

DIPLOMA PROGRAMME

HIGH ENERGY PHYSICS COURSE

2006 - 2007

First-term courses

RELATIVISTIC QUANTUM MECHANICS

Part I

1. Kets, Bras and Operators

Ket or state space, bra space and inner products. Linear operators. Product of operators, associative axiom of multiplication. Hermitian operators and observables. Representations in state space: orthonormal basis, completeness relation, representations of kets and bras, representations of operators, change of representation, unitary operators, transformation matrix. Eigenvalue equation and diagonalisation. Definition of observable, commuting observables, unitary equivalent observables, noncommuting observables.

2. Physical Postulates of Quantum Mechanics

Description of the state of the system, physical quantities, measurement of physical quantities, spectral decomposition, probability interpretation. Expectation values. Heisenberg uncertainty relation.

3. Position, momentum and translation

Continuous spectra, some useful properties of the Dirac delta function. Position eigenkets and position measurement. Translations: properties of the infinitesimal translation operator, momentum as generator of translation in quantum and classical mechanics. Finite translations. Canonical commutation relations. Wave functions in position and momentum space. Momentum operator in the position space, momentum space wave function.

4. Quantum Dynamics

Time evolution. The Schrödinger equation for the time evolution of a state ket. Schrödinger versus Heisenberg picture: state kets and observables in the Schrödinger and Heisenberg pictures. The Heisenberg equation of motion: evolution of the mean value of an observable, Ehrenfest's theorem, time evolution of base kets and transition amplitudes. Schrödinger wave equation, conservation of probability revisited.

Part II

1. Lorentz transformations

2. Relativistic Wave equations

Klein-Gordon equation, The Dirac equation: continuity equation for the Dirac wave function, covariance of Dirac equation, covariance of probability current, transformations of spinors under space reflection, bilinear covariants. Nonrelativistic limit of the Dirac equation. Solutions of the free Dirac equation: plane wave solutions, polarised electrons in

relativistic theory. Projection operators for energy and spin. Helicity, Wave packets of plane Dirac waves: Klein paradox. Negative energy solutions- the hole theory.

3. Discrete symmetries in the Dirac theory

Charge conjugation: effect of charge conjugation on a negative energy electron at rest, charge conjugation of eigenstates with arbitrary spin and momentum. Space inversion or parity transformation. Time reversal.

4. Right and left-handed electrons and neutrino equation

Right and left-handed electrons, two component neutrino theory.

INTRODUCTION TO QUANTUM FIELD THEORY

1. Classical Theory

2. Canonical quantization

3. Quantized oscillators

4. Fermion and bosons

5. The forced oscillator

6. Perturbation theory

7. Free fields

8. Green's functions and Wick's theorem

9. Interacting fields

10. Symmetries in quantum field theory I (Poncaré Group)

11. Symmetries in quantum field theory II(Internal)

12. Cross sections

13. Perturbation theory (I)

14. Perturbation theory (II) - Feynman graphs

15. Fermions and Yukawa theory

16. Momentum space

17. Electrodynamics (I)

18. Electrodynamics (II)

19. Compton scattering

INTRODUCTION TO PARTICLE PHYSICS

Part I. Discoveries of particles

1. First discoveries.

Electron, proton, neutron.
Neutrino. Fermi theory of the weak interactions.

2. Detection of particles.

Principles of detection of particles,
Particles in magnetic fields.
Radiation and ionization.
Scintillation and Cherenkov radiation

3. Cosmic rays and discoveries of particles.

Cosmic rays.
Positron.
Muon.
Pion. Yukawa theory of strong interactions.
Strange particles. Kaons.

4. Acceleration of particles.

Linear accelerators.
Cyclic accelerators.
First accelerator experiments. Muon neutrino.
Existing and future accelerators.

5. Quarks.

Classification of hadrons.
 $SU(3)$ -symmetry. Eightfold way.
u, d and s quarks.
Quark jets. Cross-section of annihilation e^+e^- - *hadrons*.
Cabibbo mixing.

6. Heavy leptons and quarks.

Observations of resonances.
Charmed particles. Discovery of J/ψ .
 τ -lepton.
Bottom quark.
Top quark.
Tau neutrino.

7. Discoveries of the gauge bosons.

Intermediate bosons, W , Z .
Introduction of Colour. Gluons.

Part II. Propagation and interaction of particles

1. Interactions

Physical picture. Potentials.
Virtual particles. Propagators.
Hamiltonian of interaction.

Evolution matrix and S-matrix
Invariant amplitude. Rates of processes.
Phase volume. Normalization of the wave functions.

2. Decay and scattering

Decay probability.
Scattering. Cross-section
Optical theorem.
Resonances.

Part III. Symmetries and interactions

1. Symmetries at classical and quantum level

Symmetries and transformations.
Conservation laws.
Properties of matrix elements.

2. Parity

Parity transformations.
Parity conservation.
Parity of pion and other particles.
Parity violation.
Pion decay.

3. Charge conjugation

C-transformation.
C-parity
Violation of C-symmetry.

4. CP-violation

CP-properties of K^0 and \bar{K}^0 mesons.
Discovery of CP-violation.
Direct CP-violation.
CP-violation in B-meson system.

5. Continuous symmetries.

Conserved currents and conserved charges,
Unitary symmetries.

6. $U(1)$ -symmetry

Baryon number.
Lepton number. Neutrinoless double beta decay.
Individual leptonic numbers. Neutrino oscillations.
B-L number.

7. $SU(2)$ -symmetry

$SU(2)$ transformations
Isospin.
Isospin symmetry in nucleon-nucleon scattering.
Isospin symmetry in pion-nucleon scattering.
Isospin and electric charge. Hypercharge.
G-parity.

8. $SU(2) \times U(1)$ -symmetry. $SU(3)$ -symmetry

Strangeness and Hypercharge.

$SU(2) \times U(1)$.

$SU(3)$ -symmetry.

Literature

D. Perkins, "Introduction to high energy physics", Addison -Wesley, 2000.

B.R. Martin and G. Shaw, "Particle physics", Wiley, 1997.

G. Kane, "Modern elementary particle physics", Addison-Wesley, 1993.

F. Halzen and A.D. Martin, "Quarks and leptons".

I.S. Hughes, "Elementary particle physics", Cambridge Univpress.

M. Fukugita and T. Yanagida, "Physics of neutrinos", Springer-Verlag, Berlin Heidelberg, 2003.

LIE GROUPS AND LIE ALGEBRAS

1. Introduction

2. Basics of Groups

What is a Group? Finite Groups. Simple Continuous Groups: $SO(2)$, $SO(1,1)$, $U(1)$.

3. The Group $SU(2)$.

$SU(2)$, its parameterization and its Lie algebra, infinitesimal generators of $SU(2)$, the Lie Algebra of $SU(2)$.

4. Representations of the Lie Algebra of $SU(2)$

Representations, reducible and irreducible representations, finite dimensional irreducible representations of Lie $SU(2)$, the quadratic Casimir, complex conjugate representation and an invariant inner product.

5. Other Groups.

Unitary Groups: $U(n)$, $SU(n)$. Orthogonal Groups: $O(n)$, $SO(n)$. The Lorentz group $SO(n,1)$. The Lie Algebra of $SO(n)$ and $SO(n,1)$.

6. Finite Dimensional Representations of the Four Dimensional Lorentz Group $SO(3,1)$.

7. Spinors and Clifford Algebras.

8. $SU(3)$ and its Lie Algebra

The Lie Algebra of $SU(3)$, the Jacobi identity and the adjoint representation, the Cartan sub-algebra and roots, fundamental representations of $SU(3)$, the eigenvectors of the adjoint action and the root diagram, some representation theory, highest weights. Relation to particle physics.

GENERAL RELATIVITY

Part I: Towards the Einstein equations

1. From the Einstein Equivalence Principle to Geodesics

Introduction, caveats and omissions, motivation: the Einstein equivalence principle, geodesics, metrics and coordinate transformation, Christoffel symbols, geodesics and coordinate transformations.

2. The Physics and Geometry of Geodesics

The geodesic equation from a variational principle, Newtonian limit, gravitational redshift, locally inertial and Riemannian normal coordinates, more on geodesics and the variational principle, affine and non-affine parametrizations.

3. Tensor Algebra

From the Einstein equivalence principle to the principle of general covariance, tensors, tensor algebra, tensor densities, a coordinate-independent interpretation of tensors.

4. Tensor Analysis

Preliminary remarks, covariant derivative for vector fields, invariant interpretation of the covariant derivative, extension of the covariant derivative to other tensor fields, main properties of the covariant derivative, principle of minimal coupling, tensor analysis - some special cases, covariant differentiation along a curve, parallel transport and geodesics, generalisations.

5. Physics in a Gravitational Field

Particle mechanics in a gravitational field revisited, electrodynamics in a gravitational field, conserved quantities from covariantly conserved currents. conserved quantities from covariantly conserved tensors?

6. Curvature I: The Riemann Curvature Tensor

Curvature: preliminary remarks, the Riemann curvature tensor from the commutator of covariant derivatives, symmetries and algebraic properties of the Riemann tensor, the Ricci tensor and the Ricci scalar. An example: the curvature tensor of the two-sphere, Bianchi identities, another look at the principle of general covariance.

7. Curvature II: Geometry and Curvature

Intrinsic geometry, curvature and parallel transport. Vanishing Riemann tensor and existence of flat coordinates. The Geodesic deviation equation.

8. Towards the Einstein Equations

Heuristics. A more systematic approach. The weak-field limit. The Einstein equations. Significance of the Bianchi identities. The cosmological constant.

Part II: Selected Applications of General Relativity

9. The Schwarzschild Metric

Introduction, Static isotropic metrics, solving the Einstein equations for a static isotropic metric. Basic properties of the Schwarzschild metric - the Schwarzschild radius. Measuring length and time in the Schwarzschild metric.

10. Particle and Photon Orbits in the Schwarzschild Geometry

From conserved quantities to the effective potential. Timelike geodesics. The anomalous precession of the perihelia of the planetary orbits. Null geodesics, bending of light by a star.

11. Approaching and Crossing the Schwarzschild Radius

Infinite gravitational redshift. Vertical free fall. Tortoise coordinates. Eddington-Finkelstein coordinates, black holes and event horizons. The Kruskal metric.

12. Einstein-Hiebert action

Derivation of the Einstein equations. Definition of the stress-energy tensor. Example of the scalar field.

13. Linearised Gravity and Gravitational Waves.

Preliminary remarks. The linearised Einstein equations. Gauge freedom and coordinate choices. The wave equation. The polarisation tensor. Physical effects of gravitational waves, detection of gravitational waves. Quadrupole formula. Energy and momentum emission.

14. Cosmology I: Maximally Symmetric Spaces

Isometries and killing vectors. Conserved quantities. Homogeneous, isotropic and maximally symmetric spaces, the curvature tensor of a maximally symmetric space. The Robertson-Walker metric.

15. Cosmology II: Basics

The Hubble expansion. Area measurements in a Robertson-Walker metric and number counts. The cosmological red-shift, the red-shift distance relation (Hubble's law).

16. Cosmology III: Basics of Friedman-Robertson-Walker Cosmology

The Ricci tensor of the Robertson-Walker metric. The matter content: a perfect fluid. Conservation laws. The Einstein and Friedman equations.

17. Cosmology IV: Qualitative Analysis

The critical density. The big bang, the age of the Universe. Long term behaviour. Density and pressure of the present universe.

18. Cosmology V: Exact Solutions

Preliminaries. The Einstein universe. The matter dominated era, age and life-time of the Universe, the radiation dominated era, the vacuum dominated era.

QUANTUM FIELD THEORY II

1. Path Integrals and Quantum Mechanics

Schödinger and Heisenberg pictures. The kernel, the kernel for small time interval. Convolution. The Path integral. Generating functional of correlation functions.

2. Many Degrees of Freedom and Field Theory Path Integrals

Generalisation to more than one degree of freedom. Quantum field theory. Euclidean space.

3. Perturbation Theory

Gaussian path integrals: finite dimensional integrals, passing to field theory. Turning on the interaction. The Feynman propagator. Leading order calculation of $Z[J]$, the normalisation. Relation to the standard perturbation theory and Wick's theorem, absence of vacuum bubbles.

4. Path Integrals for Fermions

Grassman variables, integration rules. Generating functionals. Quantum field theory of fermions.

5. Gauge Theories

Local gauge symmetry. Vector potentials and covariant derivatives, gauge covariance. Gauge invariant action and field equations. Coupling of fermions.

6. Path Integrals for Gauge Theories

Need for a gauge choice for the photon. A simple model: gauge choice independence. Gauge theory, the Fadeev-Popov trick. Fadeev-Popov ghosts.

7. The Standard Model

The Lagrangian of the Standard Model. The electroweak interaction of leptons. QCD Lagrangian and the electroweak interaction of quarks.

8. The Effective Action

Generating Function $W[J]$ for connected graphs, the effective action, effective equations of motion, leading order corrections to classical dynamics.

9. Perturbation Theory and Diagrammatics

Diagrammatics, effective action upto 2 loops, determining the solution of the effective equations of motion.

10. The Effective Action and low Energy Physics

The effective potential, 1-loop effective action, Goldstone's theorem, the mass matrix.

11. Divergences in Proper Vertices

Diagrammatic rules, diagrams and expressions for 2-pt and 4-pt vertices upto 2-loops. 1-loop renormalization.

12. Renormalization in ϕ^4 theory

2-pt vertex upto 2-loops, renormalized n-pt. vertices, renormalized action and counter-terms, 1-loop and 2-loop calculations.

13. Dimensional Regularization

1-loop 4-pt vertex in ϕ^4 theory theory, systematics of dimensional regularization, 1-loop renormalization in the MS scheme : 1-loop 2 and 4-pt vertex

14. Power Counting in Field Theory

Canonical dimensions for a scalar field theory, superficial degree of divergence, renormalizable field theories, theories with spinors, theories with vector fields.

15. Renormalization Group Equations

RG equations in ϕ^4 theory, the solution of the RG equation, a scaling argument, high energy

behaviour.

16. Analysis of the RG equations in the MS scheme

Behaviour of the solutions of the RG equations, the scaling behaviour of renormalized n-pt vertex at fixed points, independence of physical quantities on the renormalization scale, scheme independence.

THE STANDARD MODEL

1. Introduction

Overview of the Standard Model, heuristic principles for construction of Standard Model. Symmetry. Electromagnetic interactions, the Lagrangian of electrodynamics, P and C symmetry
QED Lagrangian, gauge symmetry of the EM interactions.

2. Unification of Weak and Electromagnetic Interactions

Similarity of the weak and EM interactions. Intermediate bosons. Unification and symmetry.

3. Symmetry of the Standard Model

SU(2). B_μ - Field, SU(2) x U(1)

4. γ and Z^0 boson.

Weak mixing angle,. Interaction of the right handed components. Other multiplets. Weak isospin and weak hypercharge.

5. Gauge Invariance of the Standard Model

W-bosons as gauge bosons. Gauging the symmetry, U(1) case. SU(2) gauge symmetry. Properties of Yang-Mills Lagrangian. Covariant derivative. Gauge invariant interaction of fermions. SU(2) x U(1) gauge theory. How to construct gauge invariant theory? Mass terms and symmetry breaking. Non-renormalizability due to explicit mass terms in the Lagrangian. High energy behaviour of theory.

6. Spontaneous Symmetry Breaking

Higgs mechanism. Higgs potential, two field example. U(1) invariance. Continuous set of degenerate ground states. Choice of ground state breaks symmetry. Wrong sign of mass term for scalar field. Flat direction and appearance of massless scalar field. Goldstone theorem. Spontaneous violation of discrete symmetry. More scalar fields.

7. Goldstone Theorem

Spontaneous violation of symmetry and Goldstone theorem at quantum level. Vacuum and symmetry. Vacuum expectation value (VEV). On the translation invariance of the vacuum
Spontaneous violation of symmetry and masses of particles. Proof of Goldstone theorem.

8. Higgs mechanism

Gauge interactions of the scalar boson. Adding gauge invariant Lagrangian for the scalar field

YM Lagrangian. Spontaneous violation of symmetry. Effects of gauge interactions: two important terms, mass term for gauge field, gauge condition. Gauge fixing, radiation gauge, gauge fixing Lagrangian. No massless boson in theory, would-be Goldstone boson. Unitary gauge. Summary.

9. Patterns of Symmetry Breaking

O(3) gauge symmetry breaking. Examples of symmetry breaking in SU(2) and beyond. Field counting.

10. Standard Model of the Electroweak Interactions

Higgs mechanism in the Standard Model. Spontaneous violation of SU(2) x U(1). Masses of vector bosons. Masses, VEV, mixing, weak mixing angle. Masses of fermions. Concluding remarks about Higg's mechanism in SM. Summary of SM.

11. Generations of Fermions. Fermion Mixing

Generations of fermions. Charge currents and weak mixing. GIM cancellation. How to get mixing, mixing matrices. Quark mixing matrix. Mixing matrix and CP violation. Determination of the elements of the CKM-matrix. Semileptonic decay of hadrons. Properties of CKM-matrix, unitarity, hierarchy of mixing. Mixing in Leptonic Sector: non-zero neutrino masses and mixing in leptonic sector. Lepton universality. Other processes stipulated by charge currents.

12. Phenomenology of Neutral Currents

Neutral currents in the SM. Another representation. Properties of NC couplings. 4-fermion interactions due to neutral currents. Processes stipulated by neutral currents, neutrino-electron scattering, neutrino nucleon scattering.

13. Physics of W and Z Bosons

Masses of W and Z. Decays and production of W and Z. Production of W and Z in proton-antiproton colliders. Physics of Z: Invisible width, number of generations, asymmetries and parity violation,

14. Higgs Boson Physics

Properties of the Higgs boson, mass, couplings and decays. Searches for Higgs boson, bounds on Higgs mass. Searches of Higgs at LEP. Higgs mass and precision tests of the SM. Future searches of Higgs.

15. Neutrino oscillations

SUSY FIELD THEORY

1. Introduction to Supersymmetry (SUSY)

Spinor representation of the Lorentz group. Graded Lie Algebra of (N=1) supersymmetry. Component fields. SUSY transformation of fields. Multiplets.

2. Superfields

Grassmann variables, superspace, superfields. Constraints. Scalar, chiral superfields, renormalizable supersymmetric action for scalar superfields. Field equations. R-invariance. Vector superfields.

3. Gauge transformations

Gauge invariance. SUSY and U(1) gauge invariant action

4. Supersymmetry breaking

Spontaneous SUSY breaking, O'Raifeartaigh mechanism. Fayet-Iliopoulos mechanism. SUSY QED.

5. Why SUSY?

Softening of quantum divergences. Gauge hierarchy problem, chiral protection of fermion masses, gauge protection for the gauge boson masses, scalar masses protected in SUSY. supergravity as a good candidate for unification, SUSY as most general symmetry of S-matrix, Coleman-Mandula theorem.