WAVELET ANALYSIS OF THE TURBULENT JET

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Received 9 November 1989; revised manuscript received 17 January 1990; accepted for publication 18 January 1990
Communicated by A.P. Fordy

The wavelet transform is applied to two-dimensional dye concentration data in turbulent jets at moderate Reynolds numbers. This reveals the nature and limitations of scale similarity of the inner structure of the scalar, and the stringiness associated with small scales. For comparison, two-dimensional Brownian motion is also treated.

The wavelet transform [1–3] has recently been applied to turbulence data [4,5]. It was suggested that the bifurcating patterns which resulted gave visual evidence that the Richardson cascade [6] was present. However, it was shown in another investigation [7] that a process as non-deterministic as Brownian motion produces a visually similar bifurcating pattern to that of atmospheric and other turbulence data [8,9]. Therefore, while the assertion about the cascade may be true, further analysis is clearly indicated. In the present investigation we apply the wavelet transform to the study of two-dimensional axial and meridional sections of a turbulent round jet flow [10]. (This is in contrast to the above cited works, most of which are restricted to one-dimensional records. Exceptions are the work of Argoul et al. [12] and Arneodo et al. [13] in which the wavelet transform is applied to two-dimensional aggregates and the recent treatment of two-dimensional turbulence by Farge and Rabreau [14]. We have also applied wavelet transforms to the three-dimensional jet data of ref. [11], but this will not be presented here.) As we will show, the wavelet transform, which is primarily a visual tool, is well suited to revealing the similarity and inner structure of a flow. It also leads to physical insights which only have been hinted at in previous discussions of the turbulent jet.

The wavelet transform of a function $u(x)$ is defined as

$$U(a,x)=\frac{1}{a^{N/2}}\int_{-\infty}^{\infty} g\left(\frac{x-y}{a}\right)u(y)\,dy\,, \qquad (1)$$

where $N=\dim[x]$ is the dimension of the space. $g(x)$ is localized at the origin and satisfies

$$\int g(x)\,dx=0\,. \qquad (2)$$

Most previous applications of the transform were to time records and questions of causality led to complex $g(x)$ [15]. In the present instance we consider genuine spatial records and such fine points can be disregarded. Also, while different wavelet functions have been proposed they appear to produce no truly significant variations in the results [4]. In the following we take

$$g(x)=\nabla^2\exp(-x^2/2)$$
$$=(x^2-1)\exp(-x^2/2)\,, \qquad (3)$$

which is known as the *Mexican hat*.

In fig. 1a we display a two-dimensional laser in-

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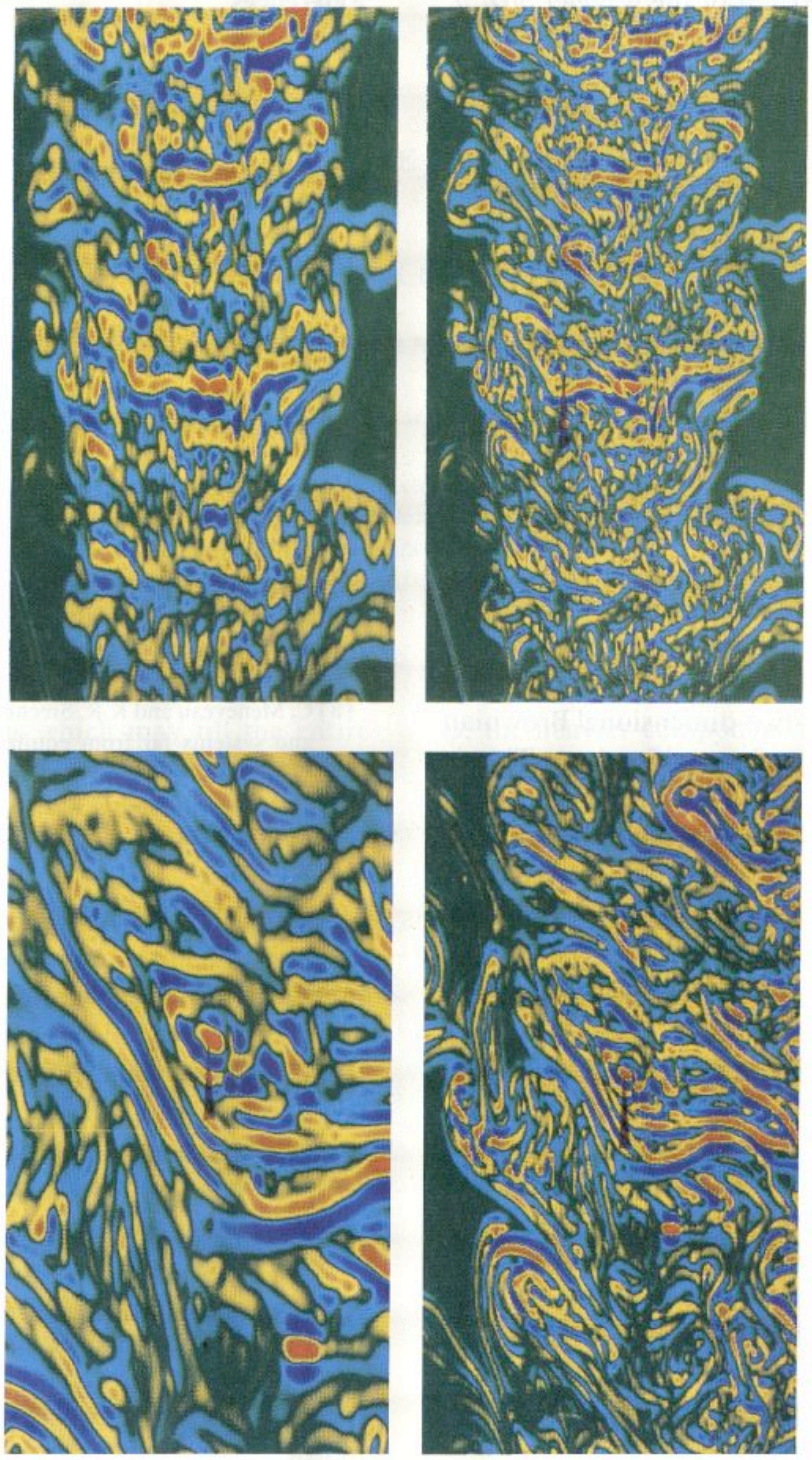

Fig. 6. (Continued).

$a=0.001$ we see a clear transition from predominantly beads to what appears to be predominantly strings. Two other features are well brought out by the wavelet transform. First, it is clear that the scalar structure is strongly anisotropic. This is perhaps best illustrated by the $a=0.001$ wavelet transform, which displays the smallest scales of the problem. The same picture also illustrates the transition from the edge region, where the *strings* are aligned at roughly 45° from the axis, to the central region where they are roughly perpendicular to the center line. We note that 45° is the direction of the principal rate of strain of the flow.

The beads are of the extent of the wavelet, viz. a. They mark regions of overall concentrations of dye, red, to regions of undyed fluid, blue. The strings are really sheet-like structures as a little thought indicates. Also a wavelet transform of a section meridional to the axis supports this assertion. Such sheet-like structures have already been indicated as spatially high frequency or dissipative structures in the velocity field [22,23]. It is of some importance to observe that sheet-like structures are all very pronounced at $a=0.001$, when the scale size is in the neightborhood of the Kolmogorov scale. This fact may be of basic importance to our understanding of mixing. Application of wavelets to the three-dimensional data [11] shows that the sheets possess strong three-dimensional convolutions.

Although the wavelet analysis appears to show the transition from *beads* to *strings*, it is necessary to directly investigate the self-similarity of the flow by proceeding, as we did for two-dimensional Brownian motion, through a series of magnifications. This is indicated in fig. 6. The arrows in the bottom four pictures correspond to that in the rightmost picture in the top row. The bottom row contains the magnified versions of the transformed pictures shown in fig. 5. Thus we see that the string-like structures persist under magnification, thus emphasizing the essential lack of self-similarity of the pictures. This is especially true when we focus on the off-axis regions of the jet.

A complete discussion of self-similarity of the inner structure of the jet is hampered by two factors: the moderate jet Reynolds number and the inherent inhomogeneity of the flow. Although we have attempted to correct for this latter aspect in the streamwise direction, it is clearly not complete; no corrections were applied to inhomogeneities in the radial direction.

We thank Rahul Prasad who obtained the jet images and Lareef Zubair for his help in handling the data, and acknowledge many useful discussions with Bruce Knight. Computing work was done at the Pittsburgh Supercomputing Center supported by the National Science Foundation. The research reported here was supported by DARPA/URI under grant number N00014-86-K0754.

References

[1] A. Grossmann and J. Morlet, Decomposition of functions into wavelets of constant shape, and related transforms, in: Mathematics and physics; lectures on recent results, ed. L. Streit (World Scientific, Singapore, 1984).

[2] J.M. Combes, A. Grossmann and Ph. Tchamitchian, eds., Wavelets (Springer, Berlin, 1989).

[3] R. Kronland-Martinet, J. Morlet and A. Grossman, Analysis of sound pattern through wavelet transform, Int. J. Patt. Recogn. Art. Intell., special issue on expert systems and pattern analysis (1987).

[4] J. Liandrat and F. Moret-Bailly, Thge wavelet transform: some applications to fluid mechanics and turbulence, Europ. J. Mech. (Fluids) (1988), submitted.

[5] F. Argoul, A. Arnéodo, G. Grasseau, Y. Cagne, E.J. Hopfinger and U. Frish, Nature 338 (1989) 51.

[6] L.F. Richardson, Weather prediction by numerical process (Cambridge Univ. Press, Cambridge, 1922).

[7] R. Everson and L. Sirovich, A survey of wavelet analysis applied to turbulence data, Center for Fluid Mechanics, Brown University, Report 89-182 (1989).

[8] C. Meneveau and K.R. Sreenivasan, in: The physics of chaos and systems far from equilibrium, ed. Minh-Doung Van (North-Holland, Amsterdam, 1987) p. 49.

[9] C. Meneveau and K.R. Sreenivasan, Phys. Rev. Lett. 59 (1987) 1424.

[10] R.R. Prasad, C. Meneveau and K.R. Sreenivasan, Phys. Rev. Lett. 61 (1988) 74.

[11] R.R. Prasad and K.R. Sreenivasan, J. Fluid Mech. (1989), to be published.

[12] F. Argoul, A. Arneodo, J. Elezgaray, G. Grasseau and R. Murenzi, Phys. Lett. A 135 (1989) 327.

[13] A. Arneodo, F. Argoul, J. Elezgaray and G. Grasseau, Wavelet analysis of fractals; application to nonequilibrium phase transitions, in: Nonlinear dynamics, ed. G. Turchetti (World Scientific, Singapore, 1989).

[14] M. Farge and G.C. Rabreau, C.R. Acad. Sci. 307 II (1988) 1479.

[15] A. Grossmann and J. Morlet, SIAM J. Math. Anal. 15 (1984) 723.

[16] B.R. Ware, in: Measurement of suspended particles by quasi-elastic light scattering, ed. B.E. Dahneke (Wiley, New York, 1983) p. 255.

[17] H. Tennekes and J.L. Lumley, A first course in turbulence (MIT Press, Cambridge, 1972).

[18] L.M. Milne-Thomson, Theoretical hydrodynamics (MacMillan, London, 1949).

[19] B.B. Mandelbrot and J.W. van Ness, SIAM Rev. 10 (1968) 422.

[20] A. Arneodo, G. Grasseau and H. Holschneider, Phys. Rev. Lett. 61 (1988) 2281.

[21] B. Julesz, Nature 290 (1981) 91.

[22] A.Y.S. Kuo and S. Corrsin, J. Fluid Mech. 56 (1972) 447.

[23] K.R. Sreenivasan and C. Meneveau, Phys. Rev. A 38 (1988) 6287.