TURBULENT FLOWS AND COUPLED MAPS

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ABSTRACT. Some remarks are made on two generic properties of turbulence, namely the scaling of the energy dissipation rate and the small-scale intermittency. The discussion pays cursory attention to the following question: Which properties, if any, of turbulence should be shared by coupled lattice maps in order for their study to meaningfully relate to turbulence? Brief remarks are also made on self-organized criticality *vis-a-vis* turbulence.

1. Introduction

Stimulating studies of coupled lattice maps were presented at the Workshop. However, it remains unclear how relevant such studies are to fluid turbulence (or, more properly, to turbulent flows). A question requiring urgent answer appears to be: What properties should such maps possess in order that conclusions drawn from their study be useful in turbulence? A second theme of the meeting was self-organized criticality, and the question here is: Is there any evidence that this idea is appropriate to turbulent flows? I shall discuss preliminary thoughts on these questions.

Different turbulent flows differ among themselves in various degrees of detail, and there have in fact been assertions that quantitative universality common to them all may not exist. If this is true, fluid turbulence acquires a status analogous to chemistry – where individual compounds have to be studied in isolation. The mystique of universality in turbulence is strong, especially among physicists and mathematicians; the Kolmogorov scenario [1] is an ingeneous characterization of possible universality. (It may be of interest to know that several articles in the July 1991 commemorative issue of the Proc. Roy. Soc. Lond. assess and reinterpret Kolmogorov's hypotheses.)

A turbulent flow near its source will be affected in a complex way by conditions specific to the manner of generating the flow. For example, the flow near a cylinder depends on the state of the boundary layer on the cylinder, on whether or not the stagnation point is fixed, on whether or not the surface of the cylinder is smooth, on how precisely the boundary layer on the cylinder interacts with the primary vortex-shedding mode, and so forth. A similar remark holds for the convection turbulence in a closed box heated from below. These are but two examples of turbulent flows that are being 'stirred' constantly and vigorously; the coupling among various scales of motion here could be quite strong. In contrast, in the cylinder problem, if one moves 'sufficiently far away' from the body, it is reasonable to think that the flow does not necessarily 'feel' all the details of the generation mechanisms of turbulence; one then believes that, in this 'far field', all details of the cylinder except the net momentum loss produced by it, or the drag force it experiences, become irrelevant: It does not matter whether the cylinder is square or round or triangular in cross-section and, by inference, whether the separation point is fixed or moving; similarly, a jet flow far away from the nozzle remembers, to a good approximation, only its net momentum. Such flows are said to be self-similar. There is some evidence that each given class of flows has a unique self-similar state [2]; that is, all wakes of two-dimensional objects have a common state to which they asymptotically tend, all round jets tend to a unique state, etc. The issue is not completely settled [3] primarily because of the following operational difficulty: How far downstream of the turbulence-producing body should be considered 'sufficiently far downstream'?

An understanding of turbulence, as in other branches of physics, is best acquired by examining a limiting situation. A limit traditionally used is that of the far field where there is no need to take into account the many details of the stirring mechanism. The far field turbulence may be called 'mature turbulence' – as opposed to 'nascent turbulence' such as obtains in the 'near field' of the cylinder or in the convection box. Conversely, the nascent turbulence is more virile and offers extraordinary richness of nonlinear phenomena typically absent in far-field situations.

The infinite Reynolds number limit is usually invoked also, chiefly because turbulence generally occurs at high Reynolds numbers. Again, how high a Reynolds number is 'high enough' is by no means obvious in practice, and the answer is clearly different for different purposes.

One generally seeks 'universal' features of turbulent flows in the 'far-field' at 'very high' Reynolds numbers. If some universal features do exist, it is clear that the coupled maps seeking to explain turbulence should incorporate them in some way.

A premise in turbulence studies is that the average value of the rate of energy dissipation remains finite even when the fluid viscosity tends to zero: Viscosity, while being essential in bringing about dissipation, does not control the rate of dissipation of turbulent energy (as long as one stays away from solid boundaries, shock waves and the like). So far, this has not been proven on a formal basis, and the empirical evidence will be examined shortly. Another feature of turbulence worth the attention is the intermittent character of small scales, which seems to become increasingly conspicuous with increasing Reynolds number. Other potentially generic features of turbulence are believed to be the spectral energy transfer across the wave-number space, the interchange of energy among different directions, etc. Something can be said about each of them, but I shall restrict myself to the scaling of the energy dissipation and intermittency.

A cursory examination reveals that in many coupled maps studied to-date there is no well-defined analog of turbulent energy and that, *ipso facto*, they cannot incorporate the notion of an energy dissipation rate which is independent of viscosity. However, most maps do possess some kind of intermittent behavior; whether or not this is important for turbulent energetics will be discussed briefly.

2. Energy dissipation rate

If the average energy dissipation rate $\langle\varepsilon\rangle$ is independent of viscosity, it follows on dimensional grounds that it should depend only on the length and velocity scales of the energy-containing motion of turbulence. The appropriate length is the correlation length scale L ($= \int dr \langle u(x)u(x+r)\rangle / \langle u^2\rangle$, r being the separation distance in the direction of the velocity component u). The velocity scale is $\mathbf{u} = (k/\rho)^{1/2}$ where k is twice the kinetic energy of turbulence; ρ is the fluid density. The behavior of $\langle\varepsilon\rangle L/\mathbf{u}^3$ will be examined below with respect to the microscale Reynolds number $R_\lambda = \mathbf{u}\lambda/\nu$, where ν is the kinematic viscosity and λ is the Taylor microscale given by $\langle\varepsilon\rangle/k = 10\nu/\lambda^2$; λ has no obvious physical meaning except that it is nearly equal to the average distance between two consecutive up (or down) zero-crossings of the fluctuating velocity signal [4]. One uses λ instead of L mainly because the former is intrinsic to a turbulence signal and therefore useful for comparing diverse flows. The data to be examined here are obtained for turbulence far downstream (but *not* too far downstream) of grids of bars across which there is a steady uniform motion. The grids are in the form of a square mesh of either circular or square rods mounted contiguously in two neighboring planes. The solidity of the grid (that is the fraction of the projected solid area) varies between 0.34 and 0.44. The source of turbulence is the grid itself; in the far field the turbulence is roughly homogeneous and isotropic. Because of viscous action, there is a monotonic (and power-law) decay of the turbulence energy as one moves away from the source in the far field. We have considered here this flow partly because it is relatively simple, and partly because the dissipation rates can be measured quite accurately without having to resort to approximations of local isotropy. For a discussion of this and other details, see [5] and references cited there.

Figure 1 shows that $\langle\varepsilon\rangle L/\mathbf{u}^3$ is sensibly independent of the Reynolds number for $R_\lambda >$ 50. The trend for lower R_λ is consistent with expectations in the limit of zero Reynolds number for which it can be shown [5] that

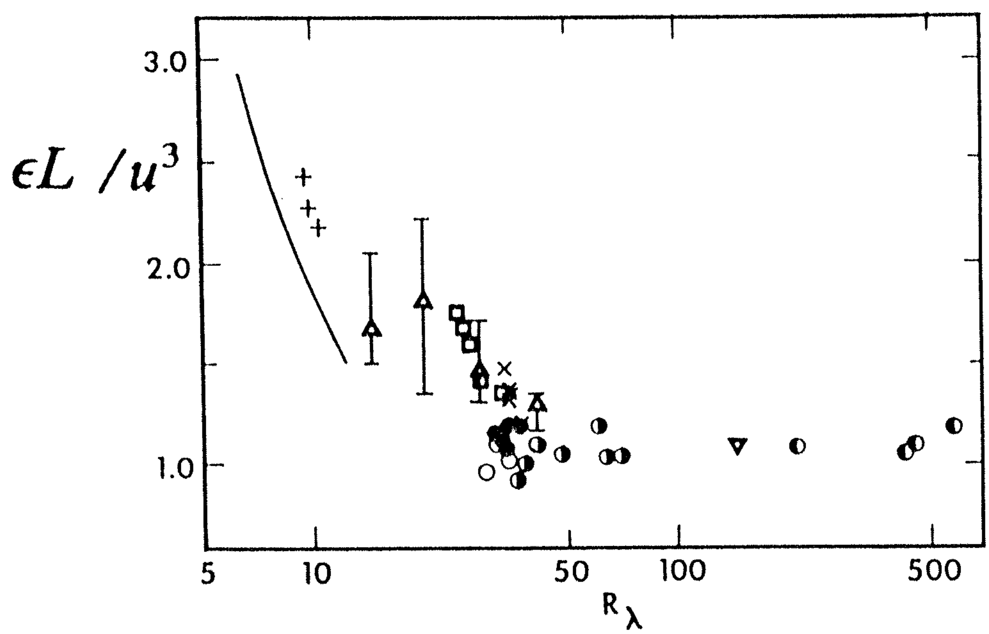

Figure 1. The average energy dissipation rate scaled on the energy-containing motion of turbulence, plotted against the microscale Reynolds number R_λ (which is proportional to the square root of the Reynolds number based on L). The data are for biplane square-mesh grids collected from a number of sources listed in [5]. The line to the left corresponds to equation (1), and is valid in the limit $R_\lambda \to 0$.

$$\langle\varepsilon\rangle L/\mathbf{u}^3 = (\pi/2)^{1/2}\,(15/R_\lambda). \tag{1}$$

The quantity $\mathbf{u}^2/\langle\varepsilon\rangle$ is the time scale for energy dissipation while $L/\mathbf{u}$ is the time scale of the energy transfer from the large to small scales, so that $\langle\varepsilon\rangle L/\mathbf{u}^3$ can be thought of as the ratio of the latter to the former. That the ratio is about unity suggests that the dissipation is intrinsically tied to the process of energy transfer across scales. Equivalently, $\langle\varepsilon\rangle L/\mathbf{u}^3$ can be thought of as the ratio of the average energy dissipation rate $\langle\varepsilon\rangle$ to $\mathbf{u}^2/(L/\mathbf{u})$, where the latter is the rate at which energy is being removed from large scales. The ratio is unity for $R_\lambda > 50$, which suggests that the large and the small scales become sensibly decoupled beyond that Reynolds number. It is perhaps not a coincidence that, when the extent of the $-5/3$ region in the power spectral density of the so-called longitudinal velocity component is extrapolated backwards, it vanishes around an $R_\lambda \cong 50$.

It would have been much more satisfactory had the Reynolds number extended to an order of magnitude or so higher in figure 1 but, even so, the conclusion seems reasonably clear that the energy dissipation rate remains finite as the Reynolds number tends to infinity. We therefore think that coupled maps would do well to exhibit this property.

Unfortunately, this conclusion becomes clouded if one examines measurements from broader class of flows. Even for grid turbulence, the precise value of the quantity $\langle\varepsilon\rangle L/\mathbf{u}^3$ even in the far field seems to depend on the type of the grid. It is not clear whether these different values converge to a common number at extremely large Reynolds numbers. It is particularly difficult to compare different classes of flows. Part of this difficulty is merely operational: In all flows except the grid turbulence considered here, $\langle\varepsilon\rangle$ cannot be measured in full because of instrumentation limitations; and so, traditionally, one estimates $\langle\varepsilon\rangle$ by assuming local isotropy. While the assumption may be good for certain purposes, the level of uncertainty in estimating $\langle\varepsilon\rangle$ may be different in different flows and vary with the flow Reynolds numbers; this may well be why a previous attempt [5] was quite inconclusive. It is deplorable that the final word is yet to be said even on this most basic feature of turbulence!

3. Intermittency

We shall now look at the spatial distribution of the energy dissipation rate. As is well known, Kolmogorov [1] assumed that this distribution is statistically uniform in space. In reality, ε is strongly intermittent (figure 2); perhaps no other aspect of turbulence has attracted so much attention in recent years. Intermittency is clearly an interesting facet of turbulence and, by its mere presence, demands an explanation. But, how justified is the enormous preoccupation with it if our goal is a practical understanding of turbulence – for example, the energetics of turbulent flows?

We have shown earlier [6] that ε can be described by a multifractal, and argued [7] that the volume V occupied by regions contributing to the average value $\langle\varepsilon\rangle$ of ε tends to zero as the Reynolds number approaches infinity. (This volume is formally that of the measure-theoretic support of the measure ε.) This implies that the original idea of Kolmogorov, which envisages ε to be statistically uniform, is incorrect in principle and that intermittency should play a very vital role. This is especially so because, from a theorist's point of view, the appropriate question to ask is: 'What happens in the infinite Reynolds number limit?' If the answer to this question becomes clear, the finite Reynolds number cases can perhaps be understood as finite size effects. In this view, the right physics does not correspond to a statistically uniform distribution of ε (with implied *only corrections* for intermittency), but something different; the appropriate first approximation is one in which the energy dissipation occurs on a set of zero volume and is singular (though not strongly so). The

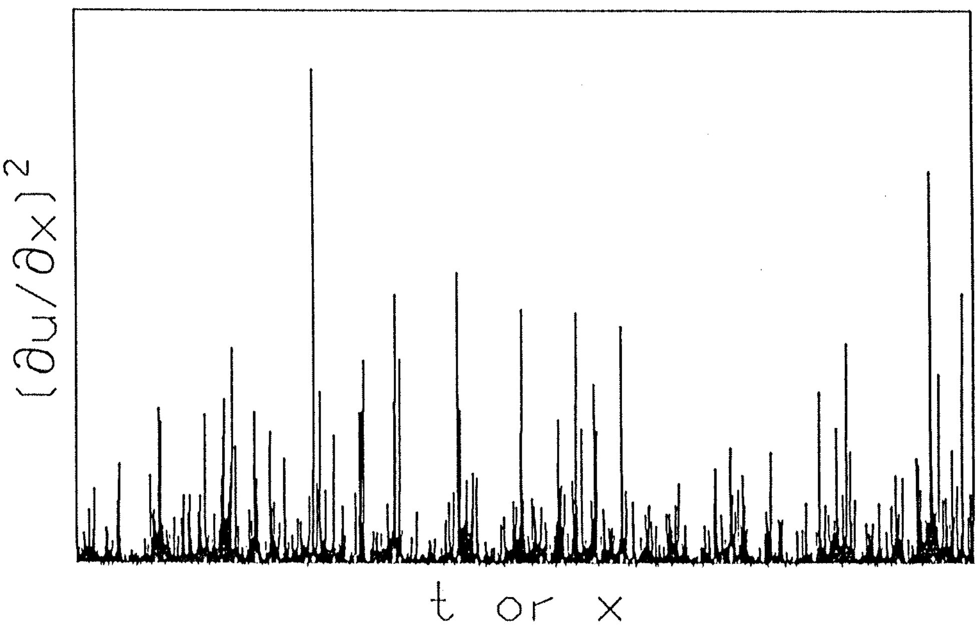

Figure 2. The quantity $(\partial u/\partial x)^2$ is a surrogate for the energy dissipation, ε. It is computed by measuring a velocity component u as a function of time t and treating the time derivative as space derivative under the assumption that turbulence is being convected unchanged at a constant velocity. This is the so-called Taylor's hypothesis. Also, $(\partial u/\partial x)^2$ is but one component of ε. We do not fully understand the limitations of these approximations/ hypotheses in inferring the intermittent properties of ε from those of $(\partial u/\partial x)^2$. However, for a passive scalar mixed by turbulence, we have measured elsewhere (Prasad & Sreenivasan, J. Fluid Mech. **216**, 1-34, 1990) *all* the components of *its* dissipation *without* the use of Taylor's hypothesis, and shown that similar approximations have no serious effects on the scaling properties of the type dicussed here. The untested hope is that the same conclusion will also hold for ε.

issue of cascades and spectral transfer of energy will then have to be discarded; it is not good physics to think of small 'corrections' modifying an inherently incorrect picture. (The fact that the intermittency 'corrections' are indeed small in practice should not diminish this asymptotic point of view.)

In practice, however, the situation is much less sharp. Figure 3 shows that the volume V goes to zero at a very slow rate. (One can work out this rate exactly for the binomial multifractal model such as used in [6].) The figure is adapted from [7] but has been redrawn under the assumption that the volume is unity at an R_λ of 50; a different assumption would merely shift the curve parallel to itself (see figure 6 of [7]) but would not change the qualitative conclusion. Note that it takes an $R_\lambda = O(10^{10})$ for the volume V to reach a 'small enough' value of order 0.1. Such high Reynolds numbers are never found in practice – not even in the outer convection zone in the Sun. In geophysical flows, one typically observes microscale Reynolds numbers ranging between several thousands to (perhaps) several ten thousands. The highest Reynolds number measurements we are aware of correspond to an $R_\lambda = O(10^4)$ for which $V \cong 0.5$. For most flows on Earth, V is

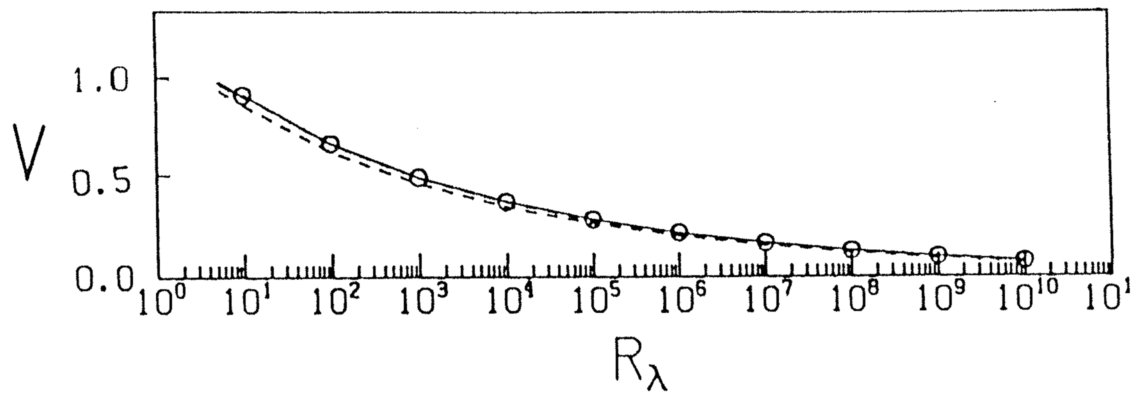

Figure 3. The volume V occupied by ε_a, the part of ε that contributes to the mean value $<\varepsilon>$ of ε. The circles are not experimental data, but computed according to the multifractal data obtained in [6]. For details of how the curve was obtained, and of what is involved in this interpretation, see [7]. The curve is drawn by adjusting (essentially arbitrarily) the volume to be unity at an R_λ of 50.

larger and closer to unity. One might say that the dissipation (in so far as its *average* value is concerned) is approximately space-filling and that, in practice, Kolmogorov's original assumption is not a bad representation.

In fact, this argument can be made somewhat stronger. It can be shown [8] that, while the energy dissipation is highly intermittent and consists of huge spikes (see figure 2), ε_a, the part of ε that contributes most to its average value is not too spiky; in fact, $\varepsilon \cong \varepsilon_a$. The basis for this conclusion is simply that, asymptotically, all the contribution to the average value of a multifractal measure comes from the set for which $f = \alpha$, where $f(\alpha)$ is its multifractal spectrum. From measurement [6], this occurs for ε when $f = \alpha = 2.87$, which is not far from 3. Thus, ε_a is only marginally singular and the set on which it lives is only marginally non-space-filling.

The conclusion, then, is that *for most practical Reynolds numbers,* a non-intermittent energy distribution is not a bad approximation, and the influence of deviations from this approximation on the energy dynamics is essentially small.

4. Do turbulent flows organize themselves to be at a 'critical' state?

An old idea in turbulence literature (see, for example, [9]) is that fully turbulent flows may be marginally stable; that, in some sense, they are always on the verge of being unstable or 'critical'. What is marginally stable is a suitably coarse-grained version of the real flow; the 'critical' Reynolds number of the latter, with the fluid viscosity replaced by a renormalized eddy viscosity, is the same as that of the relevant laminar state. The large structures are then viewed to result from the instability of this renormalized flow. This last-mentioned idea has been expressed explicitly in [10], although some version of this view appears to have been held by several others.

A more detailed version of self-organization of a turbulent flow is due to Malkus and Smith [11]. They investigate whether the observed turbulent state is the result of optimizing a certain functional of the flow properties, and indeed seem to have discovered one such for the plane Couette flow as well as the plane Poiseuille flow. The physical reasons for why their functional is the appropriate one is not fully understood.

5. Concluding remarks

Are there universal features shared by *all* turbulent flows at *all high Reynolds numbers?* The definitive answer remains elusive. Yet, in some restricted but practical sense, some degree of universality *is* manifest in turbulence. See, for example, [12]. It is therefore useful for some coupled maps to exhibit such 'universal' features. Preliminary remarks expanding this view have been made here. Unfortunately, a more serious effort will have to wait another occasion.

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* It is embarrassing to refer to my own work so much, and I beg the reader's indulgence. My excuse is that this is an informal paper.