Conditional Measurements in a Heated Axisymmetric Turbulent Mixing Layer

K. R. Sreenivasan* and R. A. Antonia†
University of Newcastle, Newcastle, N.S.W., Australia
and
S. E. Stephenson‡
University of Sydney, Sydney, N.S.W., Australia

Abstract

CONDITIONAL measurements of axial velocity u and temperature θ fluctuations are obtained in the mixing layer of a slightly heated round jet, which exhausts into still ambient air. The trimodal shape of the probability density function of temperature is used to distinguish between the turbulent and nonturbulent parts of the flow. A distinction is also made between the nonturbulent "cold" ambient fluid and the nonturbulent "hot" fluid of the potential core. Both velocity and temperature fluctuations associated with either "hot" or "cold" fluid are very nearly Gaussian. Fluid from the "hot" or "cold" side of the mixing layer is found on the opposite side of the mixing layer well beyond the central mixing region, consistent with the notion of bulk transport of nonturbulent fluid by a large-scale motion comparable to the width of the flow.

Contents

Measurements were made at two jet exit velocities U_j of 15.1 ms^{-1} and 4.8 ms^{-1}, which correspond to Reynolds numbers R_D based on the nozzle diamter ($D=48$ mm) of 4.6×10^4 and 1.4×10^4, respectively. The jet was heated to a temperature of about 13°C above the ambient temperature. The velocity fluctuation u was measured with a 5-μm-diam (platinum, 10% rhodium) hot wire operated at an overheat of 1.5 with a DISA 55M01 anemometer. For the measurement of the fluctuating temperature θ, a 0.6-μm-diam (platinum, 10% rhodium) cold wire of resistance approximately 550Ω (wire length $\simeq 0.8$ mm, thermal coefficient $\simeq 0.0014$°C^{-1}) was operated with a constant current anemometer at a current of 0.1 mA.

The rate of spread of the mixing layer is approximately twice as large as that obtained by Fiedler[1] for a two-dimensional mixing layer. As the present jet issued into the ambient air of the laboratory, it is suspected (an enlarged discussion is given in the backup paper[2]) that the large spread rate may be due to the relatively high turbulence level of the laboratory ambient air into which the jet issued. It must be noted, however, that mean velocity and temperature profiles reported in Ref. 2 are self-preserving and similar in shape to those obtained for the two-dimensional mixing layer.

A typical probability density of temperature in the central region of the mixing region [$\eta = y/(x-x_0) = 0.114$, where y is the radial distance whose origin is at the level of the nozzle lip, x is the axial distance measured from the nozzle lip, x_0 is the virtual origin] is shown in Fig. 1. The temperature signal (which is intermittent across the whole mixing layer) switches from a level corresponding to the "cold" state (still air temperature) to a level that corresponds to the "hot" state (core fluid temperature). Between these two levels, the signal may be identified with a turbulent mixed state. The corresponding probability density, therefore, shows three peaks with the left and right peaks corresponding to the "cold" and "hot" temperatures, respectively, while the central peak is associated with the turbulent state. The peaks corresponding to the hot and cold states should ideally be delta functions, but the "noise" of the electronics and that associated with freestream temperature fluctuations smear out these spikes by convolution. Bilger et al.[3] showed that these smeared out spikes may be closely fitted by Gaussian curves (see insets in Fig. 1). It was argued by Bilger et al.[1] that the areas under the Gaussian curves represent the fractions of time that the signal is either "hot" or "cold." The sum of the area under the "hot" and "cold" spikes is $1-\gamma$, where γ is the intermittency factor. Thresholds on the temperature signal can therefore be set (on both sides where necessary) such that the two areas on either side of it are equal (see insets in Fig. 1). This method determines not only the intermittency factor, but also the fractions of time the fluid is in "hot" and "cold" states. Usual conditional measurements do not distinguish between "hot" and "cold" nonturbulent states. The method has the further advantage that it is less subjective and easier to apply than that which relies on a visual setting of the threshold. However, it shares one disadvantage with all other existing methods. The probability density of temperature is terminated sharply near the hot and/or cold bounds of the temperature signal. This (artificial) abrupt cutoff may lead to possible distortion of statistics (especially high-order moments) of θ and also u.

The measured probability density distribution $p(\theta)$ for temperature can be written as

$$p(\theta) = p_c^*(\theta) + p_h^*(\theta) + p_t^*(\theta) \tag{1}$$

where $p_c^*(\theta)$, $p_h^*(\theta)$, and $P_t^*(\theta)$ are the contributions to the total density from the cold and hot nonturbulent zones and

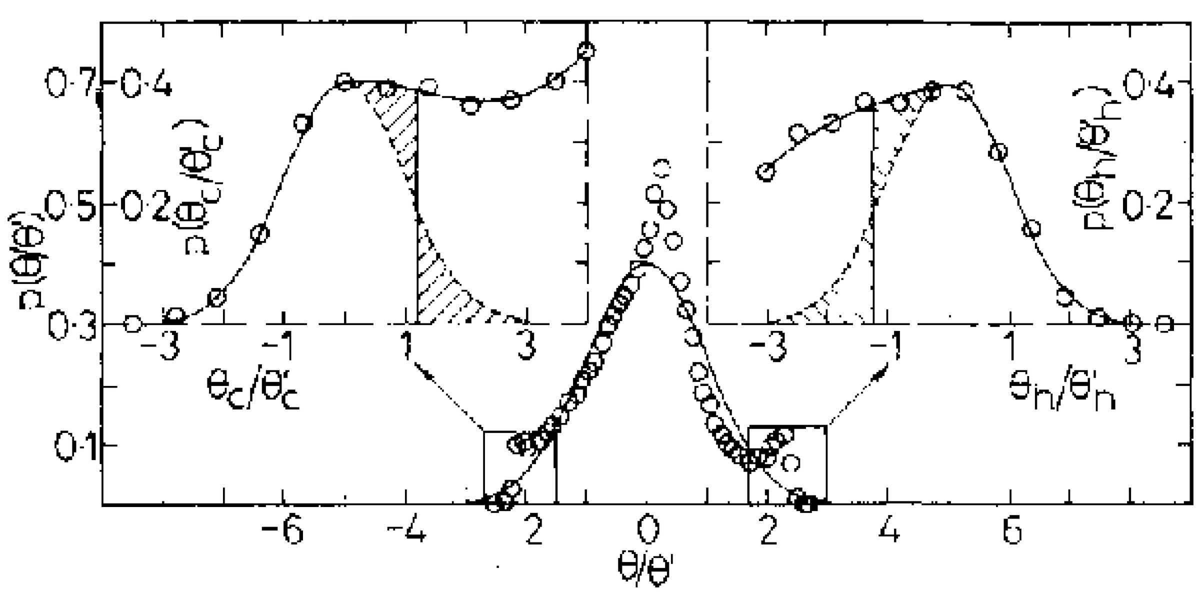

Fig. 1 Trimodal probability density of θ at $\eta = 0.114$.

Received July 25, 1977; synoptic received April 5, 1978; revision received May 17, 1978. Full paper available from National Technical Information Service, Springfield, Va., 22151, as N78-22391 at the standard price (available upon request).

Index categories: Boundary Layers and Convective Heat Transfer—Turbulent; Jets, Wakes, and Viscid-Inviscid Flow Interactions.

*Professional Officer, Dept. of Mechanical Engineering.
†Professor of Mechanical Engineering, Dept. of Mechanical Engineering.
‡Research Student, Dept. of Mechanical Engineering.

the turbulent zone, respectively. It is assumed that $p_c^*(\theta)$ and $p_h^*(\theta)$ are Gaussian in form so that,

$$p_c^*(\theta) = \frac{\alpha}{\sigma_c\sqrt{2\pi}} \exp\left(\frac{-(\theta-\bar{\theta}_c)^2}{2\sigma_c^2}\right)$$

and

$$p_h^*(\theta) = \frac{\beta}{\sigma_h\sqrt{2\pi}} \left(\frac{-(\theta-\bar{\theta}_h)^2}{2\sigma_h^2}\right)$$

where the parameters α, $\bar{\theta}_c$, σ_c, β, $\bar{\theta}_h$, and σ_h are determined each time $p(\theta)$ is analyzed. It follows that,

$$\int_{-\infty}^{\infty} p_c^*(\theta)\,d\theta = \alpha, \text{ the fraction of time temperature is in the cold region} \tag{2}$$

$$\int_{-\infty}^{\infty} p_h^*(\theta)\,d\theta = \beta, \text{ the fraction of time temperature is in the hot region} \tag{3}$$

$$\int_{-\infty}^{\infty} p_t^*(\theta)\,d\theta = \gamma, \text{ the fraction of time temperature is in the turbulent region} \tag{4}$$

and $\gamma = 1-\alpha-\beta$ since $\int p d\theta$ is, by definition, equal to unity. It also follows that,

$$\bar{\theta}_t = \gamma^{-1}(\bar{\theta} - \alpha\bar{\theta}_c - \beta\bar{\theta}_h) \tag{5}$$

All conditional moments of θ in the turbulent region of the flow can now be expressed in terms of zone averages,

$$M_{n,x} = \overline{(\theta-\bar{\theta}_x)_x^n}, \text{ with subscript } x = c,h, \text{ or } t \tag{6}$$

and of conventional averages,

$$M_n = \overline{(\theta-\bar{\theta})^n} \tag{7}$$

In general, the nth central moment $M_{n,t}$ may also be written as,

$$M_{n,t} = \frac{1}{\gamma} \sum_{r=0}^{n} \binom{n}{r} \{ (\bar{\theta}-\bar{\theta}_t)^{n-r} M_r - \alpha(\bar{\theta}_c - \bar{\theta}_t)^{n-r} M_{r,c} - \beta(\bar{\theta}_h - \bar{\theta}_t)^{n-r} M_{r,h} \} \tag{8}$$

Conditional higher moments in the turbulent zone can be found from Eq. (8) using known results for central moments of a normal distribution. Conditional probability density functions of u in the turbulent, hot, and cold states are shown in Fig. 2 at $\eta = 0.114$. As $\gamma = 0.95$ at this station, $p(u)$ and $p_t(u_t - \bar{u}_t)$ are nearly identical and are represented by the same curve. The probability density functions of $u_c - \bar{u}_c$ and $u_h - \bar{u}_h$ do not deviate significantly from the Gaussian function, but the accuracy of these results is somewhat poor, since cold and hot fluid are present for only 2% and 3%, respectively, of the time at $\eta = 0.114$. Conditional probability density functions (both marginal and joint) of u and θ at different values of η are given in Ref. 2.

The intermittency factor γ plotted in Fig. 3 was determined by the method of Bilger et al.[3] At both values of R_D, this γ is in very good agreement with that measured by applying a visually determined threshold on the temperature signal to generate the on-off intermittency signal and in reasonable agreement with Fiedler's[1] distribution. As the maximum value of γ is around 0.96, nonturbulent fluid is detected throughout the mixing layer. Especially at the maximum γ point, both hot and cold states exist, with nearly equal (though small) probability. This is consistent with the existence of a large-scale structure (e.g. Ref. 4) and the engulfment of nonturbulent fluid by this structure. It is also interesting to note that engulfment which takes place on both sides of the mixing layer is followed by transport of hot fluid beyond the central region of the mixing layer towards the cold side, and vice versa. The central peak in the probability density indicates, however, that a large part, but not all, of this engulfed fluid has been "digested." Figure 3 also indicates that α and β decrease to zero at a rate significantly slower than the error function.

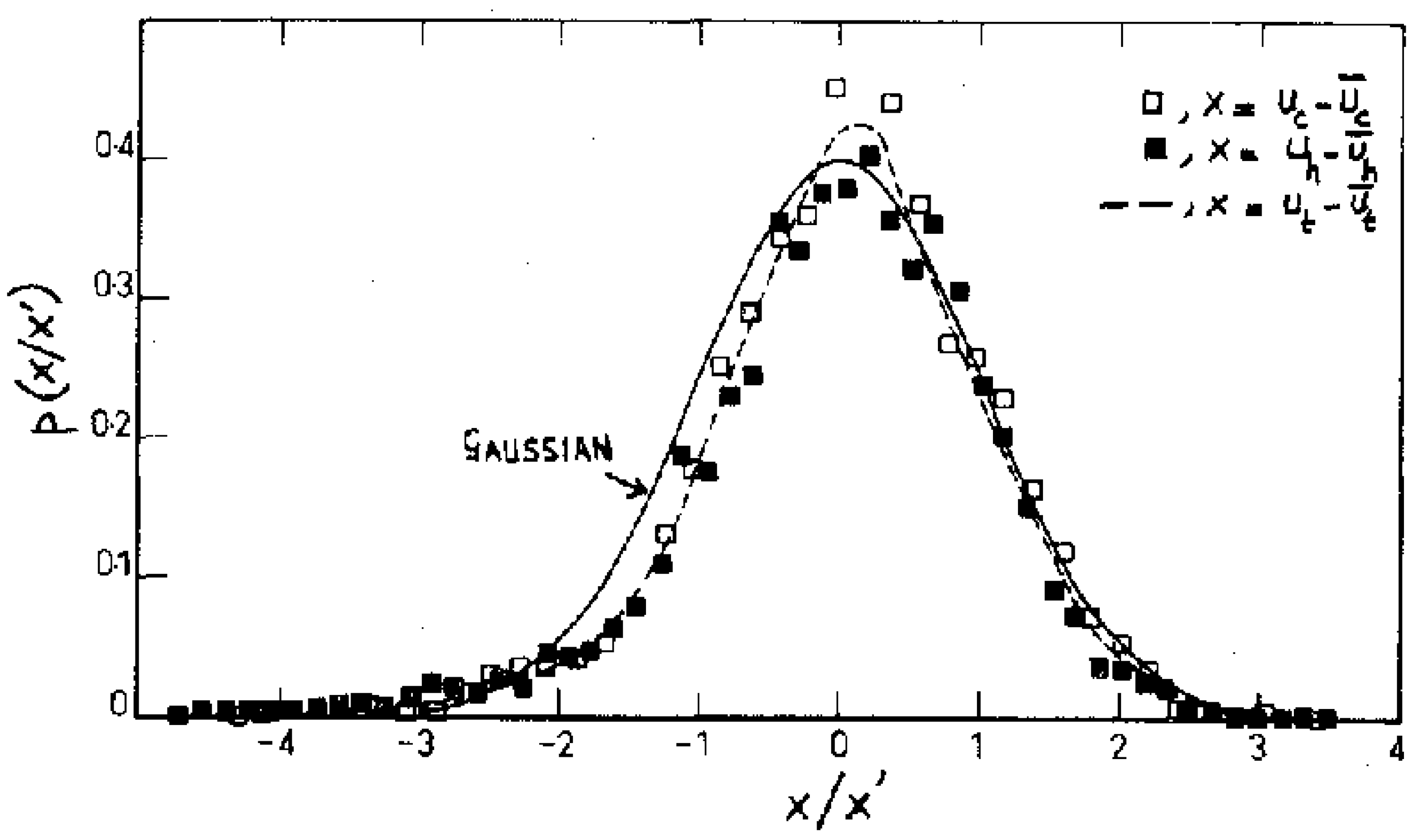

Fig. 2 Conventional and conditional probability density of u at $\eta = 0.114$.

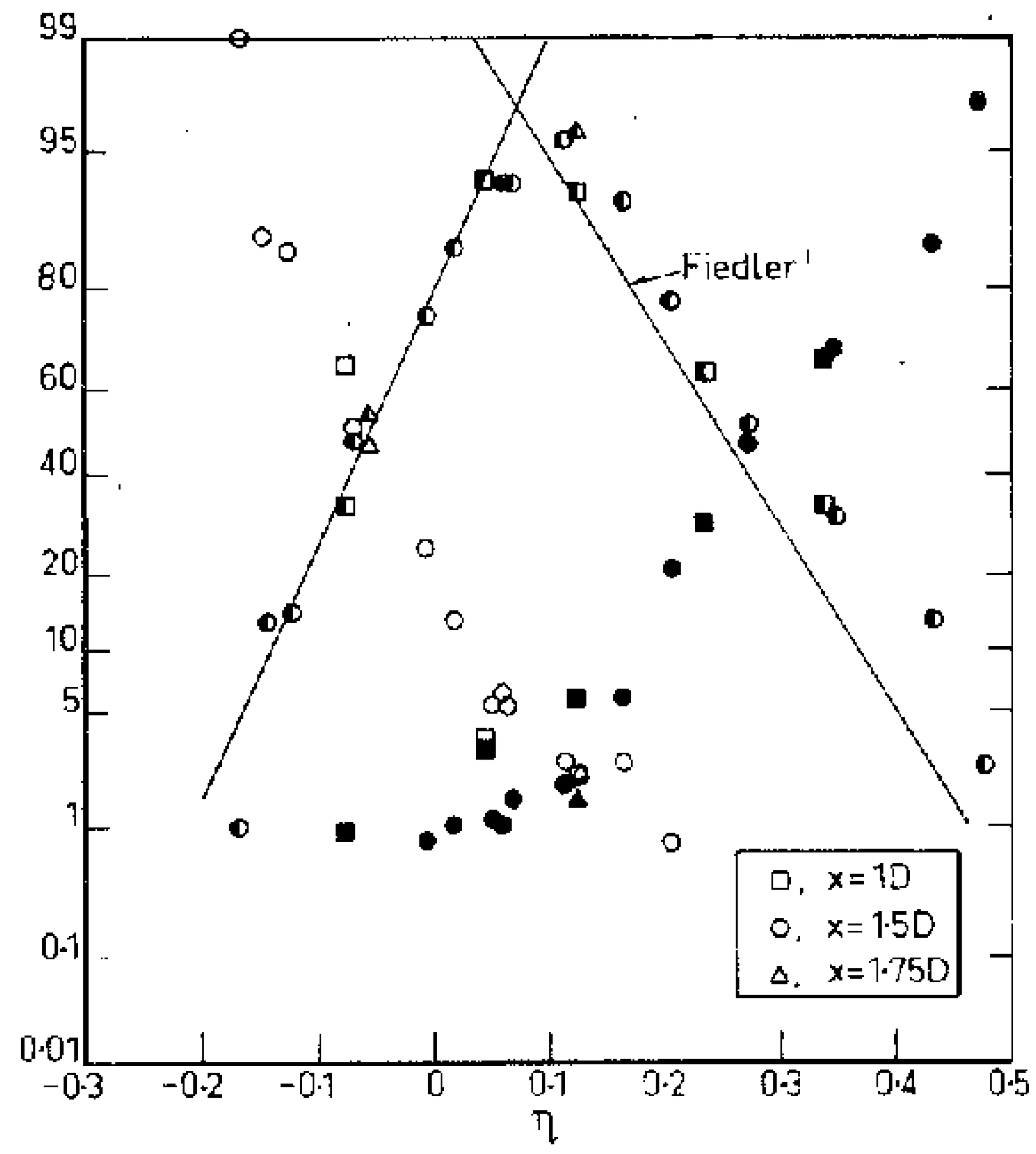

Fig. 3 Intermittency factors γ (half-filled symbols), β (open symbols), and α (filled symbols).

Acknowledgment

The support of the Australian Research Grants Committee is gratefully acknowledged.

References

[1] Fiedler, H. E., "Transport of Heat Across a Plane Turbulent Mixing Layer," *Advances in Geophysics*, Vol. 18A, 1974, pp. 93-109.

[2] Sreenivasan, K. R., Antonia, R. A., and Stephenson, S. E., "Conditional Measurements in a Heated Axisymmetric Mixing Layer," Dept. of Mechanical Engineering, University of Newcastle, T.N. F.M.5, Jan. 1977.

[3] Bilger, R. W., Antonia, R. A., and Sreenivasan, K. R., "The Determination of Intermittency from the Probability Density Function of a Passive Scalar," *Physics of Fluids*, Vol. 19, Oct. 1976, pp. 1471-1474.

[4] Winant, C. D. and Browand, F. K., "Vortex Pairing: The Mechanism of Turbulent Mixing-Layer Growth at Moderate Reynolds Number," *Journal of Fluid Mechanics*, Vol. 63, April 1974, pp. 237-255.