Schwarz,[33] or from that cautiously recommended in Ref. 7.

Before turning attention to the atmospheric measurements summarized in Table II, some older Soviet literature reviewed in Refs. 6, 7, and 8 has to be considered briefly. The *principal* source of those data appears to be Tsvang's[53] atmospheric measurements; some other early data, admittedly of dubious value, have been discussed by Yaglom.[7] Gurvich and Zubkhovskii[54] analyzed Tsvang's data and obtained a value of 0.34 for $C_\theta$ for several stability conditions. Gurvich and Zubkhovskii also evaluated $C_\theta$ from short records of their own atmospheric data, and obtained a value of 0.44. The early measurements of Tatarski,[55] which yielded a substantially higher value, were deemed unreliable by Monin and Yaglom[6] and Yaglom.[7] Besides these data, there are other spectral measurements of passive scalars in stably stratified boundary layers[56] and clear air turbulence.[57] The scatter in these data is so large that it is pointless to compute $C_\theta$ from them. Mention may also be made of ozone concentration variations due to atmospheric dispersion[58] which do not show a 5/3 region. As suggested by the authors of Ref. 58, this situation is akin to scalar mixing in two-dimensional turbulence.

The atmospheric data from Table II seem to suggest, if one momentarily sets aside the data of Boston and Burling[59] and Gibson *et al.*,[10] that $C_\theta$ lies in the range between 0.3 and 0.5. Before commenting on these two references, we should note that different authors use different methods to obtain scalar dissipation rate. Wyngaard and Cote[62] obtained the temperature dissipation by measuring all other terms in the energy budget of $\langle \theta^2 \rangle$—a procedure potentially subject to some uncertainties; Paquin and Pond[12] obtained their temperature data by a platinum resistance thermometer which did not resolve fine-scale temperature fluctuations. Gibson *et al.*[10] and Boston and Burling,[59] Champagne *et al.*,[50] Williams and Paulson,[13] and Bradley *et al.*[60] made temperature measurements with fine coldwires and obtained temperature dissipation from the well-resolved data, either by spectral integration of $k_x^2 \psi(k_x)$ over the entire wavenumber range or by directly obtaining $\langle (\partial\theta/\partial x)^2 \rangle$. Such measurements may be expected to be more precise, but they disagree among themselves. Data from Refs. 13, 50, and 60 yield a value of $C_\theta$ of about 0.4 while those from Refs. 59 and 10 are two and three times bigger, respectively. After presenting a careful discussion of the data available at the time (Refs. 13, 50, and 60 were not yet published at the time), Monin and Yaglom[6] puzzled over the peculiarity of the results of Refs. 10 and 59 but could not settle the issue. The resolution of this paradox lies in noting that the measurements of Refs. 10 and 59 were made over the ocean while the others were made over land, and that the spray of salt from ocean waters contaminated the probes significantly. This issue was explored in Ref. 66, whose conclusions are best noted by the following quote:

*...surfaces of small temperature sensors (thermistors, thermocouples and resistance wires) commonly used in marine boundary layer experiments become contaminated with salt spray when used over the ocean... the spray will exist as saline drops on the probe surfaces... The latent heat of vaporization associated with [the saline drop] evaporation and condensation processes will cool and heat the sensors, and therefore generate erroneous temperature signals ...most of the anomalous temperature results observed over the ocean... may be due to spray-induced humidity sensitivity of such temperature sensors.*

This remarkable study leaves little doubt that the $C_\theta$ results of Refs. 10 and 59, pioneering as they were in the implementation of cold wire technology to marine boundary layer research, should now be disregarded.

## IV. CONCLUSIONS

A better sense for the behavior of $C_\theta$ can be had by plotting the experimental data in their entirety as a function of $R_\lambda$. As already remarked, $C_\theta$ can be defined for grid turbulence at low Reynolds numbers and, for shear flows, only past an $R_\lambda$ of 1000 or so; for the latter, we have considered only high-Reynolds-number flows. Figure 6 shows all the relevant data plotted against $R_\lambda$. We find that there is remarkable consistency between the grid data and the high-Reynolds-number shear flow data. They lie mostly in a band between 0.3 and 0.5, suggesting a mean value of about 0.4. Combining this conclusion with the recommendation of Ref. 2 that $C_\kappa$ is about 0.5, the turbulent Prandtl number, defined as the ratio $C_\kappa / C_\theta$, is approximately 0.8.

The low Reynolds number measurements in shear flows, such as they are, do not contradict these conclusions; for example, see Ref. 67. Furthermore, Kaimal *et al.*[12] [see their

TABLE II. Sources and Reynolds number for temperature spectra in geophysical flows.

| Flow | Source | Height | $R_\lambda$ | $C_\theta$ |
|---|---|---|---|---|
| Atmospheric Surface layer | Boston and Burling[59] | 4 m above tidal mud flat. | 4400–5500 | $0.81 \pm 0.09$ [a] |
| | Bradley *et al.*[60] | Different heights from the ground. | 3800–9600 | $0.40 \pm 0.1$ [b] |
| | Champagne *et al.*[50] | 4 m above flat land. | 6200–9900 | $0.41 \pm 0.02$ [b] |
| | Gibson *et al.*[10] | Different heights over water. | 2620 | 1.2 |
| | Kaimal *et al.*[12] | Different heights. | Not specified. | $0.41 \pm 0.04$ [c] |
| | Paquin and Pond[11] | A few meters above water. | $O(10^3)$ [d] | $0.41 \pm 0.06$ [e] |
| | Williams and Paulson[13] | 2 m above rye grass. | 1310 | 0.50 |
| | | | 1410 | 0.44 |
| | | | 1780 | 0.45 |
| | | | 2130 | 0.45 |
| | | | 2250 | 0.51 |
| | | | 2600 | 0.47 |
| | | | 3280 | 0.55 |
| | | | 3960 | 0.53 |
| | | | 4150 | 0.57 |
| | | | 4170 | 0.51 |
| | | | 4280 | 0.55 |
| | Wyngaard and Cote[62] | Different heights. | 1800–10000 [f] | $0.40 \pm 0.05$ |
| Tidal channel | Grant *et al.*[64, g] | Depths of 15 and 90 m below surface. | 14,000–41,000 | $0.31 \pm 0.06$ |

[a]The values quoted correspond to the mean and standard deviation over nineteen stes of data, all taken for nominally the same conditions.

[b]There were 14 sets of data in all with only one of them at $R_\lambda = 6600$ showing a large deviation from others, contributing most to the standard deviation. Parts of these data have been reported also by Antonia.[61]

[c]The authors obtained a very low value of the order 0.015 for the analogous constant in the spectrum of humidity fluctuations.

[d]The authors do not provide adequate data for estimating the Reynolds numbers. From the familiarity with similar conditions elsewhere, $R_\lambda$ can be 'guessed' to be of the order of a few thousands. The Kolmogorov constant quoted is the average over 16 runs.

[e]The authors also measured the prefactor in the humidity spectrum and obtained an identical value. This would be the expectation from considerations of universality. See, however, footnote c above.

[f]This paper does not quote the microscale Reynolds number range covered in these experiments, and the information has been taken from Wyngaard and Tennekes.[64] The data analyzed by Wyngaard and Cote[63] are subsets of the data analyzed by Kaimal *et al.*,[12] according to this latter reference. The data were taken at heights of 5.66 m, 11.3 m and 22.6 m from a 32 m tower and encompassed different stability conditions.

[g]Grant *et al.*[64] did not quote $R_\lambda$. Those given here are estimated by assuming that $R_\lambda = 4(Uz/\nu)^{1/2}$, where $U$ is the mean velocity and $z$ is the probe depth below the surface. This seems a plausible result in the atmospheric surface layer, see for example Ref. 66, but its validity for present circumstances is not clear. The square root dependence in the above equation is probably correct, but the prefactor is uncertain. The $C_\theta$ values vary sizeably from one realization to another, and the average value given here applies to the entire range. One does not have enough confidence in each set of data to plot them individually.

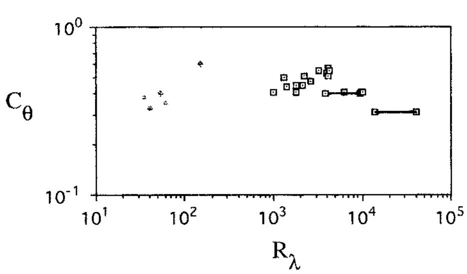


FIG. 6. The Obukhov–Corrsin constant $C_\theta$ plotted against $R_\lambda$. Diamonds are for grid data, from Table I, squares for geophysical flows, from Table II (omitting data from Refs. 10 and 60, as explained in the text). Most data form a band between 0.3 and 0.5, with 0.4 as a sensible average over the entire Reynolds number range. A reasonable supposition is that the Reynolds number trend, if it exists at all, is weak and difficult to discern with any certainty from these data.

Fig. 1) and Bradley *et al.*[65] [see their Fig. 9(b)] obtained data for several stability conditions in the atmosphere and showed that the effect of stability is negligible on the Obukhov-Corrsin constant. This conclusion is consistent with the analysis of the less detailed measurements of Tsvang[53] by Gurvich and Zubkovskii[54] (see Fig. 83b of Monin and Yaglom[6]).

There is, in general, a larger moral to this story. It appears that there is enough reason to believe that approximate spectral universality obtains at high Reynolds numbers. It is interesting that this universal behavior is already present in moderate Reynolds number grid turbulence. Thus, *if one is interested in quantifying universal properties of turbulence*, measurements in a grid experiment at a moderately large Reynolds number, say an $R_\lambda$ of about 250, would be more valuable than, say, those in a shear-flow at a comparable Reynolds number. There is thus a case to be made for detailed measurements of both velocity and scalar in a grid flow with an $R_\lambda$ of the order 250. Such experiments do not exist today.

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