The azimuthal correlations of velocity and temperature fluctuations in an axisymmetric jet

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The structure of turbulence in an axisymmetric jet at the nozzle exit Reynolds number of 4×10^4 was examined at several streamwise stations ($1.0 \leq x/D \leq 40$), by measuring, for a few fixed radial positions, two-point correlation functions of the streamwise velocity and temperature fluctuations with azimuthal separation. By an orthogonal Fourier decomposition of the azimuthal correlation functions, an interpretation of these correlations and of the large-scale evolution is provided.

I. INTRODUCTION

Several recent experimental investigations[1–8] of axisymmetric turbulent jets have revealed that the near field is characterized and dominated by well-defined instabilities and nearly periodic large-scale structural elements. It now appears[8] that linear instability analysis[9–11] can account quite well for the appearance both of axisymmetric and helical modes. The nonlinear interaction among these modes, as well as that between them and the subharmonic of the axisymmetric mode, the pairing activity[7] between vortex elements (which themselves arise when the axisymmetric mode grows to a finite amplitude), etc., have all been studied quite extensively. The various details of these interactions and of flow evolution appear to be quite sensitive to initial conditions,[8,12–15] but the phenomena listed above seem to represent the common ground, and typically cover $x/D \lesssim 3$ (where D is the diameter of the nozzle). Thereafter, the jet is prone to the so-called columnar,[4] or preferred mode,[7] instability, giving rise to long-wave modes of relatively well-defined Strouhal number. Somewhat downstream (say, $x/D \gtrsim 6$), a more complex interaction occurs among these vortex elements, and the eventual formation of what one perceives as the turbulent flow does not seem to be amenable to simple description. In this region, flow visualization studies (which have had considerable success in unraveling the details of the near field) do not show any obvious large-scale periodicity, even under controlled (acoustic) excitation of the jet.[1] However, some conditional measurements[3,5] have revealed aspects of resemblance between the large structure in the turbulent region and the vortex structure mentioned above. One obvious interpretation of such observations[2,16] seems to be that the underlying structure essentially retains its coherence (if not its periodicity) which however is obscured by the presence of the "over-riding" small-scale turbulence. Quite another interpretation is that the large-scale coherence observed through sophisticated conditional measurements is in fact a transient phenomenon somehow imposed on a wide range of the energetic small scales of turbulent motion.

In this paper, we examine the simplest way of describing the circumferentially coherent large structure in an axisymmetric jet. Measurements were made of two-point correlation functions of the streamwise velocity and temperature fluctuations with azimuthal (or circumferential) separation. Domination of the large structure implies certain azimuthal coherence: if the flow close to the nozzle exit consists only of perfectly correlated vortex rings, we expect these correlations to be independent of the azimuthal separation. If these vortex rings develop azimuthal waves, the correlation functions can be expected to show this feature too. On the other hand, a rapid falloff of the correlation functions with azimuthal separation suggests weak large-scale circumferential coherence, and a domination probably of the small-scale motion. If a coherence does exist, it clearly becomes useful to look upon the flow as consisting of various modes of motion of the ring vortex, which can be made amenable to a simple description via an orthogonal decomposition into various Fourier modes of the measured correlation functions. This is attempted here. Such decomposition is of considerable interest in jet noise studies. In Michalke's theory[17] of noise generation from round jets, the acoustic efficiency is sought for the axisymmetric and higher-order azimuthal source components, and measurements of the type discussed earlier in this section are not uncommon (although not extensive) for pressure fluctuations in jet noise studies.[18–19] When the present measurements were completed (April 1979), no such measurements for velocity or temperature fluctuations existed. Since then, however, azimuthal velocity correlations[20] for $x/D \lesssim 4.1$, and pressure-velocity correlations[8] for $x/D \gtrsim 5$ have been measured. But the present measurements go much further in that they are more detailed and extend from $x/D = 1$ to 40, and also include temperature fluctuations. For these two reasons, it was deemed that an independent presentation of these results was justified. By measuring these azimuthal correlation functions from $x/D = 1$ close to the nozzle exit to $x/D = 40$ characteristic of a fully developed jet, inferences will be drawn about the evolution of the large structure.

Because of the high contortions of the interface between the jet fluid and the ambient fluid, the potential fluctuations in the flow field make a substantive contribution to the velocity fluctuations. However, this is not true of temperature fluctuations, which are entirely limited to the jet fluid. Thus, we felt that a measurement of the azimuthal correlation of both velocity and temperature fluctuations would lead to a

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better understanding of the azimuthal structures in the jet. Conditional measurements would no doubt have been desirable, but, unfortunately, limitations of several types forced a premature halt to the study originally intended.

In this paper, no account is taken of the existence and importance of the helical mode[21,8] which, it appears,[8] becomes increasingly important with increasing distance from nozzle exit in the near field. (We suspect that the reason why the "spot" experiments of Kleis *et al.*[22] show only an azimuthally coherent mode is the acoustic excitation accompanying the generation of the spark. In our own experience, very little excitation, such as the noise due to a camera shutter near the jet, can accentuate the axisymmetric mode, and therefore dwarf the other modes otherwise possible.) An independent study attempting along similar lines to ascertain the importance of the helical mode is currently under way, but we suspect that there will be no convenient way of linking it with the present data.

II. EXPERIMENTAL FACILITIES

The jet used for the present experiments is sketched in Fig. 1(a). Briefly, the air supplied by a fan passed through a coarse screen and a diffuser to a constant area section housing three screens of increasingly finer grades. The contraction had an area ratio of about 14. The heating element was located axisymmetrically in the diffuser upstream of the screens.

The nozzle had a diameter of 4.1 cm, and the jet Reynolds number $U_j D/\nu$ (where U_j is the jet exit velocity) was maintained at 4×10^4. The jet exhausted into still ambient air. No external excitation was employed.

Measurements were made at several streamwise locations downstream of the nozzle ($0.5\leqslant x/D\leqslant 40$), but we have chosen to include data for $x/D\geqslant 1$ only. The streamwise velocity fluctuation u was obtained by means of single hot wires (5μm diam and about 0.8 mm sensitive length) aligned along a given radial line. The wires were made in the laboratory by etching the Pt–Rh Wollaston wire soldered to homemade probes. The hot wires were operated on two DISA 55D01 constant-temperature anemometers.

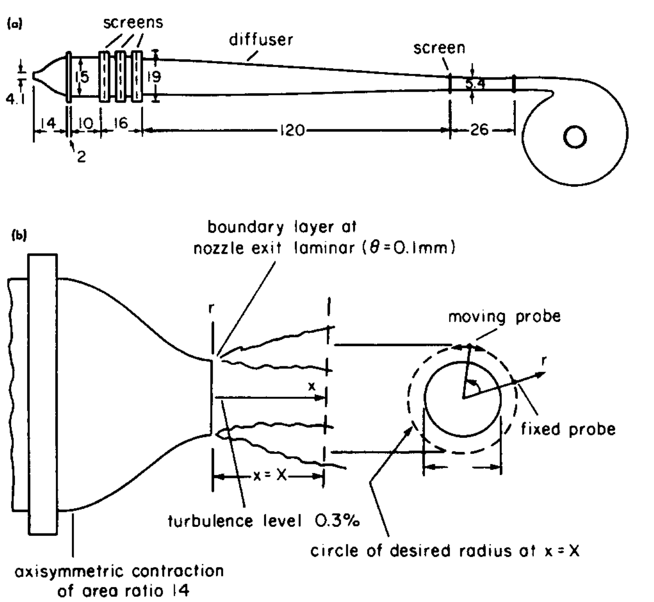


FIG. 1. (a) The jet facility, and (b) the flow and measurement description. In (a), all dimensions are in centimeters.

All signals were suitably preamplified and filtered before being digitized and processed on a PDP 11/40 computer. Typically, each data point was obtained by (ensemble) averaging over 30 record lengths each of which was about 2 sec duration.

The boundary layer at the nozzle exit was determined to be laminar; the momentum thickness, as evaluated using Thwaites' method,[23] was about 0.1 mm, so that the boundary layer Reynolds number $R_\theta \simeq 100$. The turbulence level on the jet axis at exit was about 0.3%. Because of the premature termination of this work about five years ago, relatively no effort was spent on documenting initial conditions in more detail. In the recent few years, initial conditions have come to be increasingly recognized as critical in determining the behavior of the jet in the near field (especially $x/D\leqslant 1$). This, of course, is why we present data for $x/D\geqslant 1$. Even so, two comments are in order: first, the present jet was much "noisier" than those used in the recent measurements of Drubka and Nagib,[8] and we therefore expect three-dimensionality to set in quite early in the near field. Consequently, the strength of the axisymmetric component (Sec. IV) in quieter jets should, if anything, be stronger than that revealed by the present measurements in the first one or two stations. Second, although the initial conditions have a profound influence on the details in the near-field evolution, their influence on the far-field development cannot be equally important, and so the shortcoming mentioned earlier does not necessarily extend to our far-field measurements.

III. MEASUREMENTS

The two-point azimuthal correlation coefficient of the streamwise velocity fluctuation u is defined as

$$R_{uu}(x/D;r/D;\phi;\phi_0)\equiv\frac{\overline{u(\phi_0)\,u(\phi_0+\phi)}}{[\,\overline{u^2(\phi_0)}\,]^{1/2}\,[\,\overline{u^2(\phi_0+\phi)}\,]^{1/2}}, \qquad (1)$$

where x is the streamwise distance measured from the exit of the nozzle, r is the radial distance measured from the jet axis, D is the nozzle diameter, ϕ_0 is the (arbitrary) azimuthal coordinate of the fixed wire, and ϕ is the azimuthal separation of the fixed and moving wires; both ϕ and ϕ_0 are located along the circumference of a circle with the desired radius. An overbar indicates a time average. A similar definition for the azimuthal correlation function $R_{\theta\theta}$ of the temperature fluctuation involves the replacement in Eq. (1) of u by the temperature fluctuation θ. The schematic arrangement for the measurements is shown in Fig. 1(b). It was verified that R_{uu} and $R_{\theta\theta}$ were independent of ϕ_0 (which will therefore be conveniently assumed to be zero henceforth), indicating that the experimental apparatus and the flow were satisfactorily symmetric for the present purposes. If the motion of the moving probe is constrained to a true circle, the average symmetry of the flow requires that

$$\overline{u^2(0)} = \overline{u^2(\phi)} \quad \text{and} \quad \overline{\theta^2(0)} = \overline{\theta^2(\phi)} \quad \text{for all } \phi. \qquad (2)$$

The fluid-dynamic center of the jet was determined separately by making mean velocity measurements in two orthogonal planes, and iterating on the most likely center location so determined, in such a way that Eq. (1) was satisfied as best as possible. Although considerable effort was spent in the process, we could not always make sure that Eq. (2) was satisfied to a better accuracy than about 10%; for better accuracy, it appears that a different combination of the jet and hot-wire sizes would be required. However, we believe that the correlation coefficients themselves may be reliable to a better accuracy.

A close examination of the hot-wire signals at $x/D = 1$ (the axial position closest to the nozzle for which data are presented) showed that it was well past the initial linear stage of development of the shear layer, or the stage of "surface ripples" observed by Crow and Champagne.[1] The asymptotic stage of development, namely the rollup of the vortex sheet in the form of a ring,[1–5] had apparently already occurred: detailed two-point correlations with radial separation, made for various positions of the fixed wire across the jet, were consistent with the expectation of strong vortex-like patterns at $x/D = 1$. However, by the 1000-momentum-thickness criterion either of Bradshaw[12] or Yule,[5] the flow there could not be considered a fully developed mixing layer. Figure 2 is a plot of $R_{uu}(\phi)$ at $x/D = 1$ for the three radial positions shown in the inset ($y = 0$ corresponds to $U/U_j = 0.5$); $y/D = 0.17$ corresponds to a radial position along the outer edge of the mixing region while $y/D = -0.11$ corresponds to that along the inner edge. Along both circles, the turbulence intensity is relatively low. The third position, $y/D = 0.02$ corresponds to a radial position of largest turbulence activity. It is clear that there is substantial correlation even at $\phi = 180°$ for all three radial positions, even though the actual magnitude depends on the precise radial position. We believe that the reason that R_{uu} along $y/D = 0.02$ is relatively lower for large ϕ is the large activity of small-scale turbulence there.

Apart from small differences in magnitude, the one important difference between $R_{uu}(\phi)$ for $y/D = 0.17$ and that for $y/D = -0.11$ is the appearance of waviness in the former. This waviness was repeatable from one day to another, although differences in the precise location of peaks were often observed. We (tentatively) believe that this waviness is due to the existence of azimuthal waves (see later and Sec. IV for further discussion); that this must be seen at all, in spite of the inevitable circumferential jitter that the vortex rings can be expected to have, may seem surprising, but we note that the correlation between two cosine waves with a random phase jitter between them does have a coherent part. The absence of any discernible waviness for $y/D = -0.11$ seems consistent with the view of Yule[5] that the inner side of these large eddies possesses a degree of temporal and spatial coherence even when, on the outside, they may begin to show some disorganization (of which the development of azimuthal waviness appears to be the first stage).

Figure 3 shows similar azimuthal correlations for the temperature fluctuation θ. The radial positions are identical to those of Fig. 2 for R_{uu}. Several interesting differences between Figs. 2 and 3 should be noted. Along an outside circle corresponding to $y/D = 0.17$, the waviness observed in the velocity fluctuation is not evident in the temperature correlations; the near-constant value (for $\theta \gtrsim 30°$) of the correlation coefficient is smaller for the temperature ($= 0.18$) than for the velocity. Second, along a circle corresponding to $y/D = 0.02$, where R_{uu} has a large drop for small ϕ and a subsequent overshoot, the temperature correlation $R_{\theta\theta}$ shows a smoothly varying behavior, with a value of around 0.35 for $\phi = 180°$. Along the inner circle $y/D = -0.11$, the temperature correlation is somewhat higher than that observed for the velocity fluctuation.

An obvious conclusion that follows from Figs. 2 and 3 is the existence of a basic component of motion with a strong circumferential coherence in the mean. That the temperature correlation is smaller along $y/D = 0.17$ than the corresponding velocity correlations indicates that a considerable part of the coherence observed there is due to the potential fluctuations. A similar inference was also drawn by Wygnanski and Fiedler[24] from their autocorrelation measurements. The somewhat higher temperature correlation than the corresponding velocity correlations towards the inside is indicative of a better circumferential organization of the motion within the shear layer than that in the inner potential region. Along a circle corresponding to $y/D = 0.02$, the rapid falloff for small ϕ of R_{uu}, but not $R_{\theta\theta}$, shall not be surprising if one remembers that this position corresponds to the maximum activity in velocity but not in temperature.

Further probing of the structure was made by measuring the autocorrelation function of u at several points across

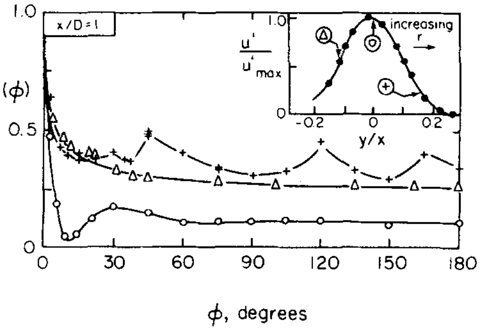


FIG. 2. Azimuthal correlation functions of velocity fluctuation for three radial positions at $x/D = 1$. The inset defines the radial positions.

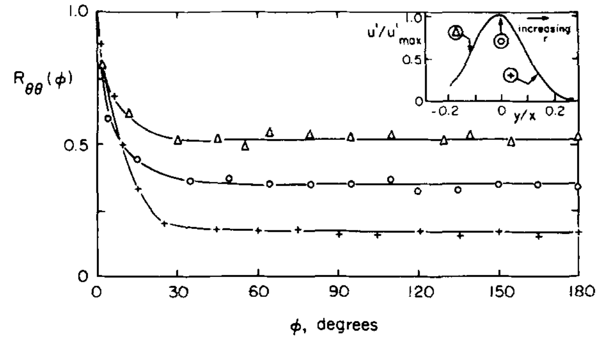


FIG. 3. Azimuthal correlation functions of temperature fluctuation for the same three radial positions as in Fig. 1. $x/D = 1$.

the shear layer; these measurements were quite detailed at $x/D = 0.5$, 1.0, 1.5, and 2, although cursory elsewhere. For brevity, however, only three representative curves for velocity fluctuations at $x/D = 1$ and 2 are given in Figs. 4(a) and 4(b), roughly corresponding to the same radial positions (i.e., circles lying along same y/x) examined in Fig. 2. Concentrating now on the region close to the potential core (typically curves with triangles in Figs. 2 and 4), we observe that the autocorrelation functions show substantial periodicity; azimuthal velocity correlations along this circle do not show any waviness at $x/D = 1$, suggesting that the vortex structures there retain their azimuthal coherence without the superposed unstable modes. Further, the Strouhal number corresponding to the second peak in the autocorrelation is in general a function of the radial position. This has been noted before,[6] and some modeling on this basis has also been attempted.[3] Towards the outer periphery, the periodicity is not even apparent in the autocorrelation functions, but there is strong azimuthal correlation $R_{uu}(\phi)$ with some waviness. This suggests that the loss of phase coherence and the onset of azimuthal instability both occur sooner (in terms of x/D) towards the outer edge of the jet than towards the potential core; this is consistent with our earlier conclusion, and also with the conclusion of Yule.[5] (Flow visualization studies[6] have shown that this picture breaks down at much higher Reynolds numbers.) There is also a suggestion in the autocorrelation function of the existence of double structure in this region, especially at $x/D = 1$. It is interesting that although the fractional energy contained in the small-scale fluctuations is large [of the order of 60%–70%—this conclusion follows from the lowest curve in Fig. 4(a)], they do not succeed in either the annihilation or masking of the circumferential coherence of the basic large scale (see Fig. 3 and the curve with crosses in Fig. 2). In contrast to the outer or inner periphery, around $y/x = 0.02$ where the turbulence energy is about the largest, the autocorrelation suggests a continuous distribution of scales, and the azimuthal correlation $R_{uu}(\phi)$ there is weak for large ϕ.

The present measurements of the azimuthal correlation $R_{uu}(\phi)$ at other x/D are somewhat less detailed, and a summary of such measurements is given in Fig. 5 which shows R_{uu} for $\phi = \pi$ as a function of x/D. These measurements were made with the two probes located diametrically opposite on circles lying roughly in $y/x = 0.17$ and 0.02. Also plotted are data for $y = 0$ from Ref. 5 at a comparable Reynolds number. In general, the azimuthal correlation drops off fairly rapidly with increasing x, and is negligible for $x/D \geqslant 4$. At $x/D = 1$, the velocity correlation is fairly large on circles to either side of $y/x = 0.02$ which, as discussed earlier, suggests the presence of well-defined vortex structure. In contrast, at $x/D = 4$ (for example), there is no radial position in the flow for which the azimuthal correlations are substantial for large ϕ. An immediate conclusion is that the basic ring structure evident at $x/D = 1$ (for example) is either absent or occurs with a large small-scale activity at larger x, say $x/D \geqslant 3$ or 4. The present measurements cannot reveal which of the two possibilities indeed occurs; in fact, no agreement on this basic issue appears to exist in other studies of the jet (e.g., Refs. 2 and 5). At higher Reynolds numbers at least, it appears that the former possibility is indeed the case.[6]

With increasing x, the azimuthal correlation functions as well as the autocorrelation functions become less sensitive to the precise radial position; for the latter at least, this is to some extent clear from a comparison of Figs. 4(a) and 4(b). At $x/D = 5$, for example, there appear to be only minor quantitative differences at different radial positions, and the general shape tends towards that of the middle curve in Figs. 4(a) and 4(b). [We note that Tso *et al.*[25] measured two-point correlation functions indirectly related to the present azimuthal velocity correlations; their short-time averages do show some radial dependence, but the implications for $R_{uu}(\phi)$ are not clear.]

Figures 6 and 7 show the evolution of the azimuthal correlation functions for velocity and temperature, respectively, measured along circles lying on $y/x = 0.02$; the data cover the range $1 \leqslant x/D \leqslant 40$. The corresponding evolution

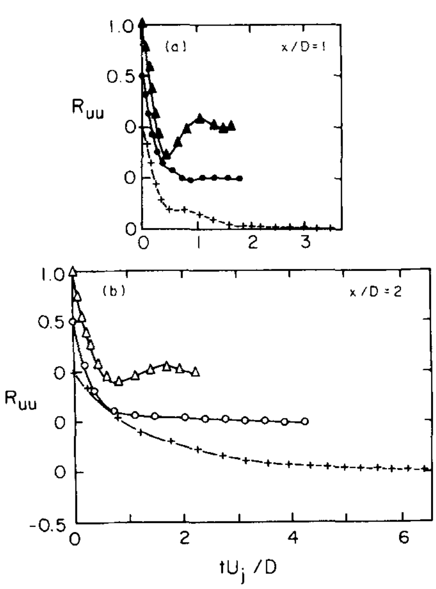


FIG. 4. Autocorrelation functions across the shear layer, (a) $x/D = 1$, and (b) $x/D = 2$. Data correspond approximately to the same radial positions as in Fig. 2. Symbols as in Fig. 2.

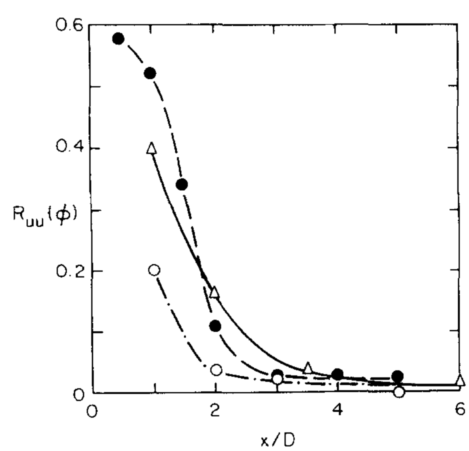


FIG. 5. Correlations of u at diametrically opposite sides of jet. ○, $y/x \simeq 0.02$; △, $y/x \simeq 0.17$; ●, from Ref. 4, $R_e = 3.5 \times 10^4$, $y/x \simeq 0$.

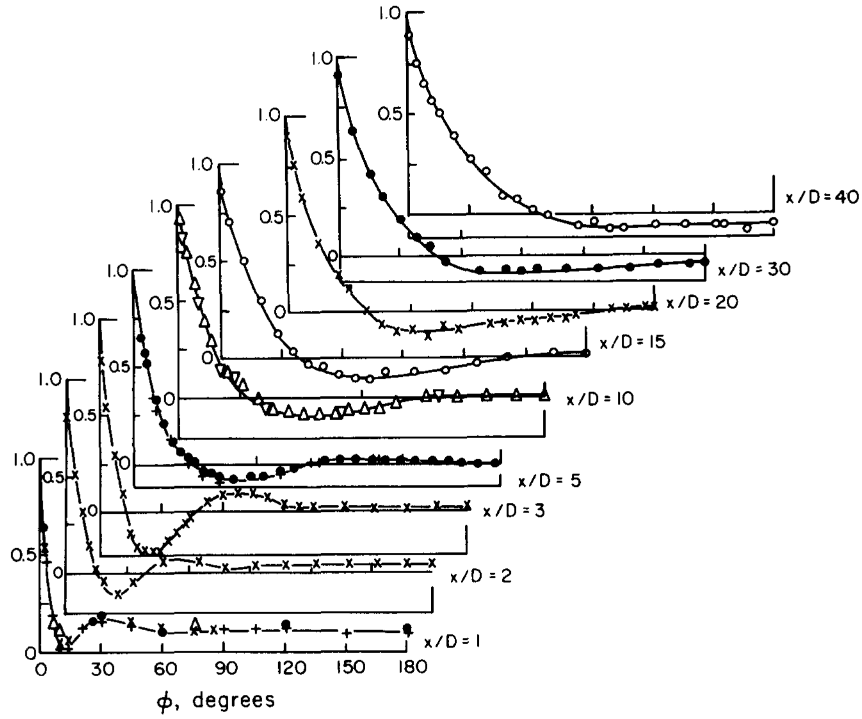


FIG. 6. Azimuthal correlation functions of the streamwise velocity fluctuation for $y/x \simeq 0.02$. From bottom to top: $x/D = 1, 2, 3, 4, 10, 15, 20, 30$, and 40. Different symbols for $x/D = 1$ and 5 are for different trials. $\triangle$ and ∇ at $x/D = 10$ indicate measurements for $0° < \phi < 180°$ and $180° < \phi < 360°$, respectively.

along other radii (say $y/x = 0.17$) can be inferred from Figs. 2 and 3 (and, for the velocity correlations, also Fig. 5) and the fact that the dependence on the radial position becomes weak for $x/D \geqslant 4$. Concentrating on the velocity correlations first, in the initial region (typified by $x/D = 1$ and 2), the correlation drops to low values for small separation distances, and goes substantially negative before reaching positive values again, somewhat like a lightly damped oscillator; the "damping" is the least in the range 3 to $3\frac{1}{2}D$. For $x/D \geqslant 5$, the second positive region is rather weak in magnitude, and disappears altogether for $x/D \geqslant 20$. In this region the range of ϕ in which the correlation function stays positive before the first zero crossing becomes increasingly larger with larger x/D. Typically, at $x/D = 40$, the correlation functions have reached their asymptotic shape in which $R_{uu} > 0$ for $\phi \leqslant \pi/3$, followed by an extended negative region where

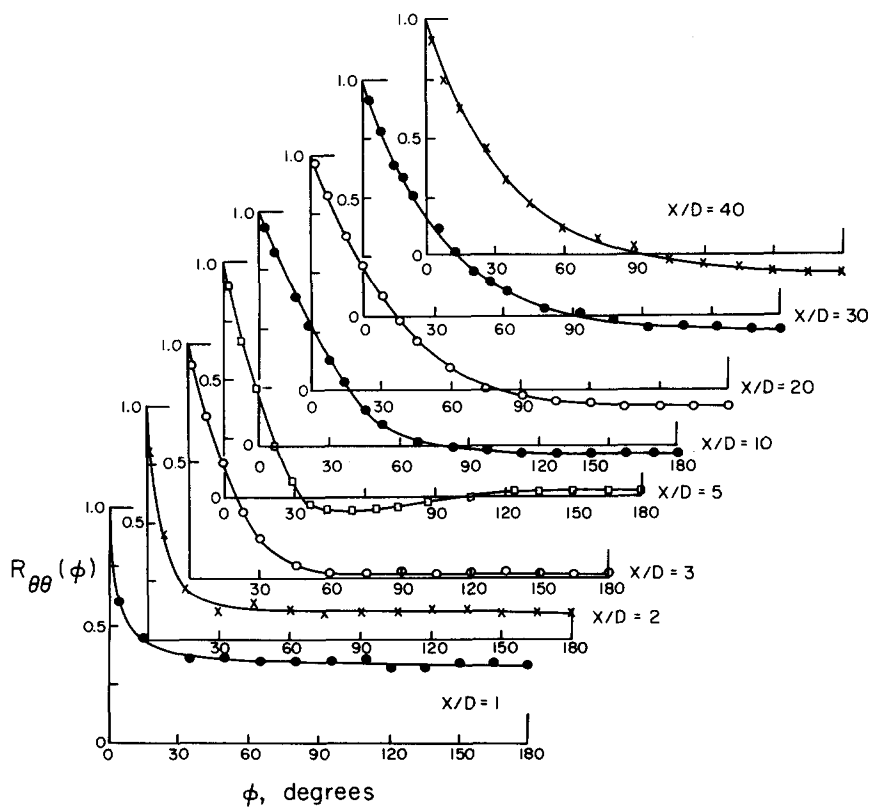


FIG. 7. Azimuthal correlation function of temperature fluctuation for $y/x \simeq 0.02$. From bottom to top: $x/D = 1, 2, 3, 5, 10, 20, 30$, and 40.

$R_{uu} \approx -0.05$. Some important characteristics of these curves are given in Table I.

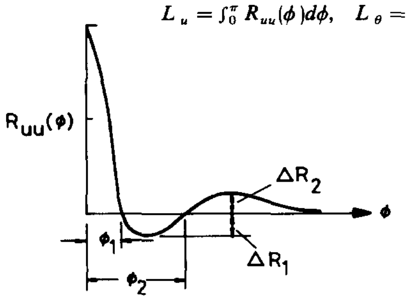
The temperature correlation function, on the other hand, shows less complex behavior. Initially, the correlation coefficient for large ϕ decreases with increasing x until, at $x/D = 3$, the correlation coefficient nearly vanishes for $\phi \geqslant 60°$. At $x/D = 5$, there is a slight negative loop followed by a region of slight positive value—essentially like the velocity correlations. Thereafter, the qualitative evolution of velocity and temperature correlation functions follows a similar pattern. (The observed differences in the two cases for $x/D \leqslant 5$, we believe, reflect the effect of potential fluctuations.) The last column in Table I shows the evaluation of the azimuthal temperature length scale for $y/x = 0.02$. These values are in some sense more representative of the large-scale coherence than the corresponding velocity length scale L_u.

It is useful to provide as reference a parallel description of the flow in terms of its other better-known features. To serve as a simple reference, the autocorrelation functions of u along the jet centerline are plotted in Fig. 8. In the region $x/D \leqslant 4$, there is a tendency towards periodicity: the Strouhal number S based on the second peak of the autocorrelation curves decreases with increasing x, and is generally attributed to vortex pairing in the initial mixing region. This decrease for the present jet does not occur in discrete steps as with an acoustically excited jet,[4] but decreases monotonically and smoothly as in unexcited jets.[8] The decrease in the Strouhal number becomes less significant with increasing x, and asymptotes to about 0.4, generally consistent with the previous data (e.g., Ref. 1, $S = 0.3$; Ref. 26, $S = 0.35$; Ref. 27, $S = 0.48$; Ref. 2, $S = 0.5$; Ref. 16, $S = 0.35$; Ref 5, $S = 0.3 - 0.35$; Ref. 7, $S = 0.4$; Ref. 8, $S = 0.42 - 0.48$, etc.). It is known that most of the vortex pairing is essentially complete (at these moderate Reynolds numbers) by $x/D \geqslant 3$ or 3.5, which is also the region beyond which very little variation occurs in S. In contrast to this initial region, no periodicity can be recognized for $x/D \geqslant 5$; this is also the region with little or no potential core.

TABLE I. Some characteristics of the azimuthal correlations.

X/D	ϕ_1 (deg)	ϕ_2 (deg)	ΔR_1	ΔR_2	$\Delta R_2 - \Delta R_1$	L_u	L_θ
1	···	···	0.02	0.17	0.15	24°	68°
2	16	37	−0.11	0.08	0.19	11°	29°
3	14	48	−0.22	0.09	0.31	5°	20°
3.5	13	60	−0.22	0.08	0.30	1°	···
5	28	87	−0.10	0.02	0.12	5°	14°
10	36	120	−0.08	0.02	0.10	8°	22°
15	38	150	−0.10	0.05	0.15	7°	···
20	40	160	−0.10	0.05	0.16	7°	22°
30	50	···	−0.08	···	···	8°	23°
40	65	···	−0.08	···	···	10°	24°

$L_u = \int_0^\pi R_{uu}(\phi)d\phi$, $L_\theta = \int_0^\pi R_{\theta\theta}(\phi)d\phi$.



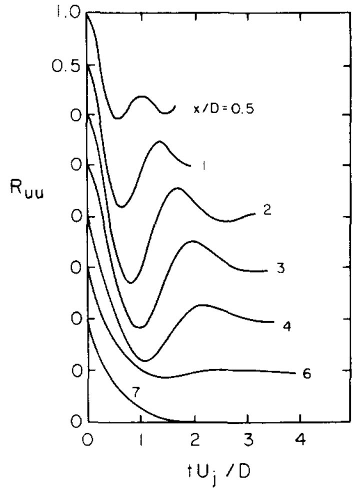


FIG. 8. Autocorrelation functions along the jet centerline. From top to bottom: $x/D = 0.5$, 1,2,3,4,6, and 7.

In summary, we may state the following.

(a) There is significant azimuthal coherence in the near field, but it is reduced significantly for a radial position corresponding to the largest turbulence activity (Fig. 2). This strong correlation at large azimuthal separation does not occur for $x/D \geqslant 2$ (Fig. 5).

(b) The azimuthally coherent structures appear to be more stable towards their inside edges than their outside edges (Fig. 2).

(c) A part of the observed nonzero correlation at large azimuthal separations is likely to be due to the induced potential fluctuations (Figs. 2 and 3).

(d) The evolution of the shape of the azimuthal velocity correlation functions with x/D does not follow a monotonic pattern, with the change in trend observed around the end of the potential core (Fig. 6).

(e) This last behavior is different for the temperature fluctuations (Fig. 7), suggesting that the nonmonotonicity mentioned in (d) above is possibly due to the potential fluctuations.

IV. ANALYSIS

Consider the decomposition of the measured azimuthal correlation functions into Fourier modes as

$$R_{uu}(\phi) = \frac{\alpha_0}{2} + \sum_n \alpha_n \cos n\phi,$$
$$R_{\theta\theta} = \frac{\beta_0}{2} + \sum_n \beta_n \cos n\phi, \tag{3}$$

for $n = 1, 2, ...,$ where

$$\alpha_n = \frac{2}{\pi}\int_0^\pi R_{uu}(\phi)\cos n\phi\, d\phi, \quad n = 0, 1, 2,...,$$

and β_n is similarly defined for the temperature correlations.

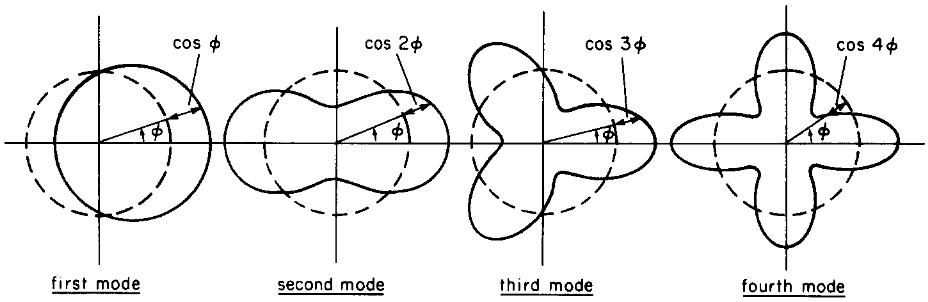


FIG. 9. A simple interpretation of the first, second, third, and fourth Fourier modes in the expansion (3).

The zeroth mode $\alpha_0/2$ is evidently the strength of the vortex ring with perfect circumferential coherence, and the α_n's ($n = 1,2,...$) represent the strength of the higher modes. The physical significance of these higher modes can be understood with reference to Fig. 9. (The significance of the temperature modes is less direct, but essentially similar.) The first mode represents the translation of the ring vortex without any substantial deformation; the higher-order modes represent various deformation geometries without substantial translation. For example, the second mode represents compression of the ring vortex in one direction, and stretching in the perpendicular direction.

Figures 10(a) and 10(b) give plots of the magnitude of α_n and β_n vs n for $x/D = 1$ (typical of the initial mixing region) and $x/D = 40$ (typical of the fully developed jet); for the velocity correlations, results are given at $x/D = 1$ for both $y/x = 0.02$ and 0.17. It is clear that the axisymmetric ring mode is quite dominant at $x/D = 1$, as seen from velocity data for $y/x = 0.17$ and the temperature data for $y/x = 0.02$. The velocity coefficients for $y/x = 0.02$ are quite different from the corresponding temperature coefficients, or those at $y/x = 0.17$ for the velocity correlations. This is not surprising in view of the fact (see earlier discussion in Sec. III) that the dominating activity of the small-scale turbulent velocity succeeds in masking the azimuthal velocity correlation for large ϕ. For $y/x = 0.17$, the next mode with the largest eigenvalue for the velocity correlation function is the sixth, suggesting a possible (although not very strong) preference for a six-lobe configuration. The detailed stability analysis leading to multiple-lobe formation has been made for isolated vortex rings.[28] This analysis shows that the number of lobes formed is a function of the circulation and the ratio of the radius of the vortex core to that of the ring. Although neither of these quantities can be estimated with any confidence for the present jet, the presence of a six-lobed structure in the present jet is not inconsistent with the Widnall–Sullivan result. The situation is somewhat similar also for $y/x = 0.02$ and $x/D = 1$, except that the zeroth mode is

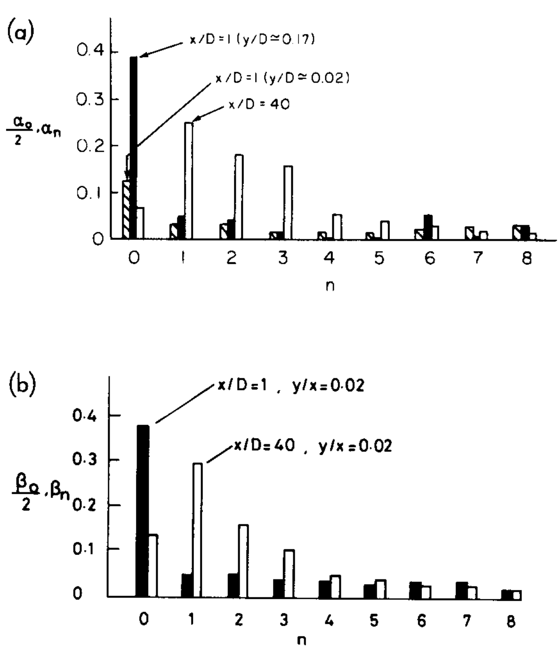


FIG. 10. The Fourier coefficients (a) α_n and (b) β_n vs n. In (a), hatched columns correspond to $x/D = 1, y/D \simeq 0.02$, filled columns to $x/D = 1, y/D \simeq 0.17$, unfilled columns to $x/D \simeq 40$, $y/x \simeq 0.17$. In (b), filled columns correspond to $x/D = 1$ and unfilled columns to $x/D = 40$, both at radial positions $y/x \simeq 0.02$. Note that for $x/D = 40$, the azimuthal correlations are not too sensitive to the precise radial position.

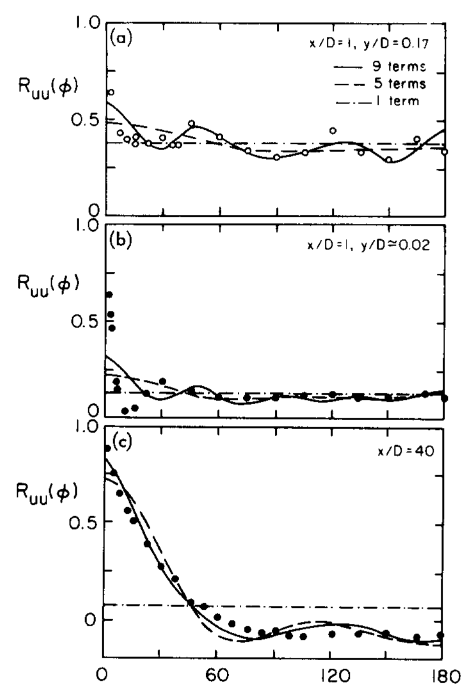


FIG. 11. Representation of the measured velocity azimuthal correlation functions by the first few terms in the expansion (2). (a) $x/D = 1, y/D \simeq 0.17$; (b) $x/D = 1$, $y/D \simeq 0.02$; and (c) $x/D = 40$. ———·———, one term; — — —, five terms; and ———, nine terms in (2).

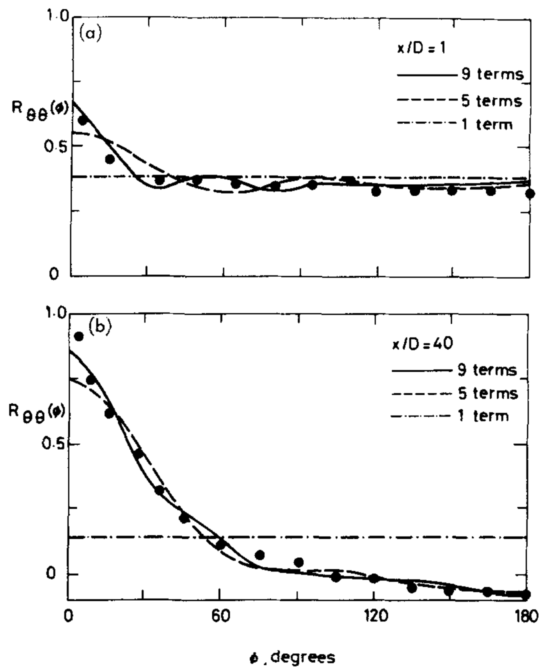


FIG. 12. Representation of the measured temperature azimuthal correlation functions by the first few terms in (2). (a) $x/D = 1$, $y/D \simeq 0.02$; and (b) $x/D = 40$, $y/x \simeq 0.02$. Other symbols as in Fig. 11.

much weaker. For the velocity correlations at $x/D = 40$, on the other hand, the most dominant modes (in that order) appear to be 1, 2, and 3 (the contribution from the axisymmetric ring mode being perhaps the next in importance); the temperature correlations are essentially similar, with perhaps a stronger contribution from the zeroth mode.

In Figs. 11 and 12, we examine the usefulness of representing the measured correlation by the first few terms in Eq. (3). At $x/D = 1$, the first term is not an unrealistic representation of both the velocity ($y/D = 0.02$ and 0.17) and temperature correlation functions for $\phi \geqslant 20°$; the subsequent inclusion of higher modes does not produce commensurate improvement especially for $y/D = 0.02$. It is clear that a true representation for small ϕ requires inclusion in some manner of the small structure. In contrast, at $x/D = 40$, the first term does not represent the true correlation function for large ϕ, but the inclusion of five or nine terms in Eq. (3) represents the overall shape reasonably well.

V. CONCLUSIONS

Many studies exist of the structure of turbulence in the near field of an axisymmetric jet, but very few (with the exception of Refs. 8 and 20) have exploited the fact the azimuthal correlation may serve as a simple quantifier of the large-scale circumferential coherence. The measurements reported here are a first step in a study originally intended to encompass both conditional measurements and high Reynolds number flows. These measurements are useful not only in the calculation of the noise production in the near field, but also (more directly and immediately) in turbulent flow calculations, and serve to complement the two-point correlation data of Ref. 25. In any calculation method of the axisymmetric jet flow, information is needed on the length scale of turbulence. Since the turbulence is highly anisotropic, length scales in different directions will be different. While the streamwise and radial length scale measurements have been made quite often (e.g., Refs. 24 and 29), the azimuthal length scale measurements are quite rare.

For a better understanding of the structure of turbulence, conditional measurements of the similar type would no doubt be more useful. But even the conventional averages are quite helpful in showing the presence of the basic ring-like structure; they also show that where the small-scale turbulence activity is large such as in the central region of the shear layer, this circumferential coherence can, however, be masked to a deceiving degree. To either side of this central region, the azimuthal correlation for separation angles greater than about 20° is entirely due to this circumferentially coherent activity. To describe the measured azimuthal correlation function for smaller separation angles, it is, however, not enough to add only the next few terms in the expansion (3) (of which the six-lobed mode appears to be next in importance to the zeroth mode), but an explicit account needs to be taken of the small structure of turbulence which has an overriding effect on the shape of the correlation functions for $\phi \leqslant 20°$.

In the far field, however, the first few modes [shown in Figs. 10(a) and 10(b)] have grown to become quite dominant. Although the zeroth mode is a poor representation of the measured azimuthal correlation function for any ϕ, that constructed from only the first five of the Fourier modes can describe the measured shape generally well. This type of Fourier decomposition thus seems to be a useful approach for the description of the large-scale features of turbulence.

On the basis of a similar analysis of the jet engine pressure field, Fuchs and Michel[30] remark that these lower-order modes are very important in aerodynamic noise generation and call to question some of the "accepted assumptions in jet noise research," although their conclusions have been doubted by Bonnet and Fisher.[31] These latter authors show that a highly correlated azimuthal pressure field is not necessarily an indication of a highly coherent source field, especially when the Helmholtz number (defined as fd/c, where f is the frequency, d is the diameter of the ring source, and c is the speed of sound) is less than about 0.5. Some of the recent work summarized by Hussain[32] appears to point to a similar conclusion for the turbulent velocity field as well, at any rate at high Reynolds numbers.

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