Zero crossings of velocity fluctuations in turbulent boundary layers

P. Kailasnath and K. R. Sreenivasan
Mason Laboratory, Yale University, New Haven, Connecticut 06520-8286

(Received 10 August 1992; accepted 3 July 1993)

In this paper some results are presented on the statistical properties of zero crossings of turbulent velocity fluctuations in boundary layers over a wide range of Reynolds numbers. The earlier finding that the probability density function (pdf) of the intervals between successive zero crossings of the streamwise velocity fluctuation u can be approximated by two exponentials, each with its own characteristic scale, is confirmed. The cross-stream variation of these characteristic scales is investigated. One of these scales, corresponding to the large zero-crossing intervals, is independent of the Reynolds number, while the other for the viscous-dominated small-scale crossings varies with as $R_\lambda^{-1/2}$, where R_λ is the Reynolds number based on the Taylor microscale, λ. The pdf's for the normal velocity component v and the fluctuating part of the Reynolds stress uv are essentially exponential over the whole range of zero-crossing scales, and each possesses just one characteristic scale. The mean and the standard deviation of the zero-crossing scales of u and v, when normalized by their respective Taylor microscales, are roughly unity and essentially independent of the cross-stream position. Similar data are also presented for the Reynolds stress fluctuations. A brief discussion of the results as well as an example of the application of the zero-crossing pdf are given.

I. INTRODUCTION

For a stationary random process $u(t)$ with zero mean, the interval between two successive up or down crossings of its zero (to be called zero crossings henceforth) is a random variable. Let us denote this variable by ζ. Much experimental work has been devoted to determining the statistical properties of ζ for turbulent signals in several flows. The most detailed work on zero crossings has occurred for velocity signals in the turbulent boundary layer.[1–3] In these papers, the temporal variable ζ has been converted to a spatial variable via Taylor's frozen-flow hypothesis (which assumes that turbulence convects with the mean velocity without distortion), and has been divided by the factor 2π. Let us denote the resulting zero-crossing variable by ξ. The following results are known about ξ.

(a) At all heights within the boundary layer,[1,3] the mean value Λ of the zero-crossing variable ξ for the streamwise velocity fluctuation u is approximately equal to the Taylor microscale, λ. The Taylor microscale is defined by

$$\lambda = U\left(\frac{\langle u^2\rangle}{\langle(\partial u/\partial t)^2\rangle}\right)^{1/2}, \tag{1}$$

where the angular brackets indicate time averages, and U is the time average velocity at the position under consideration; by definition, the fluctuation velocity u has zero mean.

(b) The pdf of the zero crossings of u in the logarithmic region of the turbulent boundary layer consists[3] essentially of two exponentials, one of which is characterized by the wall variable ν/u_τ and the other by the outer scale δ. Here, ν is the kinematic viscosity of the fluid, δ is the boundary layer thickness, and u_τ is the friction velocity. It has been pointed out[2,3] that a lognormal fit to the pdf is possible for small zero crossings, $-0.5\Lambda' < \xi - \Lambda < 0.5\Lambda'$, where $\Lambda' = \langle(\xi-\Lambda)^2\rangle^{1/2}$ is the root-mean-square (rms) value of ξ, but that the exponential models provide a better overall fit.

If we accept this conclusion, the pdf of the variable ξ for u can be written[3] as

$$p_\xi(\xi) = \exp[-(\xi-\Lambda)/K\Lambda'], \tag{2}$$

where it is known that the stretching factor K assumes two different values in the two different scale ranges, say K_L for large values of ξ and K_S for small values of ξ.

The contributions of this note are the following. First, we cover an extensive Reynolds number range to confirm the earlier results that the zero-crossing length scale Λ for the streamwise velocity fluctuation u is approximately equal to λ, and show additionally that the ratio $\Lambda/\Lambda' \approx 1$. Similar results are obtained for the normal velocity fluctuations v, as well as for the fluctuating part of the kinematic Reynolds shear stress uv. Second, only limited data are available on the Reynolds-number variation of K_L, and nothing at all is known about the Reynolds number variation of K_S. In this paper we examine the Reynolds number variation of K_S and K_L over a wide range of Reynolds numbers, and determines their scaling behaviors. Finally, the results obtained here on the pdf's of the zero crossings of the normal velocity component and of the Reynolds shear stress are also new.

The significance of zero crossings has been discussed in Refs. 1–3. The primary reason for our interest in the topic is that it provides a means for probing the distribution of length scales in a turbulent flow. A few additional points are worth noting. Figure 1 plots the average energy dissipation rate associated with various magnitudes of the velocity fluctuation u. The ordinate is normalized by the conventional average of the dissipation rate, so that the sum of the contributions over all possible values of u yields unity. The experimental conditions for the data of Fig. 1 will be

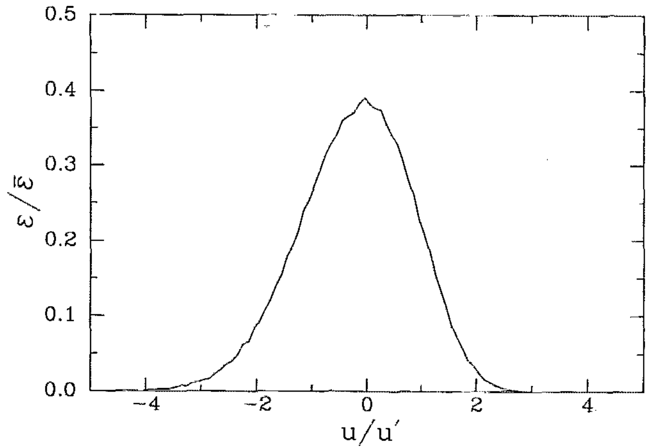

FIG. 1. The average value of the energy dissipation rate (measured under the hypothesis of local isotropy and Taylor's frozen flow hypothesis), $\langle \epsilon_u \rangle$, for various amplitudes of the streamwise velocity fluctuation, u. The velocity fluctuation is normalized by its rms value and the energy dissipation by its mean value, $\langle \epsilon \rangle$. The noteworthy feature of the plot is that the dissipation peaks around $u=0$. The boundary layer Reynolds number based on the momentum thickness is 3800 and $yu_\tau/\nu=110$.

discussed in Sec. II, but the essential details are noted in the caption. It is seen from Fig. 1 that the zeros of the velocity contribute significantly (though not entirely) to the energy dissipation rate. Now, from the definition of the fluctuating Reynolds stress, it is clear that it gets no contribution from the zero set of u (that is, the set of all zero crossings of u). Therefore, the zeros of the velocity signal constitute a part of turbulence dynamics that contributes most to the energy dissipation but nothing at all to the Reynolds stress. Second, there are several schemes[4] that attempt to reconstruct a nonperiodic signal (in one and two dimensions) from the knowledge of its zero set, and so the information on the zero crossings of turbulence signals may well find its uses in this direction. Third, in a recent paper on drag reduction in pipe flows using riblets, Liu *et al.*[5] showed that the maximum reduction in drag occurred when the spanwise spacing s of the riblets, nondimensionalized by u_τ and ν, was between 11 and 16, and that this observed spacing corresponded well to the Taylor microscale in the spanwise direction of the fluctuating streamwise velocity near the wall, which, in turn, is closely related to the fluctuating skin friction, $\partial u/\partial y$. From the result that the Taylor microscale is approximately equal to the mean zero-crossing scale, we interpret the Liu *et al.* result to mean that the drag reduction is largest when the spanwise spacing between riblets equals the mean zero-crossing scale. While more studies are required to substantiate this statement, this inference does not appear to be unreasonable because the properties of a random signal can be manipulated most easily through its zero-crossing scale. Finally, George[6] points out that the power spectra of velocity fluctuations at various Reynolds numbers in grid turbulence can be collapsed over all wave numbers by using the Taylor microscale. It appears that only the reinterpretation of Taylor's microscale as the mean zero-crossing scale helps provide some rationale for the spectral similarity claimed by George.

These are sufficient reasons for taking a new look at the zero crossings of turbulent signals. In particular, we study the variation of the statistical properties of zero crossings over a wide range of Reynolds numbers, and provide an example showing how the pdf's of zero crossings may be usefully employed.

II. EXPERIMENTS

The laboratory experiments were performed in a flat-plate boundary layer. The plate was mounted in a wind-tunnel of width 0.7 m and height 0.5 m. The distance between the test surface and the upper wall of the tunnel was 0.39 m, and the test section length was 2.5 m. The boundary layer was artificially thickened by placing strips of sandpaper and a circular rod at the leading edge of the plate. Measurements were taken at a station 1.7 m from the leading edge of the plate at tunnel speeds of 8.3 and 17.1 m/sec. For these conditions, it was verified that the boundary layer had a log-law region with the accepted constants.[7] Measurements for the higher wind speed used a single hot-wire (0.6 mm length and 5 μm diam), while those for the lower tunnel speed used a miniature X probe (0.5 mm length and 2.5 μm diam, with wire spacing of 0.3 mm). The hot wires were operated on DANTEC constant-temperature anemometers, and the signals were linearized.

The measurements in the atmosphere were taken at a height of 2 m above the roof of a four story building. One data segment of 800 000 points was acquired with a single wire. Another data segment of 250 000 points was acquired using an X probe (1.25 mm length and 5 μm diam, wire spacing 0.5 mm). The data were taken on different days, and the atmospheric turbulence levels for the X-wire data are estimated to be twice as strong as for the single wire segment.

We also used the u data from Princeton, which were acquired in a wind tunnel of width 1.2 m and height between 0.15 and 0.2 m. The test section was 5 m long and the flow speed was approximately 32 m/sec.

A summary of the conditions under which the data were acquired is given in Table I. The Taylor microscale was determined in all cases by the use of Eq. (1).

III. RESULTS

Figure 2(a) shows that the ratio of the average zero-crossing microscale Λ to the Taylor microscale λ across the logarithmic region of the turbulent boundary layer at two Reynolds numbers is close to, though slightly higher than, unity. This result agrees well with those of Refs. 1 and 3. The increase in the ratio Λ/λ in the outer part of the boundary layer is attributed to the outer intermittency in this region. Far outside the boundary layer we expect the velocity signal to be dominated by noise, and we observe that Λ/λ decreases toward unity in this region. These observations agree well with Liepmann's[8] interpretation of Rice's result[9] that $\Lambda=\lambda$ if $u(t)$ and its time derivative $\dot{u}(t)$ are both Gaussian and statistically independent. We also

TABLE I. Summary of experimental conditions. Abbreviations: na=not available; f_s=sampling frequency; $l^+=lu_\tau/\nu$ is sensor length l in units of ν/u_τ; R_θ=momentum thickness Reynolds number. Code for sensor type: 1, single wire and X, cross-wire probe.

Source	R_θ	R_λ	Sensor type	Sensor length, l^+	f_s (kHz)	u_τ (m/sec)
Present (lab. data)	2300	100	X	22	15	0.33
"	3800	150	1	22	20	0.67
Present (atm. data)	na	2000	X	6	6	0.85
"	na	1000	1	40	10	0.50
Ref. 3	~4500	350	1	40	analog	na
Princeton	13 000	400	X	96	50	1.14

note that Ylvisaker[10] proved that $\Lambda=\lambda$ for any continuous Gaussian signal $u(t)$ with finite λ without invoking statistical independence between $u(t)$ and $\dot{u}(t)$. The somewhat higher value of Λ/λ toward the wall is attributable to *large* departures of the signal from Gaussianity. (It might be added that high-probability events in u are governed, in a rough sense, by considerations of the Central Limit Theorem and that those zero crossings that correspond to such high-probability events, more or less follow Gaussian results, even though the overall pdf of u is non-Gaussian. It takes large departures from Gaussianity to produce perceptible changes in zero-crossing scales.)

Figure 2(b) shows the ratio of the rms zero-crossing scale Λ' to the mean value Λ. This ratio is also close to unity across the turbulent boundary layer. Hence both the mean and the rms of the zero-crossing scale are close to the Taylor microscale. (Note that Fig. 2 uses the suffix u to indicate explictly that the quantities plotted there refer to the streamwise velocity component u. However, this practice is not followed in the text because the meaning is always clear from the context.)

Figure 3 shows a typical probability density function of the zero crossings of u at $y^+=88$ in the logarithmic region of the turbulent boundary layer at the momentum thickness Reynolds number $R_\theta=2300$. Except for the smallest zero crossings, the pdf has two exponential regions. The dotted line represents the region for the small-scale crossings, while the dashed line represents the large-scale crossings. The inverse slope of the $\log_{10} p_\xi(\xi)$ vs $(\xi-\Lambda)/\Lambda'$ plot in each of these regions gives $\log_e(10)\times K_S$ (or K_L). The products $K_S\Lambda'$ and $K_L\Lambda'$ are representative scales for the two regions. The transition between the two regions is around unity in Λ' units.

Figures 4(a) and 4(b) show the variations of K_S and K_L for the pdf of zero crossings of u across the boundary layer at $R_\theta=2300$ and $R_\theta=3800$. In the viscous sublayer and the outer region the crossings are characterized by a unique stretching factor and is approximately unity. We interpret the sublayer results to mean that the pdf of the zero crossings in this region of intense dissipation scale uniquely on the Taylor microscale. The congruence toward the outer region simply reflects that the external stream does not possess any special length scale. In the large in-

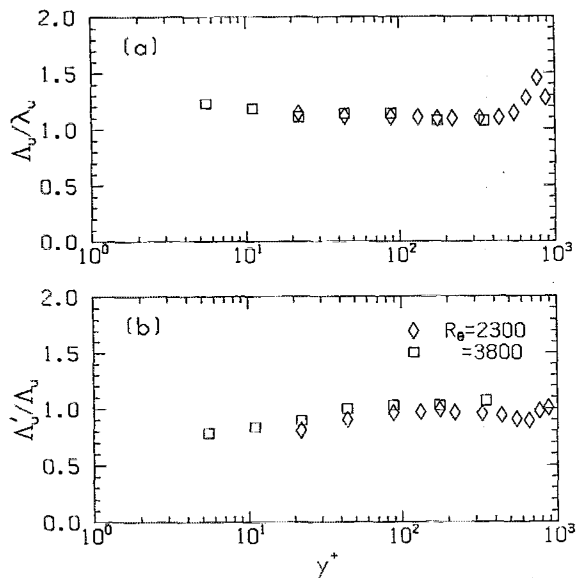

FIG. 2. (a) Ratio of the zero-crossing scale to Taylor microscale across the turbulent boundary layer. (b) Ratio of rms of the zero-crossing scale to its mean, $y^+=yu_\tau/\nu$, y being the distance from the wall.

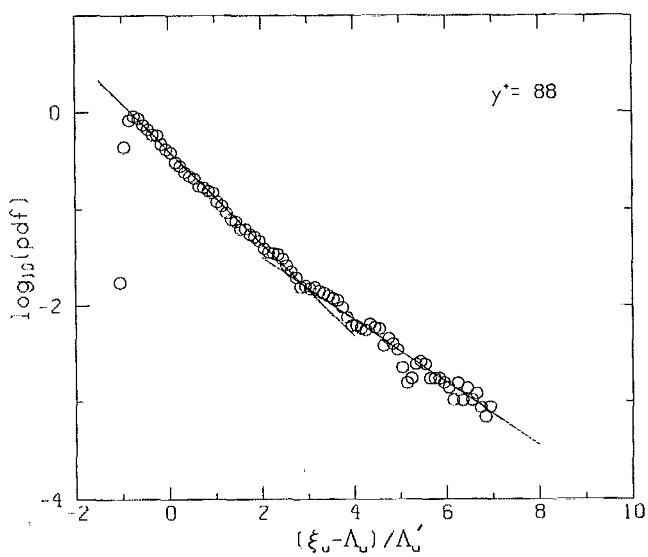

FIG. 3. Probability distribution function of the zero-crossing intervals of u in the logarithmic region of the turbulent boundary layer at $R_\theta=2300$. For clarity, exponential fits are drawn through the data.

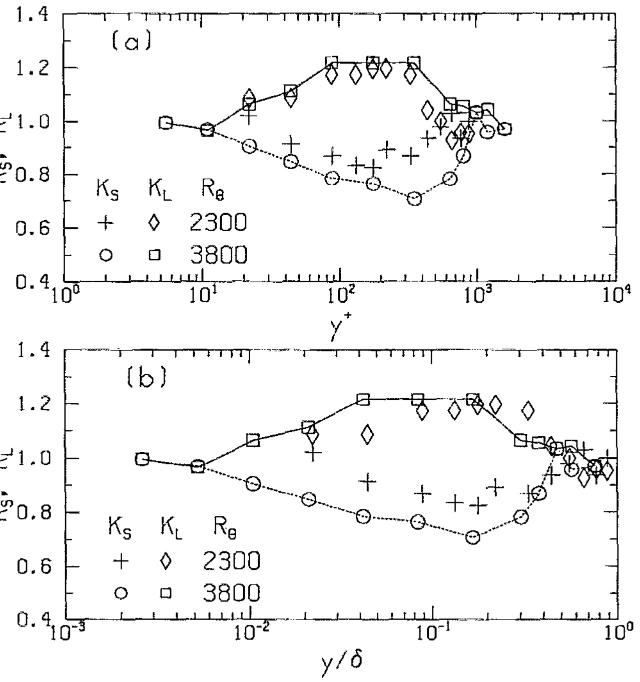

FIG. 4. Distribution of the stretching factors across the turbulent boundary layer at $R_\theta=2300$ and 3800: The normal distance y is expressed in wall variables in (a) and in outer variables in (b).

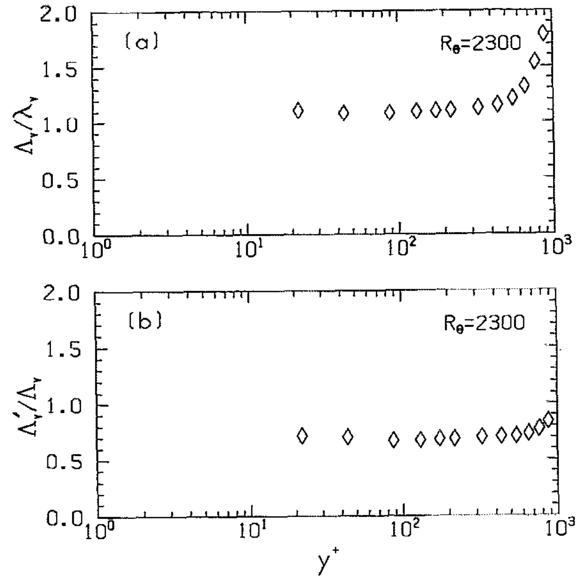

FIG. 6. Mean properties of zero crossings of v in turbulent boundary layers. (a) Ratio of the mean zero-crossing scale to the corresponding Taylor microscale across the boundary layer. (b) Ratio of the rms zero-crossing scale to the mean value.

termediate region, as typified by Fig. 3, the stretching factors K_S and K_L are different; typically, in the semilogarithmic region these numbers are approximate constants characteristic of the layer.

Figure 5 shows the Reynolds number effects on the characteristic values of the stretching factors K_S and K_L. We attempted to correlate the laboratory data against various Reynolds numbers, and found that the best collapse was obtained when the microscale Reynolds number R_λ (based on the root-mean-square u' of the streamwise velocity and the Taylor microscale λ) was used. In any case, the only other measurable Reynolds number for the atmospheric data is that based on the rms velocity and the integral scale L, but this Reynolds number is simply related[11] to R_λ. On the whole, therefore, we believe that the best choice is R_λ. The stretching factor K_L is clearly independent of R_λ and $K_S \propto R_\lambda^{-1/2}$. The characteristic length of the large-scale crossings should be independent of viscous effects, and hence K_L should be independent of the Reynolds number. If the small crossings are dominated by viscous effects, we expect K_S to be such that

$$K_S \approx \eta/\Lambda' \approx \eta/\Lambda \approx \eta/\lambda \propto R_\lambda^{-1/2}, \qquad (3)$$

as verified by the data. Here, η is the Kolmogorov scale. Notice that these simple fits to K_L and K_S intersect at an R_λ of about 50, which, according to Ref. 12, is the minimum Reynolds number required for the fully turbulent scaling to hold for the energy dissipation rate. The variation of the stretching constant K_S can also be expressed in terms of the Reynolds number $u_\tau\delta/\nu$ by noting that in the logarithmic regions of a high-Reynolds-number boundary layer,[13] $u' \approx 2u_\tau$, and that $\lambda/\delta \propto \lambda/L \propto R_\lambda^{-1}$. Thus we have, after trivial algebra, that

$$K_S \propto (u_\tau\delta/\nu)^{-1/4}. \qquad (4)$$

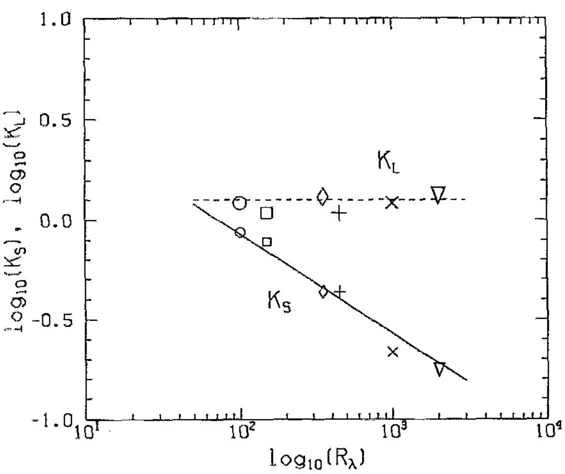

FIG. 5. Dependence of the stretching factors K_S and K_L on Reynolds number R_λ.—,$K_S \propto R_\lambda^{-1/2}$;---,$K_L$. Present data: laboratory boundary layer, ○, $R_\theta=2300$; □, $R_\theta=3800$; atmospheric surface layer: ×, 800 K points; ▽, 250 K points. Other data: ◇, Sreenivasan *et al.* (1983), laboratory boundary layer, $R_\delta=4.9\times10^4$; +, Princeton data, laboratory boundary layer, $R_\theta=13\,000$.

We now extend our study to the velocity fluctuation v normal to the wall and to the Reynolds stress uv. We have confined this study to the turbulent boundary layer at $R_\theta=2300$ (primarily because the scale resolution of the X probe was deemed unacceptable at the higher Reynolds number). Figure 6(a) shows the ratio of the mean zero-crossing scale to the Taylor microscale for v. This ratio is also close to unity over most of the boundary layer. Figure

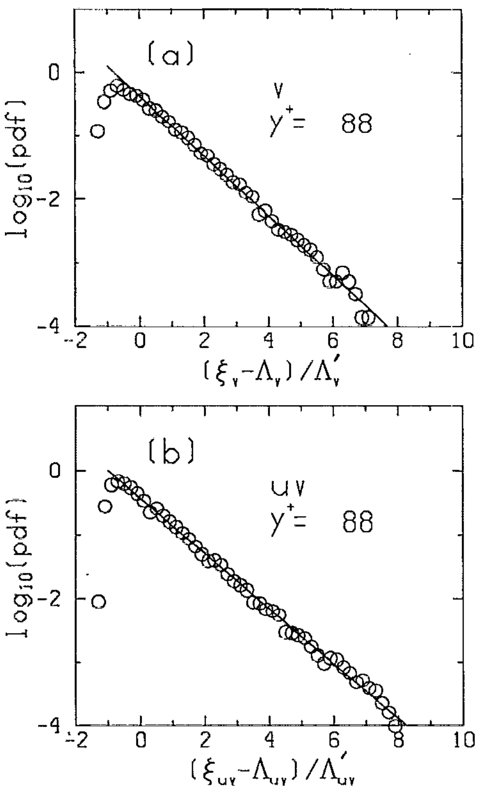

FIG. 7. Probability density function of the zero-crossing intervals in the logarithmic region of the turbulent boundary layer, $R_\theta=2300$; (a) for v and (b) for uv.

6(b) shows that the ratio of the root-mean-square zero-crossing scale to its mean is approximately 0.7 for v. Recall that this ratio for u is approximately unity. The difference between the two cases is a reflection of differences in the pdf's of u and v in the boundary layer; as a rough rule, v is farther from Gaussianity than u.

Figures 7(a) and 7(b) show that the pdf's of zero crossings of v and uv possess, unlike the case of u, more or less a single exponential. It is known[14] that the peak location in the rms profile $v'(y)$ of the transverse velocity fluctuation and in the mean profile $\langle uv\rangle$ of the Reynolds shear stress are strong functions of the Reynolds number $u_\tau\delta/\nu$, while the peak in the rms profile $u'(y)$ of the streamwise velocity fluctuation is essentially independent of $u_\tau\delta/\nu$. Also, the u' peak is sharp (i.e., it occurs over a narrow range in y^+ units), while those for v' and $\langle uv\rangle$ are relatively flat. This suggests that an interplay of small and large scales is important to u, while perhaps only the large scales are relevant for v and uv in the logarithmic and outer regions. This may well be reflected in the single exponential region in the zero crossings for v and uv.

To show that the exponential distributions for the zero crossings for v and uv are, in fact, related to large crossings, we show in Fig. 8 a comparison of the stretching factor K_L for the v and uv crossings with that for u. It is seen that they are all comparable, indicative of their common origin. Further, the K_L values for v and uv are essentially independent of the distance from the wall. Recall that, in contrast, K_L for u is approximately independent of the distance normal to the wall only in the logarithmic region.

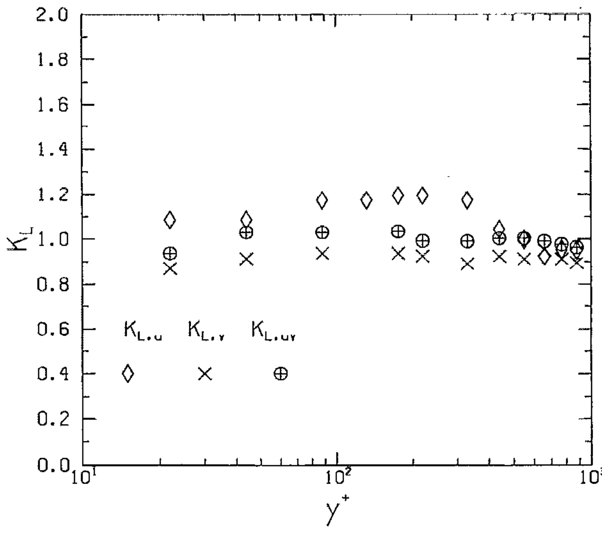

FIG. 8. Comparison of the stretching factor K_L for the zero crossings of v and uv with that for the u in the logarithmic region of the boundary layer. Here $R_\theta=2300$.

From the closeness of the zero-crossing scale to the Taylor microscale and the known empirical data on the Taylor microscales for u and v in the logarithmic region of the boundary layer,[15] one can easily show that $\Lambda_v/\Lambda_u\approx0.5$. Figure 9 shows that this is approximately true. It also shows that the ratio of the mean zero-crossing scale for uv to Λ_u is about 0.35. This is clearly related to the rms values of the Reynolds stress and its derivative, but, so far, we do not have a simple explanation for the measured numerical value.

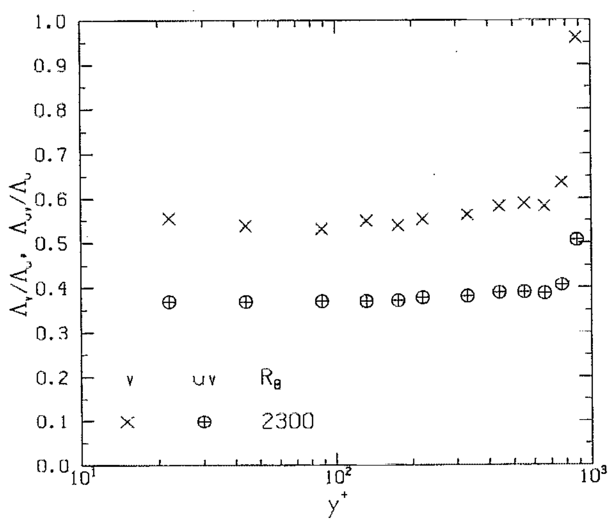

FIG. 9. Ratio of mean zero-crossing scale of v and uv to the mean zero-crossing scale for u in the logarithmic region of the boundary layer. $R_\theta=2300$.

IV. CONCLUDING REMARKS

It has been shown that the pdf of the zero-crossing intervals for the streamwise velocity fluctuation u can be approximated by two exponentials. The stretching exponent K_L for the large scales is a constant independent of the Reynolds number, while K_S varies according to the inverse half-power of the microscale Reynolds number. These facts together suggest that the variation of K_S is related to viscous effects. The two stretching factors are comparable at microscale Reynolds numbers of the order 50, and become increasingly disparate with increasing Reynolds numbers.

A rough interpretation of the two parts in the pdf of the zero crossings is as follows. The long intervals are a result of the passing of large-scale structures, each passing being independent of the next. The short intervals are due to the small-scale dissipative structures riding on the large structures. The passing of these small-scale structures is independent of the previous small-scale structure. This picture suggests that both the small-scale and the large-scale crossings would be distributed exponentially.

For the fluctuation velocity v and the Reynolds shear stress uv, the pdf of the zero-crossing intervals is essentially a single exponential, and is characterized by a unique stretching factor that is independent of the distance from the wall over most of the boundary layer thickness. Because it is characteristic of large scales, it follows that this stretching factor must be essentially independent of the Reynolds number also.

The mean zero-crossing scale for each signal is approximately equal to its Taylor microscale. The ratio of the mean zero-crossing scale to its rms value is approximately constant over most parts of the boundary layer.

As remarked in the Introduction, we believe that the data on zero crossings are useful in a variety of contexts. An application in the context of Townsend's attached eddy hypothesis[16] will now be described briefly. The hypothesis is that the boundary layer consists of a hierarchy of hairpin-type structures attached to the wall and inclined to the downstream direction at 45°. The mean velocity gradient (mean spanwise vorticity) at any given distance y from the wall is assumed to get contributions primarily from a range of scales of the attached hairpins. The model has been substantially developed by Perry and collaborators (e.g., Ref. 17 and references to the previous work cited there). In the recent form of this development, the contribution to the mean spanwise vorticity ω from one attached eddy of scale 1 is written as

$$\omega=(u_\tau/1)f(y/1), \tag{5}$$

where $f(y/1)$ is a unique function of the distance from the wall normalized by the length scale 1. For a range of geometrically similar eddies between a length $1_{\min}$ and $1_{\max}$, the total contribution to the mean velocity gradient is

$$\frac{dU}{dy}=\int_{1_{\min}}^{1_{\max}} \omega p(1)d1, \tag{6}$$

where $p(1)$ is the probability density function for the length scales. Townsend assumed $p(1)$ to be continuous and of the form

$$p(1)=(M/1), \tag{7}$$

where M is a constant. It is easy to show that these assumptions lead to a region of constant turbulent stresses for $1_{\min}<y<1_{\max}$. The constant M is determined by requiring that the velocity gradient in the logarithmic region be given by $1/\kappa$, where κ is the von Kármán constant equal to about 0.41. The defect velocity profile itself can also be computed from the same line of reasoning.

Returning to zero crossings, recall the interpretation that their pdf is also the pdf of length scales. This would suggest replacing the form (7) guessed by Townsend by the exponential form found experimentally. All further calculations can now be carried through quite simply.[18] These calculations are of specialized interest and are not repeated here, but it suffices to say that the results from these calculations are comparable to those obtained by Perry *et al.*[17]

ACKNOWLEDGMENTS

We thank Randall Smith, Amy Alving, and Lex Smits for contributing their data at $R_\theta=13\,000$ from Princeton, and Professor D. D. Joseph and Dr. C. Christodoulou for their interest in these measurements.

[1] R. A. Antonia, H. Q. Danh, and A. Prabhu, "Bursts in turbulent shear flows," Phys. Fluids **19**, 1680 (1976).

[2] M. A. Badri Narayanan, S. Rajagopalan, and R. Narasimha, "Experiments on the fine structure of turbulence," J. Fluid Mech. **80**, 237 (1977).

[3] K. R. Sreenivasan, A. Prabhu, and R. Narasimha, "Zero-crossings in turbulent signals," J. Fluid Mech. **137**, 251 (1983).

[4] A. Yuille and T. Poggio, "Scaling theorems for zero-crossings," IEEE Trans. Pattern Anal. Machine Intelligence **8**, 15 (1986); D. Roten and Y. Y. Zeevi, "Image reconstruction from zero-crossings," IEEE Trans. Acoust. Speech Signal Proc. **34**, 1269 (1986); S. R. Curtis, S. Shitz, and V. Oppenheim, "Reconstruction of non-periodic two-dimensional signals from zero-crossings," IEEE Trans. Acoust. Speech Signal Proc. **35**, 890 (1987); R. Hummel and R. Moniot, "Reconstruction from zero-crossings in scale space," IEEE Trans. Acoust. Speech Signal Proc. **37**, 2111 (1989).

[5] K. Liu, C. Christodoulou, O. Riccius, and D. D. Joseph, "Drag reduction in pipes lined with riblets," AIAA J. **28**, 1697 (1990).

[6] W. K. George, "The decay of homogeneous isotropic turbulence," Phys. Fluids **4**, 1492 (1992).

[7] T. B. Lynn, "Manipulation of the structure of the turbulent boundary layer," Ph.D. thesis, Mechanical Engineering Department, Yale University, 1987.

[8] H. W. Liepmann, "Die Anwendung eines Satzes über die Nullstellen Stochastischer Funktionen auf Turbulenzmessungen," Helv. Phys. Acta **22**, 119 (1949).

[9] S. O. Rice, "Mathematical analysis of random noise," Bell Syst. Technol. J. **24**, 46 (1945).

[10] N. D. Ylvisaker, "The expected number of zeros of a stationary Gaussian process," Annu. Math. Stat. **36**, 1043 (1965).

[11] G. K. Batchelor, *The Theory of Homogeneous Turbulence* (Cambridge University Press, Cambridge, 1953).

[12] K. R. Sreenivasan, "On the scaling of the turbulence energy dissipation rate," Phys. Fluids **27**, 1048 (1984).

[13] H. L. Petrie, A. A. Fontaine, S. T. Sommer, and T. A. Brungart, "Large flat plate turbulent boundary layer evaluation," TM 89-207, Applied Research Laboratory, State College, PA, 1990.

[14] K. R. Sreenivasan, "The turbulent boundary layer," in *Frontiers in Experimental Fluid Mechanics*, edited by M. Gad-el-Hak (Springer-Verlag, New York, 1989), pp. 159–209.

[15] P. S. Klebanoff, "Characteristics of turbulence in a boundary layer with zero pressure gradient," NACA Report No. 1247, 1955.
[16] A. A. Townsend, *The Structure of Turbulent Shear Flows* (Cambridge University Press, Cambridge, 1976).
[17] A. E. Perry, K. L. Lim, and S. Henbest, "An experimental study of the turbulence structure in smooth- and rough-wall boundary layers," J. Fluid Mech. 177, 437 (1987).
[18] P. Kailasnath, "Reynolds number effects and the momentum flux in turbulent boundary layers," Ph.D. thesis, Mechanical Engineering Department, Yale University, 1993.