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Diffusion from a Heated Wall-Cylinder Immersed in a Turbulent Boundary Layer

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SUMMARY
Measurements of both mean and fluctuating temperature are presented in the 'intermediate' region within the diffusion layer downstream of a heated circular cylinder placed spanwise on a wind-tunnel floor. The cylinder is completely submerged in the inner layer of the oncoming turbulent boundary layer (ratio of cylinder diameter to boundary layer thickness is 0.036). Mean temperature, root-mean-square temperature and temperature intermittency exhibit an approximate similarity on the same characteristic scales. The characteristic temperature scale varies inversely with the distance, and the rate of growth of the layer is comparable to that of a constant pressure turbulent boundary layer.

1 INTRODUCTION
Turbulent diffusion of heat and matter is of considerable practical interest, since most sources are close to the surface, a study of diffusion in a turbulent boundary layer is of obvious importance. Porah and Cermak (1964) studied diffusion in a turbulent boundary layer of a steady line source of smoke, which was located on the floor level. Plate (1967) extended these measurements by introducing a line fence somewhat downstream of the contaminant source. In practice, however, sources of contaminants are also sources of roughness. It is therefore useful to examine the diffusion of a scalar quantity from a line source of roughness.

Porah and Cermak's results show that the region downstream of the source can be divided into an 'initial region' close to the source where presumably the geometry of the source and other similar effects dominate, a 'final' region far away from the source where the growth of the boundary layer dominates diffusion of the scalar quantity, and an 'intermediate' region is between where initial effects are sensibly absent and diffusion is not influenced by details of boundary layer growth, One can then expect this intermediate region to possess some kind of similarity independent both of the details of the source and of the oncoming boundary layer.

The present work concerns measurements of mean and fluctuating temperature essentially in the intermediate region, downstream of a heated circular cylinder resting on the floor. Of particular interest is the relation among the similarity exhibited among different quantities, the growth rate of the diffusion layer and the rate of decay of maximum temperature in the diffusion layer. It is estimated that at the farthest downstream measuring station, the thickness of the diffusion layer is about one third of that of the momentum boundary layer and is therefore still within the fully turbulent region.

2 EXPERIMENTAL CONDITIONS
A circular steel cylinder, 1.0 mm in diameter, was placed on the floor of a wind-tunnel spanning the entire width of the test section. It was heated to about 15°C above ambient by passing a current of approximately 30 A at approximately 15 V A.C. The arrangement is schematically shown in Figure 1.

Figure 1 Schematic of experimental arrangement

The tunnel was operated at a nominal free-stream velocity of 9.0 m/s. The boundary layer just upstream of the cylinder was fully turbulent and had a thickness of (99.5% of free-stream velocity) of 45 mm. The cylinder was fully submerged in the inner layer of the boundary layer.

All temperature measurements were made by 0.06-mm diameter Wollaston wires about 0.7 mm in length (resistance approximately 500 ohm). They were operated 'cold' at a constant current of 0.1 mA on a two-channel constant current oximeter described by Stellena et al. (1975). The instrument gives two outputs, one of which between 4.0 and a few
(1)\[ \text{op} = \text{in} \times 2 \left(1 + \text{p}^2\right) \]

For values of \( p \) between 1.1 and 1.6, Figure 2 shows that (1) is a reasonable
approximation to mean temperature profiles in spite, except at \( x/d = 8 \) and \( 149 \). Classically, \( \text{op} \approx \text{in} \), similarity is not achieved at \( x/d = 149 \) because the diffusion layer to comparable in thickness to the fully turbulent region of the boundary layer, it is possible that this similarity breaks down. Note that the data in measurements is maximum very close to the wall. Also plotted for comparison are the Gaussian and the mean temperature profiles appropriate to a self-preserving two-dimensional wake (Townend, 1956).

3.2 Root-mean-square Profiles

The temperature also shows a similarity (for \( x/d > 16 \)) when its maximum value \( T_{\text{rms}} \) and the distance \( x/d = 0 \). Two times \( x/d = 0 \) a frequency response curve which was flat up to 60 Hz. A dynamic range of 1.97 and a central signal/noise ratio of 60 dB. Root-mean-square (rms) values were read on a DSA 5502 E rms meter.

The recorded signals were later played back at a speed of 23.85 mm/s and, after suitably filtering the signals to avoid aliasing, processed on a HP1000 computer at the University of Sydney.

RESULTS

Measurements were confined to the region \( 6 < x/d < 150 \), giving the diameter of the cylinder. No corresponding velocity measurements were made.

1.1 Mean Temperature Profiles

Measurements show that the mean temperature is maximum at the wall and decreases monotonically towards the free-stream value. If the maximum temperature (above ambient) \( T_{\text{rms}} \) and the normal \( x/d = 16 \) from the wall where \( x/d = 8 \) occurs as shown in the temperature and length scales respectively, all measurements (except at \( x/d = 6 \)) exhibited a well-defined similarity (Figure 3).

It is interesting to note that the rms temperature takes longer to attain similarity than the mean temperature.

3.3 Probability Density

Figure 4 shows a typical probability density of temperature fluctuations at a point \( x/d = 63/3 \). The probability density function is defined as the fraction of time for which the flow is turbulent is 0.95.

4 DISCUSSION

4.1 Similarity

It can be argued that any bounded, monotonically decreasing function will exhibit an approximate similarity when normalised in the manner described in Section 3.1. A more sensitive check on the possibility of similarity is to calculate the moments of the temperature profile and examine their variation with \( x/d \). Here, we have computed only the quantities

\[ \mu_{n} = \int y^n \rho(y) dy \quad \text{and} \quad \mu_{n}^{*} = \int y^n \rho^{*}(y) dy \]

and \( \mu_n^{**} = \int \left(y - \mu_n^{**}\right)^n \rho^{**}(y) dy \).

4.2 Relation among different scales

It was shown previously that both mean and rms temperature profiles and the intermittency profile each exhibit similarity when non-dimensionalized by the appropriate scales. The relation among the different scales is shown on Figure 7. Here, \( T_{\text{rms}} \) and \( \rho_{\text{rms}} \) are respectively the position of the mean interface and its standard deviation.

A characteristic feature is the bimodal nature of the distribution, with a sharp peak centered roughly around the ambient temperature. Bilger et al. (1975) argued that the peak is associated with the 'breaks' in ambient conditions and electronic instruments, and showed that they can be fitted rather closely by a Gaussian whose area is equal to 1.7. These arguments should in principle be applicable to any passive contaminant in any intermittent shear flow. Indeed, probability density functions of a temperature measured in a heated wake (Laffue and Smith, 1974), heated jet (Antonia and Greenshields, 1970), and of concentration in mixing layers (Rothko, 1976) clearly show this behaviour in the intermittent region. However, temperature fluctuations in a constant heat flux boundary layer do not show this feature. A probability density function measured in such a boundary layer under conditions (such as Reynolds number, intermittency, signal/noise ratio) comparable to those in the present case, is also plotted in Figure 4.

Intermittency was obtained by two methods: the conventional method of setting an appropriate threshold and by the method of Bilger et al. (1975) mentioned above. The agreement between the two methods was found to be very good. Figure 5 shows that the distribution of \( y \) is nearly Gaussian.

The total enstrophy at a station, centre of gravity of the mean temperature profile and its standard deviation is shown in Figure 6 shows that, for \( x/d > 30 \), within the scatter of the data, all quantities are sensibly invariant with \( x/d \), which suggests that similarity is well established in this region. Also indicated in Figure 6 are the corresponding quantities evaluated using \( y_{\text{rms}}^{\text{enst}} \) for 1.7, 1.6 and 1.5. The agreement with measurement is reasonably good in this range of \( y_{\text{rms}}^{\text{enst}} \), although no single value is similarly good for all the three quantities. This figure is not surprising, as (1) does not fit the measurements uniformly well for a given value of \( y_{\text{rms}}^{\text{enst}} \). For example, some departures between measurement and (1) with \( y_{\text{rms}}^{\text{enst}} \) are noticeable in Figure 7 for \( y_{\text{rms}}^{\text{enst}} < 0.5 \). Note incidentally that a moment of given order \( n \) can be computed for (1) from

\[ y_{\text{rms}}^{(n)} = \int y^n \rho(y) dy \]

where the Gamma function is defined as

\[ \Gamma(x) = \int_0^\infty e^{-x} x^{x-1} dx \].
5 CONCLUSIONS

It is shown here that, in the intermediate region, the use of the same length and temperature scales produces similarity in both mean and flare temperature profiles. The length scale is proportional to the mean position of the interface or its standard deviation. Further, the similarity profile for the mean temperature is essentially the same as that of concentration profiles of ammonia, although, in the latter case, the diffusion layer develops from somewhat different initial conditions. The rate of spread of the diffusion layer is comparable to that of a constant pressure turbulent boundary layer.

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7 REFERENCES


