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TRANSITION INTERMITTENCY IN OPEN FLOWS, AND INTERMITTENCY ROUTES TO CHAOS

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The intermittent transition to turbulence in open flows (mainly pipe flows) is examined in the context of intermittency routes to chaos. Preliminary conclusions are that some quantitative connections can be discerned, but that they are incomplete. In a similar manner, connections with phase transition and other critical phenomena are also imperfect. Some measurements which we hope will be helpful in developing alternative models describing the essentials of the phenomenon are described. Some difficulties are highlighted.

1. Introduction

This paper is a part of an overall effort related to the exploration of quantitative connections between chaos in dissipative dynamical systems on the one hand, and transition and turbulence in the so-called open flow systems on the other. Open flows by definition possess a preferred direction, and there is a flux of mass across its boundaries. At least in some circumstances this elementary feature of open flows renders the nature of flow instability convective, as opposed to being absolute, which is the case observed in closed flow systems. This can have profound consequences on the origin of turbulence in open flow systems, which may in turn render our task quite difficult.

It has been known for over a hundred years now [1] that transition to turbulence in pipe flows occurs intermittently. For example, the velocity measured on the centerline at a fixed axial location in the pipe is typically as shown in fig. 1. It is this intermittent transition to turbulence that is our concern here. With increase in Reynolds number, the fraction of time that the flow is in the turbulent state increases, until eventually the flow is continuously turbulent. One observes qualitatively similar intermittency in the advanced stages of transition to turbulence in boundary layers (fig. 2), and channel (i.e., plane Poiseuille) flows, to which also we shall make a brief reference here.

Equally well known now is that many low-dimensional dynamical systems approach a chaotic state in an intermittent fashion, qualitatively similar to the intermittent transition to turbulence just

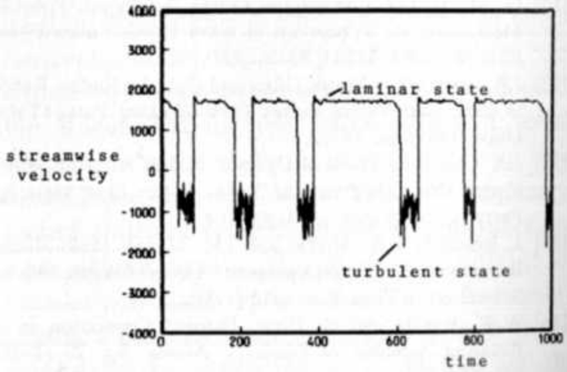

Fig. 1. The streamwise (or axial) velocity measured as a function of time on the centerline of a pipe flow. The measuring tool is a standard hot wire operated on a constant temperature mode. The signal oscillates apparently randomly between an essentially steady laminar state and a turbulent state. For a given axial position, a velocity trace obtained simultaneously at another radial position will show a coincident alternation between the two states, but the amplitude difference between the two states is a function of radial position. Both the ordinate and the abscissa are drawn to arbitrary scales.

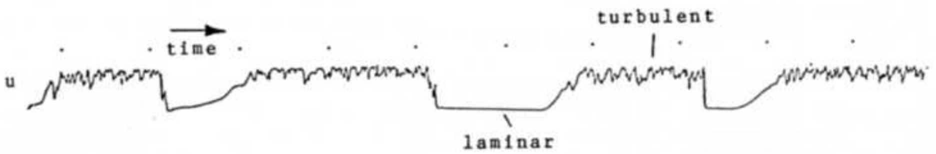

Fig. 2. Oscillograms of velocity fluctuations in the advanced stages of transition to turbulence in a constant-pressure boundary layer. Time interval between markers is 1/60 s. Source: ref. 17.

discussed. Fig. 3 is an example. The Lorenz equations [2], the Logistic map [3], and the RCL oscillator [4] are some of the other simple examples. Pomeau and Manneville [5] identified three generic intermittency routes which they called Type I, Type II and Type III – each differing from the other in terms of how the eigenvalues of the Floquet matrix, describing the return map linearized around a closed trajectory, cross the unit circle. Type I intermittency occurs when the linear stability of the limit cycle is lost by an eigenvalue of the Floquet matrix leaving the unit circle at +1, Type III when the crossing occurs at −1, and Type II when two complex conjugate eigenvalues simultaneously cross the unit circle. That these intermittent routes to chaos are relevant to fluid flow phenomena governed by partial differential equations has been demonstrated, for example, by Bergé et al. [6] and Dubois et al. [7] in the

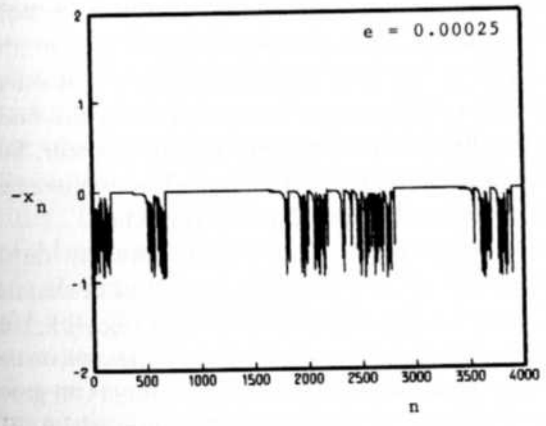

Fig. 3. Iterations of the one-dimensional map $x_{n+1} = x_n(1 + e) + (1 - e)x_n^2$ mod 1; $e = 0.00025$ for this plot. The map is the result of a reduction [24] from the so-called Type II intermittency [5]. The relevant aspects of this type of intermittency route to chaos are mentioned in the text. The qualitative analogy with fig. 1 is quite obvious.

Rayleigh–Benard experiment, and by Pomeau et al. [8] in the Belousov–Zhabotinsky reaction.

It may be argued on the basis of these interesting findings that intermittent transition in open flows may belong to some kind of universality class: Even though as already mentioned open flows are different in several non-trivial ways from the highly confined flows (see also [9]), it looks reasonable to ask whether there are any connections between the intermittency routes to chaos mentioned above and the intermittency routes to turbulence in open flows. As we shall see, the process involved in the latter are more complex; it is to their partial characterization that this paper is devoted.

2. The physical phenomenon

It is useful to recapitulate briefly the physical mechanism responsible for the temporal intermittency observed in fig. 1. Evidence from our own work – at least in pipes whose length is of the order of a few hundred diameters – as well as that of others (chiefly Wygnanski and Champagne, [10]), suggests that 'disturbances', whose developed state corresponds to the turbulent regions in the intermittent signal, arise locally (in radial, azimuthal as well as axial directions) in the entrance region of the pipe where the flow is laminar and steady, and is not fully developed (see fig. 4a). Once created in the boundary layer region, the disturbance quickly spreads over the entire cross-section of the pipe, and moves like an independent entity within the pipe; laminar regions are present both upstream and downstream of this entity, which now goes by the name 'slug'. (They have

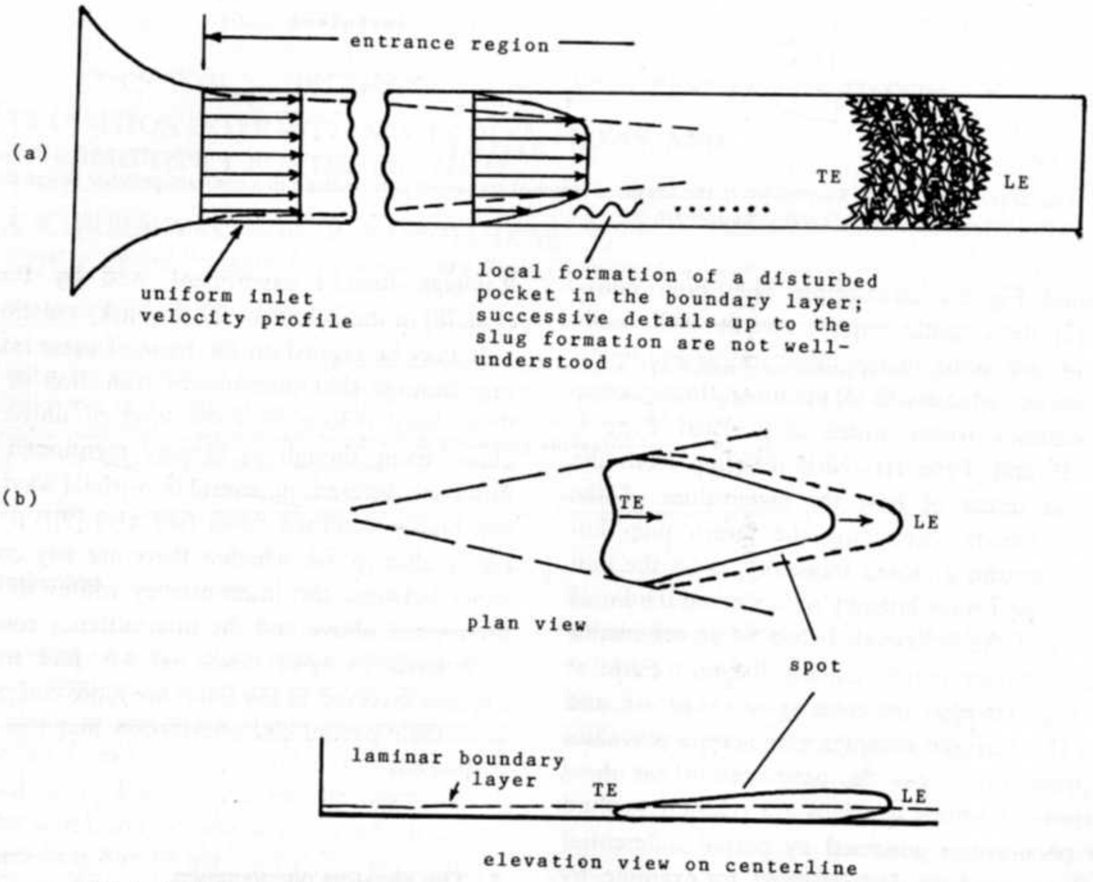

Fig. 4. A schematic of intermittent transition structures in (a) pipe flows, (b) boundary layers.

been called 'flashes of turbulence' by Reynolds [1] and 'plugs' by several workers, for example, Tritton [11].) A probe fixed at any point in the flow alternately sees the procession of slugs with laminar regions interspersed between them; the output signal consists of intermittent excursions from the laminar to the turbulent state, followed by the return (in the Eulerian frame of reference) in some stochastic manner to the former. The physical reasons for the return to the laminar state are the following. Usually in most pipe flows, it is the pressure difference between the inlet and the exit that is held constant. A given pressure difference can support a larger mass flow when the flow is laminar than when it is turbulent. As the slugs form and grow, the increased friction due to the turbulent flow in them produces a reduction in mass flux, thus inhibiting the instabilities at the inlet. New slugs are most likely to be born only after the first slug completely passes out of the pipe. It is not hard to argue similarly that intermittent transition can occur also when the mass flux is fixed but momentum variations occur, but it is possible that the two types of intermittencies do not share the same detailed properties.

Corresponding to slugs in pipe flows, the transitional structure in boundary layers and in channel flows is the so-called turbulent spot (fig. 4b). Unlike the slugs which are constrained except in the axial direction, the boundary layer spots can grow in all directions. (There are some non-trivial differences between spots in channel flows and boundary layers, but these details are not relevant here.)

It turns out that the speed of propagation U_{le} of the leading edge of the slug or a spot is different

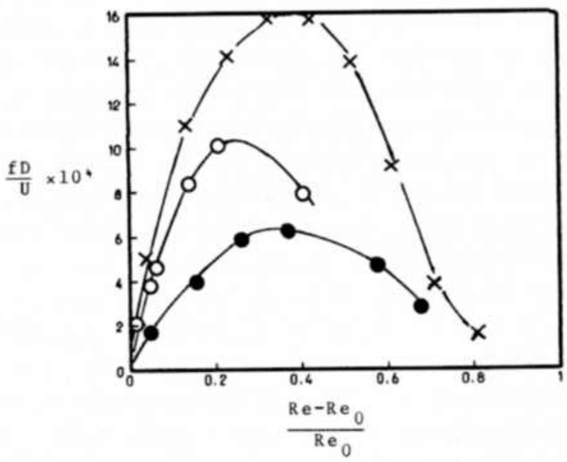

Fig. 5. Typical variation of the frequency of slugs as a function of Re for three streamwise locations. The topmost and lowermost curves are from Wygnanski and Champagne [10] corresponding to (Re_0, x/D) of (21×10^3, 220) and (19×10^3, 395), respectively. The middle curve, $Re_0 = 3900$ and $x/D = 335$, is from present measurements. U is the so-called bulk velocity (= flow rate/cross-sectional area of the pipe). $Re = UD/\nu$.

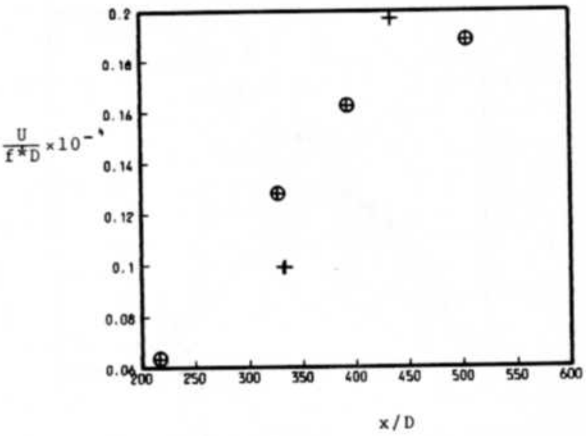

Fig. 6. Inverse of the peak slug frequency (f^*D/U) as a function of the streamwise distance. Noting that all measurements were made at the exit of pipes of different lengths, the streamwise distance must really be understood to mean the pipe length. The circled points are from Wygnanski and Champagne [10].

from the trailing edge speed U_{te}. Clearly, slugs (or spots) grow with distance if $U_{le} > U_{te}$. In the following we concentrate on the slugs. If the slugs are generated at some mean frequency, and if more than one slug resides in the pipe at the same time, the leading edge of a slug could catch up with the trailing edge of the preceding one, resulting in merger and a consequent reduction in their passage frequency with axial distance. These two factors could then provide a plausible mechanism for the streamwise dependence, at any given Reynolds number, of all measured temporal quantities. The two most important parameters in the problem are thus the Reynolds number Re and the (normalized) axial distance x/D, where D is the pipe diameter. The frequency of the slugs, for example, depends on both of these parameters, as shown in fig. 5. A characteristic value of this frequency (say, the peak value) varies inversely with x/D (fig. 6), and seems to be independent of the Reynolds number. (Strictly, one must plot on the abscissa the quantity $x - x_0$, where x_0 is a virtual origin for the slugs; it is possible that the scatter in the plot partly originates from this source. However, since x is relatively large compared to x_0, this is believed to be of small consequence.)

Several points must be made explicit. Both in figs. 5 and 6 (and in the others to follow), x/D is really L/D, where L is the pipe length. That is, measurements were actually made at the exit of pipes of different lengths. Although we have retained the notation x/D above in conformity with previous practice, it is not clear to us that measurements made at different axial locations of the same long pipe will show the behaviors of figs. 5 and 6. A look at fig. 5 shows that the mean length of slugs ($\sim U/f$) is greater than the pipe length, suggesting that more than one fully developed slug is unlikely to reside in a pipe at any given instant of time. Thus, the probability of merger is quite small. The reason for the observed reduction in the characteristic frequency with pipe length (fig. 6) must then be attributed largely to the reduction of the formation frequency of slugs with pipe length. This makes sense if we remember that longer pipes mean longer slugs which take longer to pass the entire pipe. We conclude that detailed and careful measurements at several stations in

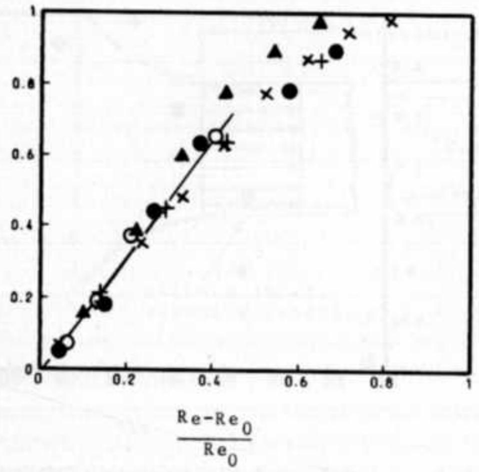

Fig. 7. Typical data on the intermittency factor γ as a function of $(\mathrm{Re}-\mathrm{Re}_0)/\mathrm{Re}_0$. ▲: $L/D=505$ [10]; +: $L/D=435$ (present). Other symbols as in fig. 5.

extremely long pipes (say, length to diameter ratio $>10^4$) is overdue.

As one varies the flow Reynolds number, the appearance of the intermittent state is quite abrupt. The intermittency factor γ, defined as the fraction of time the flow is turbulent, appears to vary approximately linearly with the Reynolds number. By a backward extrapolation to zero of the measured intermittency factor, one can define a unique value of the onset Reynolds number Re_0. Fig. 7 shows that γ is a unique linear function of $\mathrm{Re}-\mathrm{Re}_0$ in a certain non-trivial neighborhood of Re_0; x/D or L/D is thus an inconsequential parameter for this quantity.

A reasonable goal now is to describe in phase space the main features of these processes. Returning now to fig. 1, it appears plausible to think that the steady laminar state is essentially zero-dimensional – that is, a proper orthogonal decomposition of the temporal signal contains no time dependent function. (Unfortunately, estimates of statistical properties such as the entropy and dimension from the velocity signal obtained entirely in the laminar state, for example just before the onset of intermittency, is dominated by the high-dimensional, low-amplitude noise overriding the laminar motion. The noise here does not arise merely from instrumentation or other 'purely random' fluctuations in the background; as mentioned elsewhere [9], the background 'noise' in most open flow systems is usually dominated by large-scale pressure fluctuations which are far from being structureless.) From this fixed point, the motion escapes to an attractor representing the turbulent state, and gets reinjected near the fixed point at apparently random intervals. Two relevant questions can be asked: 1) Can one quantitatively capture by a low-dimensional map the essential dynamics of this intermittent motion from the fixed point? 2) What are the characteristics of the chaotic attractor? Answers to these questions are attempted below.

3. The route to chaos

Fig. 8 shows a close-up of the vicinity of the velocity signal near the leading edge of a typical slug. Corresponding to the laminar as well as this interface regions, we have constructed by discretization a return map of u_{n+1} vs u_n (fig. 9). A close look in the vicinity of the fixed point shows that the map is much like that from which fig. 3 was constructed. Secondly, the slope of the return map near the fixed point is close to but greater than unity. This shows that the fixed point is unstable once the onset of intermittency occurs; the laminar and interface regions are thus merely a reflection of the duration spent in the narrow channel in the vicinity of the fixed point. There is some hope, then, that the dynamics of the leading edge interface can be described (approximately) by a one dimensional map of some kind, for example that used in fig. 3 (for small x_n). This observation lends some emphasis to our original question of possible connections to the generic intermittency routes to chaos. We must right away note a simple fact: Pipe flows strictly belong to neither type of intermittency mentioned in section 1, an obvious reason being that, unlike in the Pomeau–Manneville formulation, intermittent transition in open flows (see especially fig. 2) occurs from a *steady state* and not from a limit cycle. (For a

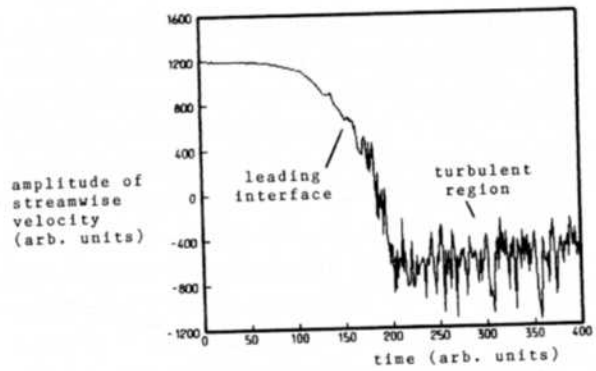

Fig. 8. A close-up of the velocity signal near the leading edge of a typical slug.

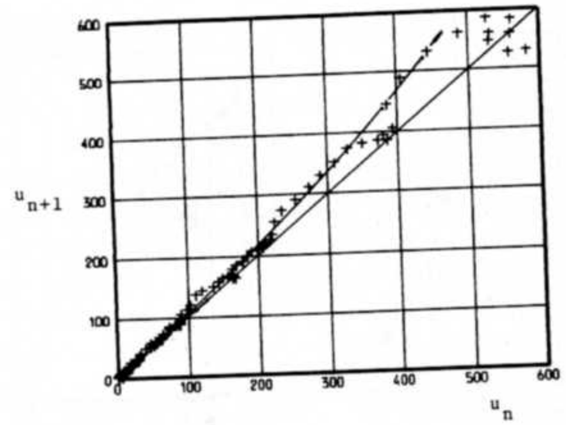

Fig. 9. The return map of u_{n+1} vs u_n for the interface region shown in fig. 8, obtained by the discretization of the signal. The origin is the fixed point representing the laminar state.

brief reexamination of this point see section 6.) This may be interpreted to mean that Poincaré sections of the Pomeau–Manneville intermittencies have a direct bearing on pipe flow transition, but it will unfold that this is not the entire story. As we shall show soon, this is related to the non-uniform manner in which the motion in phase space gets reinjected to the vicinity of the unstable fixed point. (We take the view that to label reinjection by 'relaminarization' – as is often done – is to miss the point altogether. While in the Eulerian frame of reference one sees an alternation between laminar and turbulent states, this does not imply relaminarization of fluid that was once turbulent. As must be clear from section 2, in the Lagrangian frame of reference, there is no relaminarization of fluid entrained by a slug: one is talking merely about the slug/no-slug situation.)

All three types of intermittencies mentioned in section 1 make definite predictions for certain statistical quantities of the intermittent signals, against which the outcomes of experiments can be tested. Apart from the nature of the return maps themselves, the important predictions concern the probability distribution for the duration of the laminar regions; from this distribution one can in particular calculate their mean duration as a function of the departure from the critical value of the control parameter, $\mathrm{Re}-\mathrm{Re}_0$ here. At any rate, it is useful to measure these quantities in the hope that they will help us build alternative models.

Fig. 10 shows a plot of the average length L_l of the laminar regions as a function of $\mathrm{Re}-\mathrm{Re}_0$. The data for several experimental conditions all tend to show that $L_l \sim (\mathrm{Re}-\mathrm{Re}_0)^{-1}$. This behavior is common to both Type II and Type III intermittencies. The measured inverse cumulative distributions for the length of the laminar intervals (fig. 11) follows the expression

$$P(l>l_0) \sim [e/\exp(4el_0-1)]^{1/2}, \tag{1}$$

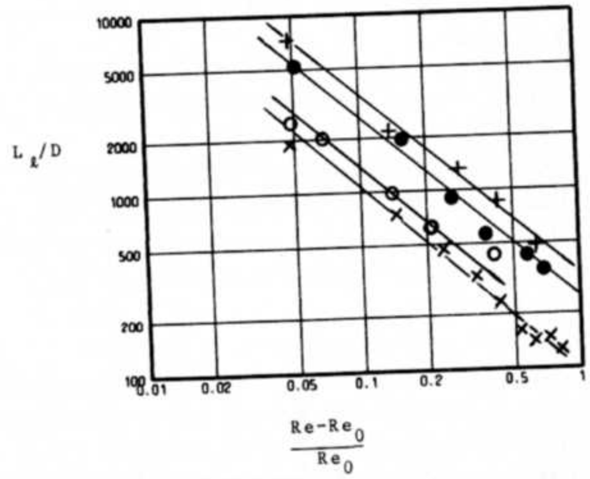

Fig. 10. The mean length of the laminar regions in the measured velocity signals, plotted as a function of the distance from the critical Reynolds number. Lines correspond to the −1 power predicted for Type II and Type III intermittencies. +, $L/D=435$, $\mathrm{Re}_0=4480$. Other symbols as in fig. 5.