Twelfth Australasian Fluid Mechanics Conference
The University of Sydney, Australia 1995

SMALL-SCALE INTERMITTENCY IN TURBULENCE

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ABSTRACT

This note summarizes the experimental evidence in support of small-scale intermittency. Separate consideration will be given to the intermittency in dissipative and inertial ranges. Various measures of quantification proposed in the literature will be highlighted, and the relations between intermittency, anomalous exponents and multifractality will be discussed briefly.

INTRODUCTION

Many years after the discovery of small-scale intermittency (Batchelor and Townsend 1949), doubts continue to be expressed about its very existence in the inertial range (e.g., Grossmann and Lohse 1993, 1994). Paradoxically, there is at the same time a proliferation of measures proposed to quantify intermittency. The relations among these various measures are not always well appreciated. This note will focus on two aspects: (a) an assessment of the experimental evidence for intermittency, and (b) the interrelation among the various measures of intermittency. Some attention will be paid to anomalous exponents and multifractality.

The Central Limit Theorem (Feller 1966, section VIII.4) guarantees that the central part of the probability density function (pdf) of all processes with nearly independent and small increments is Gaussian. Many processes have non-Gaussian tails, and often the only relevant question is this: how far from the mean does one have to reach to observe departures from Gaussianity? Equivalently, how high a moment should one compute in order to observe deviations from Gaussianity? If one has to consider very high-order moments to find sizeable departures from Gaussianity, one can say that no significant non-Gaussian behavior exists, or that the non-Gaussian behavior that may exist is benign.

In turbulence literature, one usually measures the normalized third and fourth moments (the skewness and flatness factor respectively), occasionally the fifth and the sixth. Substantially higher-order moments have been measured for inertial range structure functions, which we shall not discuss explicitly. Suppose that a measured quantity, such as a velocity increment in the inertial range, has non-Gaussian tails and yields a flatness factor that is significantly higher than the Gaussian value of 3. We are entitled to say that the quantity is non-Gaussian, but nothing more can be said with assurance. In particular, the immediate association of a long-tail behavior (responsible for large flatness) with intermittency is unwarranted in general. For example, any non-Gaussian noise (such as a squared Gaussian, see Kennedy and Corrsin 1961) has a long-tail behavior, but one does not *a priori* know if such noise signals should be considered intermittent.

In small-scale intermittency, one always considers its scale-dependence. There are various other forms of intermittency which we will not discuss here. For example, a benign form is manifested in an on-off process where the "on" process could well be Gaussian and the "off" process might be zero. This results in a Gaussian distribution with a delta-function at the origin. (If the "off" process is affected by noise, it can be treated by the technique described by Bilger et al. 1976.)

INTERMITTENCY OF THE DISSIPATION SCALES

From the time of the experimental work of Batchelor and Townsend (1949), the evidence for the intermittency of dissipative quantities has been accumulating considerably. Specifically, one may wish to inquire into the fraction of the spatial volume occupied by vorticity or energy dissipation. It should be stressed that even if that fraction is smaller than unity for some chosen experimental conditions, it does not follow that there is scale-dependent intermittency. For the latter to exist, *this fraction should become smaller as the Reynolds number increases* (since we are interested in the properties of turbulence in the high-Reynolds-number limit). Equivalently, the long-tail behavior in the pdf of either of these two quantities (and of other related quantities) should become increasingly conspicuous with increasing Reynolds number. If one takes the square of a single velocity derivative as illustrative of the behavior of the energy dissipation itself, one should observe its flatness to increase with Reynolds number.

The best quantitative evidence for intermittency comes in the form of the statistics of velocity derivatives. For a recent collection of available data, see Antonia and Sreenivasan (1996). The evidence appears incontrovertible that the flatness of velocity derivative (also its skewness) increases with Reynolds number at high Reynolds numbers. The recent measurements of Tabeling et al. (1994), which suggested the existence of a more complex behavior at much higher R_λ, seem to have been affected by probe resolution problems (see Appendix). Quite aside from the visual evidence that the square of the velocity derivative becomes more spiky and spotty as the Reynolds number increases—such plots were shown by Meneveau and Sreenivasan (1991)—the observed flatness behavior is a clear indication that the dissipative scales are indeed intermittent.

INERTIAL-RANGE INTERMITTENCY

What can be said about intermittency in the inertial range? The Reynolds number variation is irrelevant in the inertial range, at any rate as we understand the inertial range presently. Again, the existence of non-Gaussian tails for some fixed scale in the inertial range (or, for one wavenumber band in the inertial range) is not a sufficient indicator of intermittency. What is required is that the pdf of wavenumber bands become increasingly stretched out with increasing midband value, or that the flatness of velocity increments increase with decreasing scale size. This latter can be seen to be true from experimental observations that

$$K(r) \sim r^{-\alpha}, \; H(r) \sim r^{-\beta}, \; E(r) \sim r^{-\gamma}, \quad (1)$$

where $K(r)$, $H(r)$ and $E(r)$ are the normalized fourth, sixth and eighth moments of the velocity increment Δu_r over an inertial-range separation distance r, and α, β and γ are approximately 0.1, 0.29 and 0.47, respectively, independent of Reynolds number (Sreenivasan 1995).

Equivalently, if one considers velocity increments for increasingly smaller separation distances, all lying in the inertial range, one should see their pdfs develop more and more flaring tails. This aspect has been examined for the high-Reynolds-number atmospheric boundary layer; the measured pdfs of inertial-range velocity increments were found to develop stronger tails (Kailasnath et al. 1992) as the separation distance became smaller. To see this more clearly, the tails were fitted with stretched exponentials and the stretching exponent was plotted as a function of the scale size. The exponent became larger for smaller scales (until, in the dissipative range, it attained a value of about 0.5 and remained unchanged). These data, then, are in strong support of inertial-range intermittency.

THE RELATION BETWEEN INERTIAL RANGE INTERMITTENCY AND ANOMALOUS SCALING

From Kolomogorov's (1941a) second similarity hypotheses proposed for the inertial range, it follows that

$$\langle \Delta u_r^n \rangle \propto r^{\zeta_n} \quad (2)$$

where $\langle \Delta u_r^n \rangle$ represents the n-th order structure function and the scaling exponents $\zeta_n = n/3$. Here, $\langle . \rangle$ represents a suitable average and n is any positive integer. Any fractional Brownian process with a Hurst exponent H possesses scaling exponents for even-order moments, and they vary with the order of the moment n as nH. (The special case of the classical Brownian motion corresponds to the case $H = 1/2$.) Thus, if the scaling exponents simply vary with the order of the moment, n, as a linear power, one does not have any intermittency. Such linear variation would be considered trivial scaling. *Intermittency demands that the scaling exponents vary with the order of the moment in some nonlinear fashion.* Because of the Hölder inequality which guarantees the concavity of the ζ_n-n curve, the difference between the Kolmogorov value of $n/3$ and the actual exponent should increase with the order of the moment, if there is intermittency.

We shall now remark on the experimental evidence for anomalous scaling in the inertial range. Here, there are several difficulties of interpretation. The first is the finite Reynolds effect—which is not understood and cannot be calculated without assumptions. There is therefore no guide to an experimentalist as to how one might deduce this correction. Just the same way, there are effects of finite shear, whose effects on inertial-range quantities are much more pronounced than on dissipative ones. There are purely practical questions on how one obtains scaling exponents from log-log plots which are not as straight as desired. These questions have been addressed in various ways by various authors, among them by Anselmet et al. (1984), Benzi et al. (1993, 1995), Stolovitzky and Sreenivasan (1993), Stolovitzky et al. (1993), and so forth. There are differences among these various authors (and these differences need a separate discussion), but all of them agree that there is anomalous scaling in turbulent flows, and that deviations of the scaling exponents from $n/3$ are real, non-trivial and increase with the order of the moment.

As argued by Frisch and Parisi (1985), non-trivial scaling of the structure functions implies multifractality: that there is an infinity of local Hölder exponents and a corresponding infinity of dimensions associated with the Hölder exponents.

THE RELATION BETWEEN INERTIAL RANGE INTERMITTENCY AND DISSIPATION SCALE INTERMITTENCY

Define the generalized dimensions D_q of the dissipation field as follows. Denote by ϵ_r the dissipation ϵ averaged over a box of side r lying in the inertial range. Write

$$\Sigma(r\epsilon_r)^q \propto r^{(q-1)D_q}, \tag{3}$$

where the sum is taken over all intervals of size r, and q is a real number. This defines the generalized dimensions D_q, whose meaning has been discussed at some length by Meneveau and Sreenivasan (1991) and need not be repeated here. Using the Refined Similarity Hypotheses of Kolmogorov (1962), it can be shown that the generalized dimensions of the dissipation field and the scaling exponents of the dissipation field are related according to

$$\zeta_q = (q/3 - 1)D_{q/3} + D \tag{4}$$

where D is the space dimension and q is taken as an integer. It is easily seen that if $D_q = D$, that is, the dissipation field is space-filling, ζ_q is a linear function of q and there is no intermittency. Further, if $D - D_q$ is a constant independent of q, one does not have intermittency either. *Only if the generalized dimensions vary with q does one have intermittency.* This variation could well be linear (which, incidentally, corresponds to a log-normal distribution).

Thus, if the Refined Similarity Hypotheses are valid, inertial range intermittency is inseparably tied to dissipative scale intermittency: *if the various moments of velocity derivatives increase as varying powers of the Reynolds number, one can say that this reflects scale-dependent intermittency in the inertial range as well.* There are strong indications that Refined Similarity Hypotheses are plausible approximations to reality (Stolovitzky et al. 1992, Praskovsky 1992, Thoroddsen and Van Atta 1992, Chen et al. 1993, 1994, Zhu et al. 1995, Wang et al. 1995, Borue and Orszag 1995, and so forth). However, it is not certain how seriously one should take them in the context of high-order moments of energy dissipation or of high-order structure functions. It is thus not clear that one can relate simply the intermittencies in the two ranges.

INTERMITTENCY AND ODD-ORDER STRUCTURE FUNCTIONS

In previous paragraphs, we glossed over one essential fact about Kolmogorov turbulence, namely that exponents for odd moments of its increments scale in a continuous fashion with even ones. This would not be true of models of the type of fractional Brownian motion, for which odd moments are zero by construction. All elementary multifractal models (such as the p-model of Meneveau and Sreenivasan 1987) which focus on the intermittency of the dissipation scales do not, and cannot, say anything about the odd moments of velocity increments. It is therefore intriguing to ask if the odd moments tell us something about inertial-range intermittency. Imagine a situation in which odd moments of some high order are large. The entire contribution to moments, both positive and negative, can then come from one side of the pdf. This *is* intermittency, although it may not be the source of all the intermittency just discussed.

The remarkable thing about turbulence is that the scaling exponent for the third-order moment is exactly unity (provided local isotropy holds); we know not only the scaling exponent but also the coefficient in front. The question as to whether this relation, known as Kolmogorov's structure equation (Kolmogorov 1941b), can tell us anything about intermittency, was explored by Vainshtein and Sreenivasan (1995). An important conclusion reached was that it would be useful to examine the scaling of the positive and negative parts of velocity increments separately. Other inferences were inconclusive. For example, it was thought that one could not construct non-intermittent stochastic functions whose odd moments, in particular the third, were identical to that in turbulence. This is apparently not the case in general (Kraichnan 1995).

THE EVIDENCE AGAINST SMALL-SCALE INTERMITTENCY

A thorough discussion of various scenarios against intermittency was given by Kraichnan (1974), but it must be said that their status is one of plausibility—no more. Grossmann and Lohse (1993, 1994) have used the so-called reduced wave vector representation of the velocity field and calculated the scaling exponents for "very high Reynolds numbers". Their approach admits only a geometrically scaling subset of wave vectors in the Fourier representation of the velocity field, and the population of wavevectors becomes increasingly sparse for large wavenumbers. They find that the corrections to Kolmogorov scaling, if any, are very small. In another paper, Grossmann et al. (1994) had in fact found that these corrections, small as they are, were Reynolds-number-dependent and vanished at large Reynolds numbers. They are interesting conclusions, but their relevance to actual turbulence remains obscure. In fact, there are indications that the reduced wave-vector representation suffers from various shortcomings. For example, the normalized energy dissipation rate is smaller by about two orders of magnitude compared to experimental values, and does not asymptote to a constant until the effective Reynolds number is about two or three orders of magnitude larger than that found experimentally (Sreenivasan 1985). It is thus unclear how seriously one should take the conclusions that follow from such calculations.

Yet another indication that there might be no corrections to the Kolmogorov scaling came from the recent diagrammatic perturbation theory of L'vov and Procaccia (1994). They found that a diagrammatic expansion converged order by order, yielding no divergences at infrared or ultraviolet ends. This leaves no reasons to expect the appearance in inertial-range scaling of either the large scale or the Kolmogorov scale, thus eliminating any possibility of anomalous corrections. However, the work has now undergone revisions (essentially because of the realization that order by order convergence does not guarantee the correct final result), and allows for anomalous scaling.

CONCLUSIONS

It appears that there is indeed intermittency in both dissipative and inertial ranges. However, this conclusion should not mask potential uncertainties from an experimental perspective: finite Reynolds number effects, finite shear effects, ambiguities associated with the precise identification of a scaling region when it is not convincingly large, are all serious factors which one hopes will be addressed adequately sometime in the near future. Perhaps when all of these factors are well understood, one may be led eventually to conclude that there is no intermittency in the inertial range. But it is fair to say that such a conclusion would come as a surprise from the considered perspective available today.

ACKNOWLEDGEMENTS

I am grateful to R.H. Kraichnan and R.A. Antonia for comments on the draft, and to P. Tabeling for comments on an oral presentation of Appendix. Financial assistance from the Sloan Foundation is appreciated.

APPENDIX: A REEXAMINATION OF THE SKEWNESS AND FLATNESS DATA OF VELOCITY DERIVATIVES

As already mentioned, one manifestation of dissipation-scale intermittency is the increase with Reynolds numbers of high-order moments of velocity derivatives. Van Atta and Antonia (1980) compiled much of the data on the skewness and flatness factor and provided tangible evidence that they continually increase with Reynolds number. However, recent measurements of Tabeling et al. (1995) have thrown some doubt into the situation. These authors made measurements in a closed flow created by two counter-rotating discs of 20 cm diameter enclosed in a cylindrical envelope, with the discs 13 cm apart vertically. The fluid was helium gas at 5 deg K. The microscale Reynolds number, R_λ, varied from about 180 to about 5,000. An elegant feature was that this vast Reynolds number range was covered in exactly the same apparatus by changing carefully the temperature and pressure of the working fluid. The derivative skewness in these measurements followed the trend of other measurements up to an R_λ of about 900 and decreased thereafter, falling up to about 0.22 at the highest R_λ. The flatness too started a downward trend from about 15 at $R_\lambda \approx 900$ to about 6.5 at 5000. The authors inferred that, around an R_λ of 900, either some new transition to a different state of turbulence was occurring or the behavior of skewness and flatness was non-universal. They argued that the data complied by Van Atta and Antonia were from several different flows under uncontrolled conditions, and hence less reliable than their own.

If Tabeling et al. are right, the implications are far reaching and our general understanding of intermittency will have to undergo major modification. We have therefore spent much time trying to answer the following questions. Are there some obvious objections that can be levelled against previous data? Are there are some problems with the new data? Our tentative conclusion is that the previous trends are essentially correct (subject to considerations such as Taylor's hypothesis), and that the conclusions of Tabeling et al. are likely to be artifacts of the probe inadequacy at higher Reynolds numbers. We present some considerations below; since these considerations were first spelled out in February 1995, Lohse (1995) has made a more detailed analysis and confirmed them.

In their measurements, Tabeling et al. used a 7 μm thick carbon fiber whose sensitive length, l_w, varied between 7 and 25 μm in length. The ratio η/l_w varied between 45 at the lowest Reynolds number and 0.3 at the highest Reynolds number. The length to "diameter" ratio for the "hot-wire" probe was of the order unity, while the recommended ratio is generally of the order 100 or larger. The precise effects of this non-standard ratio are unknown, but it is almost certain that high frequencies (as opposed to small spatial scales) could be affected strongly. Large overheats were used, and their effects are not understood either. Finally, the hot-wire was operated beyond the usually accepted range of Reynolds number (the probe shed its own vortices), and these effects could be rather important. The probe was operated at constant temperature on homemade electronics, and the frequency response was deemed to be good up to 50 kHz. Data processing seemed free of any problems. Velocity was digitized by a 16 bit A/D converter with a "typical" sampling rate of 125 kHz. A typical acquisition time was 5 minutes, so the typical sample size was 10 million.

Some remarks now follow about the older measurements. They are presented here to show that the results, while by no means fool-proof, are reasonable.

Wyngaard and Tennekes (1970), and Wyngaard and Cote (1971) Note: Some additional details were supplied by J.C. Wyngaard in recent conversations.

Measurements were made in the atmospheric surface layer at heights of 5.7, 11.3 and 22.6 m above flat ground. It is not clear if the authors distinguished among the different stability conditions of the atmosphere. Wind speeds varied over a wide range, but are not given. The friction velocity varied, among different experimental runs, from 10 cm/s to about 50 cm/s. The sensor was a hotwire, 5 μm in diameter, 1.2 mm long. In the worst case, the Kolmogorov length scale, η, is about half the wire length, l_w; it is larger for most cases. DISA 55D05 constant temperature units with linearizers were used. Velocity was recorded on analogue tape. Data were digitized subsequently at 3000 Hz, and analyzed up to about 2000 Hz. Velocity signal was differentiated, filtered and recorded on analogue tape recorder for later processing. Some prewhitening of the velocity derivative was done, and its effects are difficult to estimate. Some correction for Taylor's hypothesis was made according to Heskestad (1965).

Data records were about an hour long each, and consisted of ten million samples. The microscale Reynolds number R_λ varied between about 4,000 and 25,000 (after multiplying the authors' numbers by a factor 2.5 to bring them in conformity with the usual definition of the Reynolds number).

Derivative skewness varied from about -0.6 on the low end to about -1.0 on the high end. Derivative flatness factor varied from about 20 on the low end to about 50 on the upper end.

Gibson et al. (1970)

Measurements were made at heights between 2 and 12 meters above the mean water level in the Atlantic ocean. The boundary layer, formed by steady light wind characteristic of Caribbean trade winds, was nearly neutral. Wind speed was of the order of 5 m/s. The sensor was a hotwire, 3.8μm diameter, 1.25 mm long. The Kolmogorov scale varied between about 0.9 and 1.5 times the wirelength. Velocity was measured by a Thermo-Systems constant temperature anemometer. Linearizers were used as well. Frequency response of the set-up was good up to 5 kHz. Data were filtered between 2 Hz and 2 kHz. Hotwire signals were recorded on analogue tape, and later digitized at about 2000 Hz. Roughly speaking, each sampling unit, translated via Taylor's hypothesis, was about 2 Kolmogorov scales long. Velocity signals were differentiated analogue and were also recorded on the tape. Differentiator introduced some phase shift at higher frequencies. Its effects are unknown. They were later digitized and analyzed. A total of about 20 minutes worth of data was used. This translates to about 2 million data points.

The authors quote Reynolds numbers, R_z, based on local mean velocity and the height from the water surface. Translating this Reynolds number to R_λ is non-trivial. For a smooth surface over a modest Reynolds number range, the relation $R_z/R_\lambda^{\frac{1}{2}}$ can be shown to be roughly constant, which the authors assumed was 0.48. The constant is now known to be of the order 5 (Bradley et al. 1981). The revised R_λ varies between about 2,600 to 8,700. The use of these numbers completely removes the anomalous points in the Van Atta/Antonia compilation and puts them right on the general trend.

Champagne et al. (1977), Champagne (1978)

Two experiments are described in the 1978 paper. The GUMBO experiment was over a flat land in the atmospheric surface layer in Minnesota, $4m$ above the ground, described in greater detail in Champagne et al. (1977). The other, called FSII, was also in the atmosphere in Denmark, at a height of 56 m above the ground. No information is available in the 1978 paper about the stability of either boundary layer, but the 1977 paper has some details. It appears that the conditions were neutral to unstable. Wind speed was of the order of 9 m/s in GUMBO (as inferred from other data provided) and 11 m/s in FSII. Some corrections were applied for Taylor's hypothesis. Although they seem to have 10 or 15 percent correction, it is hard to estimate the precise effects.

The sensors were hotwires, 2.3 μm diameter 0.4 mm long (FSII), or 5 μm diameter and 1.25 mm long (GUMBO). In the GUMBO experiment, the ratio of the Kolmogorov scale to wirelength was about 0.68; this ratio in FSII was 0.82. Velocity measured by DISA 55M01 constant temperature anemometer, in conjunction with DISA 55D10 linearizer. Frequency response was good up to 2 kHz. Data were filtered between dc and 5 kHz. (Different values appear in different places but these are typical.) Hotwire signals were recorded on analogue tape, later digitized at about 4170 Hz. (Some slow sampled data were also acquired.) Roughly speaking, each sampling unit, translated via Taylor's hypothesis, was less than a Kolmogorov scale. Velocity signals were differentiated analogue, and were also recorded: a total of about 15 minutes in GUMBO, and 40 minutes in FSII.

Williams and Paulson (1977)

This paper gives no derivative flatness and skewness data directly, but Van Atta and Antonia obtained an estimate by using the isotropic relation for the skewness from the spectral data, but they appear to have recorded the Reynolds number incorrectly. For example, according to Williams and Paulson, the spectral data (set RY25c) given in their figure 2 corresponds to an R_λ of 4,170–see their Table 1–instead of about 1000 used by Van Atta and Antonia.

Yale experiments

Measurements were made in the atmospheric surface layer 6 m above the ground at the edge of a uniform wheat canopy. Wind speed varied between 4 and 8 m/s; only records corresponding to constant velocity were used for later analysis. Some information was also available about the stability of the boundary layer, which was nearly neutral. For preliminary investigations, some data were also acquired on the roof of a four-storey building, about 18 m above the ground. Velocity data were recorded with hotwires 5 μm diameter, 0.6 mm long. The ratio η/l_w varied between 0.5 and 1.1. DISA 55M01 constant temperature anemometer was used for velocity measurements. Mean velocity was also recorded nearby with a cup anemometer. Hot wires were calibrated before measurements, and their output was later checked against the cup anemometer output. The signal from the hotwire anemometer was passed through a buck and gain amplifier and filtered from dc to 3 kHz, and were constantly monitored on an oscilloscope. Digitization was done with a 12 bit digitizer at 6,000 Hz or 10,000 Hz, and the data were stored on a magnetic tape. Some data were also digitized at 200 Hz, but were not used for derivative statistics. Linearization was done subsequently on the computer. Various record lengths were used at various times from a total recorded duration of several hours. Most of the time, 3 or 5 minute records were used (roughly on the order of a million samples).

The microscale Reynolds number varied between 1,500 and 2,500. The derivative skewness and flatness were computed from the digitized data. The skewness varied between −0.5 and −0.6, while the flatness was about 20. If the derivatives were performed analogue and analyzed, there are reasons to think that one would have obtained slightly higher numerical values for both.

Different types of corrections were applied for Taylor's hypothesis as described by Zubair (1993) and Stolovitzky (1994), but they seem to have no big effect on the results. Since the corrections are somewhat arbitrary, only uncorrected skewness and flatness data are provided.

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