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An instability associated with a sudden expansion in a pipe flow

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An instability characteristic of a fully developed laminar flow encountering a sudden expansion in a circular pipe is briefly described.

Consider a sudden expansion in a circular pipe shown in Fig. 1. A hot wire located on the centerline some distance downstream of the sudden expansion will register, beyond a threshold value of the Reynolds number, oscillations of the type shown in Fig. 2. The regularity of these oscillations is so remarkable, and their general repeatability so good, that a brief exploration of the phenomenon seemed worthwhile. This letter is a short report of a preliminary effort.

In our initial setup, oscillations which would appear at a threshold Reynolds number R_1 (based on the upstream section average velocity $\langle U \rangle$ and the upstream pipe diameter d_1) of about 1500 would disappear completely when R_1 exceeded a value of about 1700. Also a 0.24 mm diam needle inserted along a diameter through a hole slightly upstream of the expansion would destroy the oscillations everywhere in the pipe; removing the wire and resealing the hole with scotch tape (for example) would restore them exactly. On the other hand, a slightly thinner wire (0.17 mm diam) would not at all affect the occurrence of the oscillations.

It is soon realized that the 0.24 mm needle was of sufficiently large diameter (Reynolds number based on the maximum velocity in the upstream pipe and the wire diameter $\sim$48) to shed Kármán–Bénard vortices which could indeed be observed. These vortices were probably of sufficiently large magnitude to prevent the oscillations (for reasons to be explained below) from occurring. The 0.17 mm wire shed no vortices—the wire Reynolds number of 35 being lower than the critical value of about 40 (Ref. 1)—and would leave the oscillations quite intact. In fact, we found that fairly low levels of turbulence created at the expansion would disrupt the oscillations totally. This immediately suggested to us that the disappearance at $R_1 = 1700$ of these oscillations had to do with the upstream disturbances whose residue at the expansion remained sufficiently strong for destroying the oscillations mentioned earlier. We then built a new pipe of the same nominal dimensions but with more carefully designed inlet conditions having a significantly lower disturbance level. For this setup, the oscillations at a certain axial location appeared at around $R_1 = 1500$ as before, but persisted in varying forms up to at least twice that value.

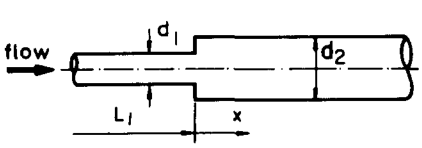

FIG. 1. The experimental configuration.

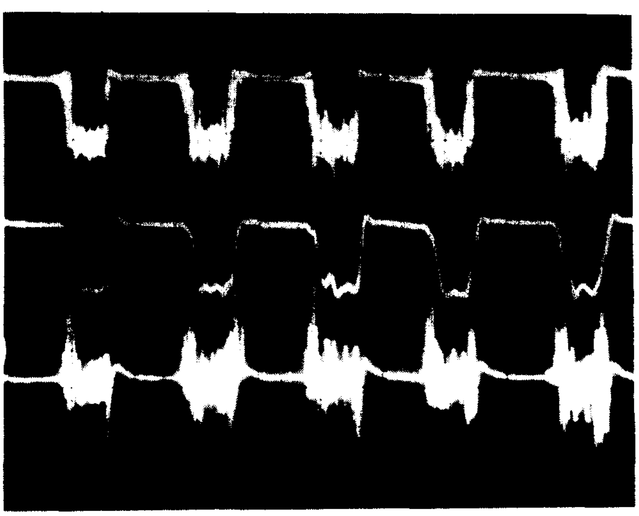

FIG. 2. Oscillations seen by a hot wire located on the pipe axis at $x/d_2 = 11$. Corresponding experimental conditions are $R_1 = \langle U \rangle d_1/\nu = 1500$, $d_1 = 0.635$ cm, $d_2 = 1.27$ cm, and $L_1 = 425\, d_1$. The uppermost trace is the unfiltered signal, the midtrace is low-pass filtered below 10 Hz, the lowermost trace being high-pass filtered above 10 Hz. Most of the fluctuations seen in the last trace are below about 500 Hz. Time scale: from left to right of figure, 4.8 sec.

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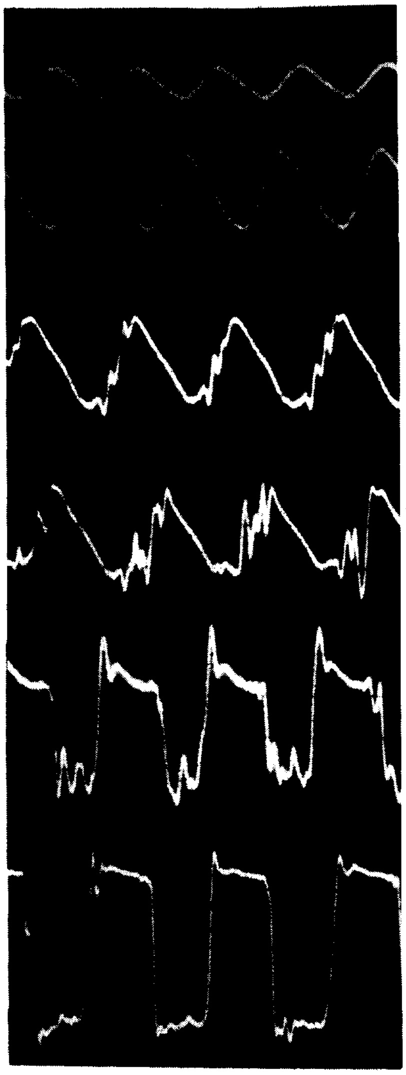

FIG. 3. Development and growth of the oscillations along the pipe axis downstream of expansion. From top to bottom, the oscillograms correspond to $x/d_2 = 5.8$, 7, 8, 9, 10, and 22, respectively. Signals low-pass filtered to 10 Hz. Unfiltered signals at x/d_2 of 5.8 and 7 are no different from the filtered ones; at other x/d_2, however, the signals do develop an increasingly higher frequency content. Time scale: from left to right of figure, 4 sec.

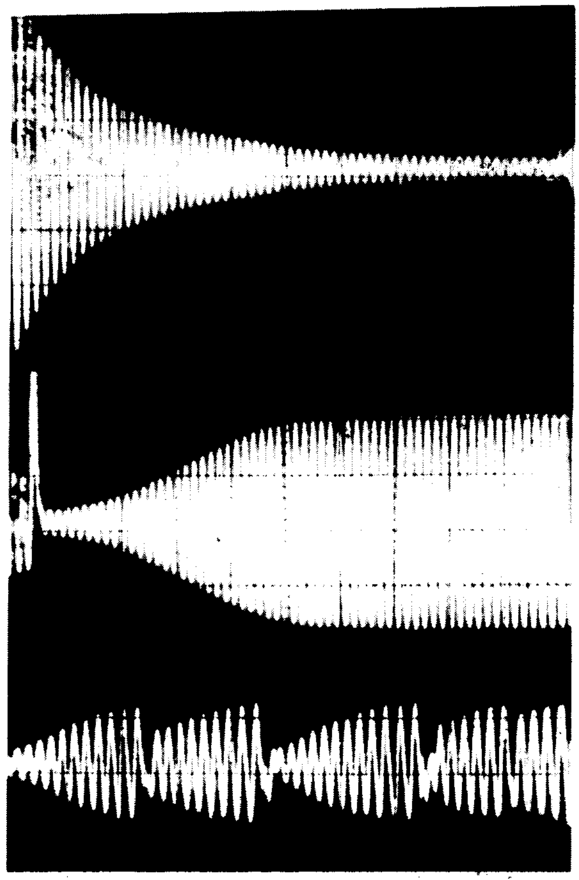

FIG. 4. Artificially triggered oscillations at $x/d_2 = 4.5$. In (a) the oscillations eventually decay, while in (b), they build up to a self-sustained state. In (c), they grow initially to a saturation amplitude and then decay abruptly before growing again. The cycle repeats indefinitely. Time scale: from left to right of figure, 50 sec.

Figure 3 is a record of the development and growth of the oscillations along the pipe axis. It is immediately clear that they are not the result of oscillations in mass flux (for, if they were, they should be seen with nearly the same amplitude everywhere axially), but must be characteristic of an instability of the oncoming flow. For $x/d_2 \lesssim 4$, no natural oscillations are seen; they can however be excited artificially by giving, for example, an impulsive but small motion to the hot-wire probe. This is sufficient to trigger oscillations (arising probably from probe–flow interactions) which may either decay with time [Fig. 4(a)], or grow into self-sustained state [Fig. 4(b)] depending probably on the initial amplitude of the impulse and the precise location of the probe in the flow. In certain cases, the oscillations grow to a saturation amplitude, decay abruptly to smaller amplitude, and build up again [Fig. 4(c)].

To better determine the nature of these oscillations, we set up a simple flow visualization experiment in water. To eliminate the possibility that the dye-introducing device placed upstream of the expansion would produce enough disturbances to destroy the flow oscillations, we introduced the dye at the inlet to the smaller pipe itself upstream of the contraction (as in the original experiments of Reynolds[2]). The contraction (area ratio ≈ 150) would damp out the disturbances produced by the dye-injecting needle to sufficiently small magnitude so as not to be disruptive to the process that resulted in the oscillations in the first place.

The dye streak downstream of the expansion would remain straight and smooth for x/d_2 of the order of about 5, apparently unaffected by the expansion. Thereafter, it would develop rapidly growing oscillations [see Fig. 5(a) and compare with Fig. 3], and would abruptly break down at some point depending on the Reynolds number; when this breakdown occurred, the dye filled the entire pipe crosssection downstream, suggesting that the breakdown and the reattachment of the oncoming flow occur essentially simultaneously. Just as abruptly, however, the reattachment "point" would move back, only to return to its original location, resulting in an essentially periodic back and forth oscil-

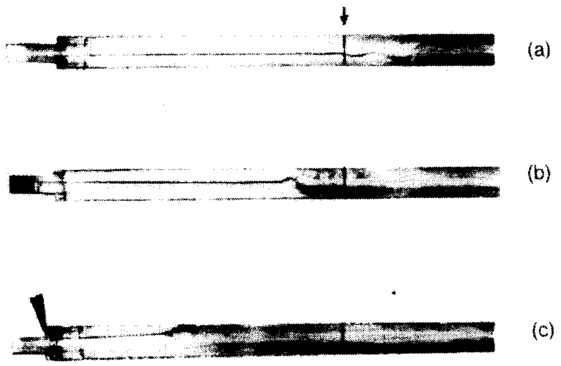

FIG. 5. Flow visualization results for $R_1 = 1600$. In (a), the breakdown of the oncoming dye streak occurs downstream of the mark indicated by the arrow, while in (b), this breakdown occurs upstream of the mark. In (c), it is seen that a needle placed just upstream of the expansion anchors the breakdown point.

lation along the pipe. Thus, if one concentrated at a fixed observation station along the pipe axis (such as the mark in Fig. 5), one would alternately see an unruffled dye streak or a situation in which the broken-up dye streak filled the entire cross section. An unruffled dye streak at the observation station implies a velocity there that is characteristic of the jet-like oncoming flow from the upstream smaller pipe whereas, once the reattachment occurred, the flow would fill up the entire pipe thus reducing the average velocity. This is essentially what makes a hot wire record (as in Fig. 2) two different levels of velocity with periodic alternation between them. In fact, we noticed that the upper level of velocity in the oscillations of Fig. 2 corresponded roughly to the centerline velocity in the smaller upstream pipe, while the lower one corresponded approximately to the average velocity that would result if the flow coming out of the smaller pipe filled the entire downstream pipe uniformly. Further, it may be seen (cf. the uppermost trace of Fig. 2) that the upper velocity level is essentially laminar-like, while the lower one is somewhat turbulent-looking, reflecting the fact that the lower level in the velocity oscillation of Fig. 2 represents a turbulent situation downstream of reattachment.

Why does the reattachment point oscillate back and forth so regularly? The answer probably lies in the complex interaction between the velocity field downstream of the expansion and the oscillatory pressure field further downstream. Presumably, the velocity distribution downstream of the expansion would be nearly parabolic in the core, but would be surrounded by a region of reverse flow. The resulting complex velocity distribution has several inflection points, and is obviously prone to instabilities which are quite possibly excited in phase by the downstream pressure field, thus providing the mechanism for the regularity of the oscillations. These instabilities grow and eventually lead to the breakdown of the flow at some point downstream. When this occurs, the turbulence that develops and the consequently increased pressure drop would shift the reattachment point upstream. One may surmise that this upstream shift of the reattachment point would restore the stability of the flow by altering its velocity distribution just enough, so that the reattachment point would move downstream to its original position; this self-perpetuating act repeats itself.

Inserting a small wire slightly upstream of the expansion [see Fig. 5(c), where the head of the needle can be seen], which in the air experiments had the effect of destroying the oscillations, always resulted in a premature breakdown and reattachment of the flow at around $x/d_2 = 4$. Disturbances because of the wire upstream, or any other artificially created disturbance, would hasten the breakdown by bypassing the normal oscillatory growth stage, and anchor so well the reattachment point at around $x/d_2 = 4$ that, upstream of this point, the flow would simply be a laminar "jet" of fluid coming from the upstream pipe. Here, a hot wire located along the pipe axis would continuously record very nearly the peak velocity $2\langle U \rangle$ in the upstream parabolic distribution; whereas downstream of this point, it would simply record continuously the lower velocity corresponding approximately to $(d_1/d_2)^2\langle U \rangle$.

Now we may note a few vagrancies of this flow. Under nominally identical circumstances, the velocity trace would sometimes deviate in shape from that shown in Fig. 2. For example, the time spent in any cycle in each of the two states discussed above could be unequal (i.e., the duty cycle of the signal of Fig. 2 would be different from 0.5); or, the velocity would not be constant in the upper and lower states but very gradually (see, for example, the lowest trace in Fig. 3). Sometimes, the small-scale oscillations superposed on the upper state (see the lowest trace of Fig. 3) would not be easily discerned. We found that small levels of turbulence or some asymmetric constraints imposed at the expansion would destroy the phenomenon or alter it to varying degrees. The extraordinary sensitivity of the phenomenon to these various details, and the narrow range of Reynolds number within which it seems to occur unless special care is taken, may well explain why it has not been noticed before. However, we believe that it is not an uncommon phenomenon altogether; for example, something similar could be occurring downstream of a sharp orifice in enclosed flow measuring devices.

Finally, we might mention the practical relevance of the sudden expansion configuration in the context of ram jets and dump combustors.

ACKNOWLEDGMENTS

We would like to thank Professor B. T. Chu and Professor M. V. Morkovin for their many useful comments. If all their many critical questions have not been answered here, it is because this study was a byproduct of another investigation on turbulence control.

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