A Tick-Borne Disease Model

Holly D. Gaff

hgaff@epi.umaryland.edu

University of Maryland, Baltimore

Overview

- Model Description
- Fairfield Glades Application
- Model Exploration
- Optimal Control
- Conclusions

Motivation for Project

- Fairfield Glade, Tennessee
- 1993 outbreak of Human Ehrlichiosis
- Tick population survey and control efforts
- 3 years of data available (1994-1996)

Amblyomma americanum

Lone Star Tick

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Lone Star Tick Distribution

Basic Model

$$\frac{dN}{dt} = \beta\left(\frac{K-N}{K}\right)N - bN$$

$$\frac{dV}{dt} = \hat{\beta}V\left(\frac{MN-V}{MN}\right) - \hat{b}V$$

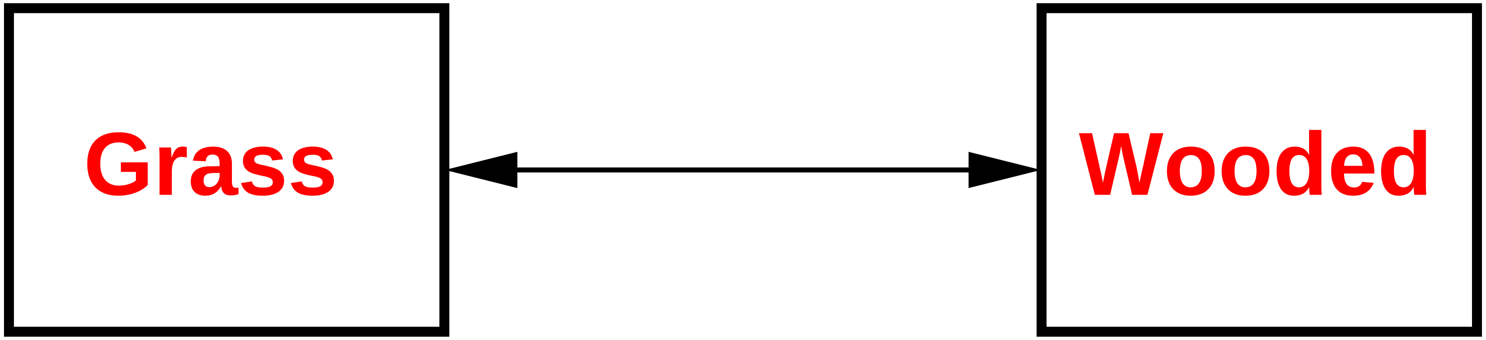
$$\frac{dY}{dt} = A\left(\frac{N-Y}{N}\right)X - \beta\frac{NY}{K} - (b+\nu)Y$$

$$\frac{dX}{dt} = \hat{A}\left(\frac{Y}{N}\right)(V-X) - \hat{\beta}\frac{VX}{MN} - \hat{b}X$$

Spatial Component

- Host diversity and community composition have been found to significantly affect tick population dynamics
- Need to add spatial component to model to evaluate these effects

Two Patch System



- Assume higher deer population in wooded area
- Assume lower tick mortality in wooded area

Multiple Patch Model

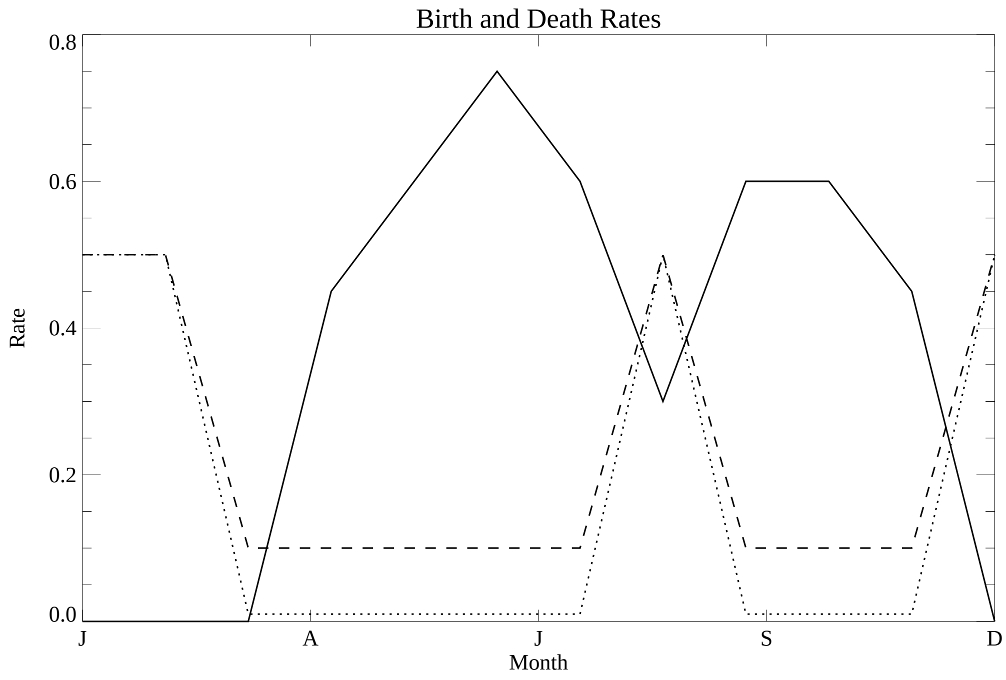
$$\frac{dN_i}{dt} = \beta_i \left(\frac{K_i - N_i}{K_i} \right) N_i - b_i N_i + \sum_j m_{ij} \left(N_j - N_i \right)$$

$$\frac{dV_i}{dt} = \hat{\beta}_i V_i \left(\frac{M_i N_i - V_i}{M_i N_i} \right) - \hat{b}_i V_i + \sum_j m_{ij} \left(V_j - V_i \right)$$

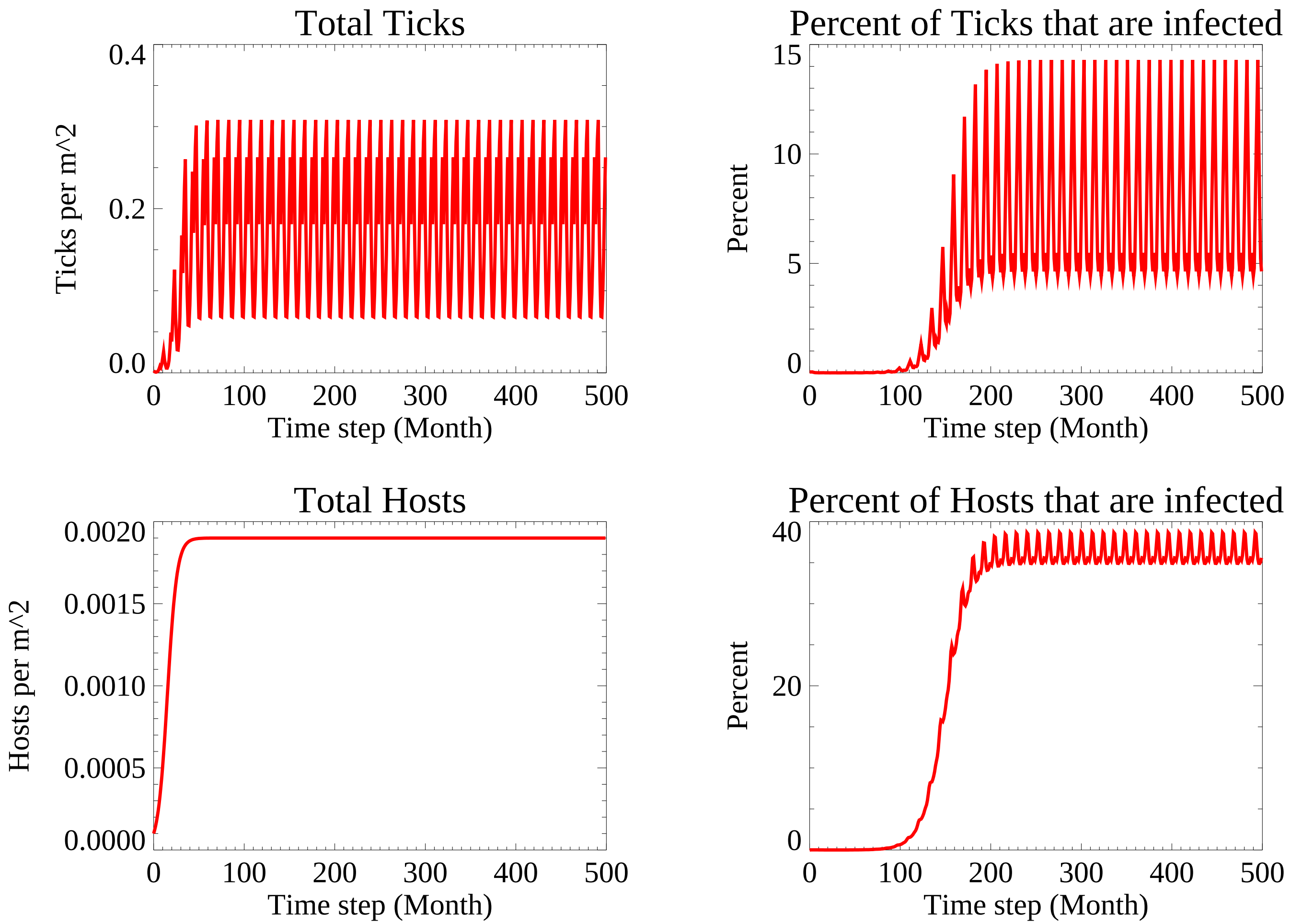
Multiple Patch Model

$$
\begin{aligned}
\frac{dY_i}{dt} &= A_i\left(\frac{N_i - Y_i}{N_i}\right) X_i - \beta_i \frac{N_i Y_i}{K_i} - (b_i + \nu_i) Y_i \\
&+ \sum_{N_j < N_i} m_{ij} \frac{Y_i}{N_i} (N_j - N_i) + \sum_{N_j > N_i} m_{ij} \frac{Y_j}{N_j} (N_j - N \\
\frac{dX_i}{dt} &= \hat{A}_i \left(\frac{Y_i}{N_i}\right) (V_i - X_i) - \hat{\beta}_i \frac{V_i X_i}{M_i N_i} - \hat{b}_i X_i \\
&+ \sum_{V_j < V_i} m_{ij} \frac{X_i}{V_i} (V_j - V_i) + \sum_{V_j > V_i} m_{ij} \frac{X_j}{V_j} (V_j - V_i)
\end{aligned}
$$

Variable “Birth” and Death Rates



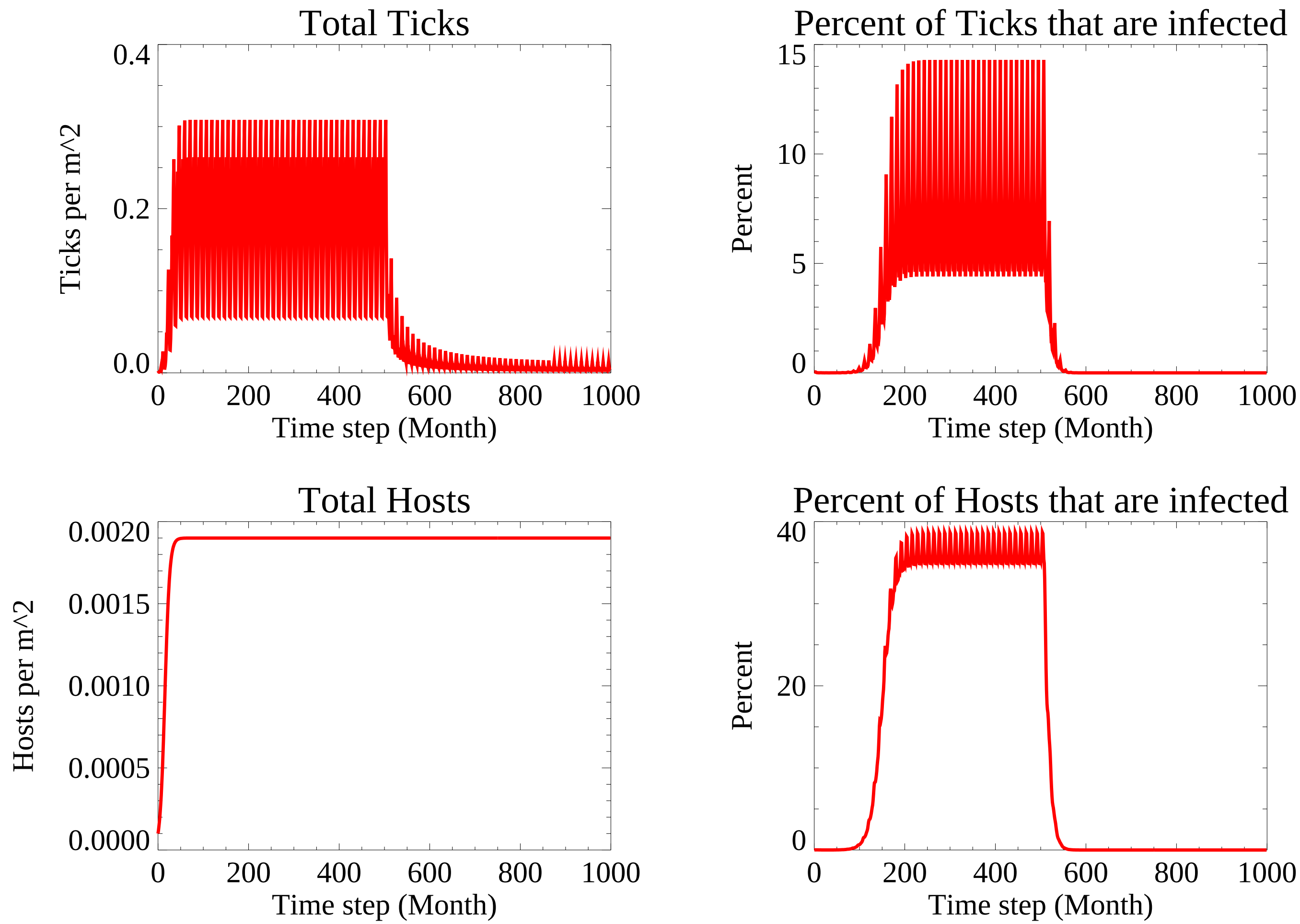
Single Wooded Patch



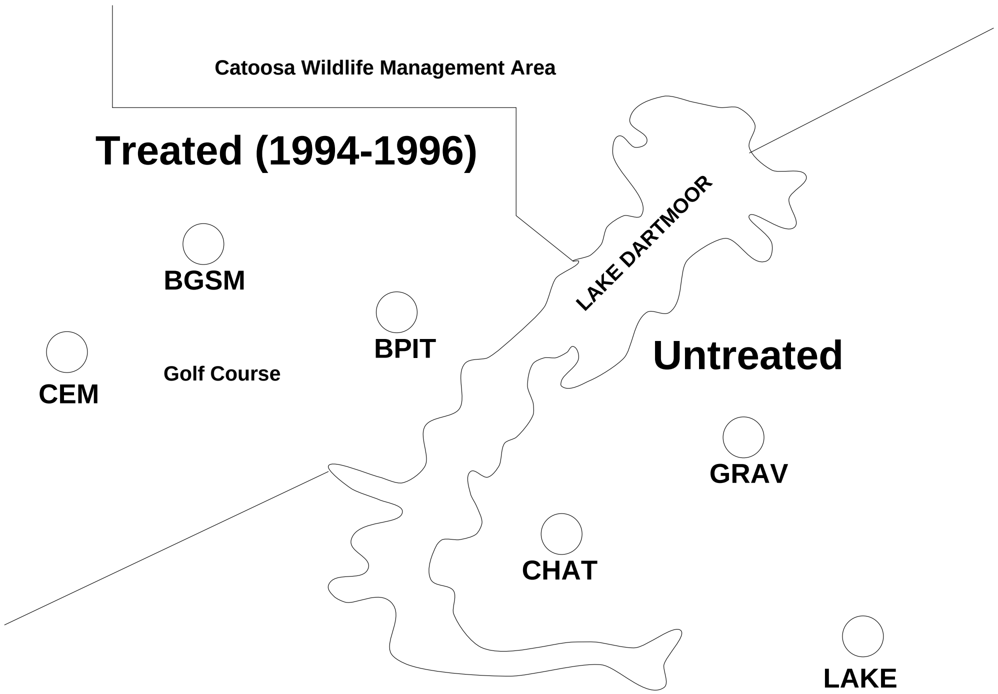
Types of Control

- Systemic Acaricide
  - Ivermectin laced corn fed to deer
  - April through July or August
  - In isolation can reduce tick populations to zero
  - Kills ticks and reduces female reproductive capability
- Contact Acaricide

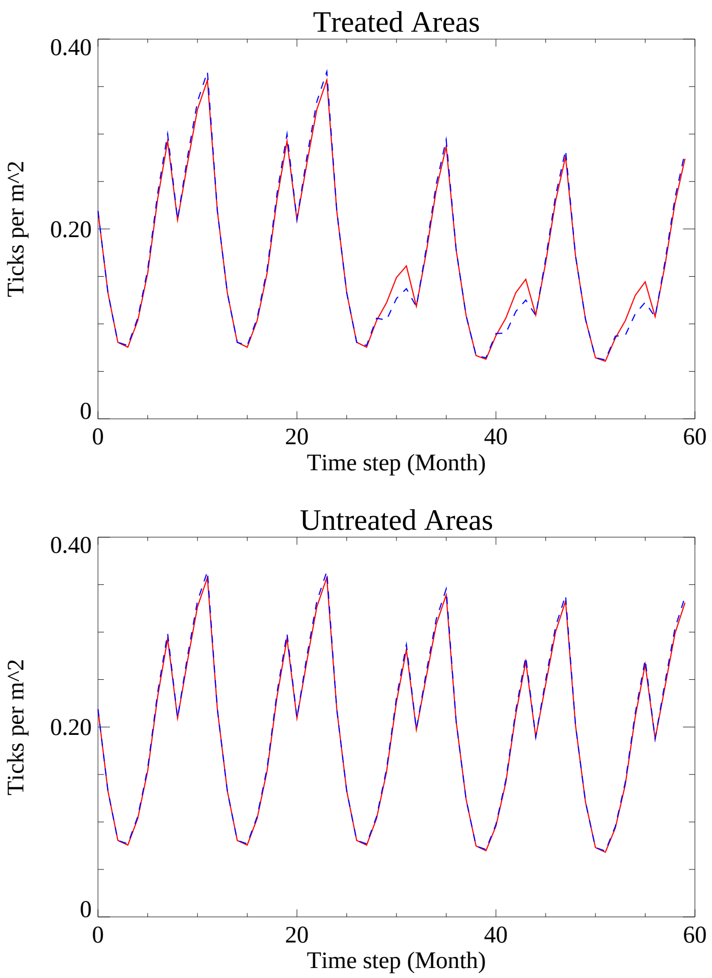
Single Wooded Patch with Control



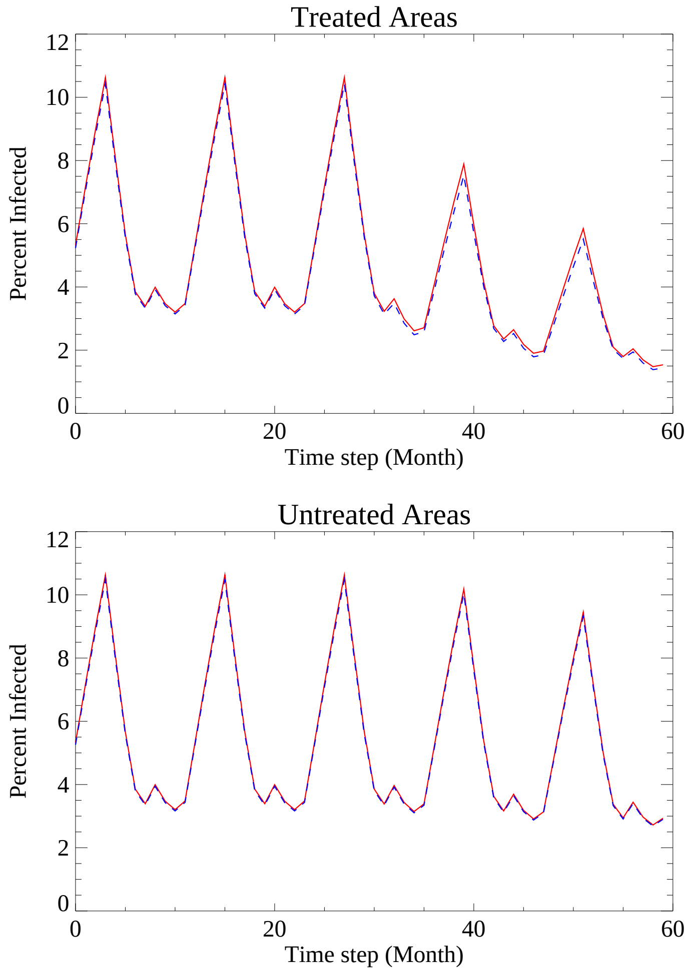
Fairfield Glades Layout



Results - Tick Population



Results - Percent Infected Ticks

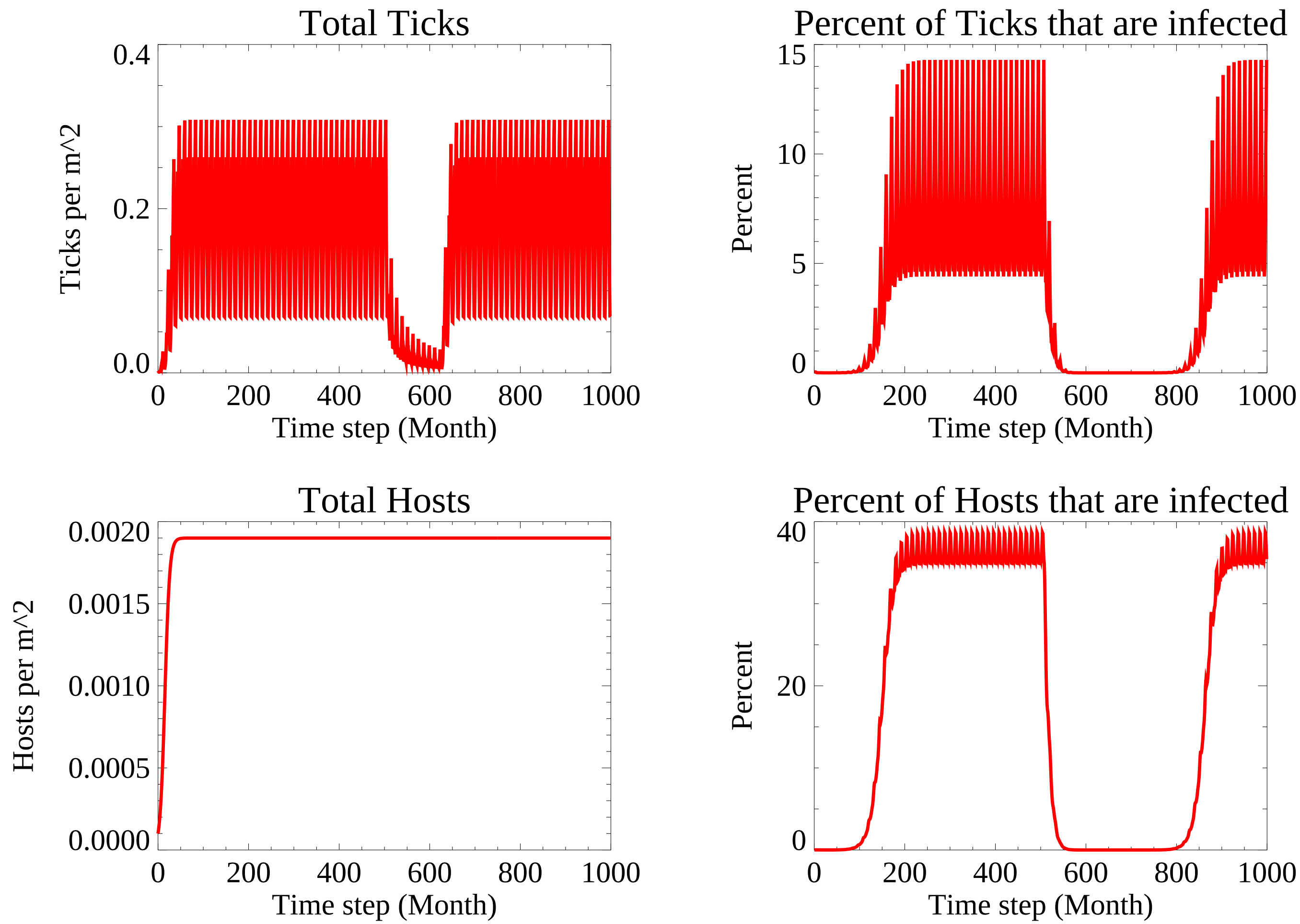


Next

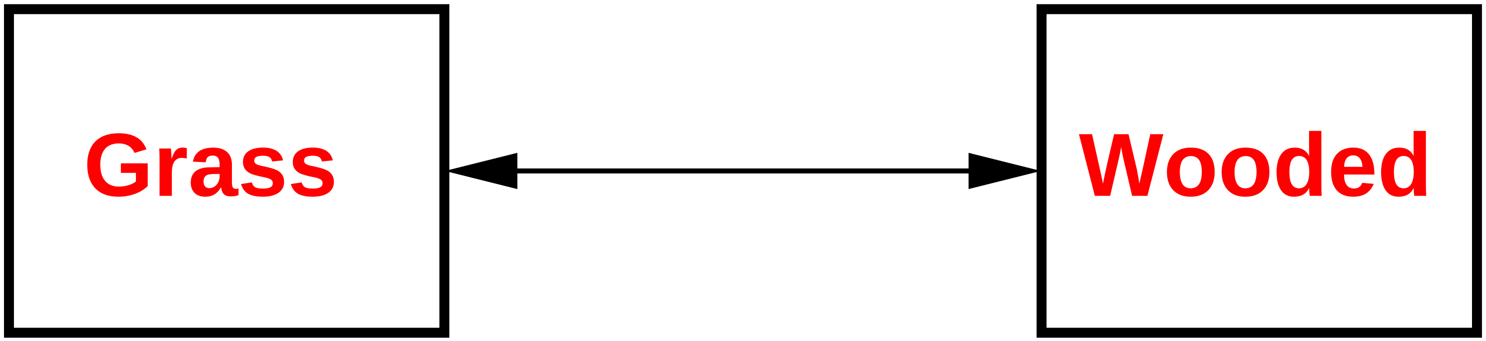
So what else can we learn with this model?

- What happens after treatment stops?
- Does it matter where we treat?

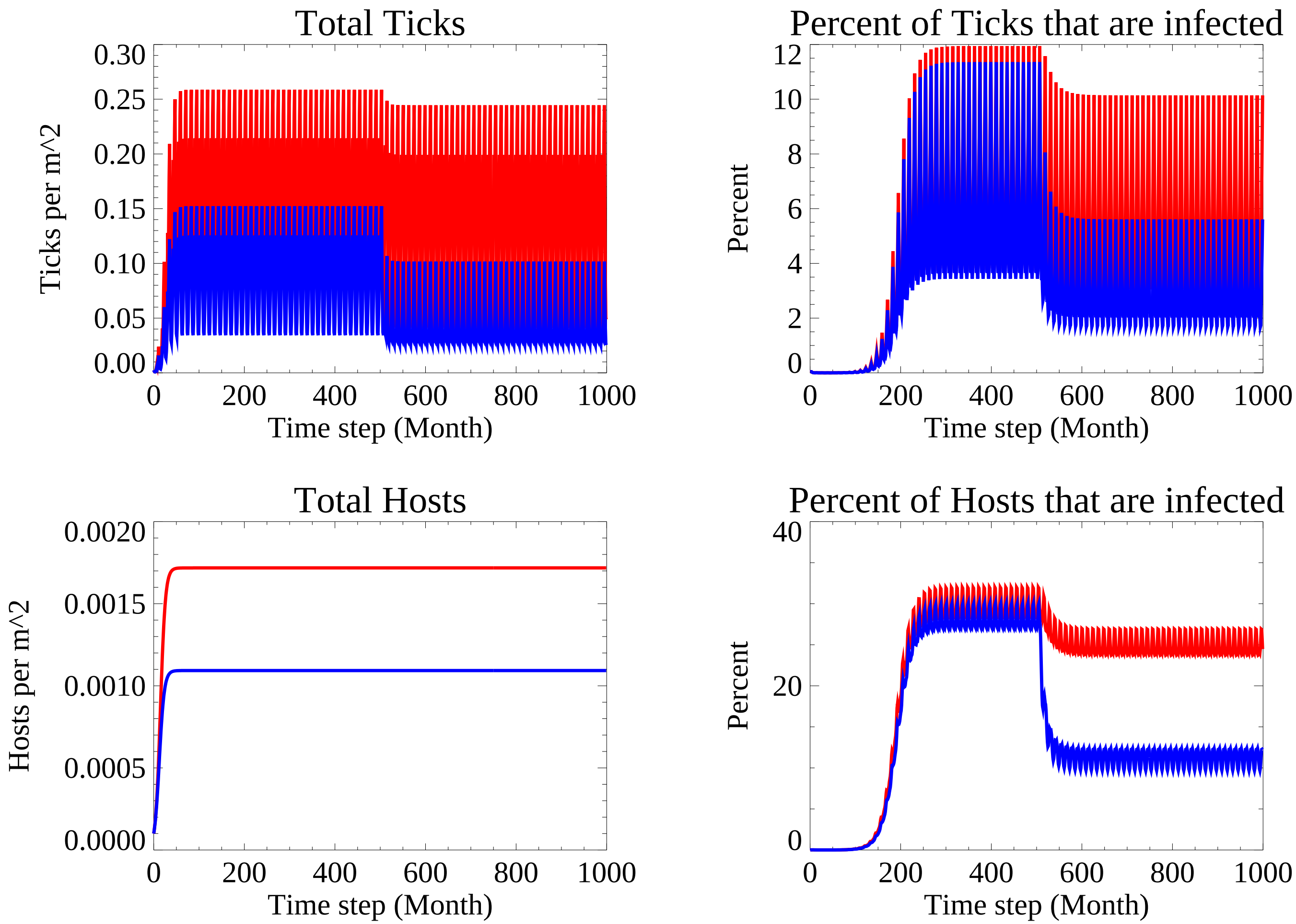
Wooded Patch - Control for 10 Years



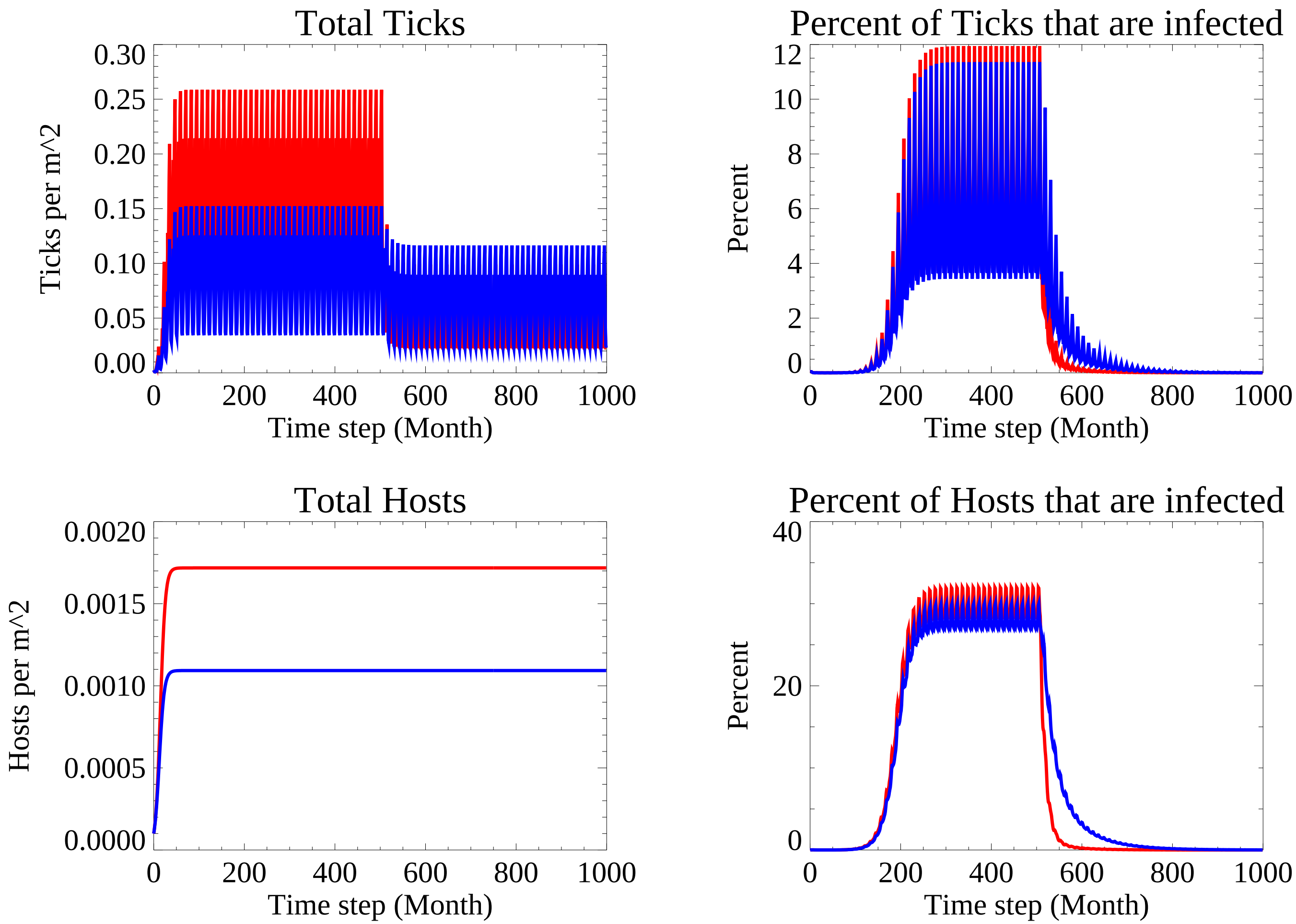
Two Patch System



Two Patches - Control in Grass



Two Patches - Control in Wooded



Conclusions - Part 1

- Tick population levels will return to equilibrium values much faster than infection levels.
- Location of control efforts can produce varying levels of ticks and the percent infected.

Optimal Control

So can we use the techniques of optimal control and Pontryagin's maximum principle to find the "when", "where" and "how much" control needed to eliminate the disease from a given system.

Single Patch Optimal Control

Goal: Maximize disease free ticks

$$J = \int_0^{T-1} C_0 V - C_1 X - \frac{C_2}{2}\hat{b}^2 dt$$

Multiple patch Optimal Control

Same procedure except now multiple control variables, $\hat{b}_i$ is optimized.

$$J = \int_0^{T-1} \sum_i \left[C_0 V_i - C_1 X_i - \frac{C_2}{2} \hat{b_i}^2 \right] dt$$

Results

To be continued.....

Conclusions

- This simple model produces interesting dynamics when controlling for disease.
- Control can remove disease and ticks or just disease
- With multiple patches connected by migration, location of control application can impact results

Acknowledgements

- Dr. Louis Gross
- Dr. Reid Gerhardt, Mr. Chris Morris