

Local spin glass order in 1D

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Abstract. – We study the behavior of one-dimensional Kac spin glasses as a function of the interaction range. We verify by Monte Carlo numerical simulations the crossover from local mean-field behavior to global paramagnetism. We investigate the behavior of correlations and find that in the low-temperature phase correlations grow at a faster rate than the interaction range. We completely characterize the growth of correlations in the vicinity of the mean-field critical region.

Introduction. – Spin glasses are well understood at the mean-field level. After more than twenty years from the physical clarification of the nature of the spin glass phase of the Sherrington-Kirkpatrick model [1], recent progress in mathematical physics [2, 3] is rapidly leading to a complete mathematical confirmation of the physical implications of replica symmetry breaking (RSB). Unfortunately, as soon as one goes beyond mean field, our ability to make predictions becomes more limited. Large theoretical efforts devoted to extend the replica symmetry breaking theory to finite-dimensional systems [4], have not led to an unanimous consensus on the nature of the spin glass phase in finite dimension. Approximate renormalization schemes [5] and phenomenological theories commonly known as droplet models [6], suggest that glassiness in finite dimension could be very different from the mean field, and indeed much simpler. Rigorous attempts to describe low-temperature spin glasses are fully compatible both with the replica symmetry breaking scenario and with droplet-like spin glass phases [7]. Numerical simulations [8] while giving strong indications that replica symmetry breaking might extend down to dimension three, do not solve the controversy, since it is always possible to argue that the systems are not large enough, the samples are not equilibrated etc. It has been recently suggested that the nature of the low-temperature finite-D spin glasses can be studied in an asymptotic expansion around mean field using models with long but

finite interactions of the Kac kind [9]. Some progress has been achieved, proving that for large enough interaction ranges, the free energy is close to the mean field. Moreover, if one admits a mild hypothesis of stochastic stability of the Gibbs metastate w.r.t. some random perturbations, mean-field spin glass order holds at least on a local level: the distribution of overlaps on scales of the order of the interaction range is close to its mean-field limit. The problem about the nature of the spin glass phase in finite dimension can be rephrased as the question if the mean-field order can become long range above a finite lower critical dimension. It is clear that in low enough dimension ($D = 1, 2$) long-range order is not possible and local mean-field order should cross over to paramagnetic behavior on large scales. Even if one feels that this case is much simpler than the high-dimensional one, a theoretical approach to this crossover is at present still to be developed. In this letter we initiate the study of the crossover from spin glass to paramagnetic behavior in a 1D spin glass model with variable interactions via Monte Carlo numerical simulations.

We first verify in explicit simulations the expected property of genericity of the unperturbed model, and show that local overlaps approach mean-field behavior. Then we investigate the growth of the correlation length for spin glass order with the interaction range, and we find that, analogously to non-disordered models, the correlation length grows more rapidly than the interaction range.

The model. – The model we consider consists in a chain of spins σ_i ($i = 1, \dots, L$) with periodic boundary conditions, interacting through the Hamiltonian

$$H(\sigma) = - \sum_{\mu=1}^M J^\mu \sigma_{i^\mu} \sigma_{j^\mu}, \quad (1)$$

where $M = (z/2)L$, the indexes (i^μ, j^μ) are chosen independently from term with uniform probability among the couples such that $|i^\mu - j^\mu| \leq R$ for some R , while the J^μ are i.i.d.r.v. equal to ± 1 with equal probability [10]. If $R = L$ the model reduces to the Viana-Bray diluted spin glass, that for $z > 2$ admits a low-temperature mean-field spin glass phase [11]. The phase diagram of the model is very simple: for finite R , the model is paramagnetic at any positive temperature. On the other hand it has been shown [12] that in the Kac limit, $R \rightarrow \infty$ after the thermodynamic limit $L \rightarrow \infty$, one recovers the mean-field phase diagram, with a second-order phase transition at a temperature $T_c = 1/\tanh^{-1}(\sqrt{z})$, below which the system is in a spin glass phase with full RSB [11].

Results. – In order to characterize the behavior of the system we study the local overlap between configurations on a scale R . We partition the line $\{1, \dots, L\}$ into disjoint, contiguous boxes B_x , ($x = 1, \dots, L/R$) of size R and consider the local overlap between spin configurations σ_i , and τ_i as $q_x(\sigma, \tau) = \frac{1}{R} \sum_{i \in B_x} \sigma_i \tau_i$. A simple generalization of the proof given in [13] shows that if one couples the original Hamiltonian with suitable finite-range random perturbations, one generically has that the probability distribution of the local overlap, induced by the Boltzmann distribution on spin configurations σ and τ and the quenched random couplings, is close to the overlap distribution function (ODF) of the infinite-range Viana-Bray model for the same temperature and value of z . This has the characteristic shape of mean-field spin glasses with full RSB, with two delta peaks at the extremal values $\pm q_{EA}$ and a smooth part in between [1]. “Generically” refers here to the fact that the property is proven almost everywhere in an interval of values of the couplings with the perturbations. The stochastic stability property amounts to say that the case of zero couplings is not a singular exception. In order to check that indeed this is the case, we simulated the model in 1D for a value of $z = 3$, where the mean-field critical temperature is $T_c = 1.5186$. In order to equilibrate the system for

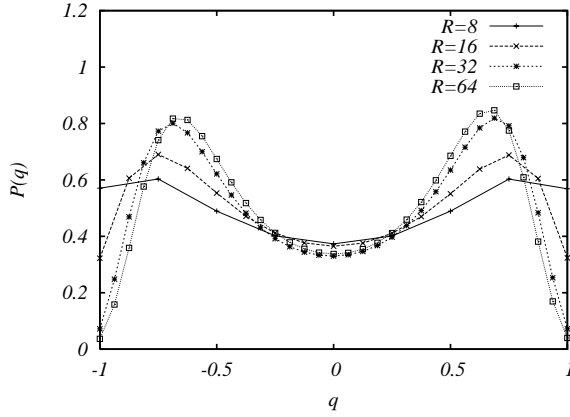


Fig. 1 – Distribution of the overlap on scales R for $T = 0.714$ and $R = 8, 16, 32, 64$. Increasing R , the overlap PDF approaches the mean-field distribution.

large samples we used parallel tempering [14]. In this way we could reach interaction ranges $R = 256$ for system sizes of $L = 8192$ without appreciable finite- L effects. All quantities we study are averaged over 100 different samples, and we have checked the stability of the average, comparing with the average over 50 samples.

In fig. 1 we show the behavior of the function of $P_R(q)$ for $T = 0.714$ and various values of R . It is apparent that in both cases, increasing the interaction range, the function $P_R(q)$ approaches the characteristic mean-field shape. This contrasts with the behavior of the PDF of the global overlap, which for a paramagnet in the thermodynamic limit has a single delta peak in zero. We then study the crossover to paramagnetic behavior of the overlap on increasing the scale of observation. To this scope, we define overlaps as before, but on boxes of size ℓR and study the distribution as a function of ℓ . In fig. 2 this is done for $R = 16$ and $T = 0.714$, where we see a clear passage from spin glass behavior at short lengths to paramagnetic behavior at large scale.

In order to study more quantitatively this crossover, we considered the overlap-overlap correlation function

$$C(x) = \langle q_y q_{x+y} \rangle, \quad (2)$$

where $\langle \cdot \rangle$ denotes the average over the Boltzmann and quenched coupling distribution. Paramagnetic behavior means that this function should tend to zero at large distance, and in one dimension one can expect the behavior

$$C(x) = C(0) \exp[-x/\xi], \quad (3)$$

where $C(0) = \langle q_y^2 \rangle$ is the local average of the overlap square, and ξ is the correlation length of the system, which is expected to diverge in the low-temperature region $T \leq T_c$ for $R \rightarrow \infty$.

In this letter we concentrate on the critical region where $\tau = \beta - \beta_c$ is small. In that case one should cross over to paramagnetic behavior for finite R to mean-field critical behavior for $R \rightarrow \infty$, in which case $\xi \approx \frac{1}{\sqrt{|\tau|}}$ and, for positive τ , $C(0) = \langle q^2 \rangle \approx \tau^2$. Analogously to the well-studied case of non-disordered systems [15], the crossover will be described by scaling

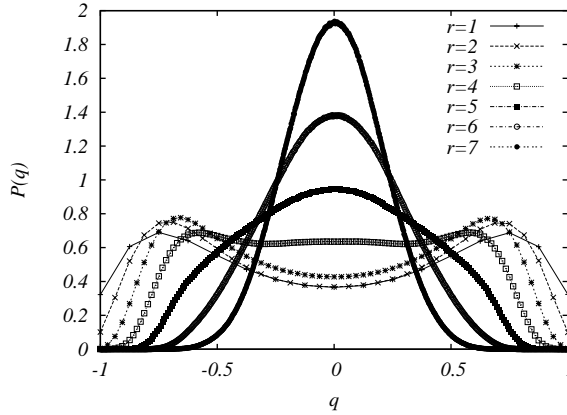


Fig. 2 – Distribution of the overlap on scales ℓR for $T = 0.714$ and $R = 16$, $\ell = 2^r$ with $r = 1, 2, 3, 4, 5, 6, 7$. It is apparent a crossover from mean-field-like two-peak behavior to paramagnetic Gaussian behavior.

functions:

$$C(0) = \langle q^2 \rangle(\tau, R) = \tau^2 g(\tau R^\alpha), \quad (4)$$

$$\xi(\tau, R) = \frac{1}{|\tau|^{1/2}} h(\tau R^\alpha), \quad (5)$$

where the properties of the functions g and h for large and small argument τR^α should be compatible with the expected behavior: namely h should go to positive constants for large negative values of x while it should behave as \sqrt{x} for small argument to cut off the mean-field singularity. Analogously, g should go to positive constants for large positive values of x , to zero for large negative values and behave as $\frac{1}{x^2}$ for small x .

The value of exponent α can be guessed through the observation that RSB effects should not affect the critical properties of the system for $T > T_c$. In that case, one can argue that the fluctuations of the order parameter are captured by a cubic field theory [16], which in the case of the 1D Kac model, reads

$$F[q] = R \int dz \left((\nabla q(z))^2 - \tau q(z)^2 + \lambda q(z)^3 \right), \quad (6)$$

where λ is a positive constant. Simple scaling analysis predicts then the value $\alpha = 2/5$. This field theory should also in principle suggest the behavior of the function $h(x)$ for large negative arguments, but we did not attempt to follow this route.

In order to confirm our predictions about the critical exponents and determine the scaling functions we simulated the model for $z = 3$ and various values of R and temperatures. We first investigated the behavior of the overlap correlation function at the critical temperature T_c ; in fig. 3 we plot the behavior of $C(0)$ as a function of R showing that the expected behavior $C(0) \approx 1/R^{4/5}$ is very well respected: we find small corrections to scaling that only affect the data points for $R = 4$ and 8 . In the inset, we plot the scaled function $P(q)$, showing that the expected scaling holds for the whole probability distribution. We next investigated the behavior of the correlation function. In fig. 4 we show the collapse of $C(x)/C(0)$ when plotted as a function of x/ξ with $\xi = \frac{R^{1/5}}{(1-2/R)}$. Again, we verify our scaling form, assuming small

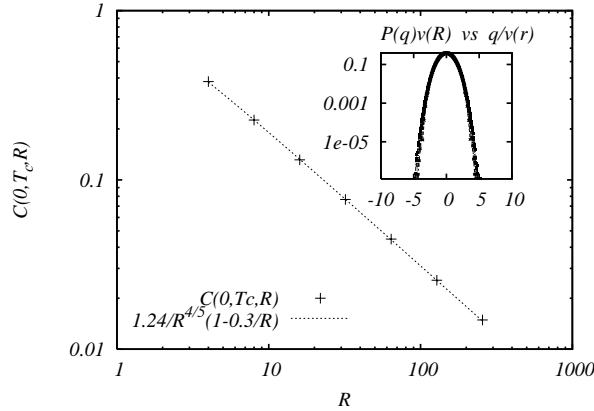


Fig. 3 – $C(0)$ as a function of R for $T = T_c$ and best fit of the kind $v(R) = \frac{a}{R^{4/5}} \left(1 + \frac{b}{R}\right)$. In the inset, scaling plot of the whole local overlap PDF $P(q)v(R)$ vs. $\frac{q}{v(R)}$.

deviations from scaling. Notice that the assumed exponential form for the correlations is very well verified for all $x \geq 1$. The same holds at all temperatures we looked at.

We then pass to the task of evaluating the scaling functions. In fig. 5 we plot, for various temperatures, $C(0)R^{4/5}$ as a function of $\tau R^{2/5}$ which gives the function $\tilde{g}(x) = x^2 g(x)$. As expected, $\tilde{g}(x)$ behaves quadratically for large positive values of x . The behavior at large negative values can also be understood, since this is the regime where the effective coupling constant in the cubic theory should tend to zero and $C(0) \approx \frac{1}{R\sqrt{|\tau|}}$, leading to $\tilde{g}(x) \approx \frac{1}{\sqrt{x}}$.

Analogously, we can understand the behavior of the scaling function $h(x)$ (see fig. 6): for small x , $h(x)$ has the expected square-root singularity with a different prefactor above and below the critical temperature, as is usual, while it goes to a constant for large negative values. The

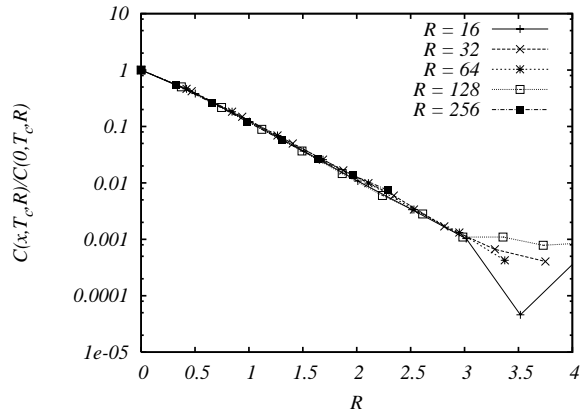


Fig. 4 – Collapse of $C(x)/C(0) = f(x/\xi)$ for $T = T_c$, $R = 16, 32, 64, 128, 256$. The collapse is obtained for $\xi = \frac{R^{1/5}}{1 - 2.0/R}$.

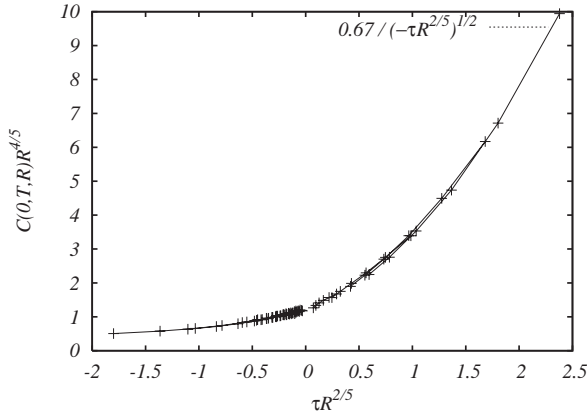


Fig. 5 – Scaling of the $C(0, T, R)$ close to T_c . In the high negative-argument region we show the fit $\tilde{g}(x) = 0.67/\sqrt{-x}$ in the region $x < -0.5$, which is indistinguishable from the data.

behavior of the function h for large positive values should describe the crossover from critical to low-temperature behavior; in particular, it should determine the behavior of the correlation length in the low-temperature phase. Unfortunately, the data we have, though they indicate that a power law behavior $\xi \sim R^\omega$ may persist at low temperature with an exponent ω larger than $1/5$, do not allow a precise determination of the exponent ω which would require larger interactions ranges R .

Summary. – Summarizing, we have studied the crossover from paramagnetic to mean-field behavior in a 1D Kac spin glass model. Our work has a qualitative aspect, from which we get evidence that stochastic stability holds at low temperature, and a quantitative aspect where we study the crossover from paramagnetic to mean-field behavior close to T_c .

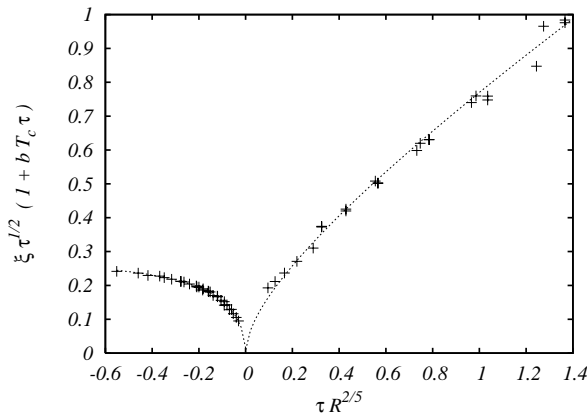


Fig. 6 – Critical scaling of the correlation length. The curves are fitted by $h(x) = 0.3\sqrt{\frac{-x}{0.277-x}}$ for negative x and by $h(x) = 0.42\sqrt{x} - 0.35x$ for positive x .

We characterize this crossover through scaling functions describing the behavior of the local Edwards-Anderson parameter $C(0)$ and the correlation length ξ . We find that while in the high-temperature phase the correlation length is, in units of the interaction range R , independent of R , in the low-temperature phase a dependence on R sets in. The correlation length grows as $R^{1/5}$ in units of R at the critical point, while it grows faster at lower temperatures. This means that in units of lattice constant the correlation length grows as $R^{1+1/5}$ or faster. This result shows that the rigorous analysis of [13] just provides a lower bound to the size of the regions where local mean-field order holds. Further studies will be necessary to make a quantitative analysis deep in the low-temperature region and to go to higher dimension.

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